11th ANKARA INTERNATIONAL AEROSPACE CONFERENCE   **AIAC-2021-132**
8-10 September 2021 - METU, Ankara TURKEY

NURBS BASED OPTIMIZATION OF VARIABLE STIFNESS COMPOSITE STRUCTURES

Kaan Çimen[1] and Altan Kayran[2]
Middle East Technical University, Aerospace Engineering Department
Ankara, Turkey

ABSTRACT

Variable stiffness composite structure concept is studied since 1972, however this concept is stepped into a new age after the automated fiber placement manufacturing machines are started to be used in industry. In this study a three dimensional (3D) variable stiffness composite structure optimization method is presented. The main difference of the current study from previous studies is the definition of the reference lay-up path by Non-uniform Rational B Splines (NURBS) in 3D. The methodology is based on defining a reference path by NURBS and optimizing the NURBS parameters by using Particle Swarm Optimization (PSO) method to minimize the strain energy of the 3D composite structure, which is selected as the objective function in the current study. The initial results show that reference lay-up path defined by NURBS is successfully optimized such that the strain energy of the final design is minimized.

INTRODUCTION

Due to their superior mechanical properties with respect to their unit weight, composite structures are widely used in aerospace industry as well as in other areas.   Automatization of the manufacturing process allowed the composite structures to step into a new age [Lukaszewicz et al., 2012]. Besides the manufacturing speed, AFP (Automated Fiber Placement) enables to manufacture variable stiffness composite (VSC) structures [Wu & Gurdal, 2001].

A composite material is made from strong fibers placed within a resin structure resulting in a material called prepreg. A material is produced from prepreg by placing multiple layers of the material on top of each other with different fiber angles, then heat processing the multi layered semi product in autoclaves. The most time consuming and expensive part of the production is the layup process. Automation of this process increases the production speed and quality as well as allowing more complex geometries to be produced from prepreg [Lukaszewicz et al., 2012]. By AFP machines, manufacturing of VSC structures are easier.

The first study on VSC with computer based automatized lay-up system is made by Cooper in 1972 and a computer-aided laying system for analogue fiber lay-up is proposed [Cooper, 1972]. Following this study, curvilinear fiber format and VSC studies are started for structural efficiency of composite structures[Hyer & Charette, 1989; Hyer & Lee, 1991]. By optimizing the lay-up path of the composite structure, observable performance increments, in terms of mechanical properties can be achieved [Yamanaka et al., 2016]. Also, in aerospace structures weight and drag can be reduced and aerodynamic ability/flight performance can be increased

[1] Kaan ÇİMEN, MSc. Student in METU Aerospace Department, Email: cimen.kaan@metu.edu.tr
[2] Altan Kayran, Professor in Aerospace Engineering Department, METU, Email: akayran@metu.edu.tr

[Zhao, 2017]. Experimental validation of the performance increments in the variable stiffness composite is solid. In a typical study, it is showed by the experiments that strength in a plate with a hole is increased by 34.4% [Matsuzaki et al., 2021].

To manufacture VSC structures the reference lay-up path should be defined and the composite lay-up is done with respect to this reference path. One of the path definition method is proposed in 1993. In this study, the lay-up path is defined as a smooth antisymmetric function of the x coordinate and it passes through the origin of the cartesian coordinate frame [Gürdal & Olmedo, 1993]. By this method, defining a path on a flat surface is very efficient and easy to use [Waldhart, 1996]. Defining the reference path based on the smooth antisymmetric function of the x coordinate is used to structurally optimize flat plate composite structures [Nagendra et al., 1995]. Another method is defining the reference lay-up path as a Non-uniform Rational B Spline (NURBS)[Nagendra et al., 1995]. Due to its flexibility and superior control capability of the spline, by changing few parameters, NURBS are used in commercial design tools [Nguyen, 2011].

Based on the previous research background presented above, the focus of this paper is developing a structural optimization method and tool for the curved composite structures by defining the lay-up path on a curved 3D structure. Defining a path on the curved structure has not been studied in-depth yet. To define the reference path on a curve surface, NURBS based Rhinoceros software [Rhinoceros, 2020] is used. Optimization process consists of several parts. Structural modelling, structural analysis, and design variable optimization with respect to analysis results, are the main parts. To model the geometry, MSC.PATRAN finite element modeler is used, and as the finite element solver MSC.NASTRAN finite element analysis program is used. Optimization of the NURBS parameters, modelling the VSC structure and finite element analysis of the model is automatized by MATLAB. With the developed tool, reference layup path can be defined and optimized directly on the designed curved structure as a spline without converting it onto a flat plate.

METHOD

In the current optimization process, one of the main differences from previous studies is the definition of the reference fiber path in 3D by NURBS and optimization of NURBS control points by Particle Swarm Optimization (PSO) to determine the optimum fiber path.

Non-uniform Rational B Spline (NURBS)

In this study, the NURBS concept is used to define the fiber path. Firstly, an initial reference fiber path guess is made, and it is drawn in Rhinoceros. This path is constructed as a NURBS by Rhinoceros.
NURBS is defined with three parameters. They are the weight (w_i), knots (u), and control points (P_i) which are used to construct the control polygon. NURBS curve ($C(u)$) is defined in Eq.(1),

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,P}(u)w_i}{\sum_{i=0}^{n} N_{i,p}(u)w_i} \, P_i \quad a < u < b \tag{1}$$

where "a"and "b" are the limit values of the knot vector and $N_{i,P}(u)$ is the ith basis function and t "a" and "b"are taken as 0 and 1 unless otherwise specified [Piegl & Tiller, 1997.] .The rational part in Eqn. (1) can be written as,

$$R_{i,p} = \frac{\sum_{i=0}^{n} N_{i,P}(u)w_i}{\sum_{i=0}^{n} N_{i,p}(u)w_i} \tag{2}$$

So, the NURBS formulation can be rewritten as,

$$C(u) = R_{i,p}(u)\boldsymbol{P}_i \tag{3}$$

The ith basis functions of a NURBS of pth degree is defined by Eqns. 4 and 5.

$$N_{i,0} = \left\{ \begin{array}{ll} 1 & if\ u_i \leq u \leq u_{i+1} \\ 0 & otherwise \end{array} \right\} \tag{4}$$

$$N_{i,p} = \frac{u - u_i}{u_{i+P} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+P+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{5}$$
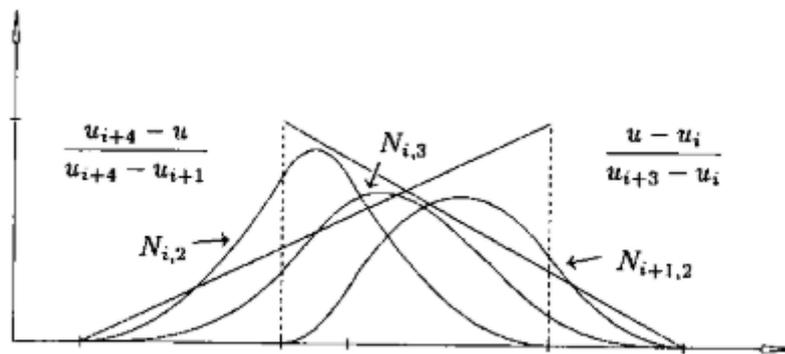
As an example, basis functions of the 4th degree NURBS curve is shown in Fig.1.

