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STRUCTURAL ANALYSIS OF COMPOSITE FOLDED BOX BEAMS

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ABSTRACT

In this study, structural analysis of thin and thick composite box beams, which are accepted as folded plates, are evaluated using the finite element method to get the vibrational properties and an in-house computer code is developed. In order to make this assessment, folded plates and box beams that have different material properties, boundary conditions and variable plate thickness values are analyzed by applying certain plate theories. Effects of several parameters on the vibrational properties are examined and tabulated in tables and figures. When it is available, the calculated results are compared with the ones in open literature. Otherwise, the folded plate structures are modeled in ABAQUS and the ABAQUS results are used for validation of the in-house FEM code.

INTRODUCTION

The changeable structural properties of composites give advantage about weight and strength in fiber reinforced layer composite applications by controlling the layer angle and order, and with this adaptive structure, the folded plate structural applications have increased [Niyogi, Laha, & Sinha, 1999]. Vibration is an important phenomenon in terms of structural integrity, strength and efficiency for box beams, shells and panels that make up the majority of aircraft today.

All plate theories provide help to understand thin or thick walled frameworks. Plates are classified according to their length-to-thickness ratio and while Kirchhoff-Love assumptions are suitable for thin plates (L/h>10), Reissner-Mindlin assumptions are suitable for the thick ones [Oñate, 2013].

The main purpose of this study is to examine the folded plates structurally and evaluate them in terms of vibration using a folded plate approach by FEM coding. In this study, Kirchhoff-Love Plate Theory (KLPT) and Reissner-Mindlin Plate Theory (RMPT) are applied to plates

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that have different thickness values. Box beams are represented by folded plates with drilling degree due to having local to global axes transformation.

The application of the Reissner-Mindlin theory yields closer results, especially since thick and composite plates also have a shear effect. In order to eliminate the disadvantage of this theory in thin structures, shear locking, various shear stress fields are adopted or the reduced integration method is used. Gauss integration method is most preferred stabilization technique on plates to eliminate the deviations caused by the dominance of the shear effects.

METHOD

The Kirchhoff-Love plate is explained by three assumptions as follows [Oñate, 2013];

- The particles of the middle plane move vertically. (u = v = 0)
- The plate middle surface has the identical vertical displacement through the normal line.
- The normal stress is negligible.
- The normal line, pointing out of the plate middle plane, stay on the same line as orthogonal to the middle plane.

The RMPT has same first three assumptions with KLPT but fourth assumption is different as the following;

- The normal line points of the plate middle plane stay on the same spline but does not have to remain orthogonal under deformation.

Equation of motion is obtained by combination of deformation functions and energy equations;

$$u(x,y,z)=z\theta_y(x,y) \quad v(x,y,z)=-z\theta_x(x,y) \quad w(x,y,z)=w \tag{1}$$

$$T_e=\frac{1}{2}\int \rho\left(h\dot{w}^2+h\dot{v}^2+h\dot{u}^2\right)dV=\frac{1}{2}\int\left(\{\dot{\delta}\}_e^{\ T}[m_e]\{\dot{\delta}\}_e\right) \tag{2a}$$

$$U_e=\frac{1}{2}\int\frac{h^3}{12}\{X\}^T[D]\{X\}\,dA+\frac{1}{2}\int\kappa h\{Y\}^T\left[D^s\right]\{Y\}\,dA=\frac{1}{2}\int\left(\{\dot{\delta}\}_e^{\ T}[k_e]\{\dot{\delta}\}_e\right) \tag{2b}$$

$$[K]_e-[M]_e\{w^2\}=0 \tag{3}$$

K is the stiffness, M is the mass, T is the kinetic energy and U is the potenrial energy matrices where w is the natural frequency.The in-house code is developed in Mathematica program based on the finite element method and the model is analyzed in the light of these information. For the modelling of the plates, 4-noded quadrilateral 2D elements which are called as Q4 element are used and their shape functions are obtained based on types of elements. The laminated plates have in plane and out of plane elongations so, there are 5 DOF, u, v, w, θx and θy for each node of the accepted element. Niyogi, et. al. (1999), states that the relationship between displacements can be expressed as follow.

