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ADAPTIVE PARAMETER ESTIMATION FOR AIR VEHICLE SYSTEM IDENTIFICATION USING FLIGHT TEST DATA

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ABSTRACT

Adaptive parameter estimation is a method that improves convergence in linear model identification for both fixed-wing and rotary-wing aircraft system identification. Recent advances showed that it is possible to guarantee a unique parameter convergence in the estimation problem by collecting linearly independent time history data points and using them in the offline adaptive parameter update. This paper uses adaptive parameter estimation techniques to estimate linear stability and control derivatives using flight test data. A high fidelity nonlinear helicopter model is used to generate flight test maneuvers around hover, such as 3211-type inputs and frequency sweeps, later to be used for identification and validation. Validation of the obtained models is done both in the Time-Domain and in the Frequency-Domain. The effectiveness of the proposed adaptive method in estimating linear stability and control derivatives is shown through simulations.

INTRODUCTION

Estimating linear models using flight test data is one of the research topics in the aerospace engineering community. Linear models include stability and control derivatives; therefore, their estimation using flight test data becomes necessary for handling quality analyses, automatic flight control system design/tuning, and nonlinear model fidelity enhancements. To estimate linear models, for example, for rotorcraft, traditional time-domain and frequency-domain system identification methods are used and well documented in the literature; a few examples are cited in Refs.[Tischler and Remple, 2012; Jategaonkar, 2015; Seher-Weiß, Tischler, Scepanovic and Gubbels, 2021; Zivan, Lior and Tischler, 2010; Cheng, Rendy, Tischler and Schulein, 2006; Tischler and Tumashofski, 2002; Seher-Weiss, 2019; Conroy, Joseph, Humbert and Pines, 2011].

This paper uses an alternative time-domain method [Gursoy, Aslandogan and Yavrucuk, 2021] to fit a linear model, hence estimating the stability and control derivatives. The methodology employs recent advances in Model Reference Adaptive Control (Refs. [Chowdhary, Yucelen, Muhlegg and Johnson, 2013; Chowdhary, 2010; Chowdhary and Johnson, 2010]), where it is shown that the uncertainty

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arising from the unmodelled dynamics could be linearly parameterized, and the convergence of the adaptive weights around the optimal locations is possible. Moreover, a unique optimal solution exists if the basis of the adaptive element is composed of a minimal representation of the dynamic system. In Refs. [Yavrucuk and Prasad, 2012; Gursoy and Yavrucuk, 2016, 2015] algorithms are proposed to represent model uncertainty using sensor measurement during flight.

In this paper, using the algorithms of Ref. [Gursoy, Aslandogan and Yavrucuk, 2021], the linear model of a helicopter is identified. A high fidelity nonlinear model of the EC135 twin-engine helicopter is used as a truth model. The model is a high fidelity nonlinear physics-based flight model using Aerotim Engineering's [Aerotim, 2021] core model components, intended to develop flight models for EASA Level D certified full flight simulators. The model employs a Blade Element Rotor Model (BERM) with virtual blades, 2nd order flapping, Pitt-Peters inflow model, aerodynamic derivatives for the fuselage, vertical tail, horizontal tail, and fenestron model. Those model components have been used in Level D certifiable simulators for helicopters of similar classes and have been verified with flight test data. The nonlinear model runs 3211-type inputs and frequency sweep type maneuvers to represent flight test data. 3211-type input inputs are used in adaptive identification of the dynamic stability and control derivatives of that model in hover. The frequency sweep tests are used for the verification of the final identified model in the frequency domain.

Estimating the stability and control derivatives from flight test data is typically challenging since multiple solutions for the identified models exist. This problem is generally solved by reducing the model structure by eliminating unnecessary derivatives and converging the remaining ones to a unique solution This process is called model reduction and is a significant part of the identification process. In Ref. [Gursoy, Aslandogan and Yavrucuk, 2021], a recent publication of the authors, the adaptive identification process with model reduction is applied, and linear models with minimum stability and control derivatives are estimated. Yet, in this paper, the model reduction process is not applied. The focus is given to parameter convergence when no derivatives are eliminated in the linear models. Identified models found using the proposed adaptive learning method are validated in both time-domain and frequency-domain, and the necessity of potential improvements in the derivative estimates are discussed.