Figure 1: The recursive definition of B spline basis functions of the p=4th degree curve [Piegl & Tiller,1997.]

The knot vector is a non-decreasing vector (i.e. $u_i < u_{i+1}$) and it consist of knot values $\boldsymbol{U} = \{u_0,..,u_m\}$.The main parameter that affects the shape of the NURBS is control points (fig. 2). Weight and knots can also change the shape of the NURBS as shown in Figs.3-4. By modifying the knot vector significant shape changes can be achieved [Hoffmann & Juhasz, 2001].
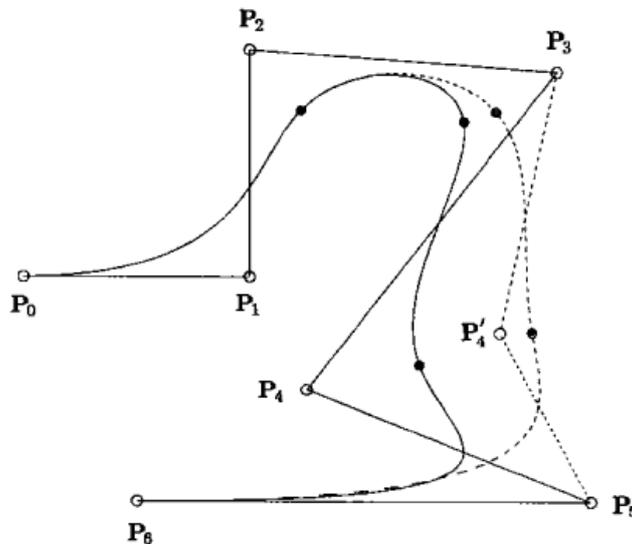

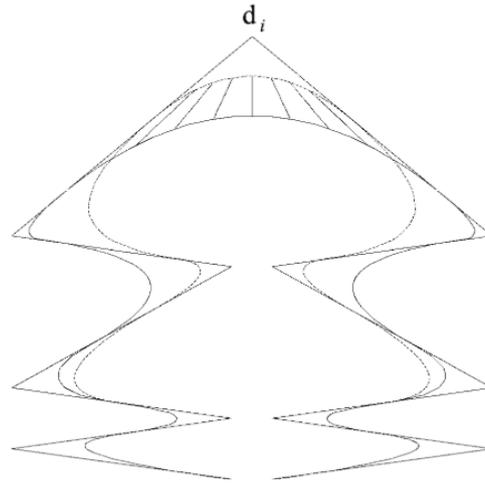Figure 2: Splines with two different P4 control points [Piegl & Tiller,1997.]

Figure 3: Splines With Two Different Knot Values In Possible NURBS Envelope [Hoffmann & Juhasz, 2001]
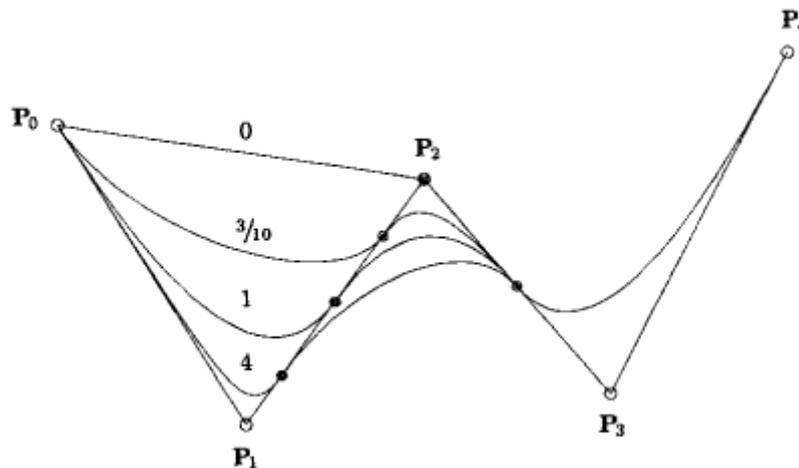

Figure 4: Splines with different weights for the same control point [Piegl & Tiller,1997.]

Particle Swarm Optimization (PSO)

In this study, Particle Swarm Optimization (PSO) is used as the optimizer. PSO is introduced by Kennedy & Eberhart in 1995. PSO is a swarm intelligence method that is inspired by social behavior and movement dynamics of insects, birds and fish [Talukder, 2011]. In a swarm, each individual behaves not only based on their intelligence/experience, but also on the intelligence/experience of the group. Thus, each individual observes others' behaviour and act either based on its own or on groups' intelligence/experience or combination of these two, to achieve their goal [Arora, 2012].

PSO is a gradient-less stochastic search method that can be applied to the continuous variable problems. It is very similar to Genetic Algorithm (GA), however it has some advantages compared to the GA. PSO is easier to implement, more effective and efficient [Parsopoulos,

2016]. The group does not have negative effect on the progress of evaluation during iteration. Also, PSO does not have evolution operations and has memory [Güldü, 2014].

In PSO there are n number of particles which iteratively change their location in the search space to find best objective function solution. The number of particles is a design parameter of the PSO algorithm and identified by the user. At each particle's location, objective function is evaluated and the value of the objective function is a potential solution of the optimization problem [Arora, 2012; Parsopoulos, 2016]. In each iteration, based on the particle's best value yet and the global best achieved,the location of each particle is improved towards better a location until the termination of the algorithm. This is done by Eqns. (6) and (7).

$$v_{k+1}^i = v_k^i + c_1 r_1 \left(p_k^i - x_k^i \right) + c_2 r_2 \left(p_k^g - x_k^i \right) \tag{6}$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{7}$$

where,

x_k^i - Particle position
v_k^i - Particle velocity
p_k^i - Best "remembered" individual particle position
p_k^g - Best "remembered" swarm position
c_1, c_2 - Cognitive and social parameters
r_1, r_2 - random numbers from uniform distribution between 0 and 1

The objective function of the optimization problem is defined as;

Minimize the objective function, $f(x)$

$$f(x) = strain\ energy\ of\ the\ system \tag{8}$$

such that radius of curvature of the NURBS curve (fiber path) is greater than the allowable radius of curvature R in mm. In Eqn. (8), x is control points .The minimum radius of curvature (R) constraint is a manufacturing constraint based on the allowable radius of curvature depending on the tow width of the prepreg[Güldü, 2014]. The curvature constraint (K) of the NURBS ($C(u)$) is defined as;

$$radius\ of\ curvature = K = \left[\frac{|C'(u)\ X\ C''(u)|}{|C'(u)|^3} \right]^{-1} < R$$

$$\tag{9}$$

Since the problem is a constrained optimization problem and the basic algorithm of PSO does not consider the constraints, penalty function has to be introduced to the objective function to handle the constraints. The aim of the penalty function is penalize all solutions when the objective function solution become infeasible [Parsopoulos, 2016] . Equation (10) defines how the objective function is penalized.

$$f(x) = F(x) + P(x) \tag{10}$$

where F($\boldsymbol{x}$) is the objective function and P($\boldsymbol{x}$) is the penalty function.

$$P(\boldsymbol{x}) = \begin{cases} A > 0, & K \geq R \; mm \\ 0, & otherwise \end{cases} \tag{11}$$

where "A " is very large value to penalize the objective function solution [Parsopoulos, 2016].