$$\left\{\begin{matrix}u\\v\\w\\\theta_x\\\theta_y\end{matrix}\right\}=\sum_{j=1}^{4}N_j[I_5]\left\{\begin{matrix}u_j\\v_j\\w_j\\\theta_{x_j}\\\theta_{y_j}\end{matrix}\right\} \tag{4}$$

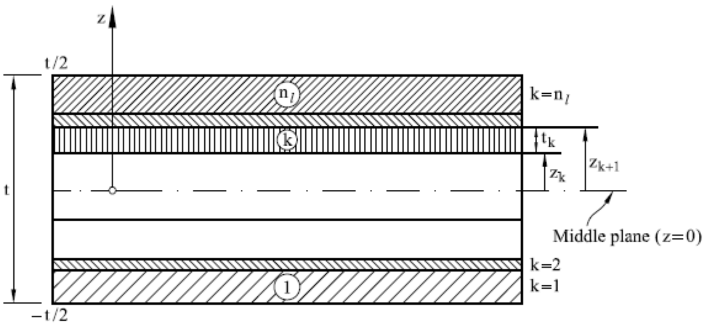

Figure 1: Lamination representation with thicknesses [Oñate, 2013].

$$D=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix} \qquad B=\begin{bmatrix} N_{j,x} & 0 & 0 & 0 & 0 \\ 0 & N_{j,y} & 0 & 0 & 0 \\ N_{j,y} & N_{j,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{j,x} & 0 \\ 0 & 0 & 0 & 0 & N_{j,y} \\ 0 & 0 & 0 & N_{j,y} & N_{j,x} \\ 0 & 0 & N_{j,y} & 0 & N_j \\ 0 & 0 & N_{j,x} & N_j & 0 \end{bmatrix} \quad \textbf{(5a)}$$

$$[K]_m=\int[B_m]^T\left[A_{ij}\right][B_m]dA \qquad [K]_b=\int[B_b]^T\left[D_{ij}\right][B_b]dA \qquad i,j=1,2,6 \quad \textbf{(5b)}$$

$$[K]_s=\int[B_s]^T\left[A_{ij}\right][B_s]dA \quad i,j=4,5 \qquad [K]_{mb}=\int[B_m]^T\left[B_{ij}\right][B_b]+[B_b]^T\left[B_{ij}\right][B_m]dA \quad \textbf{(5c)}$$

$$[K]_e=[K]_m+[K]_b+[K]_s+[K]_{mb} \quad \textbf{(5d)}$$

Aij, Bij and Dij matrices are depend on composite layers properties like angle and stacking sequence. Nj is shape function for membrane and plate bending element according to elongation type. Figure 1 represents composite laminate sequence and modelling system [Reddy,2004].

The box beams are represented by folded plates with 90° for first fold, 180° for the second fold and 270° for the third fold. To analyze the folded structures, transformation matrices between local axes to global axes are used by relocated each face is in same plane (ie, a kind of open dice). Because of the transformation of axes, drilling DOF, θz, should be added to 5 DOF of composite flat plates. Also, these additional DOF blows up the stiffness and mass matrix. To simplify these matrices, diagonal terms of drilling degree are accepted as 0.0000001. Normally, θz can be accepted as zero, but this cause singularity errors [Niyogi, et al.,1999].

In this context, natural frequency values of one folded composite plate and composite folded box beams were obtained by using the finite element method and Q4 elements. For thin plates, the reduction method called Gaussian Integration method is used on Reisner-Mindlin theory. The folded box beam as shown in the Figure 2.