METHODOLOGY

Using Ref. [Gursoy and Yavrucuk, 2016] and its assumptions, the helicopter dynamics can be represented with the following nonlinear state equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}); \quad \mathbf{x}(t_0) = \mathbf{x}_0; \tag{1}$$

where $\mathbf{x} \in \Re^n$ is the state vector with known initial condition, $\mathbf{x}_0$, and $\mathbf{u} \in \Re^p$ is a known control input vector. Here, $\mathbf{x}$ and $\mathbf{u}$ are assumed to be measured. Let's, re-write Eq. (1) as the summation of a linear approximation and modeling error as

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + \xi(\mathbf{x}, \mathbf{u}). \tag{2}$$

Here, $\xi(\mathbf{x}, \mathbf{u})$ is the modeling vector due to the linear approximation, $A\mathbf{x} + B\mathbf{u}$. Note that, if the optimal values of A and B $(A^\star, B^\star)$ were known, the norm of the modeling error would be the smallest possible value. Now, assume that $A^\star$ and $B^\star$ is not known and only an approximate $[A, B]$ pair is used to generate a model representation.

Assume a model is formed to represent Eq. (2) as follows:

$$\dot{\hat{\mathbf{x}}} = A\mathbf{x} + B\mathbf{u} + \Delta(\mathbf{x}, \mathbf{u}) \tag{3}$$

where $\Delta : \Re^{n+p} \to \Re^n$ is an adaptive element that estimates ξ.

Therefore, the goal of the adaptive element Δ is to cancel out the modeling error ξ with optimal adaptive parameters so-called adaptation weights. Here, it is assumed that the uncertainty, ξ, can be linearly parameterized, that is, there exists a set of unique optimal weights with a reconstruction error, ϵ, such that,

$$\xi(\mathbf{x},\mathbf{u}) = W^{*T}\bar{\mathbf{x}} + \epsilon \tag{4}$$

where, $\bar{\mathbf{x}} = [\mathbf{x}\ \mathbf{u}]^T \in \Re^{n+p}$ is the basis vector. $W^* \in \Re^{(n+p)\times n}$ represents a unique set of optimal weights. In order to make an approximation to ξ, the same basis vector as in Eq. (4) can be constructed for Δ of Eq. (3):

$$\Delta(\mathbf{x},\mathbf{u}) = W^T\bar{\mathbf{x}}. \tag{5}$$

If a gain term $K \in \Re^{n\times n}$ is added to Eq. (3), noting that $\hat{\mathbf{e}} = \mathbf{x} - \hat{\mathbf{x}}$, the error dynamics can be obtained by subtracting Eq. (3) from Eq. (2) as

$$\dot{\hat{\mathbf{e}}} = -K\mathbf{e} + \xi(\mathbf{x},\mathbf{u}) - \Delta(\mathbf{x},\mathbf{u}). \tag{6}$$

Using Eqs. (4) and (5), the error dynamics of Eq. (6) takes the following final form:

$$\dot{\hat{\mathbf{e}}} = -K\mathbf{e} + \tilde{W}^T\bar{\mathbf{x}} + \epsilon \tag{7}$$

where, $\tilde{W} = W^* - W$ is the error between the optimal and the approximate weights. Note that, the observer gain matrix K shall converge to a modeling error, but used to make error dynamics stable. As a result, the final form of the estimator, that is Eq. (3), would be:

$$\dot{\hat{\mathbf{x}}} = A\mathbf{x} + B\mathbf{u} + W^T\bar{\mathbf{x}} + K(\mathbf{x} - \hat{\mathbf{x}}). \tag{8}$$

Since the dynamic system of Eq.(8) is linear in $\mathbf{x}$ and $\mathbf{u}$, the resulting system after the convergence of weights to the optimal locations would the system that is closest to $[A^\star, B^\star]$. Moreover, the pair $[A^\star, B^\star]$ should be unique since the basis $\bar{x}$ used in Eq. (5) is chosen as a minimal representation of the dynamic system. When the convergence is completed, the following becomes the final form of the linear approximate system that will represent Eq. (2):

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + B\mathbf{u} + [W_A\ W_B]\begin{bmatrix}\hat{\mathbf{x}}\\ \mathbf{u}\end{bmatrix}. \tag{9}$$

where, W_A and W_B are the sub matrices of W^T at the end of convergence. Therefore, $[A^\star, B^\star]$ pair is predicted at the end as:

$$[A^\star, B^\star] = [A + W_A, B + W_B]. \tag{10}$$

Adaptive Learning Implementation

The error dynamics of Eq. (7) can be minimized to ϵ only by taking the weight errors $\tilde{W}$, to zero. Therefore, an adaptive learning law which will drive the weights, W, to the optimal locations, W^*, is required. The following weight update law is proposed:

$$\dot{W}(t) = \Gamma\bar{\mathbf{x}}\mathbf{e}^T P + \sum_{j=1}^{p}\bar{\mathbf{x}}_j(\xi_j - W^T\bar{\mathbf{x}}_j). \tag{11}$$