Flow Chart of the Optimization Process

The flow chart of the developed structural optimization algorithm is given in Figure 5. After creating an initial reference NURBS path in Rhino CAD software as seen in Fig.6, the parameters of the NURBS are read from the saved step file of the 3D data via MATLAB. By using these parameters, the same NURBS is recreated in MATLAB. Figure 6 gives a sample NURBS generated on a flat surface both by Rhino and MATLAB.

The design geometry, mesh and boundary conditions are modeled in MSC. PATRAN. The initial fiber angles of the finite elements are taken as zero degree in the finite element model. After creating the finite element model, input file of the finite element software Nastran is created, and this input file is read by MATLAB and the center of each element is determined by the interface code generated in MATLAB.

Figure 7 shows a sample NURBS generated on a curved surface. Implementation of the reference NURBS path to the finite element model is done by assigning fiber angles to each element. To do that, corresponding fiber angles at the element centers on the reference NURBS path, are determined. Fiber angles are then assigned to the corresponding finite elements in the model by shifting the reference fiber path along the y direction, as shown in Fig.8.

Following the generation of an initial model, PSO algorithm creates n number of particles which are the control points (design variables) in this problem. Particles are initialized randomly, except one of the n number of particles. This particular particle is initialized utilizing the reference path's control points. The other n-1 particles are randomly initialized.

