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SUPERSONIC WIND TUNNEL DESIGN

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ABSTRACT

The main objective of the project is to study the aerodynamics of small-scale aircraft by designing and fabricating a supersonic wind tunnel for testing test vehicles. The designed indraft supersonic tunnel uses the pressure difference between a low-pressure tank and the atmosphere to store energy. In the project design, the Mach number and the test area were determined as variable parameters. The primary criterion for the design was to achieve the maximum range of Mach numbers and test areas with the minimum cost. The wind tunnel design involved three integrated studies. Firstly, a convergent-divergent nozzle was designed to increase the fluid velocity and achieve supersonic speeds. Secondly, a test section was included for placing and testing the test object (small-scale aircraft). Finally, an expanding diffuser design was implemented to decrease the fluid velocity and increase the pressure, helping to balance the pressure around the tested object. These components collectively formed the basis of the wind tunnel design. In the project, Python software was utilized to calculate the supersonic wind tunnel nozzle and test area, while 3D drawings were created using Catia software. Computational Fluid Dynamics (CFD) simulations were also conducted to validate the design. The wind tunnel is designed to have a test section of 11cm x 10cm and a test zone with Mach number 2.1. Simulations were performed for both 3D non-viscous and viscous cases. Overall, the project aimed to create an efficient and cost-effective supersonic wind tunnel for studying the aerodynamics of small-scale aircraft. The integrated studies, calculations, 3D modeling, and simulations contributed to the successful design and validation of the wind tunnel.

INTRODUCTION

Wind tunnel is a device designed to produce air flows at various speeds from a test section. Wind tunnels are typically used in aerodynamic research to analyze the behavior of flows under varying conditions, both inside channels and on solid surfaces. Aerodynamicists can use the controlled environment of the wind tunnel to measure flow conditions and forces in

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aircraft models for which they are designed. Being able to collect diagnostic information from models allows engineers to change designs cheaply for aerodynamic performance without creating a large number of fully functional prototypes [Barlow, Rae and Pope, 2015].

Supersonic wind tunnels are experimental facilities used to study the behavior of objects and materials in supersonic flows. They generate high-speed airflows by compressing and heating air to create a gas at temperatures and pressures similar to those encountered in flight at supersonic speeds. By subjecting objects and materials to these high-speed flows, researchers can test their performance and behavior under extreme conditions [Berry, Rhode and Edquist, 2011].

A supersonic wind tunnel is a type of wind tunnel that is designed to simulate high-speed flows, typically those that occur at speeds greater than the speed of sound. In a supersonic wind tunnel, air is accelerated to supersonic speeds using a series of nozzles, and then directed towards a test section where models or prototypes can be tested. There are two main types of supersonic wind tunnels: shock tunnels and expansion tunnels. Shock tunnels use a series of shock waves to decelerate the flow of air to subsonic speeds, while expansion tunnels use a series of expansion waves to decelerate the flow. Both types of tunnels can be used to generate supersonic flows, but each has its own advantages and disadvantages. The high speeds generated in a supersonic wind tunnel make it possible to study phenomena such as shock waves, boundary layer separation, and other complex flow behaviors that occur at high speeds [Anderson, 2003].

The basic problems in the design of any high-speed wind tunnel are always those of providing suitable duct work and flow control devices to ensure that air will pass through the test section of the tunnel at the desired flow conditions. Going one step further, we can say that these problems always include those of providing air (1) with enough pressure ratio across the tunnel to achieve the desired flow velocity, (2) with enough mass per second and total mass to meet the tunnel size and run-time requirements, (3) dry enough to avoid condensation, and (4) hot enough to avoid liquefaction. The ways of solving these problems result in four basic types of wind tunnels: blowdown, indraft, pressure-vacuum, and continuous [Pope and Goin, 1965].

For this project, the indraft supersonic wind tunnel was considered for cost and manufacturability. During operation, air flows from the atmosphere through the tunnel and into the vacuum tank, causing the tank pressure to decrease. In the production, a vacuum pump and tank suitable for the design parameters and calculations are the requirements of the tunnel.

METHOD

Four different supersonic wind tunnel configurations for producing the proper pressure ratio across the supersonic nozzle are blowdown, indraft, pressure-vacuum and closed-circuit supersonic wind tunnel. A thorough discussion can be found in Pope and Goin, High-Speed Wind Tunnel Testing [Pope and Goin. 1965].

As mentioned earlier, the tunnel is selected as indraft supersonic wind tunnel due to is simpler and easier to design and build, and hence are particular favorites for academic institutions. On the other hand, their limited running times can restrict the amount and type of data to be taken. On the whole, intermittent facilities are much less expensive. Continuous flow supersonic tunnels tend to be large and expensive; for the most part they are found at large government laboratories [Anderson, 2017].

A basic sketch of a supersonic wind tunnel is given in Figure 1 that illustrates the essential components of nozzle, test section, and diffuser. The pressure ratio from the inlet to the nozzle to the exit of the diffuser is what makes the tunnel run. How this pressure ratio is generated is an essential first step in the conceptual design of a supersonic wind tunnel [Anderson, 2017].

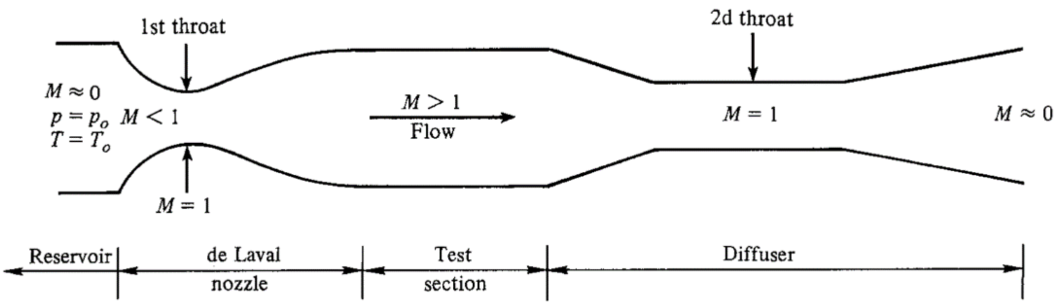

Figure 1: Schematic of the main components of a supersonic wind tunnel [Anderson, 2003].

This study aims to design a small-scale, supersonic wind tunnel prototype. The Design Requirements and Objectives of supersonic wind tunnel to be designed is as listed in Table 1.

Table 1 : Design parameters of the wind tunnel.

Requirements	Objectives
Having test section up to	11 cm x 10 cm
Operating Mach Number	2.1

The first step in designing a supersonic wind tunnel is to determine the test conditions that need to be repeated. This will include parameters such as Mach number, Reynolds number, pressure and temperature. These conditions will determine the size and specifications of the tunnel. The upper end of the tunnel is open to the atmosphere and is a high pressure zone, while at the lower end, the low pressure is provided by pumping the vacuum tank with a vacuum pump. The next crucial step is to choose a nozzle design. The nozzle is a critical component of a supersonic wind tunnel, as it is responsible for accelerating the air to the desired speed. Nozzle geometry (contour) plays a very important role in achieving a smooth flow within the test section. In the nozzle throat, the flow accelerates from subsonic to the desired supersonic speed and enters the test section. The test section is where the model or sample under test is located. It must be designed to accommodate the size and shape of the object, as well as any instrumentation required for data collection.

Principles Used in Supersonic Flow

The most important component of the wind tunnel would be the convergent-divergent duct as this is where the supersonic flow is produced. In the design, three main issues were mentioned; isentropic flow, normal shock relations and boundary layer.

Isentropic flow is useful for preliminary design of the convergent-divergent channel, assuming the fluid is adiabatic and reversible. This assumption is important such as to calculate the area ratios (A/A*) and properties such as pressure, density and temperature. If the normal shock is positioned before the test section, the flow will substantially reduce the Mach number thus not achieving the desired test section Mach number. Assuming a non-slip wall and a high velocity flow, it is clear that we would expect a boundary layer to form. The issue at hand is that the inflation of area from the divergent nozzle to the test section sees an increasing height in the boundary layer. Resulting to a decrease in the effective area thus reducing the Mach number away from the desired [Von-Karman, 1947].

When a supersonic flow encounters a normal shock, there are losses in total pressure, which account for a significant portion of the power requirements in higher Mach number supersonic tunnel operations. To minimize these losses, diffusers are used in the tunnel design.

The process starts with a low subsonic speed throughout the tunnel circuit, and the power required corresponds to the subsonic drag of the complete circuit. As power is increased, the speed throughout the circuit rises, and the Mach number at the nozzle throat reaches 1.0, causing a normal shock to form a short distance downstream of the throat. Further increases in power move the normal shock downstream through the nozzle, occurring at progressively higher Mach numbers. The added shock losses contribute to increased power requirements. The normal shock eventually moves into the test section, occurring at the test section's Mach number, and power requirements correspond to the normal shock losses at the design Mach number. Diffuser design does not influence power requirements during the tunnel starting process since the flow in the diffuser remains subsonic. Therefore, the power required to start a supersonic tunnel corresponds to normal shock losses at the design Mach number, which can be significant at higher Mach numbers [Pope and Goin, 1965].

Tunnel engineers typically use the concept of "pressure ratio" instead of "power," which is the ratio of stagnation pressure to diffuser exit pressure, and it is related to mass flow. With the normal shock in the test section, only a slight increase in power should be required to move the shock through the second throat of the diffuser, as the normal shock Mach number and losses decrease in the converging section of the diffuser. Downstream of the normal shock, the flow is subsonic. Hence the flow velocity in the converging section of the diffuser must be in- creasing, until a maximum velocity is reached in the second throat [Pope and Goin, 1965].

Design of a Supersonic Wind Tunnel

As design inputs, the Mach number and the area of the test section are the parameters determined by the designer. Using the inputs, pressure, density, temperature and area ratios, which are isentropic flow properties, were calculated. Since the tunnel to be produced is an indraft wind tunnel, the parameters in the test area were calculated using atmospheric conditions.

With the backround in compressible flow theory, air flow in general is governed by the five following laws. Moreover, the flow is assumed to be adiabatic inside the tunnel. Using these assumptions and the following equations the nozzle and the diffuser throat areas are calculated.

At any point in a flow field, the pressure, density, and temperature are related by the equation of state:

$$\boldsymbol{p = \rho\, R\, T} \tag{1}$$

For continuous flow in a duct or stream tube, the equivalence of mass flow at any two stations is specified by the continuity equation:

$$\boldsymbol{\rho_1\, A_1\, U_1 = \rho_2\, A_2\, U_2} \tag{2}$$

where A is the cross-sectional area of the duct at a given station, U is the flow velocity , and subscripts 1 and 2 denote two stations in the duct.

