ACTUATOR FAULT TOLERANT TERMINAL SLIDING MODE GUIDANCE LAW WITH IMPACT ANGLE AND ACCELERATION CONSTRAINTS

Fatih Kırmılioğlu\textsuperscript{1}  
Roketsan Missiles Inc.  
Ankara, Turkey

Emir Kutluay\textsuperscript{2}  
Hacettepe University  
Ankara, Turkey

ABSTRACT

A terminal sliding-mode guidance law is proposed in order to achieve actuator fault tolerance and desired terminal impact angle for an anti-tank missile. Impact angle is an important property for anti-tank missiles; however, this property is not considered together with fault tolerance in the literature. In order to increase robustness of the guidance, target acceleration is covered in guidance law design. During the design process of the guidance law, a sliding surface is determined from the engagement dynamics. Reaching law is designed by considering finite-time convergence and disturbance rejection. After completing the design process, simulations are performed to test guidance law’s performance.

INTRODUCTION

Considering missile’s objective of intercepting the target accurately, comprehensive design of guidance law is significant. On the flight, missiles are exposed of various disturbances and uncertainties. These terms are generally unmeasurable and they affect the interception performance negatively. In order to handle disturbances and uncertainties, the guidance system is need to have sufficient robustness. Another property of the guidance system can be defined as achieving an engagement constraint. Sliding mode control has become popular due to having robustness properties and being able to handle engagement constraints.

Actuator failure is an important topic to be considered for the missiles, since every missile can suffer from this problem and these failures have a significant effect on missile performance. Damages on control surfaces or redundancies on actuator’s mechanical performance makes the missile control difficult. Since it is hard to detect the failure type or it is hard to know how much the failure affects system properties, guidance system needs sufficient robustness to handle these disturbances. There are various studies on fault tolerant sliding mode control guidance in the literature. A fault tolerant sliding mode control guidance with state feedback and output feedback policies are studied in [Corradini and Orlando 2003]. An adaptive fault tolerant guidance law with backstepping and sliding mode control is studied in [Jegarkandi, Ashrafifar and Mohsenipour, 2015].

\textsuperscript{1} R&D Engineer in System Simulation and Modeling Department, Email: fatih.kirimlioglu@roketsan.com.tr

\textsuperscript{2} Assist. Prof Dr.-Ing in Mechanical Engineering Department, Email: kutluay@hacettepe.edu.tr
The target acceleration is another important property, that affects the missile-target engagement geometry. Since it is hard to estimate the target acceleration, the property is generally accepted as a disturbance. In the literature, finite-time convergence is studied along with sliding mode control, since it is significant in terms of sliding mode convergence speed and the stability. In [Zhu, Xia, Fu and Wang, 2011], a fault tolerant SMC guidance is proposed by considering actuator faults, target acceleration and finite-time convergence of the controller. In anti-tank missiles, impact angle of the missile is significant for the warhead performance. Thus, achieving a desired impact angle at the end of the engagement is an important role. Sliding mode control is a suitable robust control technique to implement an engagement constraint in guidance problems. In [Wang, Zhang and Wu, 2016; He and Li, n 2014] a SMC guidance law is proposed which can handle impact angle and field-of-view constraints. Impact angle and target acceleration are studied in [Wang, Tang, Shang and Guo, 2018; Li, Zhang, Han and Xie, 2016] with fuzzy sliding-mode controllers. In [Kumar, Rao and Ghose, 2012], a sliding mode guidance law is proposed considering impact angle, autopilot dynamics and finite-time stability.

There are many studies on impact angle based sliding mode guidance by considering target acceleration. However, fault tolerance is not studied along with these constraints. Adding different design constraints to the control system is challenging in terms of the problem solution. Nevertheless, adding multiple design constraints significantly increases missile guidance performance and abilities. Thus, in this paper, an actuator fault tolerant terminal sliding mode guidance law is proposed for an anti-tank missile by considering impact angle, target acceleration and finite-time convergence property.