PSO algorithm then improves the design variables toward the best objective function value. Design variables are limited by the geometrical constraints of the part.
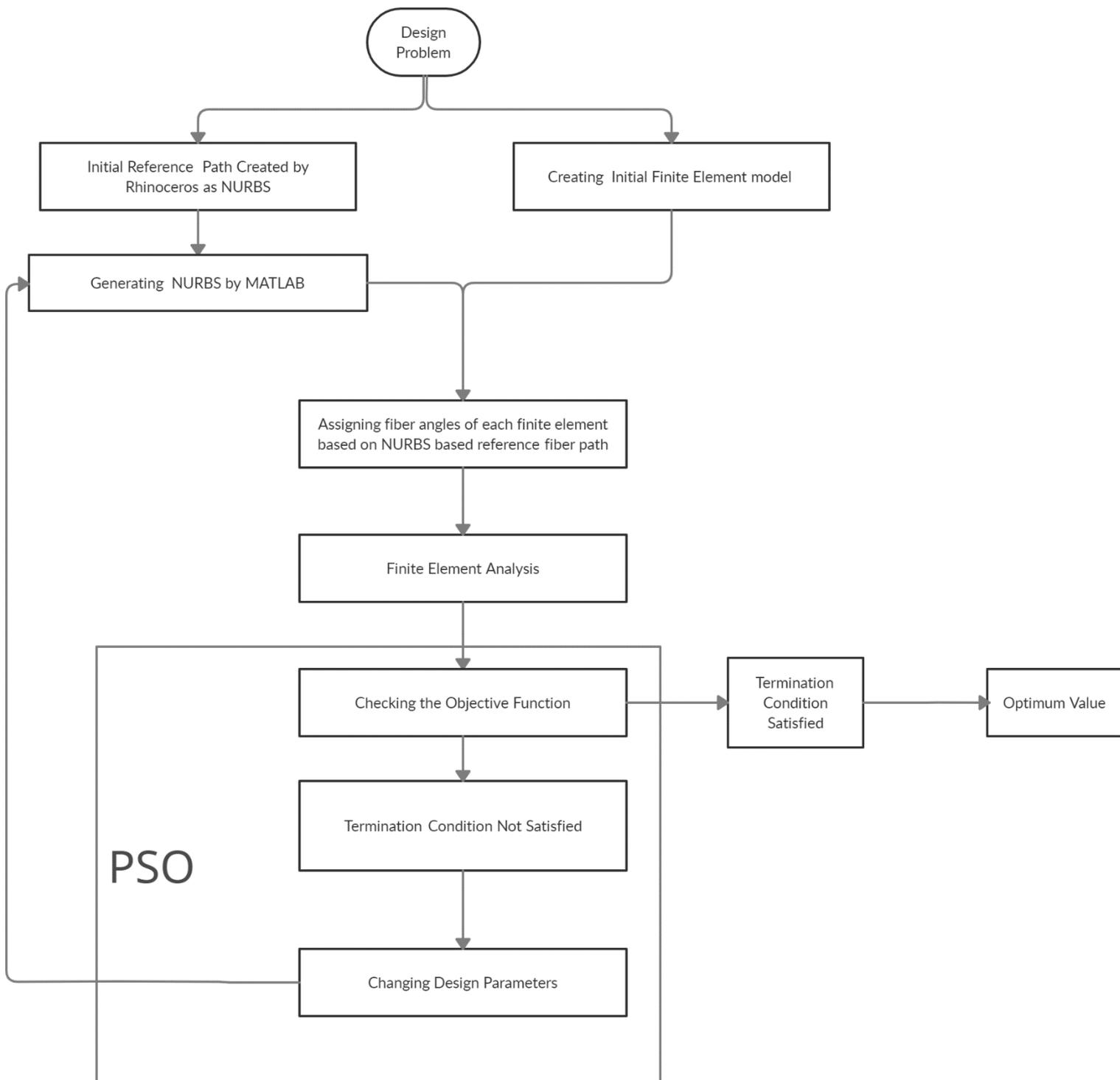
Figure 5: Flow chart of the optimization algorithm of VSC by NURBS based reference fiber path
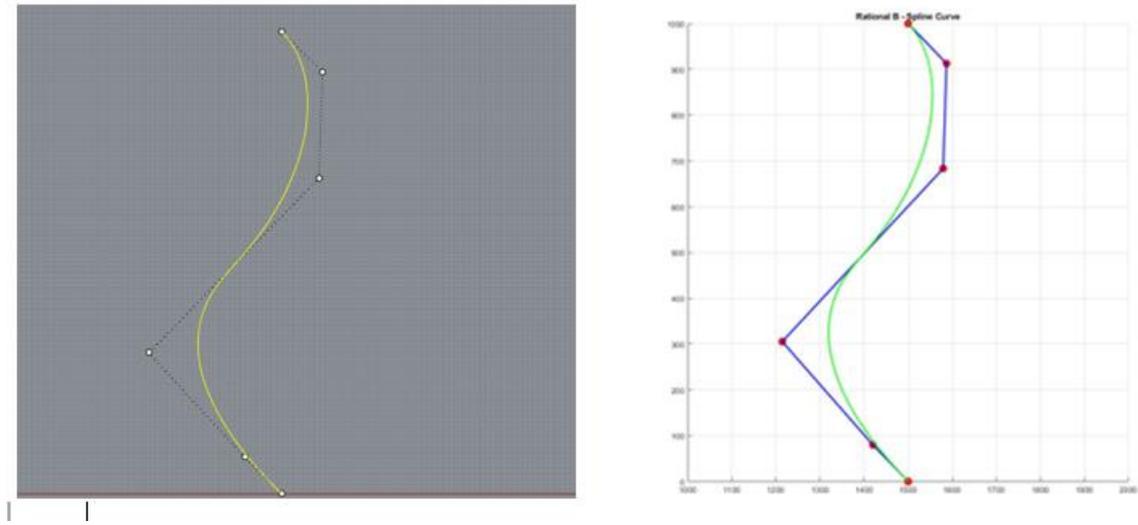
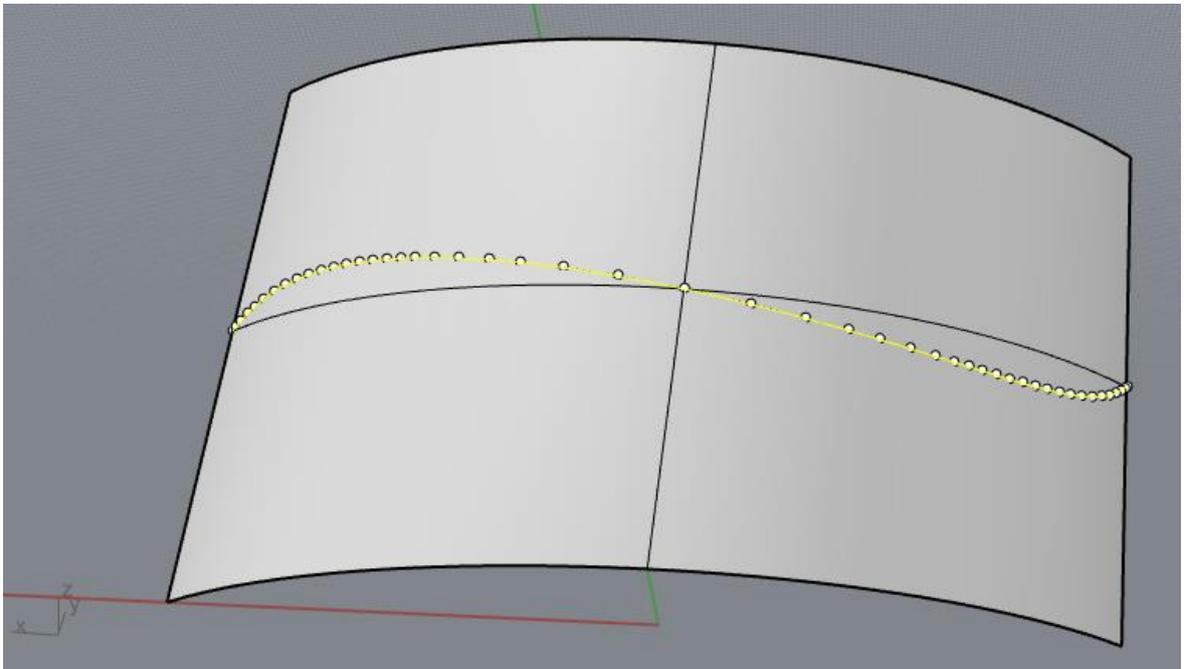Figure 6: Reference Path for flat plate in Rhino and Matlab
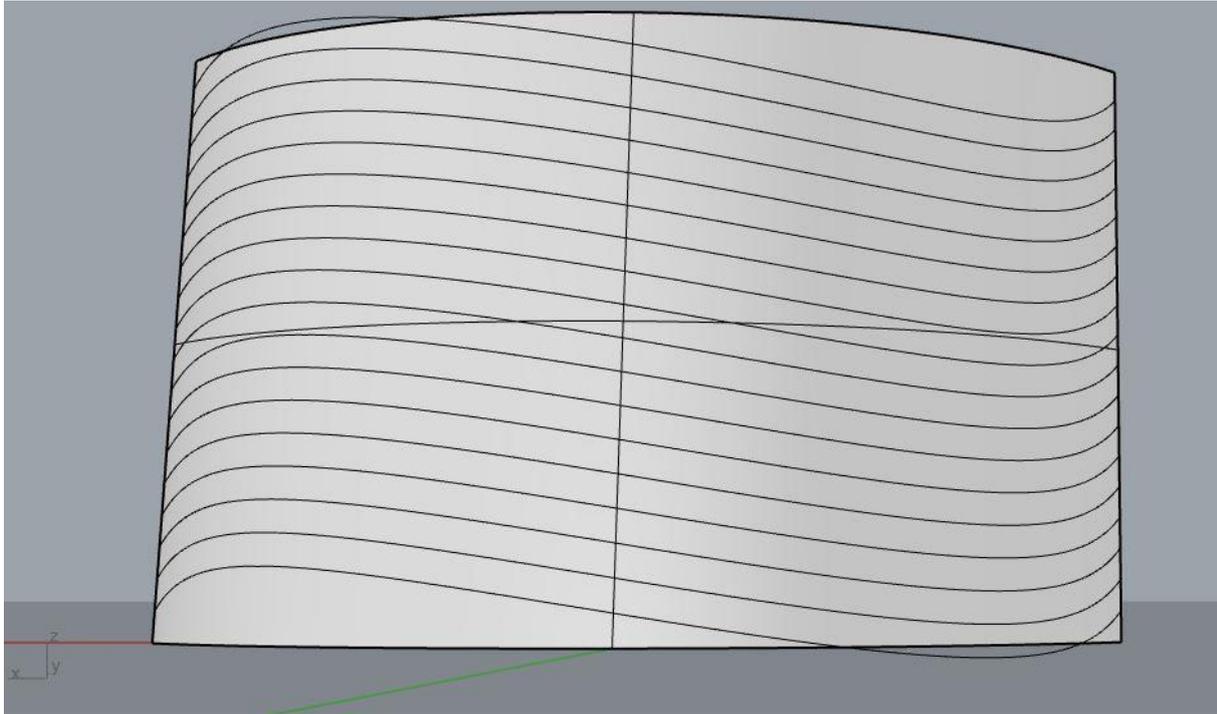

