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SMALL SPACECRAFTS IN THE SUN AND PLANET 9 SYSTEM

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ABSTRACT

A hypothetical planet in the outer solar system, so called Planet 9 is proposed to explain unexpected clustering of the Kuiper Belt Objects. In this study, the detection of Planet 9 is investigated with the transverse displacement in the trajectory of a sub-relativistic spacecraft due to gravity of Planet 9 by considering the effect of Sun. It is obtained that considering the Sun causes less deflection compared to Planet 9 and spacecraft two-body problem for the initial conditions used in the analysis.

INTRODUCTION

A possibility of a planet in the outer solar system, commonly known as Planet 9, is proposed to explain unexpected clustering of the Kuiper Belt Objects [Batygin and Brown, 2016; Batygin et al., 2019]. Telescope searches are still ongoing, although any detection has not been found yet [Naess, et al., 2021]. Since the telescope searches have been inconclusive so far, it is also proposed that this object might be a primordial black hole [Scholtz and Unwin, 2020]. To detect Planet 9, alternative ways have been suggested such as sending a cluster of small sub-relativistic spacecrafts and measuring the deflection of their trajectories due to gravity of Planet

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9 [Witten, 2020], aiming to measure the Hawking radiation spreading from the primordial black hole with a sub-relativistic spacecraft although it is mentioned that capturing the weak signal would be challenging [Arbey and Auffinger, 2021]. In addition, it is also looked for the Zwicky Transient Facility public archive but no candidates are found [Brown and Batygin, 2022]. More recently, a candidate location of Planet 9 is proposed in [Navarro, 2023] by tracing the trajectory of an interstellar meteoroid backwards and looking its coincidence with the maximum probability region of Planet 9.

In this work, it is focused on the method based on measuring the deflection due to gravity of Planet 9 in the trajectory of a spacecraft attaining sub-relativistic speed with use of advance technologies like solar sail [Macdonald et al., 2010] or laser propulsion [Parkin, 2018]. In [Witten, 2020], the deflection is studied in terms of time delay in the signal sending from spacecraft to Earth with sufficiently accurate timekeeping requirement. To avoid this requirement, measurement of the transverse displacement is proposed by using Very Long Baseline Interferometry (VLBI) [Lawrence and Rogoszinski, 2020]. In addition, effects of the drag and electromagnetic forces exerted by the interstellar medium on spacecraft's trajectory is also discussed in [Hoang and Loeb, 2020], including parameter space of the spacecraft to distinguish gravity of Planet 9 from other perturbations.

In the previous studies, calculation of the deflection is considered in the context of two-body problem, Planet 9 and spacecraft. In this paper, the deflection is investigated in the context of three-body problem, Sun, Planet 9 and spacecraft, to understand whether the three-body context provides new prospects for the detection.

EFFECT OF THE SUN

Why to consider the Sun?

The current estimates on the mass and orbit of Planet 9 given in [Batygin and Brown, 2021] suggest that the gravitational parameter is $\mu_{p9} \approx 6.2\mu_{earth}$ where $\mu_{earth} = 398600\ km^3/s^2$ and semimajor axis is $a_{p9} \approx 380$ AU. Then, the smallest impact parameter for the half sky search with 1000 spacecrafts is calculated, $\rho \cong 30$ AU, as given in [Witten, 2020]. By taking $\mu_{sun} = 132712 \times 10^6\ km^3/s^2$, the ratio of gravitational acceleration due to Planet 9 and Sun in the vicinity of Planet 9 can be calculated as:

$$\left(\frac{\mu_{sun}}{a_{p9}^2}\right) / \left(\frac{\mu_{p9}}{\rho^2}\right) \approx 335 \qquad (1)$$

which motivates us to consider the effect of the Sun besides the other perturbations.

The Circular Restricted Three-Body Problem

Motion of the spacecraft in the presence of Sun and Planet 9 is investigated by considering the Circular Restricted Three-Body Problem (CRTBP) as the underlying dynamical model. In using CRTBP, following assumptions are made:

- Sun and Planet 9 are moving in a circular orbit around their barycenter. In fact, the orbit of Planet 9 is estimated to be elliptic (e~0.2-0.5) [Batygin et al., 2019]. But it is considered that the simple circular model still may provide insight about the effect of Sun.

- To reduce the dimension of the problem, it is assumed that Sun, Planet 9 and spacecraft are moving in the same plane.