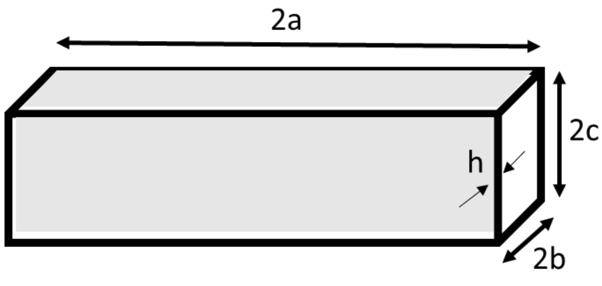

Figure 2 : The box beam representation.

When 4 noded element is chosen, selective integration can be used as integration method and Gauss integration options can be shown as Figure 3 [Hinton & Bicanic, 1979]. Also, the sampling points are required to integrate gauss quadrature matrices. According to Cook, et al [2001], gauss quadrature application expressed by

$$I=\int_{x_1}^{x_2} f\, dA=\iint_{-1}^{1} \phi(\xi,\eta)\det J\, d\eta d\xi=\sum_i \sum_j W_i W_j \phi\left(\xi_i,\eta_j\right) \quad \textbf{(6)}$$

where W is the weight factor, ξ and η are sampling points.

	Integration Gauss–Legendre product rules					
	Full		Selective		Reduced	
Element	K_f	K_s	K_f	K_s	K_f	K_s
4 noded	2×2	2×2	2×2	1×1	1×1	1×1
8 noded	3×3	3×3	3×3	2×2	2×2	2×2
9 noded	3×3	3×3	3×3	2×2	2×2	2×2
12 noded	4×4	4×4	4×4	3×3	3×3	3×3
16 noded	4×4	4×4	4×4	3×3	3×3	3×3

Figure 3 : Gauss integration schemes based on elements [Hinton & Bicanic, 1979].

The frequency values obtained by the Mathematica code prepared with the finite element method by folding a thin composite plate in Figure 4 consisting of 2 layers of equal thickness. The layer sequence 0° / 90 ° results are compared with the literature data and the results of the ABAQUS software package in Table 1. (clamped, 2b=2a/2; h=0.01a; $E_1=25E_2$; $G_{12}=G_{13}=0.5E_2$; $G_{23}=0.2E_2$; $v_{12}= v_{21}=0.25$)

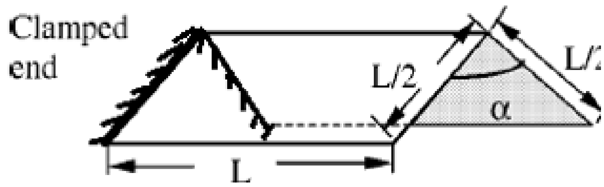

Figure 4 : Clamped one and two folded plate with crank angle α/β [Liu & Huang, 1992].

Table 1 : Comparison of dimensionless natural frequency values of a 90 ° single folded thin composite plate with 0 ° / 90 ° sequence.

Modes	Present-Kirchhoff (16x8)	Present-Mindlin (16x8)	ABAQUS (16x8)	Haldar & Sheikh (2005)
1	3.647	3.601	3.601	3.598
2	9.976	9.901	9.681	9.663
3	16.237	16.098	15.894	15.776

It can be seen from the results that the Mindlin FEM code for composite thin folded plates have has better convergence than other methods.

The layer sequence 30 ° / -30 ° / 30 ° / 30 ° / -30 ° / 30 ° composite thick one-folded-plate results are compared with the literature data and the results of the ABAQUS software package in Table 2. (clamped, 2a=1.5m; 2b=2a/2; h=0.1a; E_1=60.7 GPa;E_2=24.8 GPa; $G_{12}=G_{13}=G_{23}$=12 GPa; $v_{12}= v_{21}=0.23$; ρ=1300 kg/m^3)

Table 2: Comparison of dimensionless natural frequency values of an one-folded-thick composite plate with 30 ° / -30 ° / 30 ° / 30 ° / -30 ° / 30 ° sequence.