Here, $P \in \Re^{n\times n}$ is the positive definite solution of $(-K)^T P + P(-K) = -Q$ for any given positive definite matrix $Q \in \Re^{n\times n}$, and Γ is a positive definite learning gain matrix. The variables with subscript j refers to the recorded information. Therefore, the adaptation uses both current data, which is the first term in the update law, and the data selected previously, the second term. A slightly modified version of the given update law of Eq. (11) is used previously in the area of Model Reference Adaptive Control in Refs. [Chowdhary, Yucelen, Muhlegg and Johnson, 2013; Chowdhary, 2010], in order to achieve guaranteed boundedness around optimal weights without requiring persistency of excitation in the inputs of the adaptive element. This was achieved by using as linearly independent elements, which are $\bar{\mathbf{x}}_j$'s, as possible in the weight update. For this purpose, $\bar{\mathbf{x}}_j$'s can be recorded in a data stack matrix as follows:

$$X = [\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2, \bar{\mathbf{x}}_3, ..., \bar{\mathbf{x}}_l]. \tag{12}$$

Here, $\bar{\mathbf{x}}_j \in \Re^m$, $j = 1, 2, ..., l$, are the basis vectors recorded at different times and X is an $n \times l$ history stack matrix. Minimum singular value of X matrix can calculated and used to record linearly independent elements. Therefore, once the X matrix is full, new data will replace the old such that the minimum singular value of X increased. This was applied in Refs. [Chowdhary, Yucelen, Muhlegg and Johnson, 2013; Chowdhary, 2010] and later in Refs. [Gursoy and Yavrucuk, 2016, 2015] as being the minimum singular value maximization method to record linearly independent elements in the data stack matrix. For the details of the implementation, the reader may refer to Ref. [Gursoy, Aslandogan and Yavrucuk, 2021].

In this paper, the developed algorithm uses Eqs. (8), (11) along with the recorded data Eq. (12) at each time step of the related flight test maneuver. The validation of the resulting linear approximation is presented using Eq. (9) in both time domain and frequency domain.

SIMULATION RESULTS

A high fidelity helicopter model is treated as the "truth model" to estimate $[A^\star, B^\star]$ around a hover equilibrium point. 3211-type inputs are applied in each input channel and the input-output response is recorded. This data is later treated as if it was recorded in flight tests. Now, the goal is to obtain the relation of Eq. (3) that would fit these maneuvers. Time histories of 3211-type maneuvers for each control channel are used in the adaptation. The adaptation is run in sequence for all channels and is repeated until convergence is achieved. At the beginning of each maneuver, the adaptive weights, W, and the recorded data stack, Z, are initialized to the previous simulation's end values. Therefore a continues update on the adaptive parameters is obtained.

The state and control vectors used in Eq. (3) are as follows:

$$\mathbf{x} = [u\ w\ q\ \theta\ v\ p\ \phi\ r]^T, \tag{13}$$

$$\mathbf{u} = [\delta_{coll}\ \delta_{lat}\ \delta_{long}\ \delta_{ped}]T. \tag{14}$$

The change of adaptive weights for the number of epochs is given in Fig. 1, where one epoch of run is one set of 3211-type maneuvers in each channel. As seen from the figures, the adaptive weights reached constant values in 10 Epochs as the minimum singular value of the recorded data is maximized, which indicates that the weights are converged around their optimal values. Note that the weight update histories given in Fig. 1 represent the elements of the estimated linear model. Therefore, some elements of the linear matrices, which can be calculated using the trim conditions, are kept constant and not updated/estimated.

As seen in Fig. 1, the J_{RMS} value, an indicator of goodness of fit in the time domain, converges around 1. J_{RMS} is calculated as follows:

$$J_{RMS} = \sqrt{\frac{1}{n_t \times n_o} \sum_{i=1}^{n_t} [\mathbf{z} - \hat{\mathbf{z}}]^T [\mathbf{z} - \hat{\mathbf{z}}]} \tag{15}$$

where, $\mathbf{z} = [p\ q\ r\ a_x\ a_y\ a_z]$ is the flight test data (data of the nonlinear model in this case) and, $\hat{\mathbf{z}} = [\hat{p}\ \hat{q}\ \hat{r}\ \hat{a}_x\ \hat{a}_y\ \hat{a}_z]$ is the estimator outputs. n_t is the length of the time history points in one epoch and n_o is the size of the identification vector which is 6. J_{RMS} is calculated at the end of each epoch for all identification maneuvers (3211-type inputs). J_{RMS} values around 1 indicate a sufficient fit in the time domain (Ref.[Tischler and Remple, 2012], pages 437-438).