If no energy is added or lost during the flow of a sample of fluid between two stations in a duct (that is, if the flow is adiabatic), the following energy equation is valid:

$$\boldsymbol{c_p\, T_1 + \frac{U_1^2}{2} = c_p\, T_2 + \frac{U_2^2}{2} = c_p\, T_t} \tag{3}$$

where c_p is the specific heat at constant pressure and the subscript t denotes conditions at zero velocity or, identically, stagnation conditions.

If the change of state of a fluid during its flow from one station to another is isentropic, the following thermodynamic relation is applicable:

$$\boldsymbol{\frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}}} \tag{4}$$

where γ is the ratio of specific heat at constant pressure, c_p, to specific heat at constant volume, c_v.

From the summation of forces between two stations in a constant area stream tube or duct with no friction, the following momentum equation is obtained:

$$\boldsymbol{p_1 + \rho_1\, U_1^2 = p_2 + \rho_2\, U_2^2} \tag{5}$$

In this stage, velocity in the throat is equal to the speed of sound as the Mach number is assumed to be 2. Flow quantities are introduced; total temperature, total pressure, and speed of sound,

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{6}$$

$$a^* = \sqrt{\gamma R T^*} \tag{7}$$

which is the temperature that would exist if the flow were slowed down or speeded up to Mach 1.0; characteristic Mach number,

$$M^* = \frac{V}{a^*} \tag{8}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{9}$$

Equation (9) gives the ratio of total to static temperature at a point in a flow as a function of the Mach number M at that point.

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{10}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}} \tag{11}$$

Equations (10) and (11) give the ratios of total to static pressure and density, respectively, at a point in the flow as a function of Mach number M at that point. When the total pressure ratio is considered, this ratio is 1.893 for Mach 1, and this ratio is 9.140 since the Mach number reaches 2.1 in the test section. The density ratio is 1.577 for Mach 1 and 4.859 for Mach 2.1.

$$\frac{A}{A^*} = \frac{1}{M^2}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{\frac{\gamma + 1}{\gamma - 1}} \tag{12}$$

Equation (12) is called the area-Mach number relation, the Mach number at any location in the tunnel is a function of the ratio of the local tunnel area to the sonic throat area. When calculated according to the Mach number in the test region, the area ratio is calculated as 1.837.

When a normal shock wave exists in a flow, there is an entropy change across the shock. Consequently, the preceding isentropic flow equations are not valid. The equation of state (1), the continuity equation (2), the energy equation (3), and the momentum equation (5) are used in the derivation of normal shock flow equations.

Let subscripts 1 and 2, respectively, represent conditions upstream and downstream of a normal shock. The combination of eqs. (1) and (5) gives:

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{13}$$

The combination of continuity eqn. gives

$$M_2^2 = \frac{[2/(\gamma - 1)] + M_1^2}{[2\gamma M_1^2/(\gamma - 1)] - 1} \tag{14}$$

The stagnation pressure downstream of a normal shock is less than that upstream of the shock. The relation of static to stagnation pressure downstream of the shock is obtained from eq. (10) when the Mach number downstream of the shock is used. A relation for the total pressure downstream of a normal shock is obtained as follows:

$$\frac{p_{t2}}{p_{t1}} = \frac{(p_1/p_{t1})/(p_2/p_1)}{(p_2/p_{t2})} = \left[\frac{(\gamma+1)}{2\,\gamma\,M_1^2-(\gamma-1)}\right]^{1/(\gamma-1)}\left[\frac{(\gamma+1)\,M_1^2}{(\gamma-1)\,M_1^2+2}\right]^{\{\gamma/(\gamma-1)} \quad (15)$$

The relations of eqs. (14) to (15) are tabulated in the Table 2. for mach number of 2.1.Also included in Table 2. is the parameter p_1 /p_{t2} , which is obtained by dividing eq. (10) by eq. (15).

Table 2: Normal shock relations.

M_1	p_2/p_1	ρ_2/ρ_1	T_2/T_1	p_{t2}/p_{t1}	p_1/p_{t2}	M_2
2.0	4.500	2.667	1.688	0.7209	0.1773	0.5774
2.1	4.978	2.812	1.770	0.6742	0.1622	0.5613

However, practical considerations may limit the effectiveness of the second-throat diffuser. Since the flow downstream of the normal shock is subsonic, the flow velocity in the converging section of the diffuser must increase until it reaches a maximum velocity in the second throat. To ensure the Mach number in the second throat doesn't exceed 1.0, the second throat is sized to pass the mass flow from the nozzle, with the air's expansion downstream of the normal shock kept at a Mach number no greater than 1.0. This sizing process involves using equations to relate the Mach number downstream of the shock to the upstream Mach number and obtain the ratio of second throat area to test section area.

In conclusion, diffusers in the design of supersonic wind tunnels play a crucial role in reducing normal shock losses and improving the tunnel's efficiency. They help compress and slow down the flow, allowing the normal shock to occur at a lower Mach number with correspondingly smaller losses [Pope and Goin, 1965].

$$\frac{A_2^*}{A_T} = \frac{\left(5+M^2\right)^{0.5}\left(7M^2-1\right)}{216M^6} \quad (16)$$

In summary, when designing a supersonic wind tunnel with the requirement of being able to start smoothly, the fixed second throat size ends up being considerably larger than necessary to achieve a Mach number close to 1.0 during regular operation.

Other critical parameters are velocity in the test section, volumetric flow rate and mas flow rate. The Mach number in the test area and the velocity in the 11 cm x 10 cm test area were found to be 520.860 m/s. The mas flow rate is constant and approximately 1.445 kg/s throughout the entire tunnel, and the volumetric flow rate is 5.729 m^3/s. Based on this calculation, a vacuum source will be selected.

$$Q = VA \quad (17)$$

$$\rho = \frac{P}{RT} \quad (18)$$

$$m = \left[\frac{AP_0}{\sqrt{T_0}}\right]\sqrt{\frac{\gamma}{R}\left[\frac{2}{\gamma+1}\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \quad (19)$$

$$m = Q\rho \quad (20)$$

First stage of the design, the mass flow rate and volumetric flow rate values of the air that will pass through the tunnel made according to the Mach number 2.1, which was chosen as the design constraint. Table 3. contains the design inputs with atmospheric properties, and Table 4. contains the outputs of the theoretical calculations. All basic calculations were examined in the nozzle, test area and diffuser to be designed on the basis of the study.

Table 3: Input parameters of the system.

Pressure [Pa]	Temperature [K]	Density [kg/m^3]	Velocity [m/s]	Specific heat $[\mathbf{kJ}/(\mathbf{kg}\cdot\mathbf{K})]$
101325	288.15	1.225	0	1.005

Table 4: Design outputs.

Mach Number	Tunnel Properties	11cmx10cm Test Section Size			
		Nozzle Inlet	Throat	Test Section	Diffuser Throat
2.1	Pressure [Pa]	101325	53528.15	11080.21	36088.67
	Temperature [K]	288.15	240.125	153.1084	240.125
	Density [kg/m^3]	1.225	0.776717	0.252108	0.52366
	Velocity [m/s]	0	340.260	520.863	310.689
	Area [m^2]	0.011	0.005988	0.011	0.0088

Considering a tunnel with a cross-sectional area of 11cmx10cm and a Mach number of 2.1, the flow rate of the air passing through the test area was calculated as 5.729 m^3/s. The airflow required for supersonic speed in the test area is as Table 5.

Table 5: Calculated design parameters.

Mach Number	Tunnel Properties	11cmx10cm Test Section
2.1	Volumetric Flow Rate [m^3/s]	5.729
	Mass Flow Rate [kg/s]	1.445
	Speed of Sound [m/s]	248.030

Design of Converging Diverging Nozzle with Method of Characteristics

Method of characteristics is one of the methods used to compute supersonic irrotational flow. In supersonic nozzle design, the goal is to find a converging-diverging nozzle shape that will produce the desired supersonic flow. The method of characteristics can be used to analyze the flow through the nozzle and determine the optimal shape that will produce the desired flow conditions. This method is used to design the divergent section of the nozzle using a series of points distributed along the nozzle which we know the flow properties and lines connecting these points called characteristic lines [Anderson, 2003].

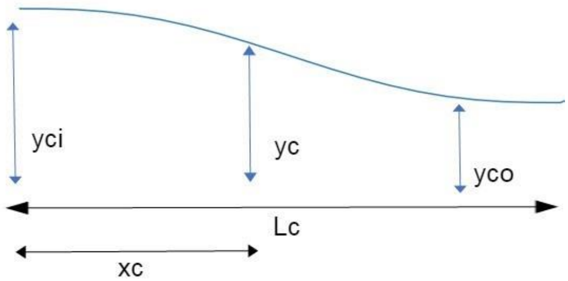

Figure 2: Schematics of the contraction shape.

Where, xc is the distance downstream, yc is the vertical distance from the axis at position xc, yci is the vertical position from the axis at the inlet position, yco is the vertical distance from the axis at the throat position, Lc is the length of the convergent passage.

Method of characteristics (moc) is a suitable numerical method for solving two-dimensional compressible flow problems. Using this technique, flow characteristics such as direction and velocity can be calculated at different points throughout the flow field [Moore, 2009].

To begin with, consider steady, adiabatic, two-dimensional, irrotational supersonic flow. Other types of flow will be considered in subsequent sections. For two-dimensional flow,

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}-\frac{2\Phi_x\Phi_y}{a^2}=0 \tag{21}$$

ϕ is the full-velocity potential.

$$\Phi_x = u \tag{22}$$

$$\Phi_y = v \tag{23}$$

$$V = ui + vj \tag{24}$$

Equation (22) is velocity vector for two-dimensional flow. Equation (23) applies to two dimensional irrotational flow and a scalar function ϕ = fcn(x,y) can be defined with relation to V shown in Eq. (24).

$$d\Phi_x = \frac{\partial \Phi_x}{\partial x}dx + \frac{\partial \Phi_x}{\partial y}dy = \Phi_{xx}dx + \Phi_{xy}dy \tag{25}$$

$$d\Phi_y = \frac{\partial \Phi_y}{\partial x}dx + \frac{\partial \Phi_y}{\partial y}dy = \Phi_{xy}dx + \Phi_{yy}dy \tag{26}$$

$$\left(1-\frac{u^2}{a^2}\right)\Phi_{xx}-\frac{2uv}{a^2}\Phi_{xy}+\left(1-\frac{v^2}{a^2}\right)\Phi_{yy}=0 \tag{27}$$

$$(dx)\Phi_{xx} + (dy)\Phi_{xy} = du \tag{28}$$

$$(dx)\Phi_{xy} + (dy)\Phi_{yy} = dv \tag{29}$$

Combining Eqs. (21), (25-26), and (27-28-29) in matrix form yields Eq. (30).

$$\begin{bmatrix} 1-\frac{u^2}{a^2} & -\frac{2uv}{a^2} & 1-\frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{bmatrix} \times \begin{bmatrix} \Phi_{xx} \\ \Phi_{xy} \\ \Phi_{yy} \end{bmatrix} = \begin{bmatrix} 0 \\ du \\ dv \end{bmatrix} \tag{30}$$