PROBLEM FORMULATION

Missile – Target Engagement Geometry

A general two-dimensional missile-target engagement geometry is considered as shown in Fig. 1, where M and T represent missile and target respectively. λ denotes the line-of-sight (LOS) angle and R is relative distance between the missile and the target. θM and θT are flight path angle of the missile and orientation of the target respectively. VM and VT are velocity of the missile and the target respectively, aM and aT are normal accelerations of the missile and target respectively.

![Figure 1: Missile-target engagement geometry](image)

Following non-linear missile-target engagement dynamic equations are derived from the geometric relationship of the missile-target engagement model.

\[
\dot{R} = V_T \cos(\lambda - \theta_T) - V_M \cos(\lambda - \theta_M) \tag{1}
\]

\[
\dot{\lambda} R = V_M \sin(\lambda - \theta_M) - V_T \sin(\lambda - \theta_T) \tag{2}
\]

\[
\dot{\theta}_M = a_M / V_M \tag{3}
\]

\[
\dot{\theta}_T = a_T / V_T \tag{4}
\]
Certain assumptions are made in order to implement proposed design. It is assumed that missile and target are point masses. Actuator and autopilot dynamics are neglected. Missile parameters $\theta_M, V_M$ are measured by the missile’s inertial navigation system. Target parameters $\theta_T, V_T$ are measured by missile sensors or transmitted from radar. Since this paper is focused on terminal guidance, it is assumed that missile is not on boosting phase and aerodynamic drag force is not big enough to change missiles velocity in terminal phase. Thus, missile velocity $V_M$ is assumed as a constant parameter. For an anti-tank missile problem, target is considered as an armored land vehicle, which is not agile in dynamic properties. Thus, $V_T$ is assumed as a constant parameter. $R, \dot{R}, \lambda, \dot{\lambda}$ are assumed to be calculated by the missile from the measurement parameters or acquired from radar.

**Impact Angle**

The impact angle is considered as the angle between missile and the target surface. Since directions of missile and target are related with their velocity vectors, it is convenient to create a relation for the impact angle by using missile flight path angle and target orientation. Thus, the impact angle $\theta_{imp}$ is defined as the angle between the velocity vectors of the target and missile at the final time of the engagement $t_f$.

$$\theta_{imp} = \theta_T(t_f) - \theta_M(t_f)$$

![Figure 2: Representation of impact angle](image)

**Actuator Failure**

Damages and deformations on the control surfaces or defects on the inner actuator mechanisms can be resulted as extra forces on the missile. Exemplary, a locked control surface is not able to operate coordinated with other control surfaces and results in unexpected forces on the missile. In addition, guidance commands may not be performed properly under these actuator failures. For instance, a small rupture on the control surface decreases the lift forces generated; consequently, normal acceleration produced becomes less than the guidance command. Thus, in this paper, actuator failures are approached as discrepancies between guidance command and actual normal acceleration implemented on the missile. The discrepancies are considered as undesired forces created by the actuator and loss of performances of the actuator. Additive term $a_F$ represents the undesired normal accelerations created as a result of the failures. Effective performance percentage of the actuator $\mu$ is represented as a multiplicative term of the guidance command. Normal acceleration command computed from the guidance law is defined as $a_{com}$. Resultant acceleration $a_M$ defines the actual normal acceleration of the missile.

$$a_M = a_F + a_{com}\mu$$

**Finite Time Stability**

In the design of the guidance law, finite-time convergence of the guidance law is considered, in order to achieve better convergence rate of the states and disturbance rejection property of the system. Following lemma is introduced to analyze finite-time stability of the system.

**Lemma:** Consider a nonlinear system [Bhat and Bernstein, 2000]

$$\dot{x} = f(s, t), \quad f(0, t) = 0, \quad x \in R^n$$

$U_0$ is an open neighborhood of the origin $s = 0$ and $f: U_0 \times R \to R^n$ is continuous on $U_0 \times R$. There exists a $C^1$ smooth and positive definite function $V(s)$ on $U \subset R^n$, and a negative semi-definite function $\dot{V}(s) + cV(s)^{\alpha}$ on $U \subset R^n$, thus system origin is finite-time stable. $c$ and $\alpha$ are
real numbers and they satisfy $c > 0$, $0 > \alpha > 1$. $V(s)$ reaches to zero from the initial value $V(s_0)$ in finite time. $T(s)$ is the settling time.

$$T(s) \leq \frac{1}{c(1-\alpha)} V(s_0)^{1-\alpha} \quad (8)$$

SLIDING MODE GUIDANCE LAW

Guidance Law Design

The goal of this design is to derive a sliding-mode guidance law that calculates normal acceleration of the missile, which steers the missile in order to achieve successful interception by satisfying desired constraints. First, states of the system are determined in order to construct a sliding surface.