In the following simulation, weights are initialized to a set of non-zero initial conditions to demonstrate the convergence capability of the proposed method. The initial state of the parameters is

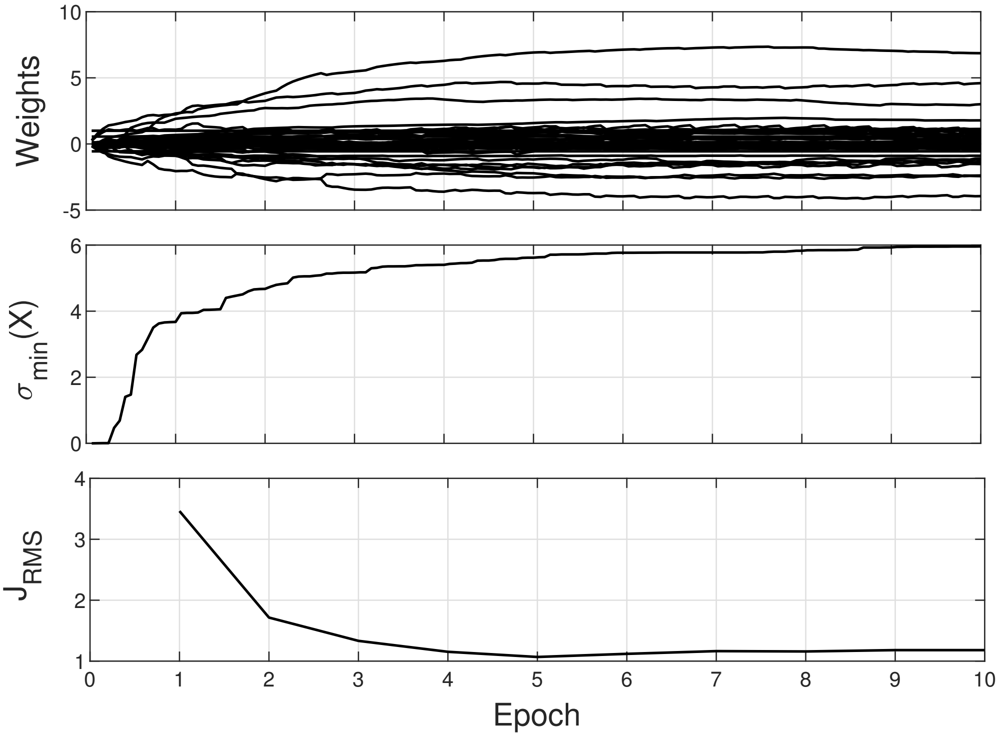

Figure 1: Evolution of Adaptive Weights for around Hover 3211-Type Inputs, Zero Initial Condition

chosen far from the converged parameters of Fig. 1. Results are presented in Fig. 2. As seen, the converged state of the parameters in Fig. 2 is almost similar to the converged state of Fig. 1. J_{RMS} values in both figures are nearly the same as well. That example indicates that the proposed method's convergence is repeatable and independent of the initial condition even if the chosen condition is far from the optimal state.

Time-domain validation of the resulting linear model after the convergence is presented in Fig. 3. 3211-type inputs different from those used in the identification are given to the nonlinear and linear models in four channels from left to right. A low-frequency ground speed controller is also used to keep the model around the original trim point at each input channel. The linear approximation tracks the nonlinear response in all four axes (p, q, r, a_z). Also, the J_{RMS} value obtained at the end of the convergence in Fig. 1 indicates a sufficient fit in the time domain [Tischler and Remple, 2012]. Note that J_{RMS} values below one would mean a better match in the time domain.

Frequency sweep inputs are applied to nonlinear and linear models for the validation task in the frequency domain. Frequency domain comparisons of both models are given in Figs. 4, 5, 6, and 7. In these figures, the most dominant response variables, i.e., q and p for the longitudinal input, for each input channel are shown. Also, the coherence parameter, which measures the linearity between the input and the state, is presented. Here, coherence values above 0.6 indicate that a linear model representing the input-output transfer function pair is possible. Although the magnitude and the phase curves give close trends in the figures, a parameter or a method to measure the closeness of the linear model to the nonlinear model is required. MUAD (Maximum Unnoticeable Added Dynamics) boundaries [Tischler and Remple, 2012] are widely used for that purpose. MUAD boundaries are the errors bounds defined for the difference (error) between the magnitude and the phase responses of the linear and nonlinear models. In Fig. 8, MUAD boundaries and the frequency domain errors (magnitude and phase errors) between the nonlinear and linear models are presented. Note that if

