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DESIGN AND PERFORMANCE ANALYSIS OF A LOW EARTH ORBIT WALKER CONSTELLATION AS A REGIONAL NAVIGATION SATELLITE SYSTEM FOR TÜRKİYE

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ABSTRACT

In this study, performance of a low earth orbit Walker satellite constellation (LWSC) is analyzed as a Regional Navigation Satellite System (RNSS) for Türkiye. A reference LWSC is designed with respect to the Walker method. Geometric dilution of precision (GDOP) is used as a figure of merit. A software is developed to design and analyze the Walker satellite constellation. Results are compared with the current Global Positioning System (GPS) constellation. Both LWSC and reference GPS constellations are visualized in Systems Tool Kit (STK). The software is also corrected with STK.

INTRODUCTION

Navigation is quite an important term for both military and civil utilization. In previous years, several systems and methods have been used for navigation, especially land-based systems. The term satellite navigation (SATNAV) came up with an idea of large service area and precise geographical positioning by using Earth orbiting satellites. The space segment of SATNAV consists of a group of satellites, called satellite constellation, and is constructed to provide autonomous precise positioning via radio signals, especially on the Earth. Global navigation satellite system (GNSS) is the most used term in this field to indicate the satellite systems providing autonomous geographical positioning and timing globally on the Earth. For GNSS systems, at least 4 visible satellites are required to communicate with the receiver via radio signals to determine the geographical position on the Earth. GPS is one of the most used satellite-based radio navigation system owned by the United States government. GLONASS and Galileo are also the most used GNSSs owned by Russian Federation and European Union, respectively.

The term RNSS is used for the systems providing positioning and timing service only on a desired region on the Earth. Since GPS and other global systems are owned by other countries which give rise to occur of military concerns, RNSS is quite important for the national security of the countries. Indian Regional Navigation Satellite System (IRNSS) can be given as an example of RNSS which are owned by India. Although Quasi-Zenith Satellite System (QZSS) is an also constructed as a space-based navigation system for regional service, It's not a

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completely RNSS but it's an augmentation system of GPS. Türkiye, due to its geopolitical location, is one of the countries that would benefit from having its own RNSS.

Various types of constellations with different configurations, number of satellites, and different size of satellites etc., have been studied so far. Walker satellite constellation and its Walker delta pattern (WDP), on the other hand, is one of the most used constellation patterns described by J.G Walker [Walker, 1970]. Jennez et al. studied a Walker constellation at a height of 23000 km with 30 satellites in 10 different orbital planes [Jennez et al., 2018]. They specified the phasing parameter (F) as 6 and inclination of $45°$ and aimed a global coverage. Beech et al. studied three different satellites constellation design algorithms [Beech et al., 1999]. They used an algorithm to optimize the Walker parameters of total number of satellites (T), total number of orbital planes (P), F, and inclination. They assumed an initial constellation and achieved the current GPS constellation since GPS is one of the constellations which has a WDP. They also studied a second algorithm based on a street of coverage (SOC) method for polar, non-symmetric constellations. As a third algorithm, they studied an adaptive random search algorithm as a variation of genetic algorithm to solve complex constellation problems with a cost function. Designing a satellite constellation is a complex and multi-objective problem since it has a lot of effective variables such as several orbital parameters, launch options, cost, link performances etc. Therefore, a lot of studies can be sorted including such an optimization algorithm. Savitri et al. used a combined genetic algorithm to design multi-objective optimization to maximize the percent coverage and minimize the revisit time for a small satellite constellation in circular low Earth orbit (LEO) [Savitri et al., 2017]. They used six orbital elements as optimization design variables. Noer et al. studied a hybrid RNSS constellation with geostationary (GEO) and geosynchronous (GSO) orbits for India [Noer et al., 2020]. They also used a genetic algorithm to optimize the constellation for least mean GDOP. They achieved that 3 GEO and 4 GSO satellites provide better service on India. Shtark et al. also looked for a solution for RNSS but using a LEO satellite constellation [Shtark et al., 2018]. They defended that a nano-satellite constellation based RNSS in LEO will significantly reduce the launching, building, and maintenance costs. They developed a LEO constellation optimization and design method, using genetic algorithm and gradient-based optimization. They used the total coverage time, revisit time, and GDOP as figures of merit. Their analysis also included solar radiation pressure, drag and the lunisolar gravitational perturbation. They analyzed 268 generated different LEO constellation at approximately 833.5 km. Perrotta and Girolamo, on the other hand, studied the performance of three different constellation configuration in a study that are circular, GSO, and high elliptic orbits in terms of accuracy, availability, continuity, and integrity [Perrotta and Girolamo, 1998]. They studied a constellation with 27 satellites located in intermediate circular orbits with a common altitude of 10,839 km and common inclination of $57°$ which is a WDP. They showed that the constellation provides global continuous coverage with lower GDOP less than 4 and at least six satellites are always available. As seen from the literature, WDP is one of the most favorable satellite constellation patterns and is studied for different purposes.