Using Cramer's rule, we get Eq. (31).

$$\Phi_{xy} = \frac{\left(1-\frac{u^2}{a^2}\right)dudy + \left(1-\frac{v^2}{a^2}\right)dvdx}{\left(1-\frac{u^2}{a^2}\right)dy^2 + \left(\frac{2uv}{a^2}\right)dxdy + \left(1-\frac{v^2}{a^2}\right)dx^2} \tag{31}$$

To keep Φ_{xy} finite, Eq. (31) must be indeterminate and therefore we make it as in Eqs. (32) and (33). Namely, test for a zero reciprocal.

$$\left(1-\frac{u^2}{a^2}\right)dudy + \left(1-\frac{v^2}{a^2}\right)dvdx = 0 \tag{32}$$

$$\left(1-\frac{u^2}{a^2}\right)dy^2 + \left(\frac{2uv}{a^2}\right)dxdy + \left(1-\frac{v^2}{a^2}\right)dx^2 = 0 \tag{33}$$

Dividing Eq. (33) with dx^2 and solving it yields Eq. (34-35).

$$\left(1-\frac{u^2}{a^2}\right)\left(\frac{dy}{dx}\right)^2 + \left(\frac{2uv}{a^2}\right)\frac{dy}{dx} + \left(1-\frac{v^2}{a^2}\right) = 0 \tag{34}$$

$$\boldsymbol{\frac{dy}{dx} = \frac{-\frac{uv}{a^2} \pm \left(\frac{u^2+v^2}{a^2} - 1\right)}{1 - \frac{u^2}{a^2}}} \tag{35}$$

Dividing Eq. (32) with $\frac{du}{dx}$ yields Eq. (36).

$$\boldsymbol{\frac{dv}{du} = \frac{uva^{-2}}{\left(1 - \frac{v^2}{a^2}\right)\left(\frac{dy}{dx}\right)}} \tag{36}$$

Substituting Eq. (36) with Eq. (35) resulted in Eq. (37).

$$\boldsymbol{\frac{dv}{du} = \frac{\frac{uv}{a^2} \pm \left(\frac{u^2+v^2}{a^2} - 1\right)^{0.5}}{1 - \frac{v^2}{a^2}}} \tag{37}$$

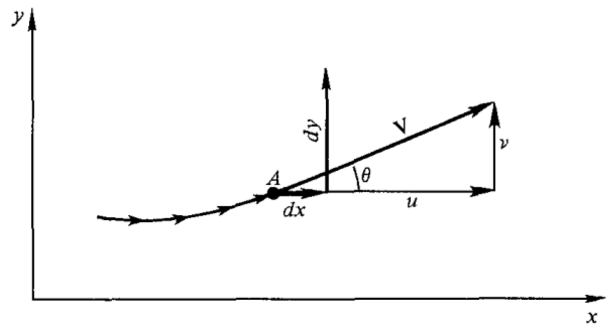

Figure 3: Streamline geometry [Anderson, 2003].

Substituting $M^2 = \frac{u^2+v^2}{a^2}$, $Mcos\theta = \frac{u^2}{a^2}, and\ Msin\theta = \frac{v^2}{a^2}$ to Eq. (37), we get Eq. (38).

$$\boldsymbol{\frac{dv}{du} = \frac{M^2 sin\theta cos\theta \pm \sqrt{M^2-1}}{1 - M^2 \sin^2\theta}} \tag{38}$$

With some algebraic manipulation on Eq. (38), we get Eq. (39).

$$\boldsymbol{d\theta = \pm\sqrt{M^2-1}\frac{dV}{V}} \tag{39}$$

Integrating Eq. (39), we get Eqs. (40) and (41).

$$\boldsymbol{\theta + v(M)} = consant = C_I \text{ (Right running characteristic)} \tag{40}$$

$$\boldsymbol{\theta - v(M)} = consant = C_{II}(\text{Left } running\ characteristic) \tag{41}$$

Equations (40) and (41) are the basis for the characteristic method. Figure 4 shows the illustration of these characteristic lines.

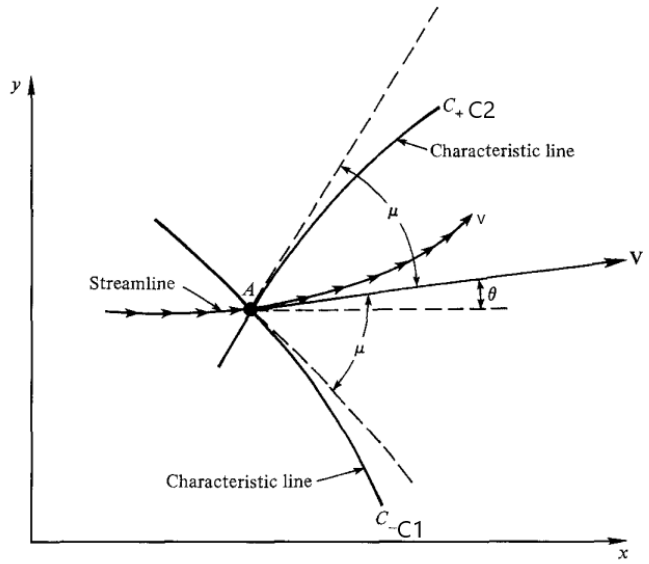

Figure 4: Illustration of the characteristic lines [Anderson, 2003].

The application of this characteristic lines is shown in Figure 5 in which the flow properties at point 3 can be determined by the right running characteristic line of point 1 and the left running characteristic line of point 2 because point 1 and point 3 have the same right running characteristic line while point 2 and 3 have the same left running characteristic line. For the case in Figure 5, Eqs. (40) and (41) can be applied.

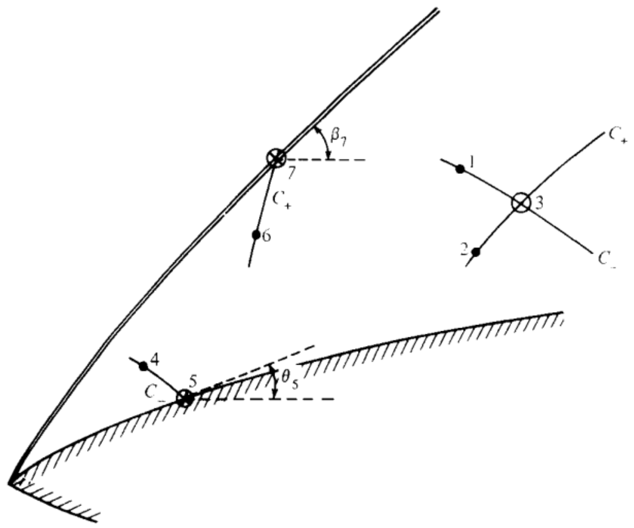

Figure 5: Unit processes for the steady-flow, two-dimensional. irrotational method of characteristics [Anderson, 2003].

To determine the coordinate of the points, the slope between point 1 to 3 and point 2 to 3 can be assumed to be constant. For the slope, it can be assumed to be the average of ($\theta - \mu$) from the 2 connected points for the right running characteristic line, and ($\theta + \mu$) for the left running characteristic line. Figure 6 shows the illustration for determining the slope.

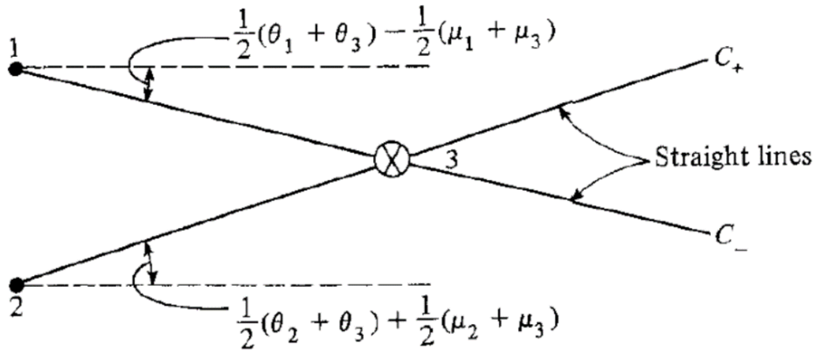

Figure 6: Determining the slope of characteristic line [Anderson, 2003].

The steps of a nozzle design with a Mach number of 2.1 and a test area of 0.011m^2 are given below, respectively. Also, some Prandtl-Meyer datas are given in Table 6.

Table 6: Prandtl-Meyer corner data.

Turning Angle of Corner [*v*, Degree]	Mach Number	Angles of Mach Lines [α_m, Degree]	Static to Total Pressure [p/p_t]
0.0	1.0000	90.00	0.5282
26.5	2.0000	29.93	0.1270
30.0	2.1336	27.97	0.1037

In this study, 5 characteristic lines will be drawn. Since 30 degrees can be divided equally into 3-degree intervals, the Mach number was chosen as 2.1336.

1. Calculate the turning angle *v* for the desired Mach number and compute the maximum wall angle Θ_{max} from $\Theta_{max} = v/2$. Maximum wall angle is 15deg.
2. The nozzle throat area calculated in Table 4 is 0.005926 m^2. The nozzle throat dimension OA is 26.63 mm. The first step turning the flow up 3 deg is arbitrarily constructed with a length AB. At B, the second step turning the flow up an additional 3 deg is constructed, respectively.
3. The network of waves will form a number of spaces, each having its individual flow angle and Mach number. It is convenient to label each space according to a coordinate system (a,b) where a denotes the number of degrees of turn produced so far by waves from the upper surface, and b is the number of degrees of turn produced so far by waves from the lower surface. Since waves from the upper wall turn the flow upward and those from the lower wall turn the flow downward, the local flow angle 0 is equal to (a-b), and the flow is hence horizontal when a is equal to b. The total v is (a+b) degrees [Pope and Goin. 1978].

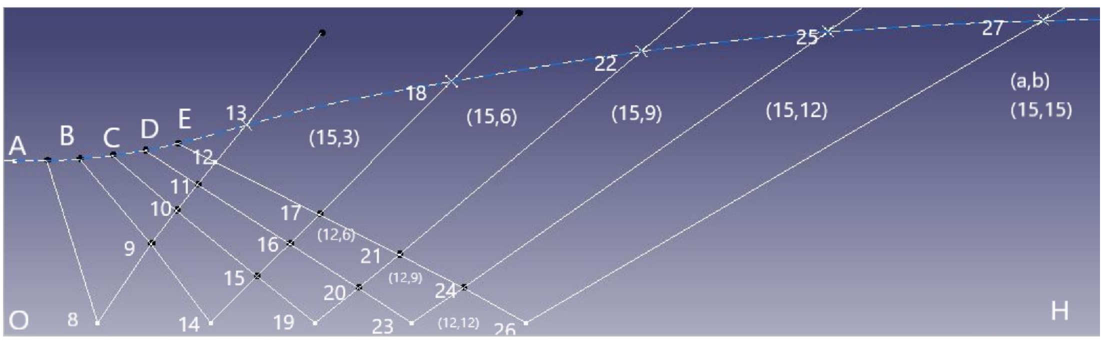

Figure 7: The characteristic network for design of a nozzle with 5 step expansion.