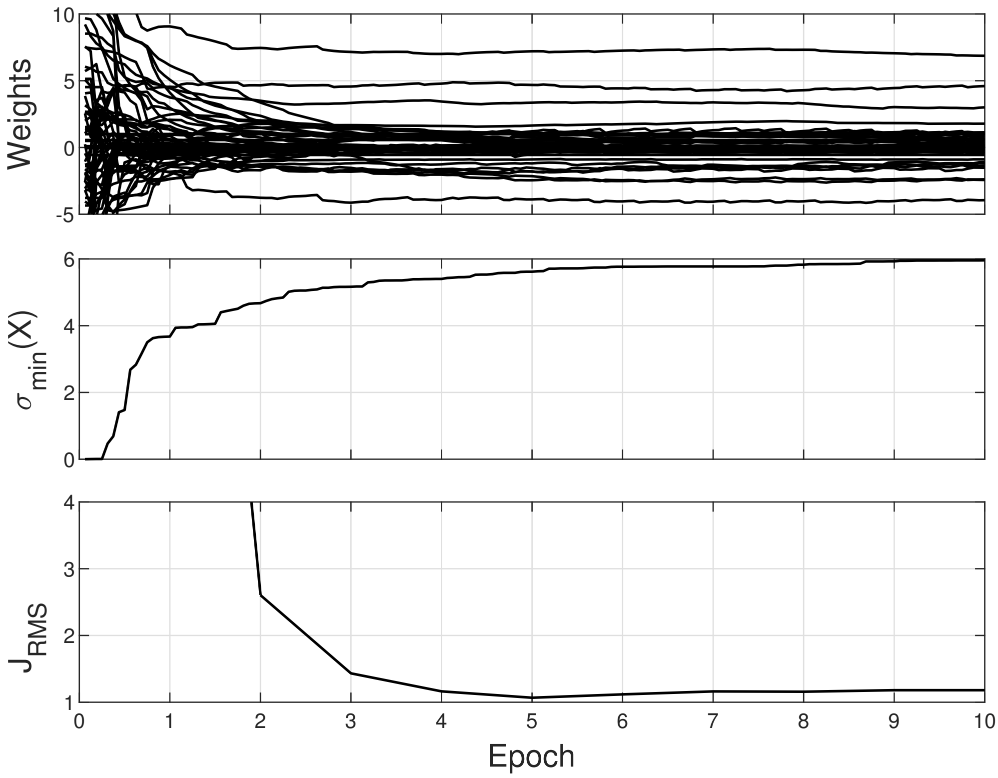

Figure 2: Evolution of Adaptive Weights around Hover 3211-Type Inputs, Non-zero Initial Condition

the difference between the frequency response of a dynamic model and the actual aircraft is within the MUAD bounds, it would be difficult for the pilot to notice a considerable difference from the real aircraft. Therefore, MUAD bounds are widely used to validate a linear model in the frequency domain [Tischler and Remple, 2012]. As seen In Fig. 8, the transfer function pairs such as q/δ_{lat} and p/δ_{long} have non-zero errors in the phase plots. Phase errors are out of the MUAD bounds for the frequency ranges between 3-10 rad/sec. Therefore, the linear models are not good enough to capture the off-axis dynamics, and improvements in the linear models are required for those transfer function pairs. The other response couples are within the bounds and indicate good fits in the frequency domain. The mismatch in the off-axis response is due to the higher-order dynamics not considered in the 6DoF linear model structure. That can be alleviated by including higher-order rotor states [Tischler and Remple, 2012] in the linear model or adding time delays [Jategaonkar, 2015] in the linear model structure. In Ref. [Gursoy, Aslandogan and Yavrucuk, 2021], time delays represent the higher-order dynamics in the linear model; therefore, a better match in the frequency domain is demonstrated. Since it is out of the scope, those improvements are not included in this paper. The reader may refer to Ref. [Gursoy, Aslandogan and Yavrucuk, 2021] to further improve the linear models.