The purpose of this study is to design and conduct a comparative performance analysis of a LWSC as an RNSS for Türkiye. Through the design and analysis of the proposed system, this study intends to assess the LWSC's ability to provide reliable and robust navigation service in terms of orbital considerations by comparing to current GPS constellation, thereby, contributing to the development of an indigenous and independent satellite-based navigation system for Türkiye. GDOP is used as a figure of merit. Preliminary design of the LWSC follows the Walker method. The accuracy of the positioning is considered through the effects of orbital configurations which are measured with GDOP.

METHOD

LWSC Design Method

A Walker constellation with its WDP is a common altitude, inclined, and symmetric satellite constellation [Walker, 1970]. All orbital planes are at same inclination angle and equally spread around the $360°$ equatorial plane which means that the angles between the adjacent planes are common. A WDP can be described with a notation of $i{:}T/P/F$ where $i, T, P,$ and F stand for inclination of all satellites in the constellation, total number of satellites in the constellation, total number of inclined orbital planes, and relative phasing parameter, respectively [Wertz, 2001; Chobotov, 2002]. Figure 1 represents the geometry of WDP. Figure 2 represents the single satellite coverage geometry. It's written from the Figure 2

$$\cos(\theta + \varepsilon) = \frac{cos\ \varepsilon}{1 + (h/R_e)} \tag{1}$$

where ε, h, and R_e are the elevation angle, satellite altitude from body to subsatellite point, and earth radius for desired ground station, respectively. Angle θ, on the other hand, is the earth central angle of the coverage. Number of satellites in each plane (S) can then be described [Saeed et al., 2020]

$$S = \left\lceil \frac{360}{2\theta} \right\rceil \tag{2}$$

where ⌈ . ⌉ is the ceiling function.

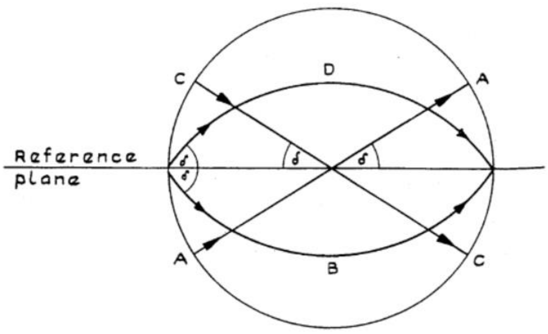


Figure 1: SOC from a single orbit plane [Beech et al., 1999]

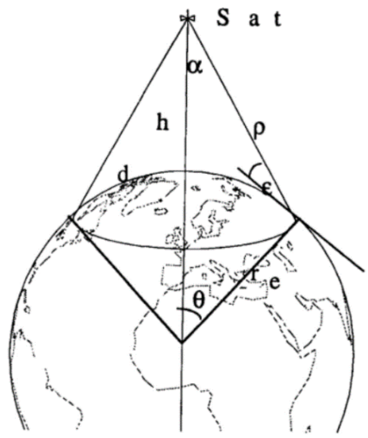


Figure 2: Satellite coverage geometry [Chobotov, 2002]

Since WDP is symmetric, S is also given by

$$S = T/P \tag{3}$$

where P can also be determined with the following equation for continuous multi-fold coverage.

$$P = \left\lceil \frac{360}{\theta} \right\rceil \tag{4}$$

The value of F relates the satellite positions in one orbital plane to those in an adjacent plane and can be taken any value from 0 to $(P-1)$ [Walker, 1970]. Relative phasing angle between satellites in adjacent planes (β) is then given by [Liang et al., 2021]

$$\beta = F * 360/T \tag{5}$$

Since WDP is a symmetric pattern with common altitude, one should consider the phasing parameter for collision avoidance.