4. An examination of the preliminary grid of Figure 7 indicates that we will be concerned with total turning angles v of 0, 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30 deg. It is convenient to list in tabular form angles of pertinent Mach waves with respect to the horizontal as an aid in determining characteristic lines. It is noted that the inclination with respect to the horizontal of downward-moving Mach waves,d_d, is the difference between the Mach angle and the upward flow angle, (α_m – Ɵ). Similarly, the inclination with respect to the horizontal of upward-moving Mach waves, is d_u, (α_m – Ɵ). Using these two relations together with Table 7 allows the following tabulation
5. The characteristic line A-8 is determined by averaging α_m for a zero turn angle with d_d for a 3-deg turn angle and 3-deg up flow. The angle of A-8 with respect to the horizontal is thus (90.00+55.17)/2 = 72.58 deg.
6. The characteristic line B-9 is determined by averaging d_d for a 3-deg turn angle and 3-deg up flow with d_d for a 6-deg turn angle and 6-deg up flow: (55.17+44.63)/2 = 49.90 deg.
7. The characteristic line 8-9 is determined by averaging d_u for a 3-deg turn angle and 3-deg up flow with α_m for a 6-deg turn angle (horizontal flow): (61.17+50.63)/2 = 55.90.
8. The characteristic line 12-13 is determined by averaging d_u for a 15-deg turn angle and 15- deg up flow with d_u for a 15-deg turn angle and 3-deg up flow: (53.54+47.88)/2 = 50.71 deg. Since the flow downstream of 12-13 is 3 deg up, the nozzle contour must turn down 3 deg to that flow direction at 13 to avoid a reflection of 12-13 from the wall.

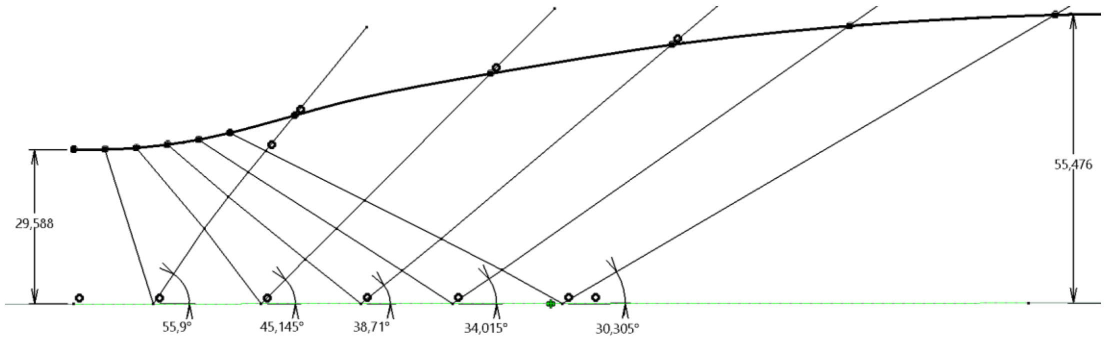

Figure 8: Construction of a five-step characteristic net for a supersonic nozzle.

The steps for supersonic nozzle design are given above with some example in detail. By applying all these steps, the angles of the characteristic lines are given in Table 7. The nozzle coordinates were created by drawing lines with these angles in the Catia Software.

Table 7: Properties of supersonic nozzle design.

No	K=Ɵ+v	K_+=Ɵ-v	Ɵ=1/2(K_-+K_+)	V=1/2(K_{--}-K_+)	M	α_m	X [mm]	Y[mm]
1	0	0	0	0	1	74.01	0	0
2	0.75	0	0.375	0.375	1.04	74.01	0	29.63
3	6	0	3	3	1.177	58.17	6.00	29.63
4	12	0	6	6	1.2935	50.63	12.00	29.94
5	18	0	9	9	1.4005	45.57	18.00	30.58
6	24	0	12	12	1.5028	41.72	24.00	31.53
7	30	0	15	15	1.6045	38.54	30.00	32.80
8	+6	-6	0	6	1.2935	50.63	15.29	0
9	12	-6	3	9	1.4005	45.57	25.07	14.43
10	18	-6	6	12	1.5028	41.72	29.73	20.53

11	24	-6	9	15	1.6045	38.54	33.56	25.28
12	30	-6	12	18	1.7061	35.88	36.89	29.35
13	30	-6	12	18	1.7061	35.88	42.45	36.14
14	12	-12	0	12	1.5028	41.72	35.98	0
15	18	-12	3	15	1.6045	38.54	44.46	8.53
16	24	-12	6	18	1.7061	35.88	50.47	14.46
17	30	-12	9	21	1.8090	33.54	55.89	19.82
18	30	-12	9	21	1.8090	33.54	80.009	44.12
19	18	-18	0	18	1.7061	35.88	55.14	0
20	24	-18	3	21	1.8090	33.54	63.07	6.36
21	30	-18	6	24	1.9150	31.49	70.54	12.45
22	30	-18	6	24	1.9150	31.49	114.83	49.64
23	24	-24	0	24	1.9150	31.49	72.79	0
24	30	-24	3	27	2.0222	29.64	82.29	6.41
25	30	-24	3	27	2.0222	29.64	148.99	53.23
26	30	-30	0	30	2.1336	27.97	93.82	0
27	30	-30	0	30	2.1336	27.97	188.417	55.292

This study calculates the Mach number for a given Prandtl-Meyer angle. The inputs to the function is the Prandtl-Meyer angle in degrees, and the output is the Mach number. The function uses the inverse Prandtl-Meyer relation to calculate the Mach number.

Table 7 calculates the geometry of the divergent section of the tunnel by dividing it by the grid. The formula calculates the angles and Mach numbers at each point on the grid using the characteristic method. The result of this problem is sketched in Catia and given as Figure 9. To begin with, the sonic line at the throat, is assumed to be straight. Then outputs a graph of the turning angle as a function of the station number along the divergent section. This graph can be used to determine the shape of the divergent section that is required to achieve the desired Mach number at the test section in the Figure 9. Finally, the output curve data in the Figure 9 is received and the curve is plotted in the design module of ANSYS Software and analyzed with inviscid condition.

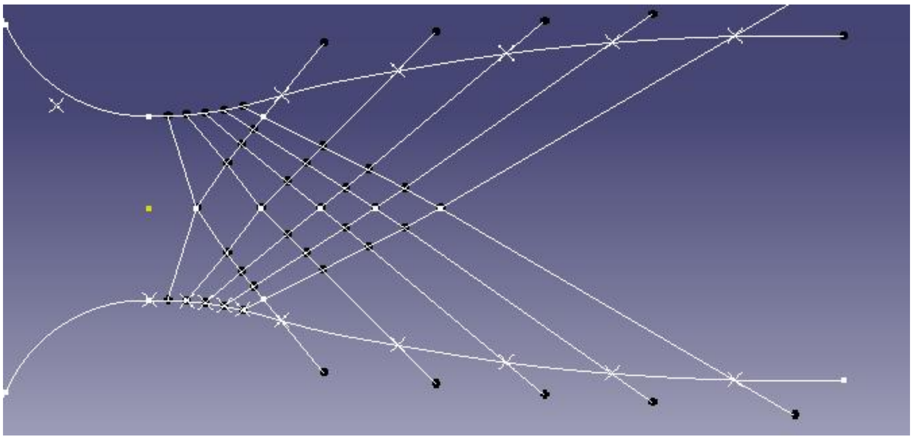

Figure 9: Schematic of minimum length nozzle.

Initially the design parameters are 110mm test height and 2.1 Mach number. The coordinates in the Table 7 were obtained using the method of characteristics for nozzle design. As a result of the calculations, the height of the test area was revised to 110.5 mm. Additionally, the Mach number was calculated as 2.1336 due to the increase in the nozzle's end area. This difference is attributed to considering the Mach number as 2.13, which had a significant impact. The accuracy of the calculations has been proven.

Design of Diffuser

Any duct designed to slow down an incoming gas flow to a lower velocity is called a diffuser. Incoming flow is supersonic based on the application. A diffuser is designed such that the loss in total pressure is minimal during the slowing down of the flow.

An actual supersonic diffuser slows down an incoming flow by a series of reflected oblique shocks in the convergent section and the throat (usually in the form of a constant area section). Interaction of shock waves with the viscous flows near the wall weakens and diffuses the reflected shock patterns, which ends up in the form of a weak normal shock wave at the end of the constant area throat. The subsonic flow downstream of the throat is subsequently slowed down via a diverging section. As the flow is no longer isentropic, the entropy at the exit is higher and the total pressure is lower [Anderson, 2017].

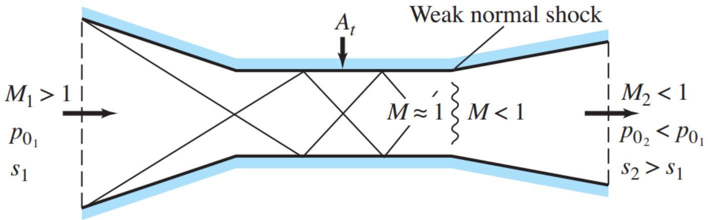

Figure 10: Actual supersonic diffuser [Anderson, 2017].

In the wind tunnel nomenclature, the nozzle throat is called the second first throat, with cross sectional area A_{t1} = A*; the diffuser throat is called the second throat, with area $A_{t2.}$ Due to entropy increase in the diffuser, $A_{t2} > A_{t1}$.

Second throat as Figure 10, the ratio of the second throat area (diffuser) to the first throat area (nozzle) is given by,

$$\frac{A_{t2}}{A_{t1}} = \frac{P_{o1}}{P_{o2}} \tag{42}$$

$$\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \tag{43}$$

$$\frac{p_{02}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \tag{44}$$

The properties in the test region are used for the diffuser input. The total pressure at the diffuser inlet is equal to atmospheric pressure, and the static pressure is accepted as 11080.21 Pa, calculated in the test area. At the exit of the diffuser, the air drops to Mach number 0.5613. The inlet and outlet static pressures and total pressures of the diffuser were recalculated using equations 43 and 44. While the total pressure at the outlet is 68311.80 Pa, the static pressure drops to 55,223.77 Pa. The temperature in this region is 271 Kelvin.

Normal Shock properties for Mach number 2.1, P_{o2}/P_{o1} is equal to 0.6742 and first throat area is 0.0059. Also second throat area is calculated as 0.0088 m^2.

