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CONTROL ALLOCATION FOR AN OVERACTUATED E-VTOL AIRCRAFT USING BLENDED-INVERSE

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ABSTRACT

In this paper, the control allocation problem is addressed in a flight control system of an overactuated multi-rotor eVTOL (electric Vertical Takeoff and Landing) aircraft with 20 rotors. The objective is to find a control solution that provides optimal flight performance and handling quality. The proposed method is applied to a nonlinear simulation model of the conceptual aircraft. Also, various simulator based rotor failure scenarios are tested on the aircraft to evaluate the flight safety. It is shown that robust and efficient control redistribution can be achieved using the proposed solution under various failure conditions.

INTRODUCTION

Problem Definition

While eVTOL (electric Vertical Takeoff and Landing) aircraft are becoming increasingly popular in civil transportation, their safety and reliability characteristics are current research topic in the rotorcraft community. Commonly those aircraft are designed with redundant rotors or control surfaces to increase safety when single or multiple of these rotors fail during flight. In case of a rotor fail, those vehicles are desired to fly and land safely with minimum degraded handling qualities and low consumed power. This paper focuses on the control allocation problem of such multi-rotor vehicles when multiple rotors fail during flight.

Control Allocation in Literature

There are various types of control allocation method in the literature and a common approach of these studies is to estimate the inverse of the control matrix. For this purpose, quite often the minimum-norm solution as known as pseudoinverse is used [E.D.F. Snell S.A. and G.W.L., 1990]. A common problem of pseudoinverse is the possibility of arising singularities during the inversion process. In [Tekinalp and Yavuzoglu, 2005], the singularities are removed by introducing a method called Blended-Inverse and applied to the attitude control problem for a satellite. The method is used in [Tekinalp, Yavrucuk and Ünlü, 2009] to find optimum control allocation strategies for a compound helicopter. In those references, a continuous update on the control allocation strategy

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is demonstrated without any singularities in the calculations. As a multi-rotor application, rotor failures are addressed for a simple VTOL octorotor model in [Marks, Whidborne and Yamamoto, 2012]. Here, a method based on control matrix dimension reduction [Oppenheimer, Doman and Bolender, 2006] is applied in case of rotor failures and it is shown that the control loss is prevented with this method.

Objective

In this paper, the methods of [Tekinalp, Yavrucuk and Ünlü, 2009; Tekinalp and Yavuzoglu, 2005; Marks, Whidborne and Yamamoto, 2012] are used to re-distribute the control law in case of rotor failure for a 20 rotor eVTOL conceptual aircraft (Figure 1). Simulation studies are performed around hover and forward flight to demonstrate the effectiveness of the combined methods in case of multiple rotor failures.

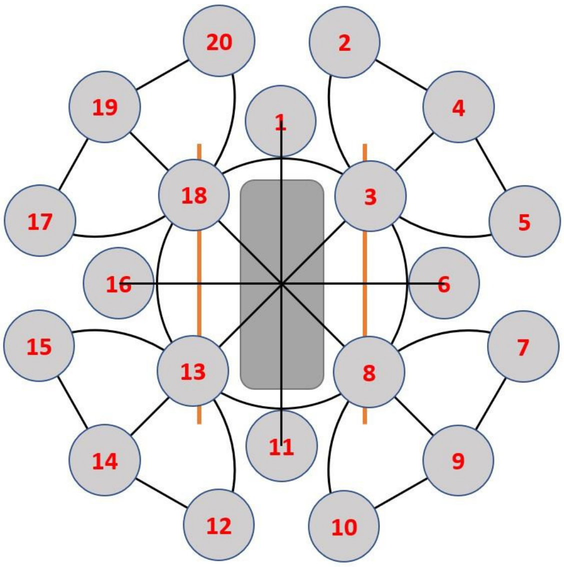


Figure 1: Conceptual eVTOL aircraft

METHOD

Nonlinear state equation of open loop dynamics:

$$\dot{x} = f(\boldsymbol{x}, \Omega_{ref}) \tag{1}$$

In (1), "Ω_{ref}" represents the rotational speed reference to control the vehicle. In the conceptual aircraft (Fig. 1), all rotors have an individual electric motor that provides power for rotation. "Ω_{ref}" in (1) are reference rotational speed commands generated by the flight control system given to the control units of electric motors for manoeuvring. Electric motor of each rotor provides torque that is required to match the corresponding rotor rotational speed with its reference command.