Impact angle constraint is satisfied by achieving desired $\theta_{imp}$ at the end of the engagement. A relationship between $\lambda$ and $\theta_{imp}$ is formed at the time of the interception, in order to represent $\theta_{imp}$ in terms of $\lambda$ [Li, Zhang, Han and Xie, 2016].

$$\lambda_f = \theta_T - \arctan\left(\frac{\sin \theta_{imp}}{\cos \theta_{imp} - \frac{V_T}{V_M}}\right) \quad (9)$$

Since $\lambda_f$ represents $\theta_{imp}$ in terms of $\lambda$, zeroing the error between $\lambda$ and $\lambda_f$ satisfies the impact angle constraint.

According to parallel navigation rule, missile and target must be on collision course in order to achieve successive interception [Yanushevsky, 2008]. Therefore, zero $\dot{\lambda}$ is desired at the end of the engagement.

To construct states of the sliding surface, time derivative of LOS rate $\dot{\lambda}$ is derived from equation (2), (3), (4) and (6) by taking into account that $\dot{V}_M = 0$ and $\dot{V}_T = 0$, since $V_M$ and $V_T$ are assumed constant parameters as mentioned before.

$$\dot{\lambda} = -\left(a_F + a_{com}\mu\right) \cos(\lambda - \theta_M) + a_F \cos(\lambda - \theta_T) - 2R\dot{\lambda} \quad (10)$$

Considering impact angle constraint, first state is defined as $x_1 = \lambda - \lambda_f$. Second state is defined as $x_2 = \dot{\lambda}$ for the success of the interception. The sliding variable is defined as follows

$$s = x_1 \frac{\varepsilon}{R^\gamma} + x_2 \quad (11)$$

The sliding surface to be tracked by the states in the sliding phase is defined as

$$0 = x_1 \frac{\varepsilon}{R^\gamma} + x_2 \quad (12)$$

Achieving the desired impact angle is only crucial at the end of the engagement. For this reason, it is more convenient for the impact angle to having less gain at the beginning and higher gain just before the interception. Thus, unnecessary control effort to achieve desired impact angle at the early flight phase is avoided and generated guidance commands are decreased in this time interval. The range variable $R$ provides time-varying slope for the sliding surface. The slope parameter $\frac{\varepsilon}{R^\gamma}$ increases with respect to range through the end of the engagement. Constant parameters $\varepsilon$ and $\gamma$ provide flexibility to manipulate $R(t)$ curve, where $\varepsilon > 0$ and $1 > \gamma > 0$. By this way, better-optimized slope parameters can be produced for the sliding surface.

The guidance law is determined by defining an equivalent control $a_{ec}$ and a reaching law $a_{rl}$.

$$a_{com} = a_{ec} + a_{rl} \quad (13)$$

In the sliding phase, states track the sliding surface. If the states are on the sliding surface equivalent control keeps the states on the sliding phase [Utkin, Guldner and Shi, 2009]. This is achieved when $s = 0$ and $\dot{s} = 0$. Thus, equivalent control is calculated by solving derivative of the sliding variable (11) for $\dot{s} = 0$ with equations (1-4) and (10). Disturbances like actuator failure and target acceleration are not considered in the solution of $a_{ec}$, since they are unknown parameters.
\[ a_{ec} = \frac{\varepsilon R^{1-\gamma} - 2R}{\cos(\lambda - \theta_M)} x_2 - \frac{\varepsilon y R^{\gamma} R}{\cos(\lambda - \theta_M)} x_1 \]  

(14)