Figure 7: Reference path on a curved surface

Figure 8: Shifted reference path along y direction of curved surface

RESULTS AND DISCUSSION

Based on the method describe above, fiber path optimization for a flat plate and a curved surface is done and results are presented below. The plate has one layer, and it is made from unidirectional carbon fiber reinforced epoxy (Inci, 2012). The material properties and thickness of composite panels are:

E_1=134 GPa E_2=16.75 GPa G_{12}=5.8625 GPa $γ_{12}$=0.3   t=200 mm

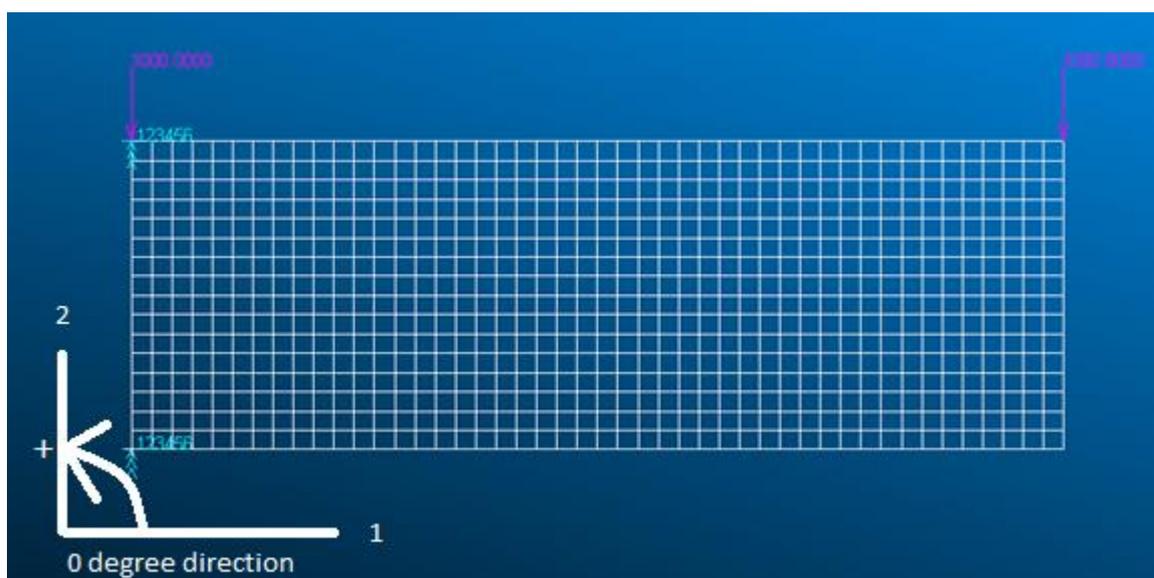**Control point optimization of the flat composite plate**

Figure 9: Cantilevered flat plate with aspect ratio=3 and the loading case

The cantilevered flat plate has an aspect ratio of three (3000mm x1000 mm). The cantilever beam is subjected to a uniformly distributed load of 1000 N/m along the top edge of the plate, as shown in Fig 9. For the cantilever flat plate, control point optimization minimizes the strain energy, so this is a minimum compliance problem.
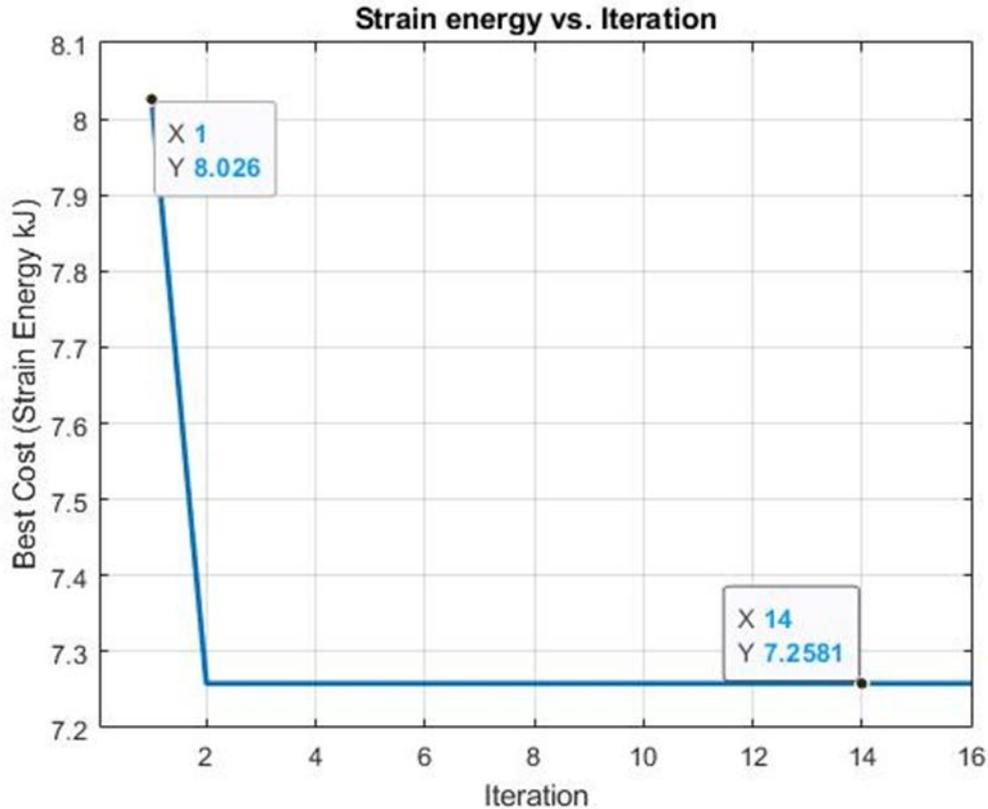

Figure 10: Control point optimization of cantilevered flat plate

Figure 10 shows that control point optimization of the reference fiber path results in minimization of the total strain energy. Figure 11 shows randomly created initial fiber path. It is seen that for the randomly created initial fiber path's total strain energy is 8.026 kJ. After the optimization, total strain energy decreases to the optimum value of 7.26 kJ. The final optimum fiber path in Fig. 12 is very close to the 0-degree fiber angle. It is known that the constant angled carbon fiber orientation 0 degree is the best layup angle for described loading(Inci, 2012). A separate analysis has been done for the flat plate with 0 degree orientation by Nastran and analysis results are consistent with the optimization results. Strain energy for the 0-degree orientation is also about 7.26 kJ. This shows that the optimization algorithm works fine for the cantilever beams control point optimization. The strain energy is minimized 18% with respect to the initial path guess.