State vector x in (1) is given as following,

$$\boldsymbol{x} = [u\ v\ w\ p\ q\ r\ \phi\ \theta\ \Omega_1\ \Omega_2\ \Omega_3\ ...\ \Omega_n]^T \tag{2}$$

In (2), rotational speed of each rotor (Ω_n) are additional states of the system. Here, a vehicle has 20 rotors, then n=20 (Figure 1). Due to redundant controls in the state equation, different combinations of Ω_{ref} result in similar time domain solutions. The issue is to find optimum allocation of Ω_{ref} for sufficient handling qualities and flight performance even in the case of rotor failures.

Control Allocation Structure

Thrust and moments produced by individual rotor are assumed to be linearly proportional with square of rotational speed. For a multi-rotor with 20 rotors, a virtual input vector composed of squares of rotational speeds is defined:

$$\Omega'_{ref} = [\Omega^2_{ref_1}\ \Omega^2_{ref_2}\ ...\ \Omega^2_{ref_{20}}]^T \tag{3}$$

Using the virtual input, the following linear relation can be defined:

$$R\Omega'_{ref} = D \tag{4}$$

where $\mathrm{D} \in \Re^4$ is desired thrust and moments vector and $\mathrm{R} \in \Re^{4\times 20}$ is control allocation matrix representing the physical relation between rotor rotational speeds and desired outputs.

D in (4) is defined as following,

$$D = [T_D\ M_{xD}\ M_{y_D}\ M_{zD}]^T \tag{5}$$

Elements of "D" are desired thrust force, roll, pitch and yaw moments. They are linearly mapped with respect to pilot control inputs to pre-defined values as shown in Figure 2.

Control allocation matrix (R) in (4) is defined as following,

$$R = \begin{bmatrix} C_T & C_T & ... & C_T \\ C_T l_{y_1} & C_T l_{y_2} & ... & C_T l_{y20} \\ C_T l_{x_1} & C_T l_{x_2} & ... & C_T l_{x_{20}} \\ C_Q \tau_1 & C_Q \tau_2 & ... & C_Q \tau_{20} \end{bmatrix} \tag{6}$$

where $\mathrm{C_T}$ and $\mathrm{C_Q}$ are thrust and torque coefficients, $\mathrm{l_x}$ and $\mathrm{l_y}$ are longitudinal and lateral moment arms of rotors with respect to aircraft's C.G., τ represents rotation direction of individual rotor (either "1" or "-1").

Each column of the control allocation matrix (6) represents single rotor properties. When open form of (4) is considered, the following relations are obtained:

- **Thrust sharing:** $C_T\Omega'_{ref_1} + C_T\Omega'_{ref_2} +\ ...\ + C_T\Omega'_{ref_{20}} = T_D$
- **Roll moment sharing:** $C_T l_{y_1}\Omega'_{ref_1} + C_T l_{y_2}\Omega'_{ref_2} +\ ...\ + C_T l_{y20}\Omega'_{ref_{20}} = M_{xD}$
- **Pitch moment sharing:** $C_T l_{x_1}\Omega'_{ref_1} + C_T l_{x_2}\Omega'_{ref_2} +\ ...\ + C_T l_{x_{20}}\Omega'_{ref_{20}} = M_{y_D}$
- **Yaw moment sharing:** $C_Q\tau_1\Omega'_{ref_1} + C_Q\tau_2\Omega'_{ref_2} +\ ...\ + C_Q\tau_{20}\Omega'_{ref_{20}} = M_{zD}$

Using (4), reference rotational speeds (Ω_{ref}) can be calculated as following:

$$\Omega'_{ref} = R^{-1}D \tag{7}$$

$$\Omega_{ref} = \sqrt{\Omega'_{ref}} \tag{8}$$

The inverse (R^{-1}) in (7) is not possible since $\mathrm{R} \in \Re^{4\times 20}$ is a non-square matrix. The proper inversion process is explained in the following section.