Reaching law is a discontinuous switching function. It forces states of the system to reach sliding surface. Disturbances tend to make the states diverge from the sliding surface. Reaching law need to be capable of compensate effects of the disturbances. Thus, a reaching law is generated, which is able to compensate bounded disturbances \( a_{\text{r, max}} > |a_f|, 1 > \mu > \mu_{\text{min}} > 0 \) and \( a_{\text{r, max}} > |a_f| \) by considering finite time stability of the guidance law

\[ a_{rl} = (k_1 |a_{ec}| + \frac{k_2}{\cos(\lambda - \theta_M)} + \frac{k_3}{\sqrt{2\cos(\lambda - \theta_M)}}) sgn[s] sgn[\cos(\lambda - \theta_M)] \]  

(15)

In the reaching law \( k_1, k_2 \) and \( k_3 \) are guidance gains related to disturbance rejection. The guidance law is derived by combining equivalent control \( a_{ec} \) (14) and reaching law \( a_{rl} \) (15) with respect to equation (13)

\[ a_{com} = a_{ec} + (k_1 |a_{ec}| + \frac{k_2}{\cos(\lambda - \theta_M)} + \frac{k_3}{\sqrt{2\cos(\lambda - \theta_M)}}) sgn[s] sgn[\cos(\lambda - \theta_M)] \]  

(16)

Signum function turns the guidance law a discontinuous function. Sign of the normal acceleration calculated from the guidance law changes as the sign of the sliding variable or \( \cos(\lambda - \theta_M) \) changes. Especially on sliding phase, sign change occurs rapidly and chattering is inevitable to observe on the output command. It is not possible to track a discontinuous command by the actuator, since real systems are continuous. In order to avoid chattering, signum functions in the guidance law (16) are replaced with a continuous sigmoid function [Shiessel, Edwards, Fridman and Levant, 2010].

\[ \sigma(z) = \frac{z}{|z|+d} \]  

(17)

Constant parameter \( d \) is a sufficiently small number. The function becomes smoother as \( d \) increases, however robustness of the controller affected negatively. When \( d = 0 \), equation (17) becomes signum function. Replacing signum functions with equation (17) in the guidance law (16) yields

\[ a_{com} = a_{ec} + (k_1 |a_{ec}| + \frac{k_2}{\cos(\lambda - \theta_M)} + \frac{k_3}{\sqrt{2\cos(\lambda - \theta_M)}}) \sigma[s] \sigma[\cos(\lambda - \theta_M)] \]  

(18)

Stability Analysis

To analyze the stability of the guidance law, a Lyapunov function is selected.

\[ V(s) = \frac{1}{2} s^2 \]  

(19)

In order to satisfy the inequality \( \dot{V}(s) + cV(s)^{\alpha} \leq 0 \) referring to the Lemma stated before, derivative of the Lyapunov function is taken by using equations (6), (10), (11) and (13).

\[ \dot{V}(s) = s(a_f \cos(\lambda - \theta_T) + (1 - \mu)a_{ec}\cos(\lambda - \theta_T) - \mu a_{rl}\cos(\lambda - \theta_T) - a_f \cos(\lambda - \theta_T)) \leq -cV(s)^{\alpha} \leq 0 \]  

(20)

After expanding the inequality (20) by using the equivalent control equation \( a_{ec} \) (14) and reaching law equation \( a_{rl} \) (15), the relation between the guidance gains and disturbances can be observed.

If \( \dot{V}(s) \) is a negative semi-definite function, guidance gains need to satisfy \( k_1 > \frac{1 - \mu_{\text{min}}}{\mu_{\text{min}}} \), \( k_2 > \frac{a_{\text{r, max}} + a_{\text{r, max}}}{\mu_{\text{min}}} \). By taking into consideration this relation of the guidance gains, inequality (20) is reduced to

\[ \dot{V}(s) \leq -V(s)^{0.5} \mu_{\text{min}} k_3 \leq -cV(s)^{\alpha} \leq 0 \]  

(21)

Since \( \alpha = 0.5 \), \( 0 > \alpha > 1 \), \( c > 0 \) and if inequality \( k_3 > \frac{c}{\mu_{\text{min}}} \) is satisfied, \( V(s) \) reaches to zero from the any initial value \( V(s_0) \) in finite time. Thus, the guidance law is finite time stable. The settling time is shown as

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The relation between the guidance gains show that $k_1$ is only related to effective performance percentage $\mu$ of the actuator. If $k_1$ is chosen high enough, the loss of performance of the actuator is tolerated. In order to compensate target acceleration $a_T$ and undesired normal accelerations created by actuator failures $a_F$ by considering loss of performance of the actuator $\mu$, $k_2$ must be chosen high enough. Gain $k_3$ is responsible of adjusting the sliding surface reaching time. The larger the gain $k_3$, settling time becomes smaller.

