ACCURACY OF EMPIRICAL CORRELATIONS FOR THE PREDICTION OF AERODYNAMIC HEATING AT SUPersonic SPEEDS

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ABSTRACT
Vehicles moving through the air are subjected to a resistance during the flight. Some heat transfer loads are generated on them. These loads are called the aerodynamic heating. By using some fast prediction techniques, aerodynamic heating rates on a vehicle body could be estimated, but these loads include some uncertainties. Hence, the level of these error bands should be known. In this study, the aerodynamic heating rates on a vehicle body flying at supersonic speeds are calculated by empirical correlations. Also, the time-dependent temperature measurements taken from several points on the inner surface of the vehicle body during a flight test are used to compute the aerodynamic heating rates by the inverse heat transfer method. By comparing the results, the accuracy of these empirical methods is determined.

INTRODUCTION
At supersonic and hypersonic regimes, air temperature around the vehicle body is increased much more than the free stream temperature due to the compression and friction in the boundary layer. The heating phenomena formed from the conversion of kinetic energy of the air molecules to heat energy is called aerodynamic heating and it is the basis of many researches since 1940s.

For vehicles that operate at high speed flows, aerodynamic heating is an important design criterion and it must be calculated accurately and efficiently. Computational fluid dynamic (CFD) simulations are frequently used in order to compute the aerodynamic heating rates in a direct manner. However, CFD is not the preferred method in the initial design phases because CFD simulations are computationally expensive and not time efficient. Moreover, some modelling assumptions and use of empirical turbulent models make the CFD results in supersonic/hypersonic flows questionable. In order to reduce time cost and expedite trade-off design studies approximate methods are commonly employed to predict the aerodynamic heating rates. Eckert’s reference temperature method is one of the most notable fast prediction techniques to obtain the aerodynamic heating rates over certain shapes such as

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flat plates, cones, and blunted noses [Eckert, 1955]. Another predicting method is the property ratio method where the aerodynamic heating is evaluated by variable fluid properties [Kays, 1966]. The effects of variable properties are considered as a function of the ratio of two properties obtained from the surface temperature and the adiabatic wall temperature. Despite the development of many numerical techniques since the 1940s, these techniques continue to be used. With their advantages, they provide benefits to many scientific studies on the heating problems of aircrafts at high speeds. In a study [Quinn, 2000], a real time aerodynamic heating algorithm has been developed by using Eckert’s reference temperature method for obtaining the surface heating rates and the surface temperatures for hypersonic vehicles. Similar fast prediction algorithm was developed [Duarte et. al. 2009] to calculate the transient missile aerodynamic heating parameters utilizing basic flight parameters such as altitude, Mach number and angle of attack. In another study [Crabtree et al. 1965], the rates of heat transfer to various bodies were estimated by using these prediction techniques. Also, wind tunnel tests were conducted and good agreement was made between analytical results and test measurements. In spite of their advantages, since both methods are based on certain assumptions and simplifications, their results are more likely to be different from the real heating rate values.

METHOD

The aerodynamic heating rates calculated by empirical correlations include some uncertainties and may lack accuracy. Thus, the level of these error bands should be known by designers. In this study, aerodynamic heating rates on a vehicle body flying at supersonic speeds are calculated by both Eckert’s reference temperature method and the property ratio method. In addition, the time-dependent temperature measurements taken from several points on the inner surface of the vehicle body during a flight test are used to compute the aerodynamic heating rates by inverse heat transfer method. The heating rates obtained from the inverse heat transfer method are then compared with the results of both empirical correlations. By comparing the results, the error bands of these methods are determined. Additionally, some modeling aspects are addressed for better estimations.

Eckert’s Temperature Method

Calculation of heating rates in a supersonic boundary layer is complicated by the dependence of fluid properties on the temperature within the boundary layer. The incompressible solution must be extended to the compressible case. Eckert’s reference temperature method is employed for this purpose. The reference temperature given in Eq. (1) is the temperature whose magnitude becomes the average of the wall temperature and the local temperature in low speed flow. The fluid properties should be evaluated at the reference temperature.

\[ T^* = T_L + 0.5(T_S - T_L) + 0.22(T_{AW} - T_L) \]  

According to the experimental data [Chapman, 1960], the Nusselt number in Eq. (2) is defined as a function of Reynolds and Prandtl numbers. They are given in Eq. (3) and Eq. (4), respectively.

\[ \text{Nu} = 0.0292R^{0.8}P^{1/3} \]  

where

\[ \text{Re}^* = \frac{\rho^*M_L[1 + 0.5(\gamma - 1)M_L^2]^{\gamma+1}/\gamma RT_0}{\mu^*} \]  

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In the view of these formulations that have been mentioned so far, the convective heat transfer coefficient within the turbulent boundary layer is written as

\[ \Pr^* = \frac{\mu^* c_p^*}{k^*} \]  

(4)

For large temperature difference through the boundary layer, the specific heat is treated as variable, so these formulations based on enthalpies rather than temperatures become more convenient.

**The Property Ratio Method**

In the property ratio method, flow properties are considered as constant and the variable property effects are evaluated as a ratio of the property at the surface temperature to the property at the adiabatic wall temperature.