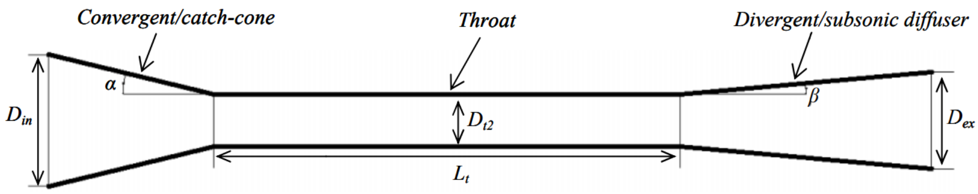

Figure 11: Schematic diagram of adjustable diffuser design [Pugazenthi and Mcintosh, 2011].

About 88 percent of theoretical pressure rise is obtained in subsonic small-angle (7° cone) diffusers at M < 0.9. Contraction angles of the order of 5° are best at M < 3 and increase to 15° and more at M = 7 to 10. Minimum pressure ratios, appreciably smaller than the normal shock values, are obtained at M > 2 and amount to only half the normal shock stagnation pressure ratio at M=7. At Mach Numbers below 4, maximum contraction of constant geometry diffusers (or starting contraction of variable diffusers) is closely predicted by the simple theory. At optimum contractions (which are practically equal to maximum contractions), normal shock pressure recoveries are achieved, although appreciably larger starting pressure ratios are usually required at M > 2. Small angles of two-dimensional contractions, of the order of 3° to 4°, are favorable at M < 4 and increase to about 9° at M = 7 to 10 [Lukasiewicz, 1953].

It has been found that the diffuser should have a throat with a cross section less than that of the test section. Included angles of convergence from quite small up to 30 deg or more have been used, as have second throat lengths of zero to ten test section lengths [Pope and Goin].

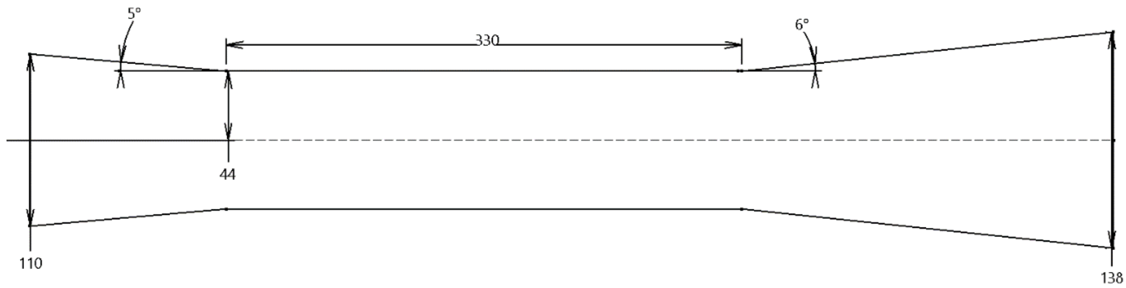

Figure 12: Calculated diffuser design.

Downstream of the second throat the diffuser of the supersonic tunnel should be kept below 6 degrees that is the angle between opposite walls as Figure 12. The alpha angle, which is the convergent catch cone angle, was determined as 5 degrees based on the references. The cone angle, which is the beta angle on the diffuser side, was taken as 6 degrees and the design was completed. The drawing is detailed in Figure 12.

Vacuum Tank and Run Time

The run time of an indraft tunnel is limited by the rising pressure in the vacuum tank. When the pressure in the vacuum tank increases to the point where the pressure ratio across the tunnel is not sufficient to operate the tunnel at the desired Mach number, the run comes to an end [Pope and Goin, 1965].

Initially, the vacuum tank is evacuated to a very low pressure, and it is connected to the exit of the tunnel. The entrance to the tunnel is open to the atmosphere, where the atmospheric pressure is 101325 Pascal. When the valve in front of the vacuum tank is opened, atmospheric air is sucked into the tunnel entrance, and flow starts through the tunnel. The pressure ratio across the tunnel is represented as ratio of atmospheric pressure and tank pressure. As the run continues, air fills the vacuum tank, and the tank pressure increases. The test run effectively ends when the pressure ratio becomes smaller than what is required to maintain isentropic flow through the nozzle. To calculate the run time, it is necessary to equate the product of weight flow of air through the tunnel and the run time to the change in the weight of air in the vacuum tank during the run.

$$\dot{m}t_{run} = V_v(\rho_e - \rho_i) \tag{45}$$

Where V_v is vacuum tank volume, ρ is the mass density in vacuum tank and *e* denotes end of the run and *i* denotes beginning of run. In this part of the study, the run time was calculated. The volume of the vacuum tank varies depending on where the supersonic wind tunnel will be placed and the vacuum tank can be purchased in the required sizes according to the budget. Therefore, run times for different tank volumes that can be used are calculated separately, as shown in Table 8.

Table 8: Run times for different volume and pressure ratios.

Mach Number	Test Section [m^2]	Min. Tank Pressure [Pa]	Tank Volume [m^3]	Run Time [sec]
2.1	0.011	110 mbar (11080 Pascal)	7	2.21
			8	2.53
			10	3.16
			12	3.79
2.3	0.011	81 mbar (8103 Pascal)	8	3.35
			10	4.19
2.5	0.011	59 mbar (5930 Pascal)	8	4.35
			10	5.44

The pressure at the diffuser exit is calculated as 55223 Pascal (-450 mbar gage pressure or 550 mbar absolute pressure). This pressure represents the maximum pressure of the vacuum tank. During the test at Mach 2.1, the pressure at the test location is equal to 11080 Pascal (110 mbar absolute pressure). If the tank pressure is reduced to 110 mbar, the test will continue until it reaches 550 mbar. When the pressure exceeds 550 mbar, the Mach number will start to decrease from 2.1. On the other hand, if the tank pressure is lowered below 110 mbar, Mach number and test times will increase. The test times at low tank pressures and the Mach number that will occur in the test area are added to Table 8.

RESULTS

At this stage of the supersonic tunnel design, all the calculated properties and dimensions of the tunnel are combined. Two-dimensional sketches were converted into three-dimension using Catia Software. All sections and lengths of the tunnel are given in Figure 13. A vacuum tank is connected to the exit of the tunnel, which is evacuated to a very low pressure (approximately 50 mbar and 110 mbar). The entrance of the tunnel is open to the atmosphere, which is atmospheric pressure. When the valve in front of the vacuum tank is opened, atmospheric air is sucked from the tunnel inlet and flow begins through the tunnel.

The separate calculations of nozzle and diffuser sections are mentioned in detail above. The accuracy of the theoretical calculations was checked using Ansys software and the results were compared in detail.

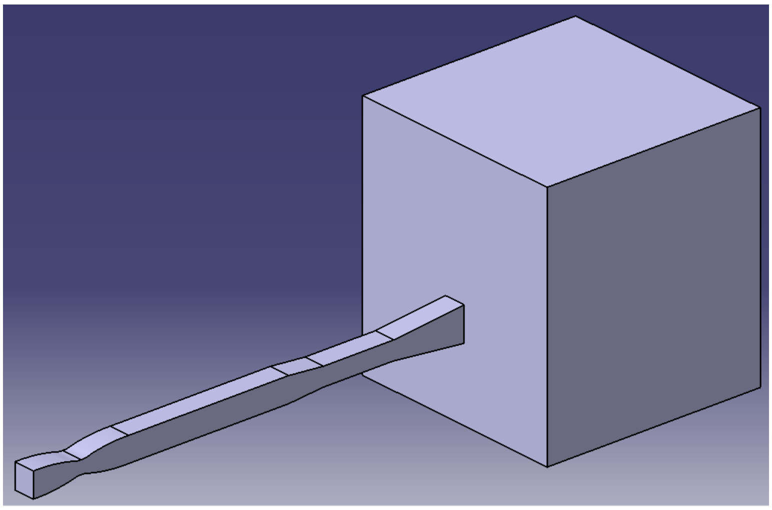

Figure 13: Full body of supersonic wind tunnel.

After the nozzle, test section and diffuser designs were made and its geometry was created, the calculated parameters of the tunnel were compared and verified with the computational fluid mechanics method. ANSYS Fluent solves the conservation of mass and moment equations for all flows. In addition, it takes into account the energy equation for flows involving heat transfer or compressibility. There are two main types of solvents in Fluent, pressure-based and density-based. The pressure-based solver traditionally has been used for incompressible and weak compressible flows. For high velocity flows, a density-based solvent is a better option due to the high compressibility of air in supersonic flows. The density-based solver simultaneously solves the equations governing continuity, momentum, and (where appropriate) energy and species transfer [Fluent Theory Guide, 2013].

For this study, two cases are conducted in ANSYS Fluent using a density-based solver, with boundary conditions set as 'pressure-inlet' on the convergent nozzle inlet face and 'pressure-outlet' on the diffuser exit. The two cases consist of an inviscid scenario and a viscous scenario, with the latter considering the boundary layer by implementing the k-ε turbulent model and Spalart-Almaras model, separately.

Inviscid Case

For an inlet total pressure of 101325 Pa and a static pressure of 98000 Pa, the flow property contours shown in Figures 14.1 to 18.1 were obtained. Two-dimensional analyses were performed for ease of design and time-saving purposes. For the boundary conditions, properties were defined for the input and output. Additionally, properties for the outer contour wall were also specified.

For the inviscid model, the effect of the boundary layer on the wall is neglected, and without the boundary layer, there are no obstacles to the flow. Consequently, a normal shockwave is expected to occur within the wind tunnel. The purpose of the normal shock is to balance the pressure at the exit of the diffuser, making it the same as the back pressure.

The location of this shockwave is determined by the total pressure at the convergent-divergent nozzle inlet, and as the total pressure increases, the shockwave moves backward toward the diffuser outlet. This phenomenon is evident in both inviscid model simulation results. No shock wave occurs up to the test area, which indicates that the test will perform better. The Mach number in the entire test section region remains unaffected for the inviscid case.

Viscous Case

In the viscous model, the boundary layer is incorporated into the simulation, and it plays a significant role in determining the flow properties. This boundary layer reduces the effective area of the nozzle's cross-section and consequently lowers the flow Mach number. The reduction in area can also lead to the occurrence of an oblique shock, which disturbs the flow downstream of the shock.

At this stage, two different models were utilized. The Spalart-Allmaras and k-epsilon models, both belonging to the viscous models, were analyzed separately. Upon examining these results, it was observed that nearly identical outputs were obtained. The results for Figures 14. to 18. were presented below.