Initial Orbit Determination Method of the LWSC

Initial orbit determination (IOD) of the LWSC for a certain epoch is required to analyze the performance of the constellation. First, a reference satellite in a reference orbital plane should be clearly determined. Since orbital planes in WDP is equally spread around the equatorial plane, angle between adjacent planes ($\Delta\Omega$) can be found as

$$\Delta\Omega = \frac{360°}{P} \tag{6}$$

Satellites in each plane is also spread equally in orbital plane that is

$$\Delta \mathrm{T} = \frac{360°}{S} \tag{7}$$

where $\Delta\mathrm{T}$ is the angle between adjacent satellites in a common plane. After determining the phasing parameter F and total number of satellites T , β can be found from the equation (5). The complete pattern and angles are given in Figure 3.

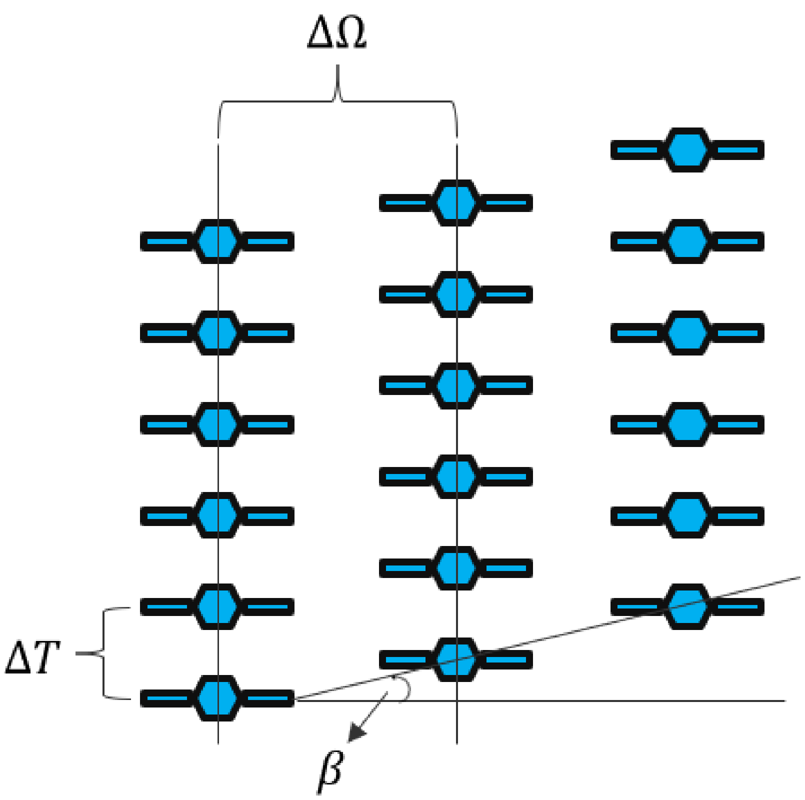


Figure 3: Phasing angles of Walker delta pattern

Since the orbital planes are circular and inclined in WDP [Walker, 1970], argument of periapsis (ω) is not defined while true anomaly is defined from node line to the satellite position in Earth-centered inertial (ECI) frame. Eccentricity of a circular plane, on the other hand, is 0 and semimajor axis of a circular orbit can be defined as a radius of the orbit with respect to the center of ECI frame as follows:

$$a = R_e + h \tag{8}$$

The orbit of each satellite in the constellation can now be described as follows:

$$[S]_{i\,x\,j} = [\Omega_{ij} \quad \mathrm{i}_{ij} \quad \upsilon_{ij}] \tag{9}$$