The random initial fiber path's control points, knot vector and weight are as follows.

$$CP = \begin{bmatrix} 0 & 500 & 1000 & 1500 & 2000 & 2500 & 3000 \\ 500 & 750 & 750 & 500 & 250 & 250 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$U = [0\ 0\ 0\ 0\ 0.250\ 0.500\ 0.750\ 1\ 1\ 1\ 1]$$

$$w = 1$$

The final optimized path's control points, knot vector and weight are as follows.

$$CP = \begin{bmatrix} 0 & 500 & 1000 & 1500 & 2000 & 2500 & 3000 \\ 500 & 498.01 & 274.9 & 539.45 & 362.42 & 352.81 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

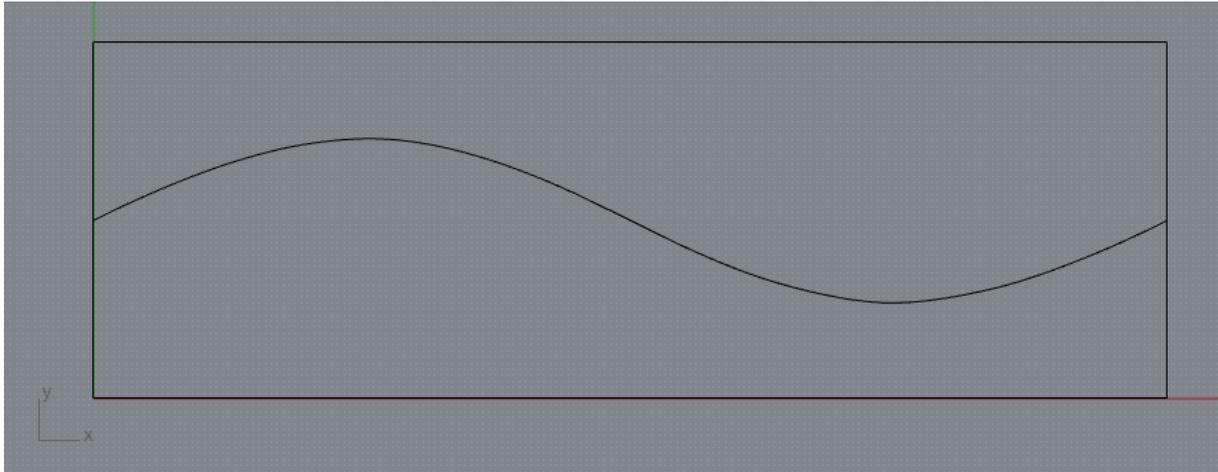$$U = [0\ 0\ 0\ 0\ 0.250\ 0.500\ 0.750\ 1\ 1\ 1\ 1]$$

$$w = 1$$

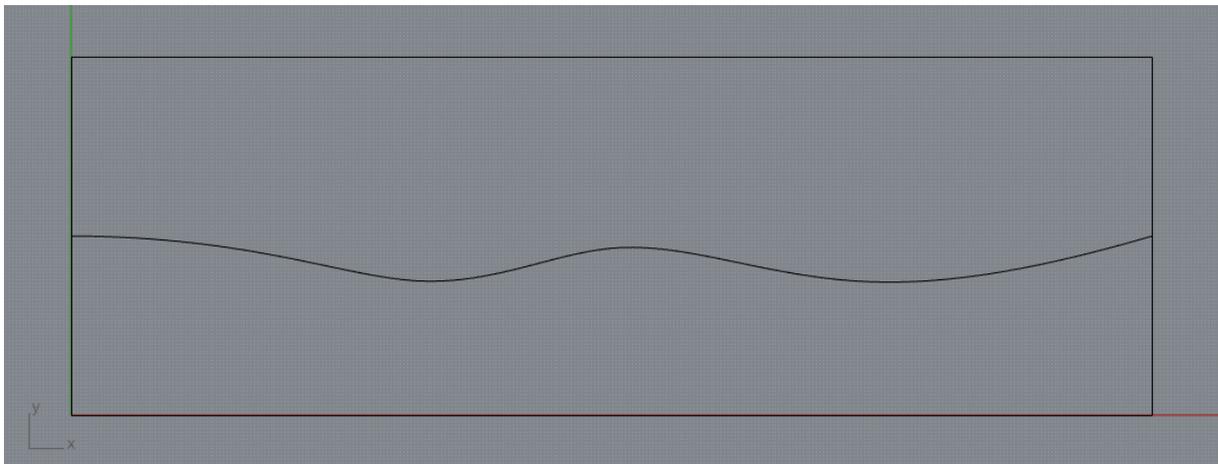Figure 11: Initial random path of cantilever beam