Modes	Present-Mindlin (32x8)	ABAQUS (32x8)
1	140.71	139.79
2	180.99	178.27
3	335.51	331.00

For thick plate Mindlin approach converges ABAQUS program but when The Kirchhoff FEM code is applied, it diverges with increased the thickness ratio of the plate. So, for thick ones just Mindlin method can be applied on the plate because of the shear effects through the thickness [Temuçin, B. T., 2020].

The natural frequency values of a box beam structure with 45 ° / 45 ° / 45 ° / 45 ° / 45 ° sequence thin clamped-free composite box beam were obtained and compared with the literature and frequency values obtained from the ABAQUS software package in Table 3. The mode shapes obtained from the modal analysis are shown in Figure 5. (clamped,2a=0.75m; 2b=2c=2a/4; h=0.004; E_1=37.78 GPa; E_2=10.9 GPa; G_{12}=G_{13}=G_{23}=4.91GPa; v_{12}= v_{21}=0.3; ρ=1870 kg/m^3) The modified Mindlin code with Gauss quadrature method for thin composite box structure gives closer results [Temuçin, B. T., 2020].

Table 3 : Comparison of the [45/45/45/45/45] composite clamped-free box beam natural frequencies.

Modes	Present-Kirchhoff (32x8)	Present-Mindlin (32x8)	ABAQUS (32x8)	Ramkumar & Kang (2013)
1	147.81	146.02	146.7	146.7
2	147.81	146.02	146.7	146.7
3	168.63	166.22	170.25	168.68

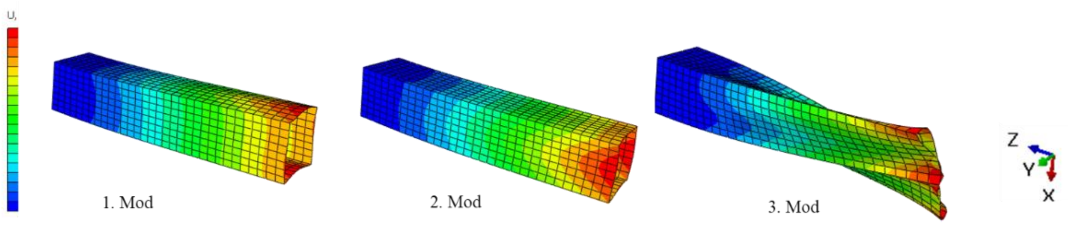

Figure 5 : Mode shapes of the [45/45/45/45/45] composite clamped-free box beam.

The vibration values of a box beam structure with $0_3/90_2/0_3$ sequence folded thin and thick composite cantilevered support are obtained and compared with the frequency values obtained from the ABAQUS software package respectively.

Table 4 consists of thin box data and Table 5 for thick one. The thin and thick composite boxes mode shapes obtained from the modal analysis prepared in ABAQUS are shown in Figure 6 and Figure 7 respectively. (cantilevered beam, 2a=0.655m; 2b=0.057m; 2c=0.019m; h_{thin}=0.00025m; h_{thick}=0.005m; E_1=23.69 GPa; E_2=7.63 GPa; G_{12}=G_{13}=G_{23}=3.37 GPa; v_{12}= v_{21}=0.26; ρ=1985 kg/m^3)

Table 4 : Comparison of natural frequencies (Hz) of the composite thin box beam [$0_3/90_2/0_3$].

Modes	Present-Kirchhoff (16x4)	Present-Mindlin (16x4)	Present-ABAQUS (16x4)
1	35.50	35.50	35.14
2	80.97	80.97	80.20
3	206.67	207.58	202.73
4	313.44	314.67	309.82
5	461.25	461.88	458.75