Inversion with Blended-Inverse

A common way of inverting a non-square matrix is to calculate the minimum-norm solution (pseudo-inverse). A major drawback of this method arises from singularities. Also, it does not yield the best performance solution [Tekinalp, Yavrucuk and Ünlü, 2009]. To overcome these issues, the Blended-Inverse [Tekinalp and Yavuzoglu, 2005] technique is used. For this purpose, the minimization problem for the control system is defined:

$$\min_{\Omega'_{ref}} \frac{1}{2}\{\Omega'^{T}_{ref_e} Q\, \Omega'_{ref_e} + D_e^{\,T} F D_e\} \tag{9}$$

where $\mathrm{Q} \in \Re^{20\times20}$ and $\mathrm{F} \in \Re^{4\times4}$ are square weight matrices of input and desired output respectively, and "e" represents errors.

Solution of this equation is the Blended-Inverse:

$$\Omega'_{ref} = [Q + R^T F R]^{-1} [Q \Omega'_{ref_d} + R^T F D] \tag{10}$$

where, $\Omega'_{\mathrm{refd}} \in \Re^{20}$ is the desired control input vector.

Rotor speed distribution of hover trim condition is assigned for Ω'_{ref_d} in (10). The aim of defining such vector is to keep rotor speeds around trim values as much as possible for efficiency.

The overall Blended-Inverse control diagram is shown in Figure. 2.

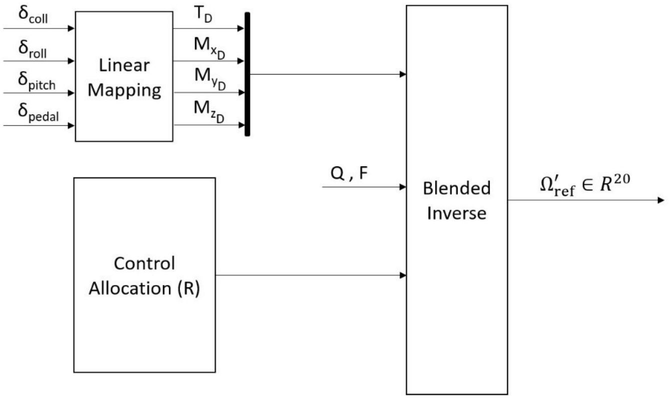


Figure 2: Blended-Inverse block diagram

Control Allocation During Rotor Failures

Control allocation using Blended-Inverse provides a solution where all rotors share total thrust, to achieve demanded lift and moments. Re-distributed pseudo-inverse in [Marks, Whidborne and Yamamoto, 2012] is proposed for the rotor failure solution. In this paper, the Blended-Inverse is used instead of pseudo-inverse. If a rotor fails, dimension of the control allocation matrix is reduced such that its corresponding column is removed, hence, not included in the control allocation. Then, the same desired thrust and moments are shared between only the operating rotors. For instance, if "k" is the number of failed rotors, the control allocation matrix becomes $\mathrm{R} \in \Re^{4\times(20-\mathrm{k})}$, input weight matrix becomes $\mathrm{Q} \in \Re^{(20-\mathrm{k})\times(20-\mathrm{k})}$ and desired input vector becomes $\Omega'_{\mathrm{refd}} \in \Re^{(20-\mathrm{k})}$.

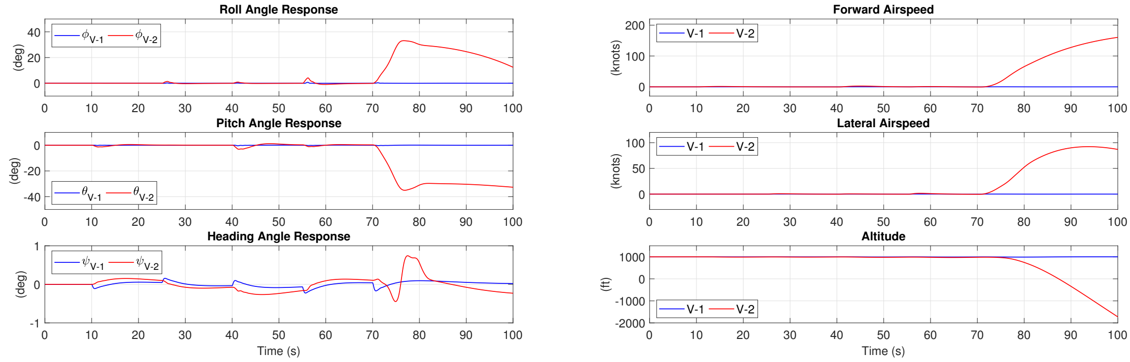
RESULTS AND DISCUSSION

In all simulation tests conducted in this chapter, intentionally chosen rotors are failed one after another and there are 15 seconds between failures. Tests are started from trim condition and a closed-loop controller provides commands to keep aircraft in the trimmed position. Also, two vehicles are compared to demonstrate the effect of the input re-distribution in all tests:

- **Vehicle-1:** Inputs are re-distributed using the proposed method in this paper
- **Vehicle-2:** Inputs are not re-distributed

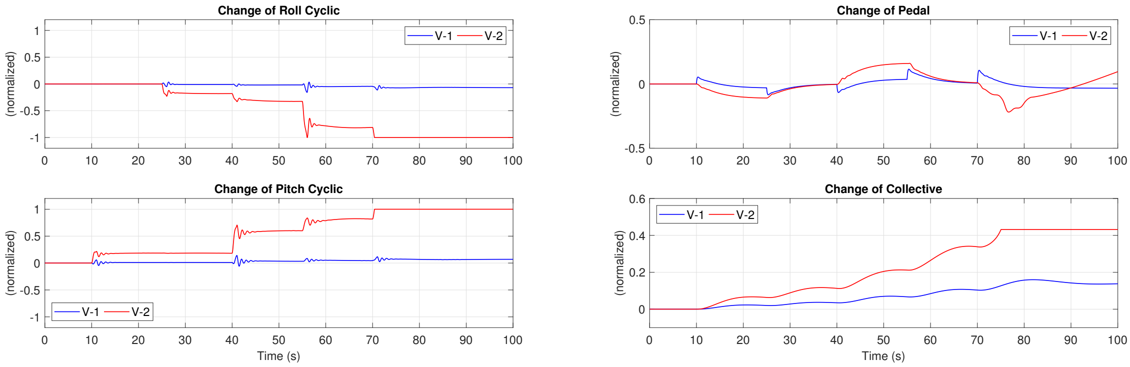
Hover/Case-1

Rotors with the following numbers are failed, respectively: 1, 6, 2, 5, 3 (Fig. 1).
The results are shown in Figures 3a, 3b, 4a, 4b, 5.



(a) (b)

Figure 3: Attitude, airspeed and altitude responses of the vehicles (Hover/Case-1)



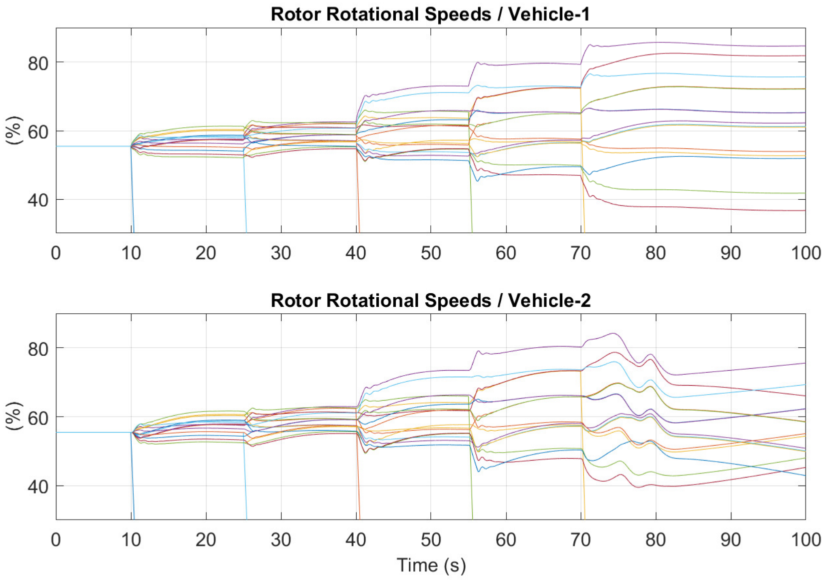
(a) (b)

Figure 4: Control inputs of the vehicles (Hover/Case-1)

By looking at the results, Vehicle-1 can keep its hover trim condition steadily with very little control effort. All rotor rotational speeds are properly re-distributed on-line, following the rotor failures. As a result, there is very little change in the controls in order to stay at hover. However, in Vehicle-2, controller work-load starts to increase with the failures (Figs. 4a, 4b). Control positions reach their limits and the control is lost in all axes. Consequently, the aircraft diverges from the hover trim condition.