where Ω_{ij}, i_{ij}, $and\, \upsilon_{ij}$ stand for right ascension of ascending node, inclination, and true anomaly of the satellite, respectively. i and j indices, on the other hand, stand for the plane number and the satellite number in that plane and is described as $i = 1,2,3,\ldots,P$ and $j = 1,2,3,\ldots,S$, respectively. Rearranging equation (9) for the reference satellite gives

$$[S]_{1\,x\,1} = [\Omega_{11} \quad \mathrm{i}_{11} \quad \upsilon_{11}] \tag{10}$$

where $[S]_{1\,x\,1}$ clearly indicates the required orbit elements of the first satellite in the first plane. Then by using equation (6) yields

$$\Omega_{ij} = \Omega_{11} + ((i-1) * \Delta\Omega), \quad i = 2,3,\ldots,P, \quad j = 1,2,\ldots,S \tag{11}$$

By using equations (5) and (7) for the true anomaly of the first satellites in each plane

$$\upsilon_{i1} = \upsilon_{(i-1)1} + \beta, \quad i = 2,3,\ldots,P, \quad j = 1 \tag{12}$$

For true anomaly of entire satellites in each plane yields

$$\upsilon_{ij} = \upsilon_{i1} + ((\mathrm{j}-1) * \Delta\mathrm{T}), \quad i = 1,2,\ldots,P, \quad j = 2,3,\ldots,S \tag{13}$$

A matrix demonstration can now be used to describe initial orbit of all satellites in the constellation at the epoch as follows:

$$[LWSC]_{P\,x\,S} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1S} \\ S_{21} & S_{22} & \cdots & S_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ S_{P1} & S_{P1} & \cdots & S_{PS} \end{bmatrix} \tag{14}$$

Force Model

Force model is selected as two-body. Orbital perturbations such as J2, solar pressure, atmospheric drag, and solar radiation are neglected for simplicity. The equation of motion then yields [Curtis, 2020]

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} \tag{15}$$

where μ and r are the Earth gravitational parameter and the range between the satellite and target ground station, respectively. Since the results of IOD at the epoch gives the satellite positions in terms of orbital elements, the algorithm to acquire the satellite positions as a vector in topocentric North-East-Zenith frame (NEZ) is given in Figure 4 where $\boldsymbol{r}_{\bar{x}}$, $\boldsymbol{r}_{X}$, and $\boldsymbol{\rho}_{x}$ stands for position vector in perifocal frame, position vector in ECI, and position vector in NEZ, respectively.

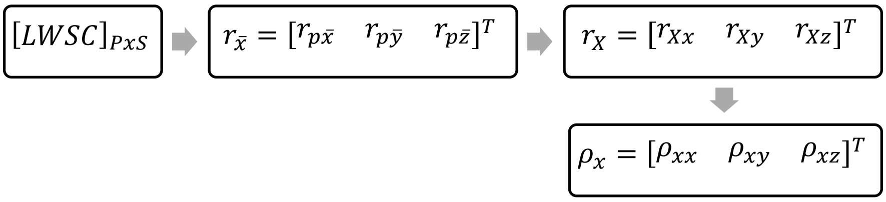


Figure 4: The overall transformation process for acquiring the satellite positions in NEZ

Dilution of Precision

Dilution of precision (DOP) is one of the most favorable performance indicators in designing satellite-based navigation systems [Zhang et al., 2020; Li et al., 2018]. DOP is used to quantify how the geometric distribution of the navigation satellites in the constellation and the position of the receiver affects the positioning accuracy. Mathematical model of the DOP and the discussion to understand the concept is given with equations (16-20) [Kaplan and Hegarty, 2018]. Solving the timing/positioning problem with linearization method yields

$$\Delta x = H^{-1}\Delta\rho \tag{16}$$

where Δx and $\Delta\rho$ are displacement and pseudorange difference between true position and approximate position. H, on the other hand, is the $n\ x\ 4$ matrix and given as follows:

$$H = \begin{bmatrix} a_{x_1} & a_{y_1} & a_{z_1} & 1 \\ a_{x_2} & a_{y_2} & a_{z_2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{x_n} & a_{y_n} & a_{z_n} & 1 \end{bmatrix} \tag{17}$$