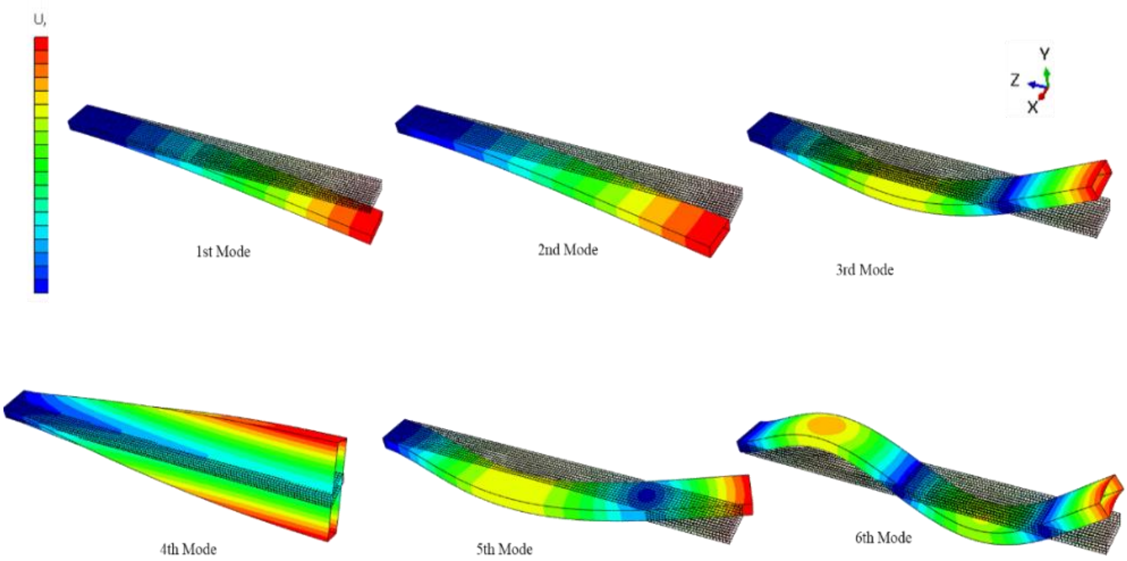

Figure 6 : Mode shapes of the thin cantilever composite box beam with undeformed and deformed shapes.

Table 5 : Comparison of natural frequencies of the composite thick box beam

Modes	Present-Mindlin (Hz)	Present-ABAQUS (Hz)
1	34.74	35.42
2	79.20	81.11
3	203.53	204.13
4	325.51	324.96
5	451.42	459.19
6	519.85	509.85

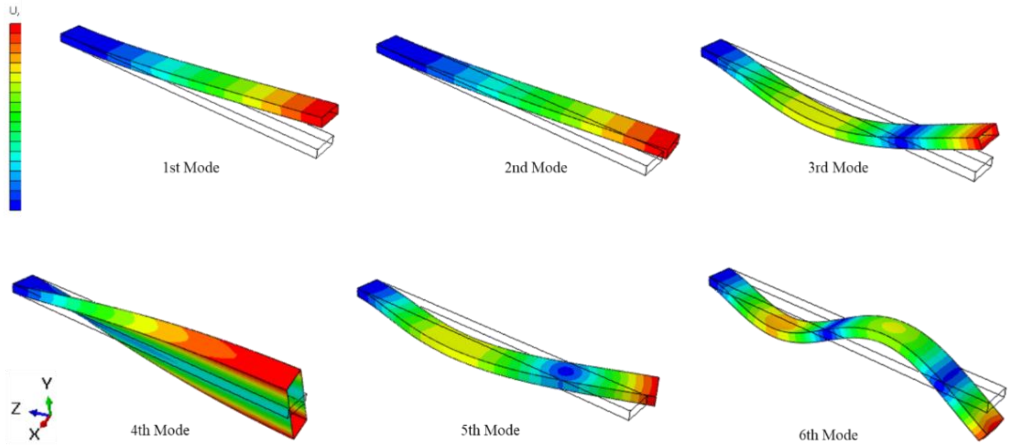

Figure 7 : Mode shapes of the thick cantilever composite box beam with undeformed and deformed shapes.

The Mindlin code should be used for thick composite box structure owing to better performance on composites and high thickness ratios. For the same mesh amount/size, gives accurate results compared to ABAQUS package program on composite boxes [Temuçin, B. T., 2020].

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