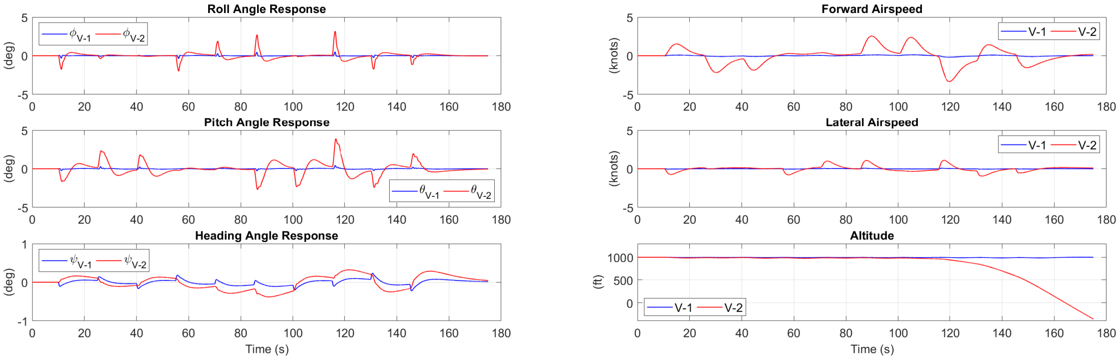
Hover/Case-2

Rotors with the following numbers are failed, respectively: 19, 12, 11, 16, 6, 4, 1, 9, 18, 13 (Fig. 1).
The results are shown in Figures 6a, 6b, 7a, 7b, 8.


Rotor Rotational Speeds / Vehicle-1

Rotor Rotational Speeds / Vehicle-2

Figure 5: Rotor rotational speed outputs of the vehicles (Hover/Case-1)


Roll Angle Response

Pitch Angle Response

Heading Angle Response

Forward Airspeed

Lateral Airspeed

Altitude

(a) (b)

Figure 6: Attitude, airspeed and altitude responses of the vehicles (Hover/Case-2)

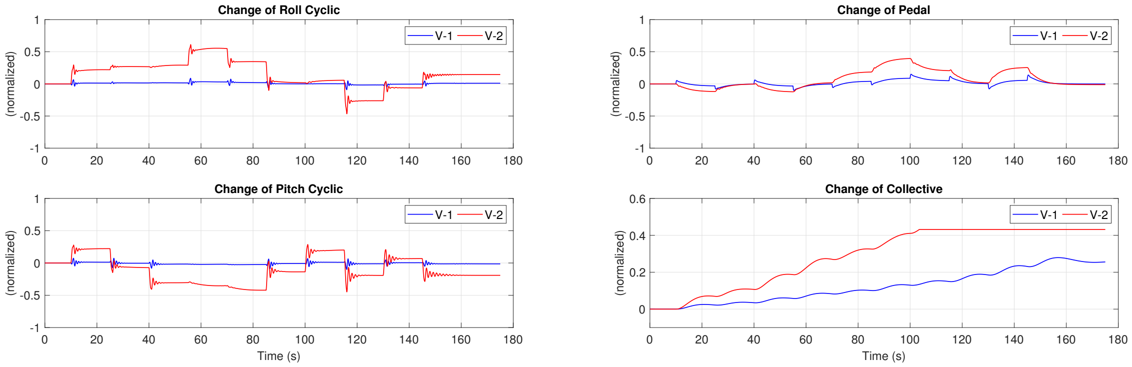
In Vehicle-1, the aircraft survives from ten random rotor failures as a result from input control re-distribution as rotors fail. Very little control position change is observed in all channels in Figs. 7a and 7b.

In Vehicle-2, the aircraft can keep euler attitudes in the trim condition (Fig. 6a) since some of the failed rotors mutually compensate their dynamic effects during failure. However, compensation of lift force is not sufficient because the collective control reaches its maximum limit (Fig. 7b) and the control is lost in the vertical channel (Fig. 6b).

Forward Flight/Case-1

Rotors with the following numbers are failed, respectively: 1, 6, 2, 5, 3 (Fig. 1).
The results are shown in Figures 9a, 9b, 10a, 10b, 11.