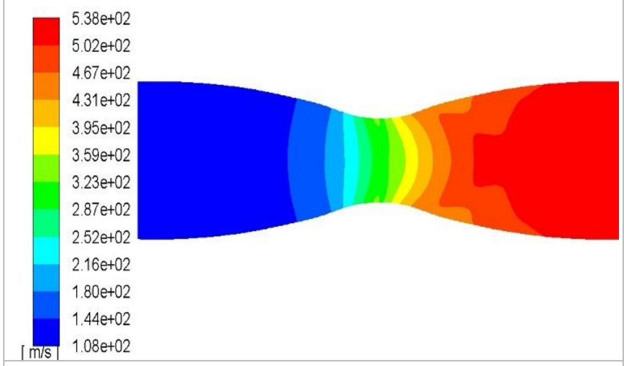

Figure 14.1: Velocity contour of inviscid flow.

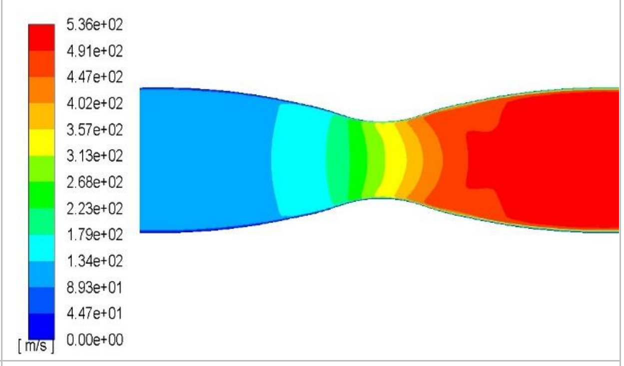

Figure 14.2: Velocity contour of viscous flow.

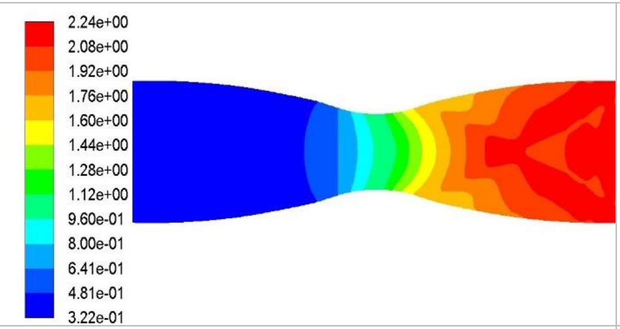

Figure 15.1: Mach-Number contour of inviscid flow.

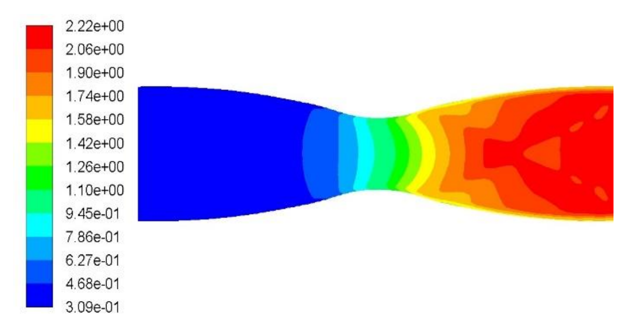

Figure 15.2: Mach-Number contour of viscous flow.

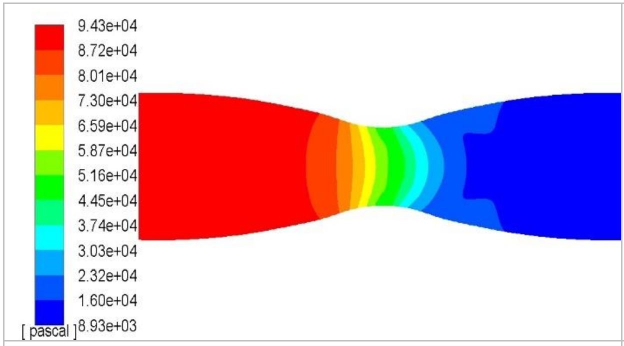

Figure 16.1: Pressure contour of inviscid flow.

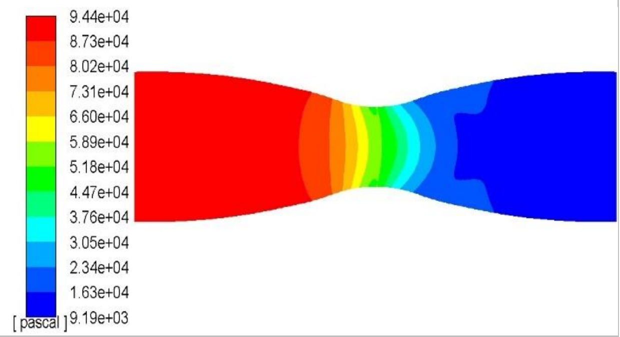

Figure 16.2: Pressure contour of viscous flow.

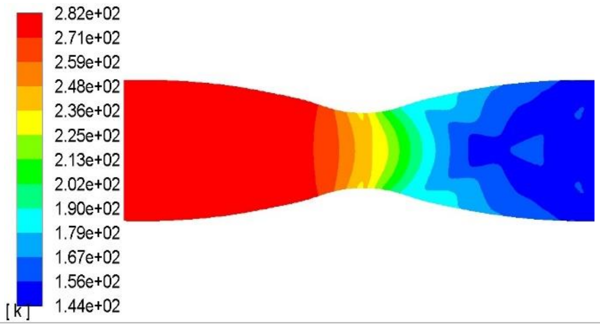

Figure 17.1: Temperature contour of inviscid flow.

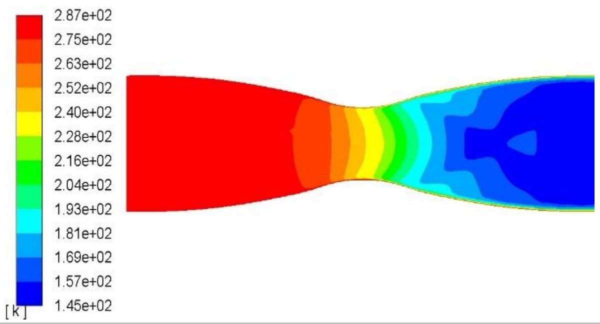

Figure 17.2: Temperature contour of viscous flow.

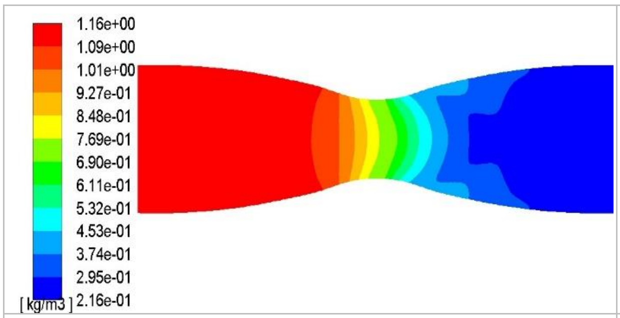

Figure 18.1: Density contour of inviscid flow.

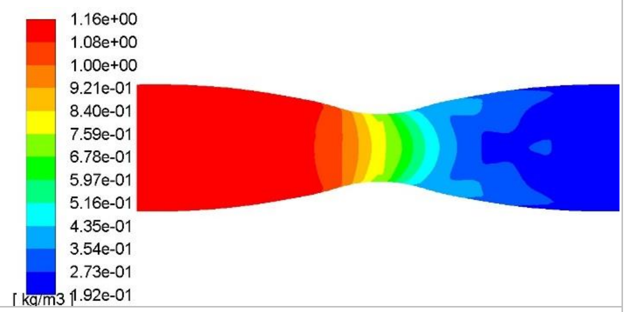

Figure 18.2: Density contour of viscous flow.

Table 9: Area and mass weighted properties of nozzle inlet to diffuser outlet for inviscid flow.

	Mass Weighted Properties			Area Weighted Properties	
Properties	Pressure [Pa]	Temperature [K]	Density [kg/m^3]	Mach Number	Velocity [m/s]
Nozzle Inlet	93972.298	281.878	1.161	0.329	110.965
Diffuser Outlet	10770.013	152.000	0.247	2.118	523.234

The analyses conducted an inviscid flow in Ansys software are presented in Figures 14.1 to 18.1, respectively, and the corresponding input and output values are listed in Table 9. The analysis input considered atmospheric values. From the velocity analysis shown in Figure 14.1, the relationship of the Mach number can be determined, as illustrated in Figure 15.1. Considering the effect of vacuum at the nozzle inlet, the velocity was approximately 110 m/s at the inlet and 523 m/s at the exit. When compared with the theoretical calculations, the velocity increases from 340.26 m/s at the throat to 520.86 m/s in the test region, corresponding to the Mach number value of 2.1 as given in Table 4. These values indicate that the analysis results are in good agreement with the theoretical predictions, with a certain margin of error. As the nozzle design was based on a Mach number of 2.1336, it is expected that both the Mach analysis and the velocity analysis would slightly exceed the theoretical calculations. This outcome is reasonable. The Mach number used in the design was found to be 2.118 as a result of the analysis, which confirms the correctness of the design and calculations.

Table 10: Area and mass weighted properties of nozzle inlet to diffuser outlet for viscous flow.

	Mass Weighted Properties			Area Weighted Properties	
Properties	Pressure [Pa]	Temperature [K]	Density [kg/m^3]	Mach Number	Velocity [m/s]
Nozzle Inlet	94033.632	281.931	1.162	0.328	110.502
Diffuser Outlet	11094.759	155.046	0.249	2.067	515.940

When conducting a viscous flow supersonic wind tunnel CFD analysis, obtaining different values on the nozzle walls is due to the effects of viscosity and boundary layer on the flow properties. The boundary layer is the region where the flow interacts with the surface, and due to viscosity, the flow slows down, reaching zero velocity at the surface. Viscosity leads to energy loss within the flow. As a result, the flow decelerates as it approaches the walls, causing pressure changes on the walls.

The diffuser analyses were repeated for both inviscid and viscous flow. Similar to the nozzle analysis, the k-epsilon model was employed. Static pressure of 11080 Pascal was specified as the input at the diffuser inlet, representing the test area's static pressure. The inlet was set with an total pressure of 101325 Pascal. Based on theoretical calculations, the output values entered were 55223 Pascal for static pressure and 68311 Pascal for total pressure. The results for Figures 19. to 23. were presented below.

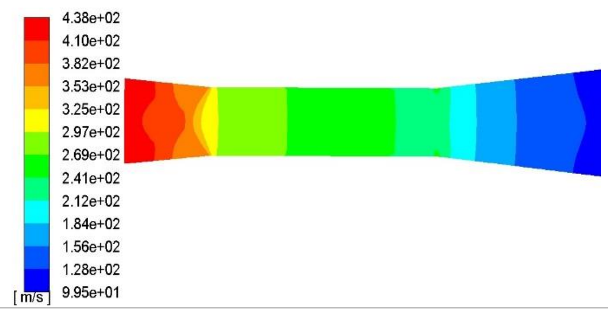

Figure 19.1: Velocity contour of inviscid flow.