The relation given in equation (16) is used to relate the covariance of the user position and time bias to the covariance of pseudorange errors. The components of H matrix, on the other hand, which represented as a_{x_n}, a_{y_n}, and a_{z_n} are the direction cosines of the unit vector pointing from the approximate position to the $n\text{th}$ available satellite. For the case $n > 4$, the relation given in equation (16) can be solved by obtaining least-squares matrix as follows:

$$\Delta x = (H^T H)^{-1} H^T \Delta\rho \tag{18}$$

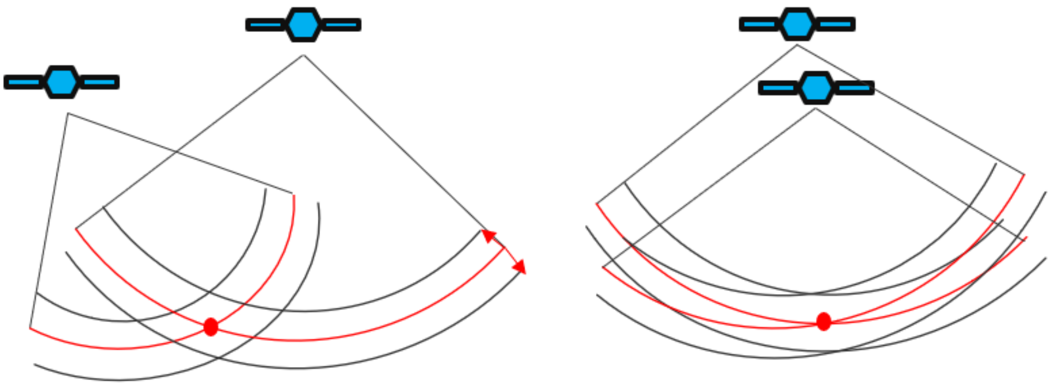
Therefore, the components of the matrix $(H^T H)^{-1}$ in equation (18) quantify how pseudorange errors translate into components of the covariance of the error in the position and time estimate. Obtaining $(H^T H)^{-1}$ in component form yields

$$(H^T H)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \tag{19}$$

Once $(H^T H)^{-1}$ is obtained, several DOP parameters can be described. GDOP, on the other hand, is the most general parameter and is given by

$$Geometric\ DOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}} \tag{20}$$

The corresponding geometries for good and poor DOP are given in Figure 5 and ratings of DOP values are given in Table 1.



(a) (b)

Figure 5: (a) Good DOP geometry, (b) Poor DOP geometry

Table 1: DOP ratings [Kaya and Saritas, 2005]

DOP	Ratings
1	Ideal
2-3	Excellent
4-6	Good
7-8	Moderate
9-20	Fair
21-50	Poor

RESULTS AND DISCUSSION

The Walker method is followed for preliminary design of the LWSC. Inclination is chosen as $55°$ same as GPS. The altitude and phasing parameter F of the LWSC, on the other hand, are chosen 1200 km and 1, respectively. Overall data required for the design and analysis processes are summarized in Table 2. Results for the LWSC and pattern of the reference GPS are given in Table 3.

Table 2: Overall data required for the design and analysis processes

Parameter	LWSC	Reference GPS
Targeted Zonal Area	$30° - 50°$	Global
Constellation Orbit Altitude	1200 km	20200 km
Elevation Constraint	$10°$	$10°$
Orbit Inclination	$55°$	
Reference Receiver Location	Ankara $(39.8911°,\ 32.7786°)$	
F	1	
Orbit Epoch	27 March 2023 09:00:00	
Scenario Timespan	27 March 2023 09:00:00 – 28 March 2023 09:00:00	
Analysis Step Size	60 sec	

Table 3: Results for the LWSC design process and reference GPS constellation pattern

Parameter	LWSC	Reference GPS
Earth Central Angle for Given Elevation Constraint	$24°$	$66.3°$
P	15	6
S	8	4
T	120	24
Walker Delta Pattern	$55°: 120/15/1$	$55°: 24/6/1$

Figure 6 and Figure 7 represent the LWSC and reference GPS constellations visualized in STK, respectively.