For property variations with temperature the relations given in Eq. (6) are used. In Eq. (6), the subscript CP refers to the appropriate constant-property solution and St is a dimensionless number that measures the ratio of heat transferred into a fluid to the thermal capacity of fluid and called the Stanton number that is given in Eq. (7).

\[ \frac{St}{St_{CP}} = \left( \frac{T_0}{T_{aw}} \right)^{-0.08} \left( \frac{T_{aw}}{T_\infty} \right)^{-0.12} \quad \text{for } T_0 > T_\infty, \quad M \leq 6 \]  

(6)

\[ St = \frac{h}{u_\infty \rho c} \]  

(7)

**Inverse Heat Transfer Method**

To acquire the real heating rates on the vehicle body, the inverse heat transfer method was employed by using the time-dependent temperature measurements on the inner surface of the vehicle body during a flight test (Figure 1). There are various inverse methods for this purpose. Detailed examination of these methods could be found in [Ozisik, 2000 and Beck, 1985].

\[ h = C R e^m P r^n \]  

Figure 1: Inverse heat transfer method

For estimation of the convective heat transfer coefficient, it was assumed that the heat transfer coefficient is a function of Reynolds and Prandtl numbers as given in Eq. (8). The constant and the exponents in Eq. (8) were tried to be guessed by inverse methods.

\[ h = C R e^m P r^n \]  

(8)
The direct heat transfer mechanism is described by formulations given between Eq. (1) and Eq. (4). The direct solution was obtained using a MATLAB code which solves the direct heat transfer equations by the finite difference method. The adiabatic wall temperature, Reynolds and Prandtl numbers are the inputs of the direct problem. In each cycle, the calculations were made with new parameter sets. The calculated temperature values at the end of each cycle were compared to the temperature measurements taken from the flight test. If there was a difference between these two values, new parameter sets were defined and calculations were repeated. This cycle repeated until the results reach the desired proximity. The flow diagram of this inverse method is given in Figure 2.

![Flow diagram of inverse method](image)

**Figure 2: Estimation of the convective heat transfer coefficient**

Inverse heat transfer studies to obtain the heat transfer coefficient were performed for two different locations on the inner surface of the vehicle. These points (P1 and P2) that are given in Figure 3 are located on the conical and cylindrical section of the vehicle body. The material of the vehicle body is Aluminum 7000 series and properties are given in Table 1.

![Simulation model and Location of P1 and P2](image)

**Figure 3: Simulation model and Location of P1 and P2**

**Table 1: Material Properties of Aluminum 7000 series**

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2800</td>
</tr>
<tr>
<td>Thermal Conductivity (W/m/K)</td>
<td>140</td>
</tr>
<tr>
<td>Specific Heat (J/kg/K)</td>
<td>840</td>
</tr>
</tbody>
</table>
RESULTS

Results for obtaining the actual convective heat transfer coefficient

The convective heat transfer coefficients at P1 and P2 were calculated by the procedure given in Figure 2. The m, n and C parameters were investigated in certain ranges and optimized until the best suited convective coefficients were obtained. Figure 4 and Figure 5 show that the temperature profiles obtained by using the optimum convective heat transfer coefficient are differed in the range of 5% from the test measurements. Figure 6 shows the estimated heat transfer coefficients. Due to confidentiality, the actual values in figures are not given. The heat transfer coefficients reach the highest value at the beginning of the flight and decrease over time and remain at a certain value.

Figure 4: Comparison between the optimization result and the test data for P1

Figure 5: Comparison between the optimization result and the test data for P2
Results for obtaining the convective heat transfer coefficients from the empirical correlations

The empirical correlations were used to calculate the convective heat transfer coefficient for the same points (conical and cylindrical). The time-dependent convective heat transfer coefficients obtained have been applied to the simulation model to calculate the temperature chance. Figure 7 and Figure 8 compare the convective heat transfer coefficient results at P1 and P2 using the heat transfer coefficient obtained by the inverse heat transfer method and the temperature data measured during a flight test to the empirical correlations. It is apparent from figures that the closest results were obtained with the constant property method in the simulations performed in the cylindrical region. On the other hand, in the conical region, the property ratio method is more consistent with the results than the constant property method. It is because the difference between the boundary layer edge temperature and the wall temperature in cylindrical region is smaller than that in the conical region.
CONCLUSION

Aerothermal loads, which vehicles that operate at high speed flows are exposed during the flight, could be calculated by empirical correlations. In this study, the accuracy of these correlations are investigated and estimated loads are compared with the actual aerothermal heating data obtained from the flight test. The actual convective heat transfer coefficients are calculated from the time-dependent temperature measurements taken during the flight test.

The convective heat transfer coefficient obtained from the constant property ratio is proportionally more than that obtained from the property ratio method and Eckert’s temperature method. In the conical region, since the difference between the boundary layer edge temperature and the wall temperature is significant, the property ratio method gives more consistent results with the actual heat transfer coefficient. The highest error rate is about 5% in that region. In cylindrical region, the constant property method predicts the convective heat transfer coefficient better than the property ratio and Eckert’s temperature method and the highest error rate observed is about 8%.

References