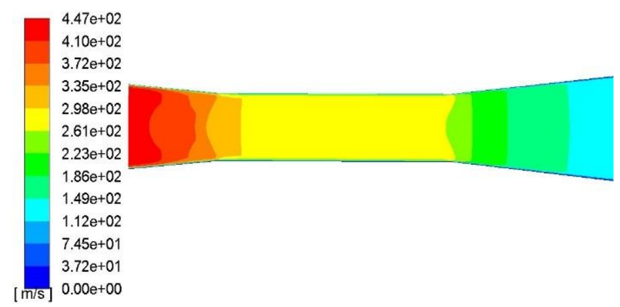

Figure 19.2: Velocity contour of viscous flow.

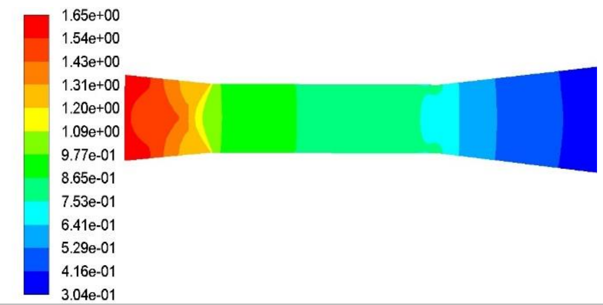

Figure 20.1: Mach-Number contour of inviscid flow.

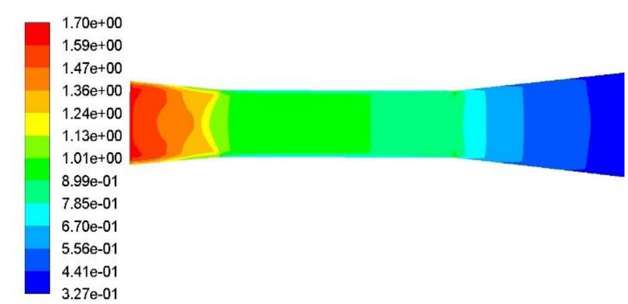

Figure 20.2: Mach-Number contour of viscous flow.

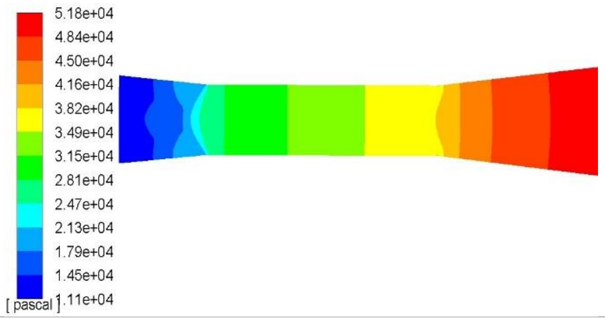

Figure 21.1: Pressure contour of inviscid flow.

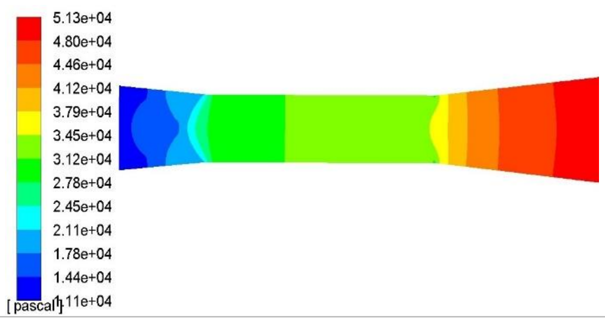

Figure 21.2: Pressure contour of viscous flow.

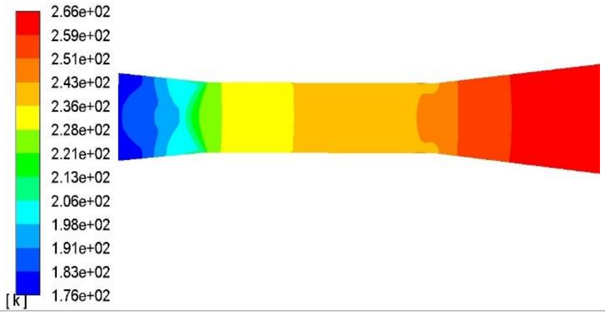

Figure 22.1: Temperature contour of inviscid flow.

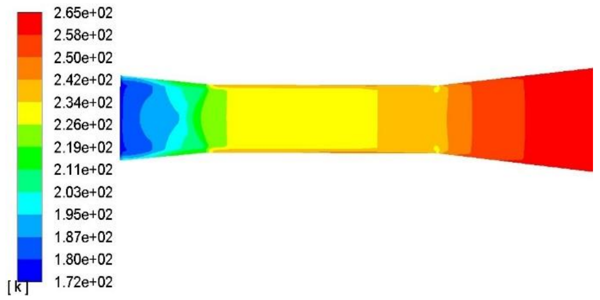

Figure 22.2: Temperature contour of viscous flow.

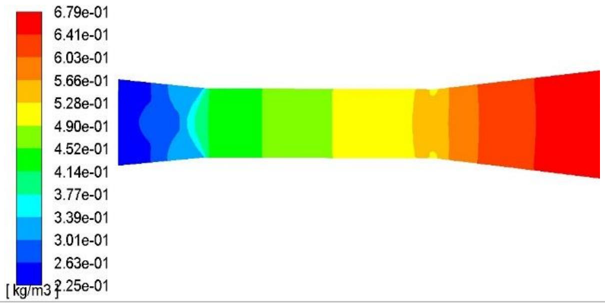

Figure 23.1: Density contour of inviscid flow.

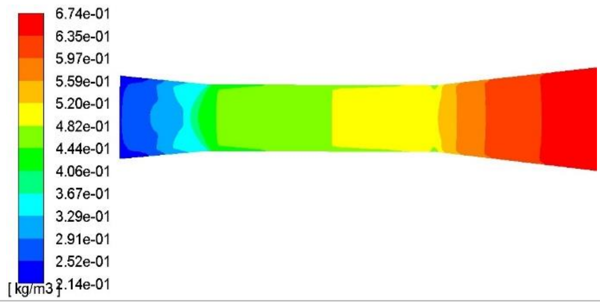

Figure 23.2: Density contour of viscous flow.

The speed of the air in the test area increases to 520 m/s. The characteristics of the test area have been entered for the diffuser analysis. According to the results in Figure 19.1 and 19.2, it is observed that the speed decreases to 439 m/s at the inlet. This decrease is expected to be due to losses in the test area. The Mach number has dropped to approximately Mach 1 at the second throat and is around 0.3 at the exit. Static pressure of 51900 Pascal has been found in Figure 21.1 and 21.2 compared to the theoretical calculations. The analyses are in agreement with the calculations.

Table 11: Area and mass weighted properties of diffuser inlet to diffuser outlet for inviscid flow.

	Mass Weighted Properties			Area Weighted Properties	
Properties	Pressure [Pa]	Temperature [K]	Density [kg/m^3]	Mach Number	Velocity [m/s]
Nozzle Inlet	11080.000	176.710	0.227	1.637	435.945
Diffuser Outlet	50360.301	263.931	0.665	0.365	118.741

In the conducted flow analyses, various flow properties at different locations within the flow domain are presented in Table 11 and 12. The average values based on the mass flow rate of the fluid and the surface area of the flow domain are compared.

Table 12: Area and mass weighted properties of diffuser inlet to diffuser outlet for viscous flow.

	Mass Weighted Properties			Area Weighted Properties	
Properties	Pressure [Pa]	Temperature [K]	Density [kg/m^3]	Mach Number	Velocity [m/s]
Nozzle Inlet	11080.000	176.700	0.228	1.633	434.997
Diffuser Outlet	50333.243	263.897	0.665	0.366	119.010

The foundation of this study is built upon the design of the nozzle, test section, and diffuser segments of a supersonic tunnel, along with the subsequent flow analyses resulting from these designs. As detailed in the preceding sections, the relevant portions of the tunnel have been individually computed. Theoretical calculations have been validated through comparison with flow analyses. As an outcome of this study, the separately designed nozzle, test section, and diffuser components of the tunnel have been integrated, and flow analyses have been conducted using ANSYS software.

The tunnel entrance was analyzed by considering atmospheric properties, and the exit of the tunnel was assessed with reference to tank pressure. Essentially, throughout the testing period, the characteristic features of the flow, including pressure, velocity, temperature, and density, have been presented in Figures 24 through 28.

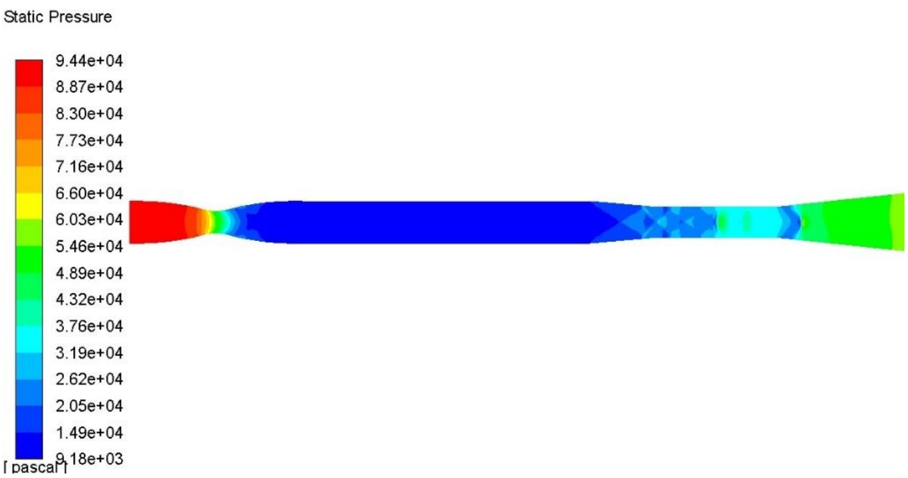

Figure 24: Pressure contour of supersonic wind tunnel.

As observed from the analysis results, within the test section, the velocity drops to 540 m/s, and the pressure reaches a minimum value of 9180 Pascal. In comparison to the theoretical calculations, the velocity in the test section is close to 520 m/s, while the pressure is approximately 11080 Pascal. At the tunnel exit, the static pressure is 54600 Pascal, and the velocity is 180 m/s and 216 m/s. As indicated in Table 13, the theoretically calculated values align with the analysis, confirming a static pressure of 55223 Pascal and a total pressure of 68311 Pascal at the diffuser outlet. At the test section exit, a Mach number of 0.56 corresponds to a velocity of 190 m/s, which is in agreement with the analysis results.