Figure 12: Optimum path of cantilever beam

Control point optimization of the curved composite panel

Curved composite panel optimized is shown in Fig. 13. As shown in Fig. 13, curved panel has the dimensions 1600 x1000X R1350 mm. The panel is subjected to a uniformly distributed load of 1000 N/m along the shorter edge of the panel, as shown in Fig. 14. Initial fiber path randomly defined for the lay up for the optimization is shown in Fig. 15.
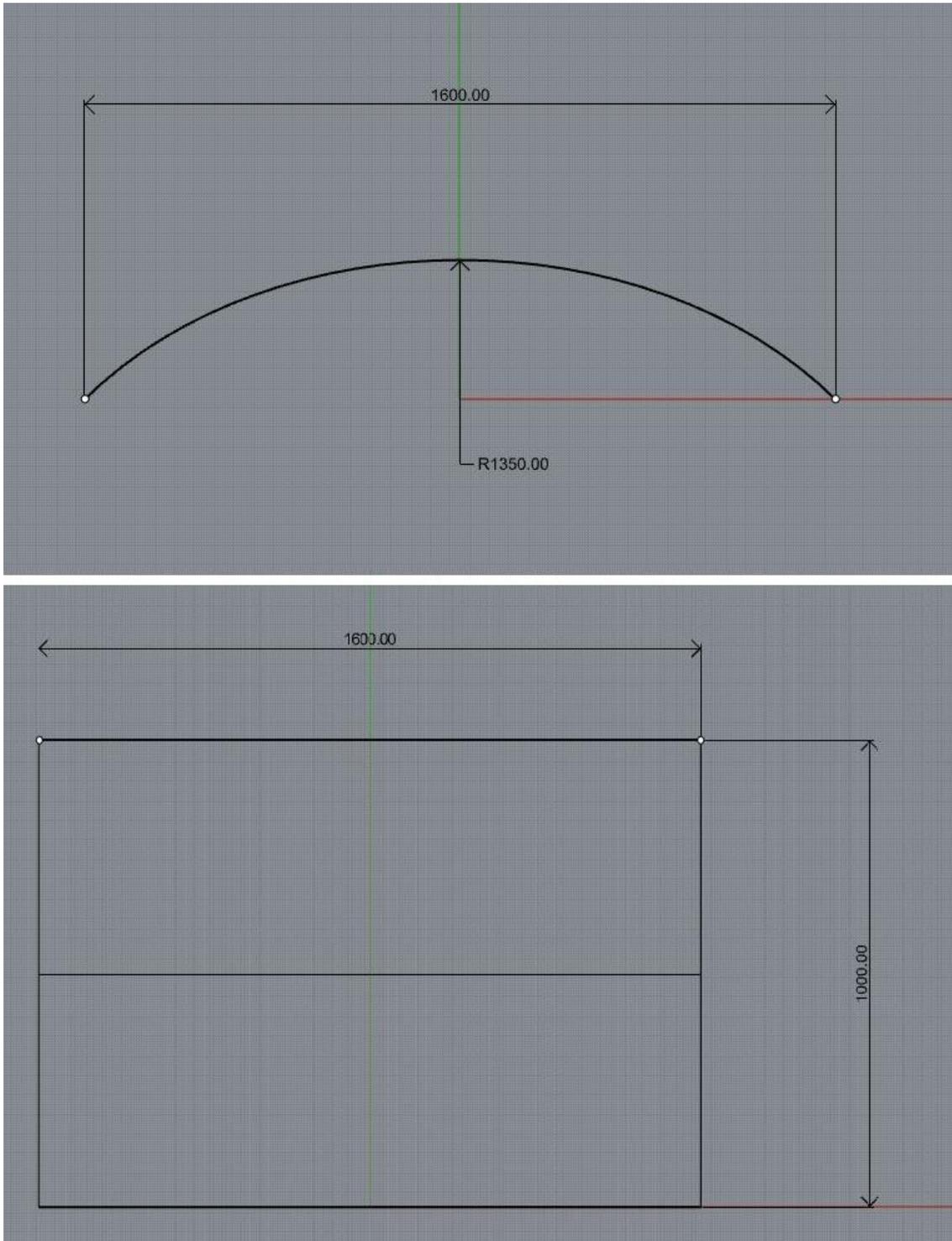
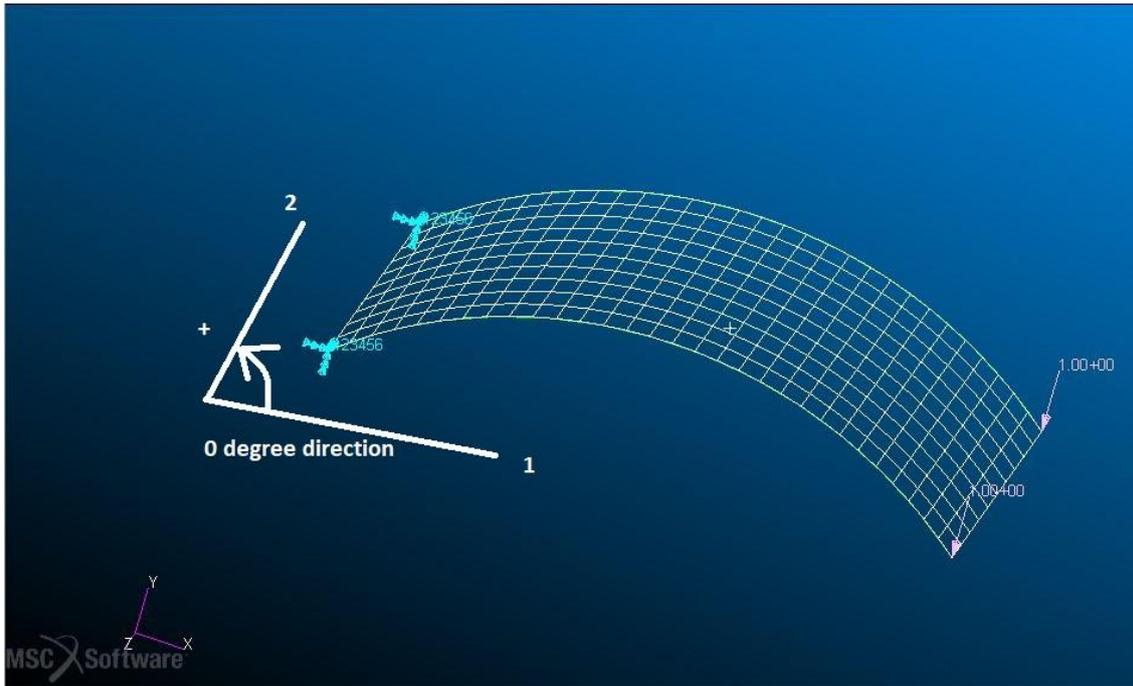Figure 13: Dimensions of the curved panel

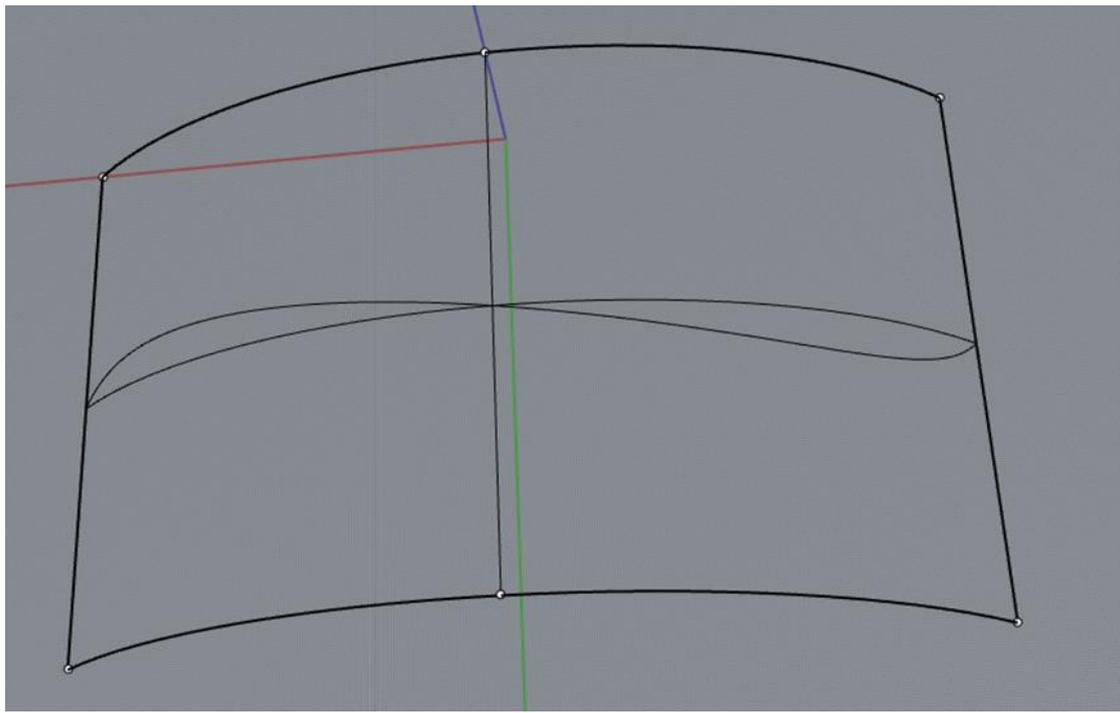Figure 14: Load applied to the curved composite panel

Figure 15: Initial random fiber path on the curved composite panel

Figure 16 shows the result of control point optimization of the curved composite panel. It is seen that, for the initial reference fiber path, for the edge loading described in Fig. 14, total strain energy is 22.05 kJ. As a result of optimization, total strain energy decreases to the optimum value of 8.43 kJ. The optimized reference fiber path in Fig. 17 is very close to the constant fiber orientation 0-degree just like the 2D cantilevered flat plate. For this curved panel,

again the 0-degree fiber angle is the best fiber orientation for described loading for minimum compliance. A separate finite element analysis has been done for the 0-degree fiber orientation case and it is found that total strain energy for the 0-degree fiber orientation case is about 8.43 kJ. This shows that the optimization algorithm works fine for the control point optimization of the curved panel.