- Mass of the spacecraft is negligible such that it does not effect the motion of Sun and Planet 9.

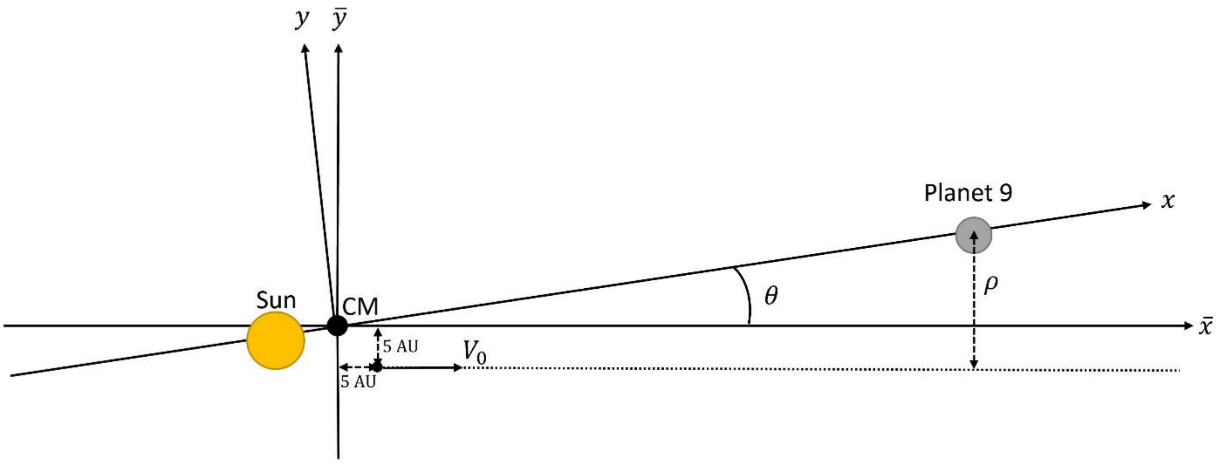

Figure 1: Ilustration of the CRTBP

In Fig. 1, $\bar{x}$-$\bar{y}$ is an arbitrary inertial frame and x-y is the Sun-Planet 9 rotating frame with an angular velocity of $\Omega = \sqrt{\frac{\mu_{sun}+\mu_{p9}}{r_{12}^3}}$. The equations of motion of CRTBP is given as [Curtis, 2013]:

$$\ddot{x}-2\Omega\dot{y}-\Omega^2 x=-\frac{\mu_{sun}}{r_1^3}(x+\pi_2 r_{12})-\frac{\mu_{p9}}{r_2^3}(x-\pi_1 r_{12}) \tag{2a}$$

$$\ddot{y}+2\Omega\dot{x}-\Omega^2 y=-\frac{\mu_{sun}}{r_1^3}y-\frac{\mu_{p9}}{r_2^3}y \tag{2b}$$

where $\pi_1=m_{sun}/(m_{sun}+m_{p9})$, $\pi_2=m_{p9}/(m_{sun}+m_{p9})$, $r_{12}=a_{p9}$, lastly r_1 and r_2 are the distances of Sun-spacecraft and Planet 9-spacecraft, respectively. It is important to note that motion of the spacecraft is calculated relative to x-y rotating frame but the deflection is investigated in the inertial frame. So, the frame transformation between $\bar{x}$-$\bar{y}$ inertial frame and x-y rotating frame is necessary.

The position vector of the spacecraft can be transformed from rotating frame to inertial frame as follows:

$$\begin{bmatrix}\bar{x}\\ \bar{y}\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \tag{3}$$

Then, the velocity vector transformation can be found by taking the time derivative of (3) as:

$$\begin{bmatrix}\dot{\bar{x}}\\ \dot{\bar{y}}\end{bmatrix}=\Omega\begin{bmatrix}-\sin\theta & -\cos\theta\\ \cos\theta & -\sin\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} \tag{4}$$

The reverse transformation can be obtained in the same manner.

Numerical Analysis Results

The equations of motion of CRTBP are solved with numerical integration for a particular initial condition. In the numerical integration, MATLAB built-in ode45 solver is used with absolute and relative tolerances of 10^{-13}.