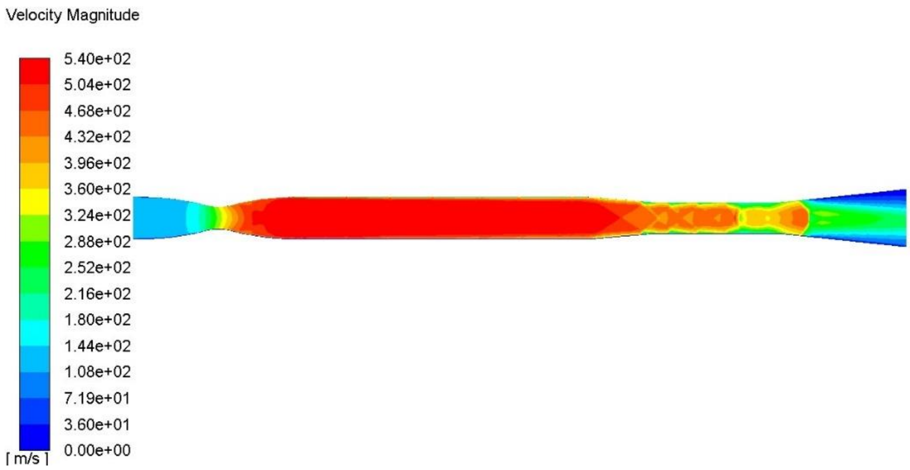

Figure 25: Velocity contour of supersonic wind tunnel.

Table 13: Diffuser region results of flow.

	Static Pressure [Pa]	Total Pressure [Pa]	Mach Number	Temperature [K]
Diffuser Inlet	11080.21	101325	2.1	153.11
Diffuser Throat	36088.67	101325	1.0	240.13
Diffuser Outlet	55223.77	68311.80	0.5613	271.00

The viscous model includes boundary layer simulation constitutes a significant factor determining the flow characteristics. This boundary layer reduces the effective area of the nozzle section and decreases the flow Mach number. The reduction in the area can also lead to the formation of an oblique shock and disrupt the downward flow. As demonstrated in viscous simulation results, the oblique shock wave will also generate a shock reflection event. The location of this oblique shock, under conditions with a total inlet pressure of 101325 Pascal, is closer to the test section outlet and diffuser inlet. The position of the shock wave will influence the Mach number of the test section since it disrupts the flow. The Mach number in the test section decreases from 2.22 and 2.07 towards the end of the test section, due to the formation of shock waves, reaching a value as low as 1.6 in the Figure 26.

According to flow analyses, due to the shock wave occurring at the exit of the test section, most of the test section region remains usable. The analyses also demonstrate the effect of the boundary layer in the wind tunnel, where a minimum total pressure of 11080 Pa is necessary to prevent the occurrence of the shock wave in most of the test section region and avoid its disruption.

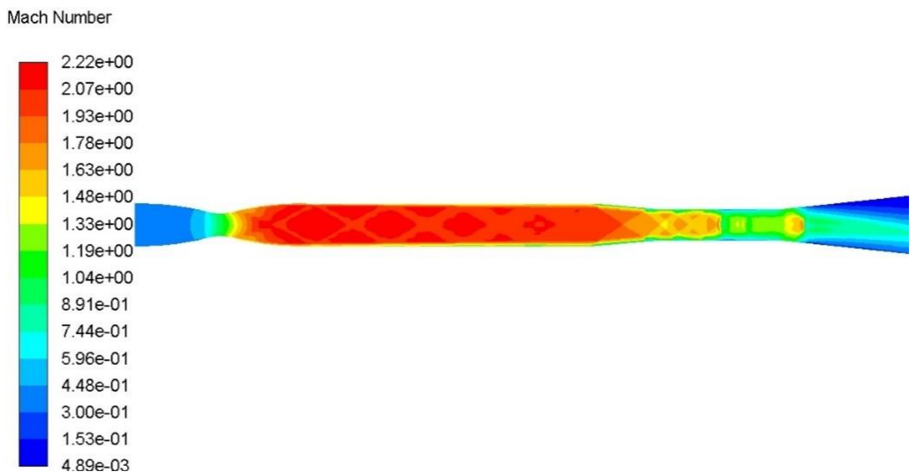

Figure 26: Mach number of supersonic wind tunnel.

When the calculated Mach number at the diffuser outlet is known, the static pressure at the exit of the diffuser is determined as the minimum vacuum pressure. For the designed diffuser outlet, as shown in Table 13, at a Mach number of 0.56, the static pressure is 55223 Pascal. However, due to losses, the total pressure is found to be lower than 101325 Pascal.

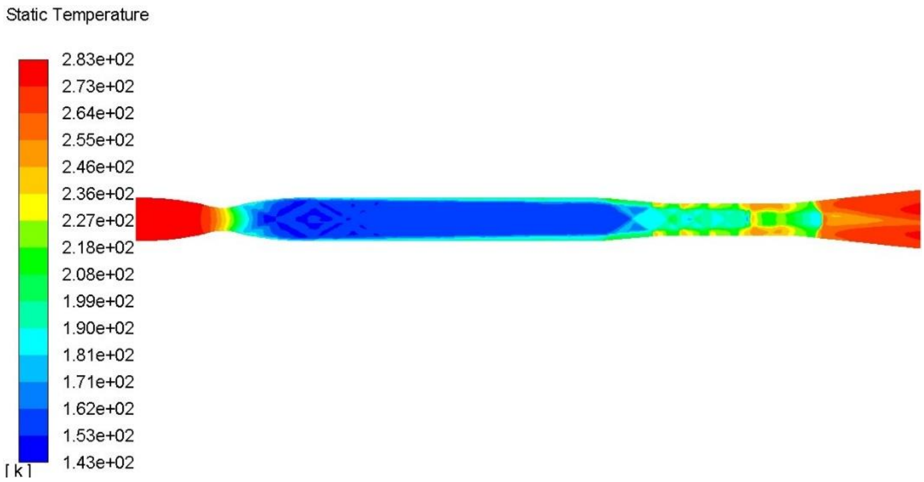

Figure 27: Temperature contour of supersonic wind tunnel.

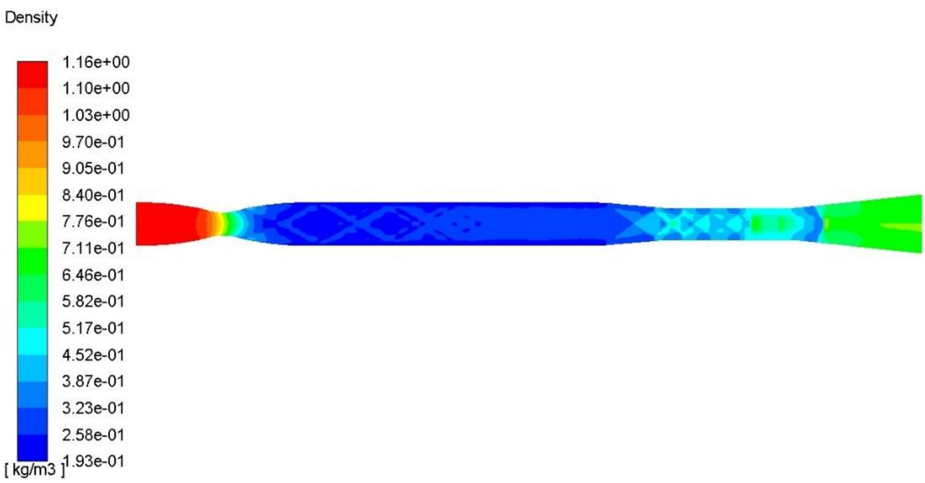

Figure 28: Density contour of supersonic wind tunnel.

The pressure will maintain the diffuser outlet in a choked state up to a certain value, thereby keeping the Mach number constant in the test section. However, if the pressure in the tank increases and disrupts the choked state of the exit diffuser, the Mach number in the test section begins to decrease. In this case, the tank pressure becomes the static pressure at which the exit diffuser is choked. Pressure values below 55223 Pascal are considered feasible.

CONCLUSION

The study aims to present the preliminary design of an indraft supersonic wind tunnel capable of reaching a maximum speed of Mach 2.1. The selected wind tunnel type is the indraft type, as it provides better value compared to the blowdown type. The nozzle geometry of the wind tunnel is designed using the characteristic method, resulting in a rectangular shape for the test section with dimensions of 11 cm x 10 cm. The number of Mach used in this design is a variable, and the study is repeated with different Mach numbers and test areas to optimize the wind tunnel's speed and cost. The necessary theoretical calculations and assumptions are made to determine the dimensions of the nozzle and test section under operating conditions. A code that combines the method of characteristics and analytical solutions is used to create the contour of the nozzle geometry. The coordinates of the nozzle structure are generated in Python software and then transferred to a CAD software for further design refinement. To validate the theoretical calculations and flow properties in the designed nozzle geometry, CFD analysis is performed using the ANSYS Fluent program. The CFD outputs are compared with analytical calculations, and the error rate is found to be negligible, indicating the accuracy of the results. The smoothness of the Mach number in the test region along the nozzle exit is found to be satisfactory in the project. Overall, the study successfully demonstrates the feasibility of the wind tunnel design, optimizing its performance while ensuring accuracy in the results.

REFERENCES

Barlow, J. B., Rae Jr, W. H., & Pope, A. (2015), *Low Speed Wind Tunnel Testing*. INCAS Bulletin, 7(1), 133.

Fluent Theory Guide (2013), *Ansys Fluent Theory Guide.* In ANSYS Inc., Vol. 15317, Issue, p: 724–746, November 2013.

Ivolya N. (2012), *Design of Contraction, Test Section, and Diffuser for a High-Speed Water Tunnel Master’s thesis in Fluid Mechanics,* Chalmers University of Technology, Sweden.

John D. Anderson, J. (2003), *Modern Compressible Flow with Historical Perspective 3rd Edition*, McGraw-Hill.

John D. Anderson, J. (2017), *Fundamentals of Aerodynamics*, In The Aeronautical Journal, Vol. 116, Issue 1176.

Lukasiewicz, J. (1953), Diffusers for Supersonic Wind Tunnels, Journal of the Aeronautical Sciences, September 1953.

Peter J. Moore (2009), *Design of a Supersonic Wind Tunnel*, Worcester Polytechnic Institute, October 2009.

Pope, A. and Goin, K. L. (1965). *High-Speed Wind Tunnel Testing.*

Pugazenthi, R. S. And McIntosh, A. C. (2011), Design and Performance Analysis of a Supersonic Diffuser for Plasma Wind Tunnel, International Journal of Aerospace and Mechanical Engineering, Vol.5, No 8.

Scott A. Berry, Matthew N. Rhode, and Karl T. Edquist (2011), *Supersonic Retropropulsion Experimental Results from the NASA Ames 9- x 7-Foot Supersonic Wind Tunnel*, NASA Langley Research Center, Hampton, VA 23681, June 2011.

Slater, J. W., Saunders and John H. Increased Mach Number Capability fort he NASA Glenn 10x10 Supersonic Wind Tunnel, Research Center Cleveland, Ohio.

Von-Karman, T. (1947), *Supersonic Aerodynamics Principles and Applications,* Journal of the Aeronautical Science, Vol. 14, Issue 7, July 1947.