SIMULATIONS AND DISCUSSION

In order to check the performance of the guidance law, several numerical simulations are made. 3 degrees of freedom simulation model that includes the pitching plane of the missile is created for the missile-target engagement geometry. Since it is not the main concern of this paper, gravity is not modeled in the numerical simulations. There are three cases for each three scenarios. In the first scenario, only actuator failures are considered. Target acceleration is considered alone in the second scenario. In the last scenario, both actuator failures and target acceleration take place. Different impact angles are chosen as constant constraint through the flight, in each case.

In all of the scenarios, initial positions of the missile and target are selected as $x_M(0) = 1 \text{ m}$, $y_M(0) = 500 \text{ m}$, $x_T(0) = 1500 \text{ m}$ and $y_T(0) = 5 \text{ m}$. Missile velocity is constant $V_M(0) = 200 \text{ m/s}$. Initial flight path angle of the missile is $\theta_M(0) = 0^\circ$ and orientation of the target is $\theta_T(0) = 0^\circ$. Desired impact angles at the end of the engagement are $20^\circ$ for Case-1, $40^\circ$ for Case-2 and $60^\circ$ for Case-3. Optimized sliding surface parameters are chosen as $\varepsilon = 100$ and $\gamma = 0.9$. These parameters are optimized for the given initial conditions by using a cost function. The cost function to be minimized is

$$CF = \int_{t_f}^{t_i} (|x_1(t)| + |x_2(t)|) \, dt$$

In the engagement scenario final values of the states are prioritized, since interception accuracy is mostly related to state conditions at time $t_f$. Thus, in this optimization, sliding surface parameters that provide least error in the last second of the engagement are desired. Although it is not the subject of this paper, a normal acceleration limit of $10 \text{ g}$ is considered for the missile, since missiles have a normal acceleration limit in reality. Therefore, in the optimization process, sliding surface parameters that make the missile exceed the acceleration limit, are eliminated. Different cost functions and limitations can be selected for desired purposes.

Guidance gains are chosen in order to satisfy the relation between bounded disturbances $k_1 = 0.5$, $k_2 = 35$ and $k_3 = 20$.

For the chattering attenuation, parameter $d$ in the sigmoid function is chosen small enough to not degrade the robustness of the guidance law significantly, $d = 0.001$.

After the range between the missile and the target falls under 5 m, latest guidance command value is used as constant. This prevents divergence of the guidance command at the end of the engagement.

Scenario - 1

In this scenario, performance of the guidance law is verified against actuator failures. A stationary target is considered. Actuator failure terms are taken as sinusoidal signals $a_F = 20 \sin(t) \text{ m/s}^2$ and $\mu = 0.15 \sin(t) + 0.85$. For example, a deformation on the control surface affects aerodynamic stability properties of the missile negatively. Thus, under these conditions, missile may experience oscillatory motion. In addition, the main aim of this scenario is the observe guidance law’s performance under bounded disturbances. Thus, failure terms are modeled as sinusoidal signals. Results are shown at Figure 3-5.
Impact angle errors and miss distances at the end of the engagement are shown at Table 1:

<table>
<thead>
<tr>
<th>Case</th>
<th>Miss distances [m]</th>
<th>Impact angle errors [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case - 1</td>
<td>0.044</td>
<td>0.002</td>
</tr>
<tr>
<td>Case - 2</td>
<td>0.064</td>
<td>0.345</td>
</tr>
<tr>
<td>Case - 3</td>
<td>0.064</td>
<td>0.645</td>
</tr>
</tbody>
</table>