Figure 6: LWSC

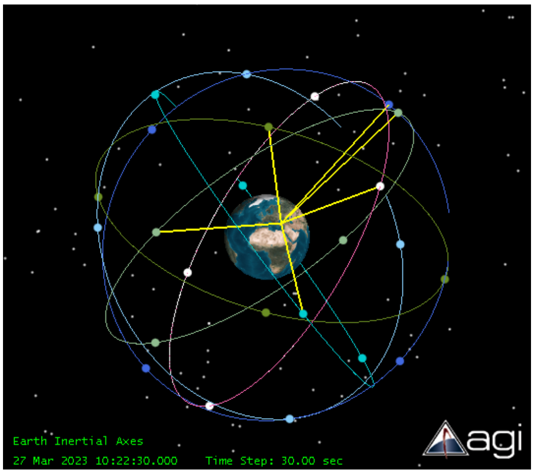


Figure 7: Reference GPS constellation

Since there is no constraint in the mathematical model to ensure that LWSC will provide for better or for worse navigation service with continuously at least 4 satellites, the number of access analysis is required to proof that LWSC can continuously provide at least 4 accessible satellites. Figure 8 and Table 4 represent the results of the analysis by comparing the LWSC and reference GPS. Although LWSC provides slightly better access performance, the difference does not cause a significant effect on the performance.

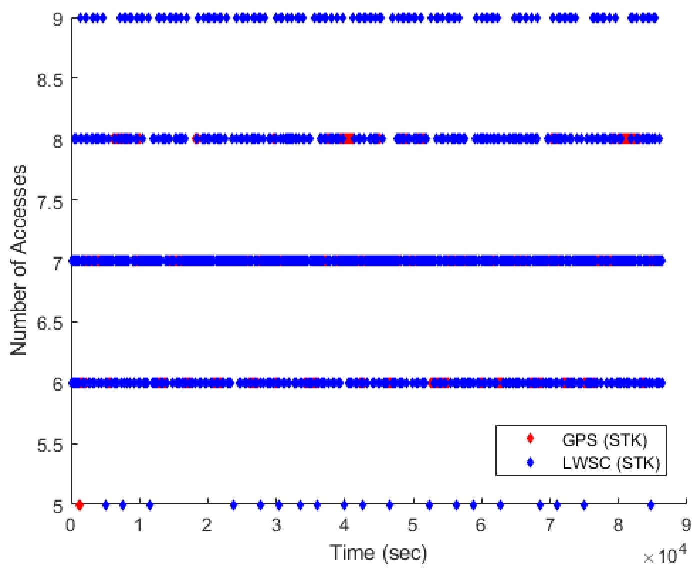


Figure 8: Number of access comparison of the LWSC and reference GPS constellation

Table 4: Number of access comparison of the LWSC and reference GPS constellation

Parameter	LWSC	Reference GPS
Average	~7.1	~6.9
Minimum	5	5
Maximum	9	8

Figure 9, on the other hand, shows that choosing inclination of 55° degree provides the least mean GDOP and is a reasonable choice.