In [Zeng et al., 2019] and [Sauer, 1999], the interstellar mission trajectories with use of solar sails are investigated. Due to the reduction of solar flux and so its propulsive effect, it is considered in [Sauer, 1999] to seperate solar sail at approximately 5 AU. It is assumed that the spacecraft reaches the sub-relativistic speed at the solar sail seperation distance, so the initial position for the numerical integration is chosen by considering this assumption. For the sub-relativistic speed, 0.001c, c=2.99792458×10^5 km/s is the speed of light, is chosen to make the mission duration reasonable. Then, the initial condition of the spacecraft is:

$$\left(\bar{x}_0,\bar{y}_0,\dot{\bar{x}}_0,\dot{\bar{y}}_0\right)=(5\text{ AU}, -5\text{ AU}, 0.001c, 0) \qquad (5)$$

Notice that the (5) is given in the inertial frame. It should be transformed to the rotating frame of CRTBP in order to obtain three-body trajectory. So, the initial phase angle of Planet 9, θ, is required to make transformation, and it is chosen such that the impact parameter would be $\rho\cong 30$ AU. Lastly, the integration duration is chosen to be $t = 2(a_{p9}-\bar{x}_0)/\dot{\bar{x}}_0$. It is assumed that measurements start when the spacecraft reaches sub-relativistic speed and continue until the given t is achieved.

In [Lawrance and Rogoszinski, 2020], the measurement of transverse displacement is proposed to avoid accurate timekeeping requirement for the time delay measurement given in [Witten, 2020]. It is also mentioned that transverse velocity change is permenant which makes the detection easier. Therefore, change in the transverse velocity between initial and final times are investigated in this paper.

In [Lawrance and Rogoszinski, 2020], it is also given an analytical expression for the transverse velocity change by considering the perturbative effect of Planet 9 on a straight line trajectory in the absence of Sun:

$$\Delta v_y=2\mu_{p9}/\rho v_x^{(0)}\approx 3.674\times 10^{-6} \qquad (6)$$

In the presence of Sun, unperturbed or nominal trajectory of the spacecraft is defined as the trajectory obtained from the Sun-spacecraft two-body problem. In calculating the Sun-spacecraft two-body trajectory, the terms with μ_{p9} and Ω are excluded in the equations of CRTBP. Then, the perturbation due to Planet 9 is calculated as the difference between Sun-Planet 9-spacecraft three-body trajectory and the Sun-spacecraft two-body trajectory. The results are given in Tab. 1.

Table 1: Comparison of two-body and three-body results

Trajectory Model	$\left\|\Delta\dot{\bar{y}}\right\|$ **(km/s)**
Planet 9+S/C	3.674×10^{-6}
(Sun+Planet 9+S/C)-(Sun+S/C)	3.454×10^{-6}

It is obtained that considering the Sun decreases the transverse velocity change. In CRTBP, Planet 9 and Sun are moving in circular orbits unlike the Planet 9 and spacecraft two body problem which considers the Planet 9 as stationary. Due to the movement of Planet 9, the distance between Planet 9 and spacecraft is larger in the flyby for the CRTBP compared to two-body problem. Therefore, less transverse velocity change is obtained for the used initial conditions. To observe the effect of Sun, different velocity scales for the same initial position are investigated in Tab. 2.

Velocity scales are decreased until 0.0004c because further reduction is resulted with longer mission duration, larger than the order of decades. As the velocity decreases, the transverse velocity change increases in both two-body and three-body systems. But still the Sun causes less deflection.

One may look at [Köprücü, 2023] for the further discussion on the observability of the deflection due to Planet 9 and the effect of Sun for the different initial conditions.

Table 2: Transverse velocity change for different velocity scales

	$\lvert \Delta\dot{y} \rvert$ (km/s)	
Velocity	**Planet 9+S/C**	**(Sun+Planet 9+S/C)-(Sun+S/C)**
0.001c	3.674×10^{-6}	3.454×10^{-6}
0.0008c	4.592×10^{-6}	4.266×10^{-6}
0.0006c	6.123×10^{-6}	5.591×10^{-6}
0.0004c	9.184×10^{-6}	8.170×10^{-6}

CONCLUSIONS

In this study, the detection of Planet 9 is investigated by measuring the transverse velocity change in the trajectory of a sub-relativistic spacecraft due Planet 9 in the presence of Sun. It is obtained that considering the Sun yields less deflection compared to Planet 9-spacecraft two-body trajectory for the initial conditions used in the analysis.

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