In this case, simulation results demonstrate that the proposed control method works properly also in the forward flight condition. Although Vehicle-2 can survive from failures, the effort to control the aircraft is higher compared to Vehicle-1 (Fig. 10a and 10b). In the previous section (Hover/Case-1), Vehicle-2 could not stay at trim with the same rotor failure configuration. The reason why it can



(a) (b)

Figure 7: Control inputs of the vehicles (Hover/Case-2)

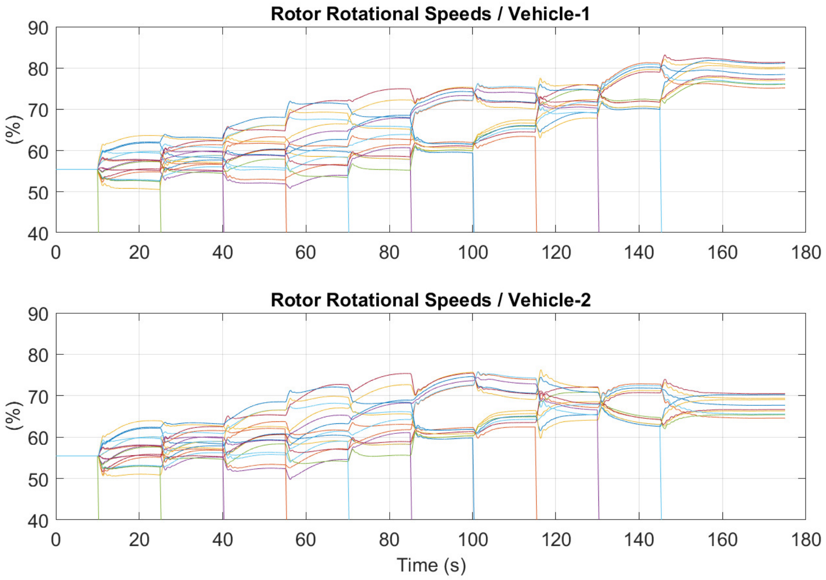


Figure 8: Rotor rotational speed outputs of the vehicles (Hover/Case-2)

survive in this case is that higher aerodynamic forces can be generated in forward flight due to higher free-stream velocity.

As control inputs are re-distributed in Vehicle-1, it can survive until the end of simulation with a very little pilot control effort. This significant difference can be seen in Figs. 10a and 10b.

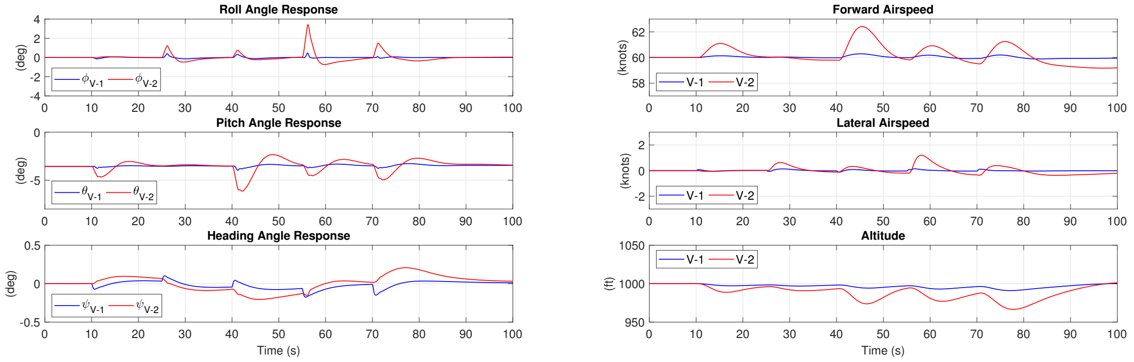
Forward Flight/Case-2

Rotors with the following numbers are failed, respectively: 19, 12, 11, 16, 6, 4, 1, 9, 18, 13 (Fig. 1). The results are shown in Figures 12a, 12b, 13a, 13b, 14.

Results of this case demonstrate that the safe flight is achievable, also in forward flight, without deviating from the trim point as 10 rotors randomly fail.