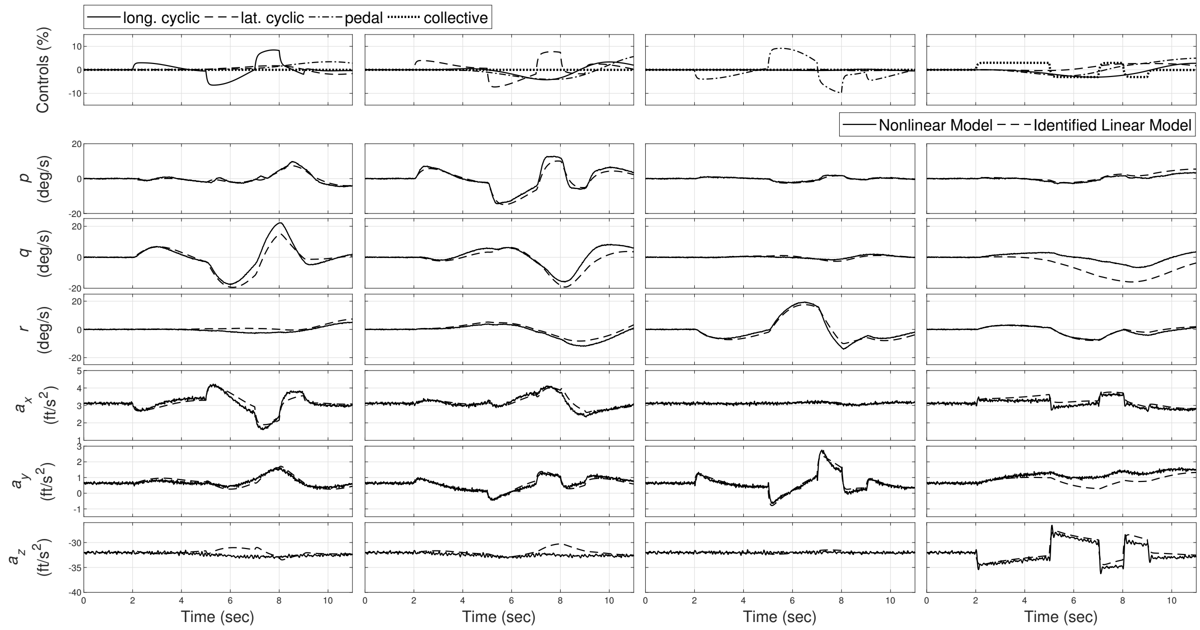

Figure 3: Time Domain Validation of States and Specific Accelerations

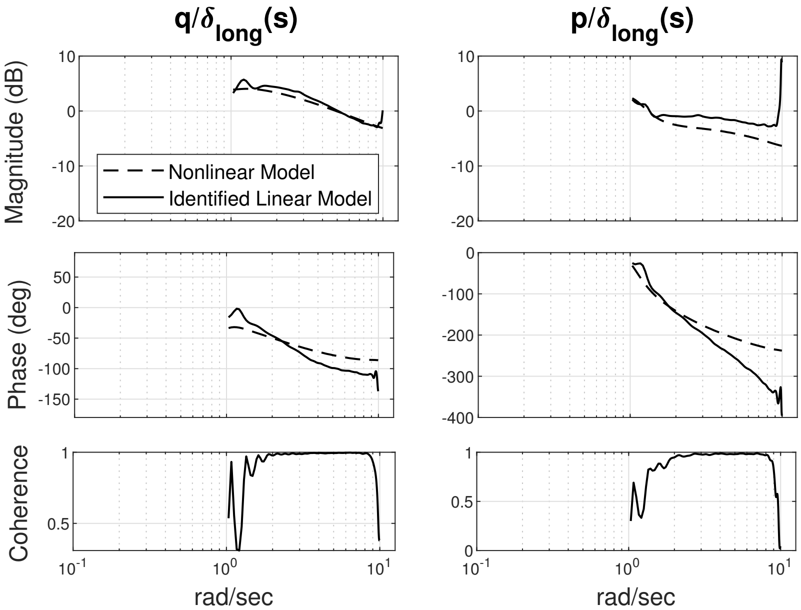

Figure 4: Frequency Domain Comparison of Longitudinal Cyclic Input

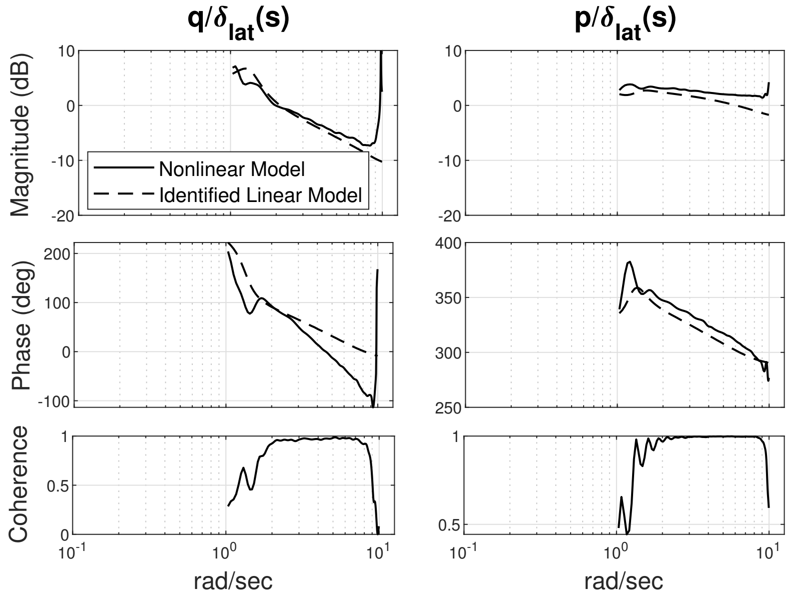

Figure 5: Frequency Domain Comparison of of Lateral Cyclic Input

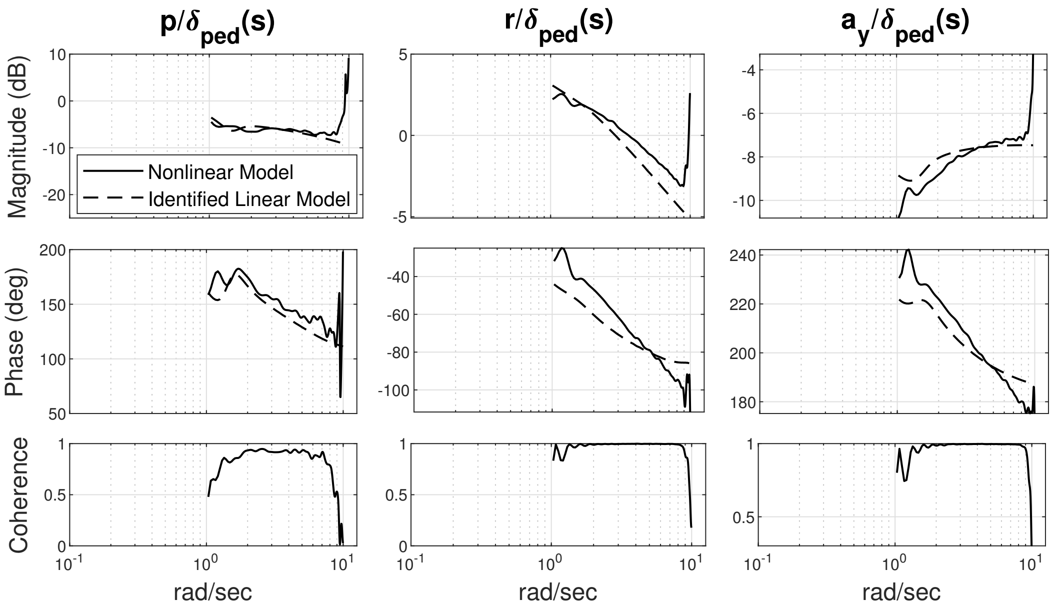

Figure 6: Frequency Domain Comparison of Pedal Input

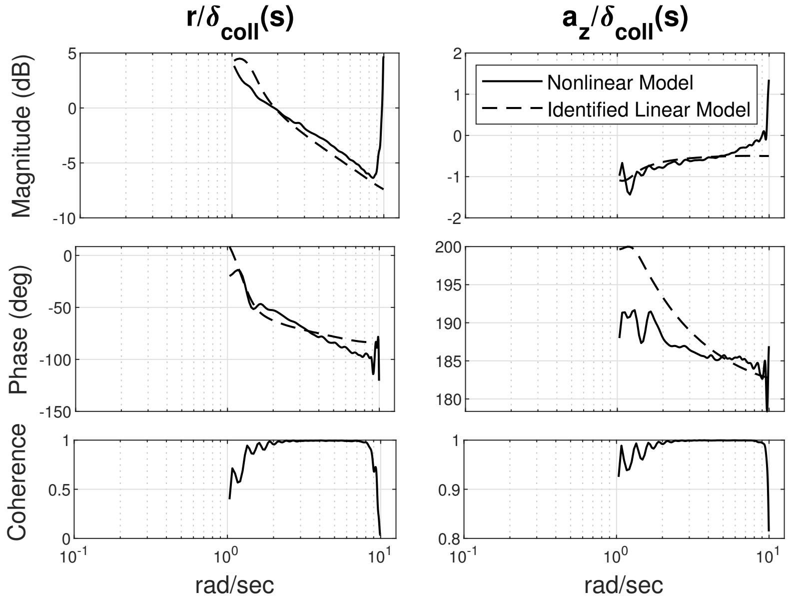

Figure 7: Frequency Domain Comparison of Collective Input

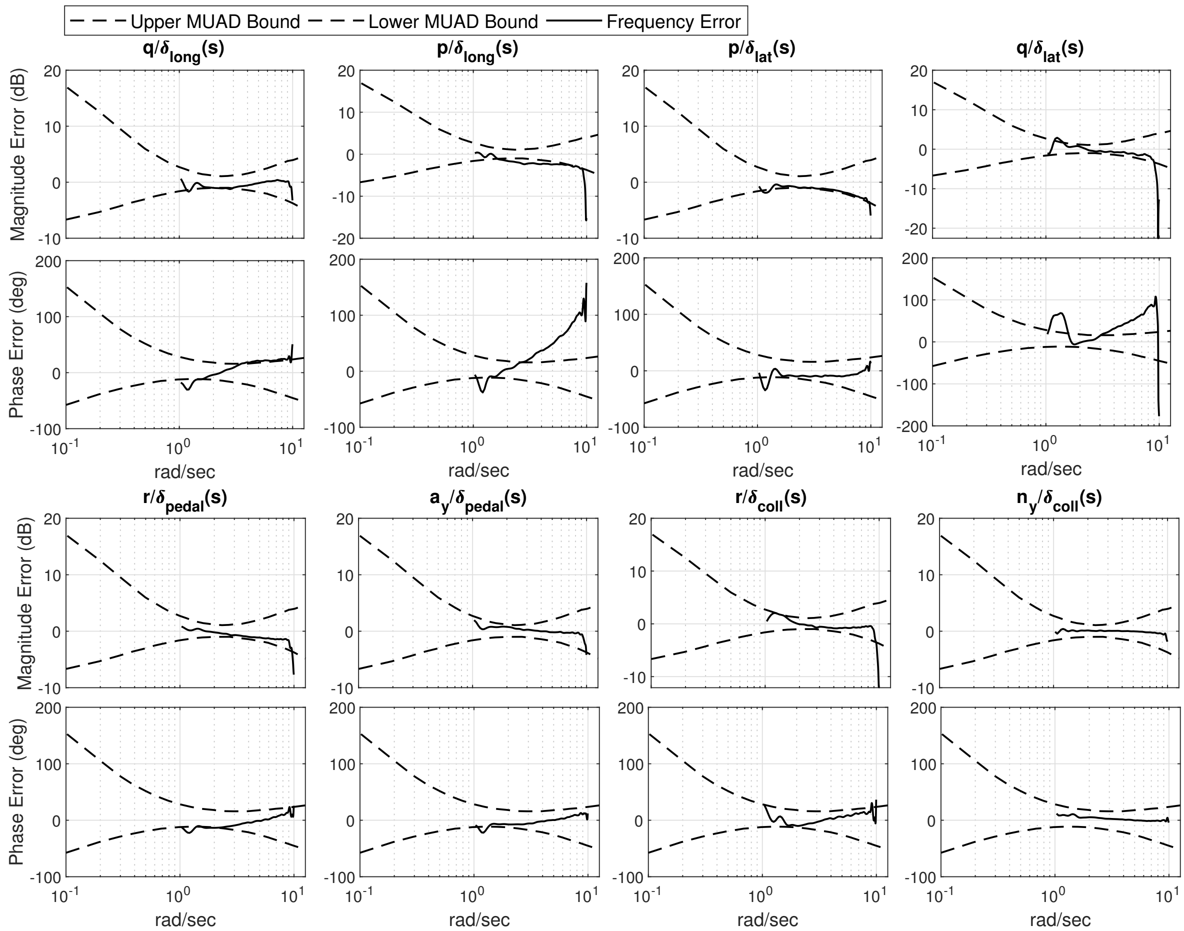

Figure 8: Frequency Domain Errors and MUAD boundaries for All Input Axes

CONCLUSION

In this paper, adaptive learning methods are applied to identify the stability and control derivatives of a nonlinear helicopter model in hover. The following conclusion can be made:

1. Bounded convergence in estimating the stability and control derivatives of a 6-Dof linear model is shown to be possible with the proposed approach. Convergence is independent of the initial condition selection; therefore, a robust initialization of the system identification problem is possible, even when the derivatives are not eliminated from the linear model structure. The method is expected to minimize the effort given for the proper initialization of the system identification problem when used as a start-up algorithm.

2. 6-Dof linear models are not sufficient to represent the higher order dynamics of the rotors. Therefore, when the method is used for rotorcraft system identification, additional states in the identification models or other procedures might be required to improve the off-axis response of the linear identified models.

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