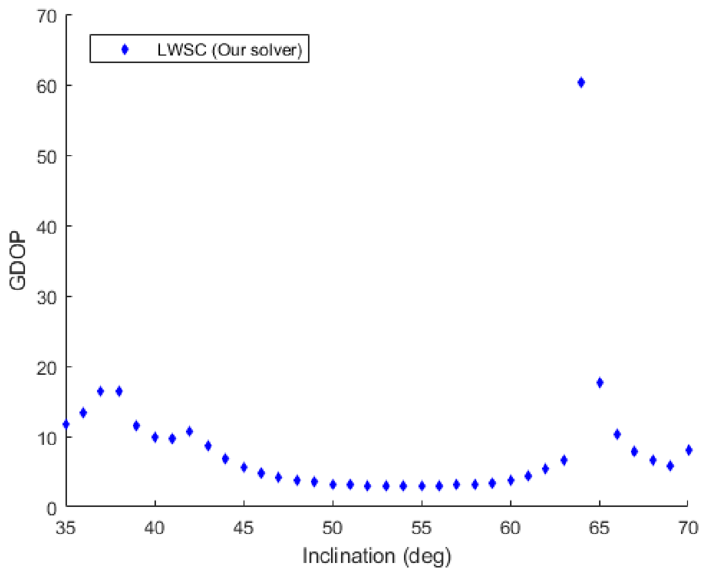


Figure 9: Average GDOP of the LWSC in given inclination range

Figure 10-11 and Table 5-6 represent the GDOP results for the LWSC and reference GPS by comparing STK and our solver. According to the results, our solver performs well and converges for all analysis. Small errors can be detected due to the accuracy level of coordinate transformations. While reference GPS provides slightly better GDOP, GDOP of LWSC is also in good limits and can provide an accurate navigation service in terms of orbital considerations. While LWSC provides better access performance, better GDOP performance of reference GPS shows that increasing of number of accessible satellites does not always increase the navigation accuracy if they are distributed too close in space.

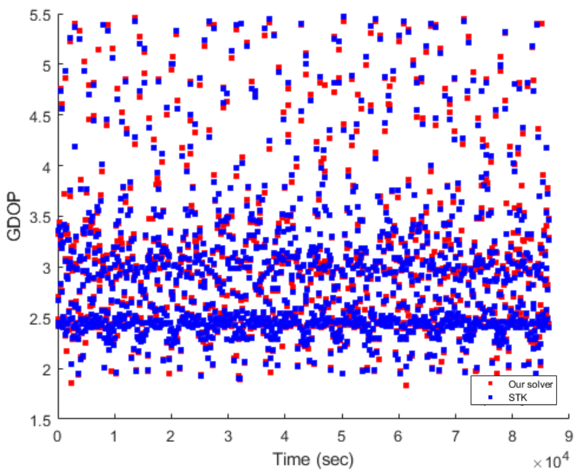


Figure 10: LWSC GDOP

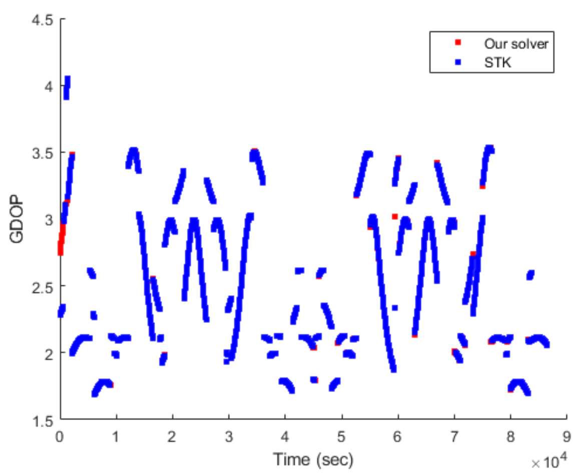


Figure 11: GPS GDOP

Table 5: GDOP results for the LWSC

Parameter	Our solver	STK
Average	2.95827	2.95821
Minimum	1.83	1.89
Maximum	5.45	5.47

Table 6: GDOP results for the GPS

Parameter	Our solver	STK
Average	2.59	2.51
Minimum	1.687	1.689
Maximum	4.13	4.01

CONCLUSION

In this study, an LWSC is designed with Walker method and is analyzed as an RNSS for Türkiye. The LWSC is compared to a reference GPS constellation. Results of the solver are also compared with STK. GDOP is used as performance indicator. The LWSC provides a good performance as an RNSS for Türkiye in terms of orbital considerations. As a future work, an optimization algorithm should be developed or used to optimize the constellation through a corresponding fitness function. Therefore, optimizing for the best options for space segment, ground segment, launch options, cost etc. can yield a recommendable result. A force model including orbital perturbations may be used for more accurate results. Different systems such as IRNSS and QZSS can also be analyzed and compared to an LWSC.

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