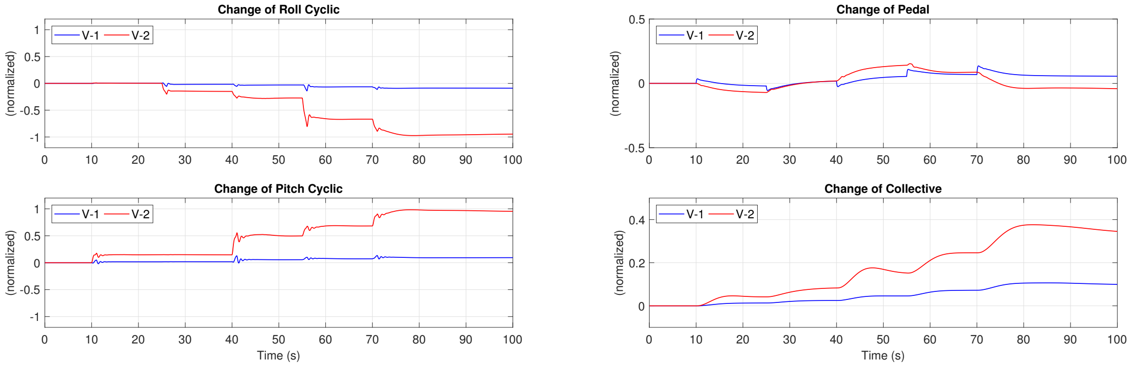
CONCLUSIONS

With the proposed method, singularities are prevented during control matrix inversion and input distribution along rotors are obtained to perform the demanded manoeuvre. In case of rotor failure,



(a) (b)

Figure 9: Attitude, airspeed and altitude responses of the vehicles (Forward/Case-1)



(a) (b)

Figure 10: Control inputs of the vehicles (Forward/Case-1)

dimension reduction technique is applied dynamically to the control allocation where aerodynamic effect of the failed rotor is removed. This yields that required thrust and moments are shared between operating rotors only. This method provides on-line input re-distribution to maintain current manoeuvre. Simulation results show that input re-distribution provides significantly reduced control effort to maintain the trim condition of the aircraft. This prevents handling quality degradation of the aircraft following the failure. Also, it is observed that safe flight is achievable at both hover and forward flight conditions using the proposed method.