Table 1: Miss distance and impact angle errors of Scenario - 1

Table 1 shows that guidance law has high precision. Desired impact angles are achieved successfully. Considering the size of the armored vehicles, miss distances are sufficiently small. It is observed from the Figure 3 that as the desired impact angle increases missile gains more altitude. For greater impact angle constraints i.e. Case-2 and Case-3, the need of maximum normal acceleration increases as the desired impact angle increases. As seen from the Figure 4, sliding variable converges to zero immediately thereby guidance law is finite time stable. Small fluctuations around zero are observed through the flight. The reasons of these fluctuations are sinusoidal actuator failure parameters. However, guidance law successively keeps the sliding variable near the zero.

**Scenario – 2**

In this case, performance of the guidance law is verified against target acceleration. A weaving target, which is moving on a barrowed road, is considered. The missile has a constant velocity of $V_T = 30 \text{ m/s}$ and because of the road profile, a normal acceleration of $a_T = 2 \sin(t) \text{ m/s}^2$. Results are presented at Figure 7-10.
Impact angle errors and miss distances at the end of the engagement are shown at Table 2:

<table>
<thead>
<tr>
<th>Case</th>
<th>Miss distances [m]</th>
<th>Impact angle errors [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case - 1</td>
<td>0.034</td>
<td>0.255</td>
</tr>
<tr>
<td>Case - 2</td>
<td>0.068</td>
<td>0.042</td>
</tr>
<tr>
<td>Case - 3</td>
<td>0.040</td>
<td>0.622</td>
</tr>
</tbody>
</table>

Table 2: Miss distance and impact angle errors of Scenario - 2

Guidance law shows a good performance against weaving target. Miss distance and impact angle error data verify the guidance law performance. As in the first scenario, maximum missile altitude is higher on larger impact angle constraints. Again, sliding variables converge to zero quickly in finite time. On Figure 10, there are fluctuations on impact angle. Desired LOS angle $\lambda_f$ changes as orientation of the vehicle changes. This means that the equation of the first state $x_1 = \lambda - \lambda_f$ continuously changes along the flight. Thus, fluctuations on impact angle curves occur.

**Scenario - 3**

Both target acceleration and actuator failures are considered in the last scenario to check all aspects of the guidance law together. Actuator failure and target parameters are chosen same as in scenarios 1 and 2 respectively. Results are shown at Figure 11-14.
Impact angle errors and miss distances at the end of the engagement are shown at Table 2:

<table>
<thead>
<tr>
<th>Case</th>
<th>Miss distances [m]</th>
<th>Impact angle errors [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.021</td>
<td>0.266</td>
</tr>
<tr>
<td>2</td>
<td>0.029</td>
<td>0.039</td>
</tr>
<tr>
<td>3</td>
<td>0.017</td>
<td>0.638</td>
</tr>
</tbody>
</table>

Table 3: Miss distance and impact angle errors of Scenario - 3

Precision of the guidance law is preserved, under both target acceleration and actuator failures. Miss distances are kept low; even there are significant number of disturbances. Fluctuations observed at previous scenarios exist on both sliding variables and impact angles. Since both target acceleration and actuator failure are appeared in this scenario, more guidance command is required to compensate disturbances. From the Figure 12 it is observed that guidance law is finite time stable under all type of disturbances considered in this paper.

**CONCLUSIONS**

In this paper, a terminal sliding mode guidance law is proposed, in order to achieve robust properties against unknown target acceleration and actuator failure effects. The guidance law also allows missile to hit the target with a desired impact angle, which is an important property for anti-tank missiles. States of the sliding surface are chosen by considering impact angle constraint and success of interception. An adjustable time varying slope parameter is used in the sliding surface to achieve a flexible sliding surface. The guidance law is derived by defining
an equivalent control and a reaching law. The proposed guidance law guarantees finite time convergence of sliding variables against bounded disturbances and it is proved by both stability analysis and simulations. Finally, performance of the proposed guidance law is tested under different scenarios with different cases. Miss distances and impact angle errors are sufficiently low in all the scenarios and cases. Thus, simulation results verify the performance of the guidance law under actuator failures and target accelerations.

References