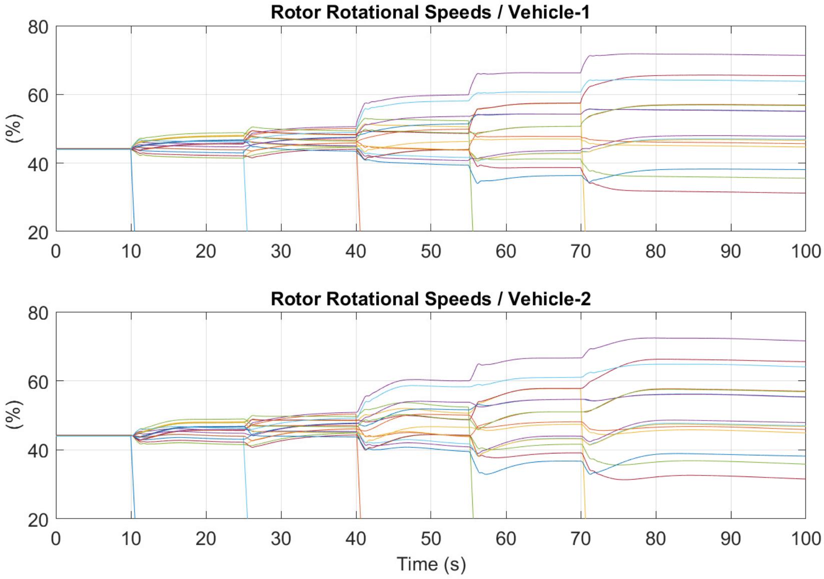
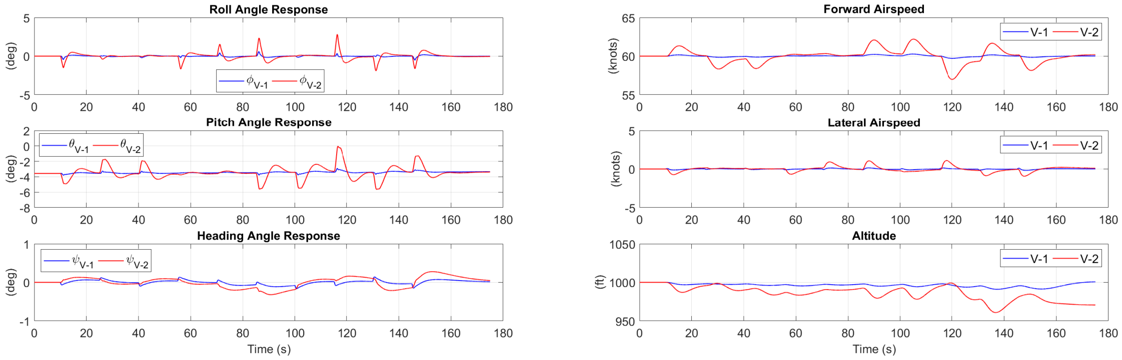
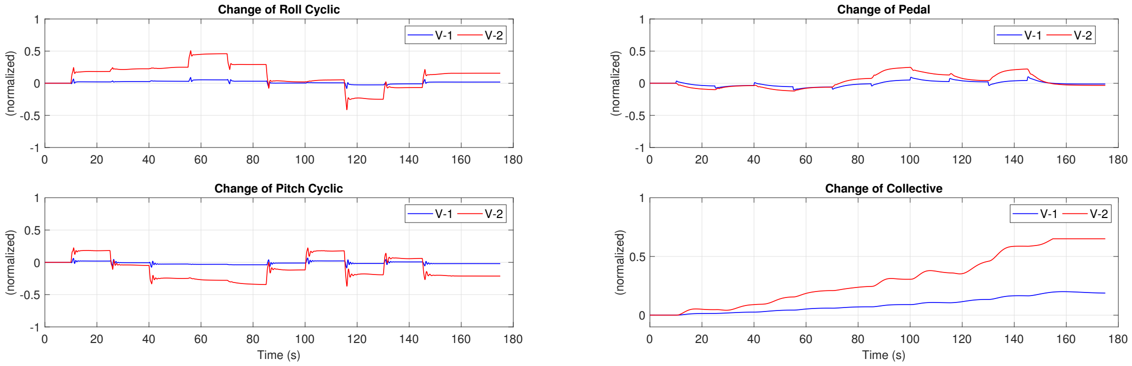


Figure 11: Rotor rotational speed outputs of the vehicles (Forward/Case-1)



(a) (b)

Figure 12: Attitude, airspeed and altitude responses of the vehicles (Forward/Case-2)



(a) (b)

Figure 13: Control inputs of the vehicles (Forward/Case-2)

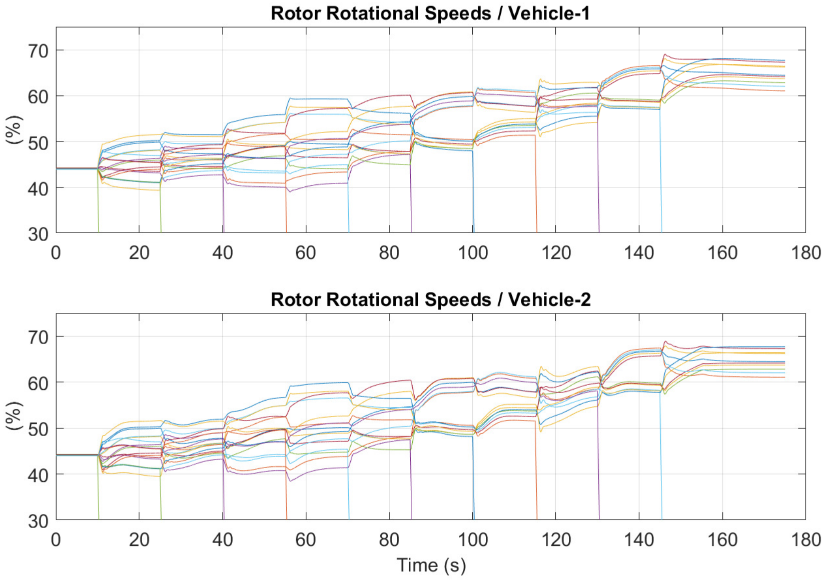


Figure 14: Rotor rotational speed outputs of the vehicles (Forward/Case-2)

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