The random initial fiber path's control points, knot vector and weight are as follows.

$$CP = \begin{bmatrix} -800 & -774.86 & -689.84 & -466.24 & -60.47 & 363.89 & 659.88 & 764.40 & 80 \\ 0 & 25.31 & 100.37 & 231.26 & 330.94 & 274.90 & 123 & 36.07 & 0 \\ 500 & 468.01 & 424.30 & 427.59 & 487.75 & 562.94 & 578.87 & 544.62 & 500 \end{bmatrix}$$

$$U = [0\ 0\ 0\ 0\ 0.0781\ 0.2030\ 0.4375\ 0.7658\ 0.8907\ 1\ 1\ 1\ 1]$$

$$w = 1$$

The final optimized path's control points, knot vector and weight are as follows.

$$CP = \begin{bmatrix} -800 & -774.86 & -689.84 & -466.24 & -60.47 & 363.89 & 659.88 & 764.40 & 80 \\ 0 & 25.31 & 100.37 & 231.26 & 330.94 & 274.90 & 123 & 36.07 & 0 \\ 500 & 517.31 & 455.80 & 637.63 & 346.29 & 569.61 & 5785.78 & 439.79 & 500 \end{bmatrix}$$

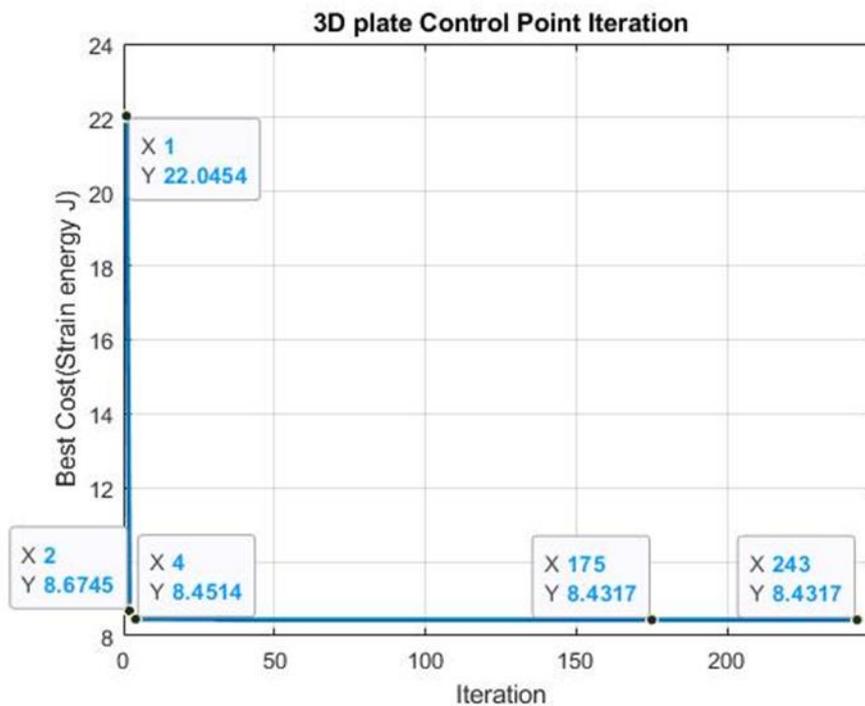$$U = [0\ 0\ 0\ 0\ 0.0781\ 0.2030\ 0.4375\ 0.7658\ 0.8907\ 1\ 1\ 1\ 1]$$

$$w = 1$$

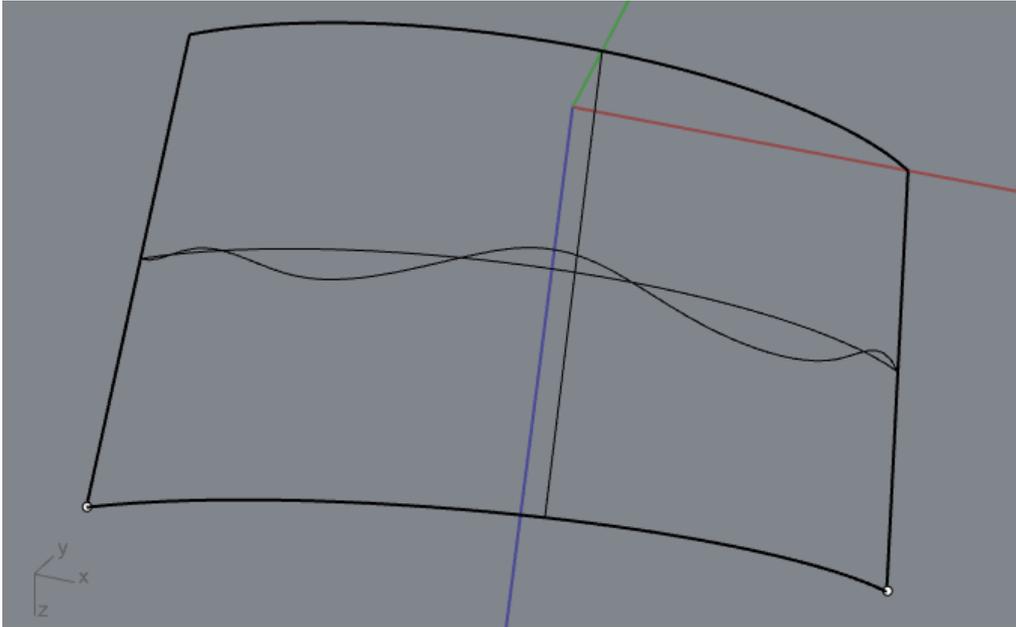Figure 16: Curved Structure Control point optimization of the curved composite panel

Figure 17: Optimized fiber path on curved composite panel

CONCLUSION

Structural performance improvements of composite structures can be achieved by optimizing the fiber path in a curvilinear fashion. Such composite structures have variable stiffness and have better performance under loading conditions. Defining a reference path on a curved composite structure can achieved by using NURBS. Optimization of the NURBS parameters, according to a desired goal is a useful method for the optimization of variable stiffness composites, since NURBS allows the selection of alternative design parameters such as the knots, control points and weights. In this study a method to optimize curved composite structures is presented. The reference fiber path of the variable stiffness composite is defined by NURBS and by using PSO, NURBS control points are optimized to achieve the minimization of strain energy. The results showed that a significant improvement in the minimization of the strain energy can be achieved.

As for the future work, reference fiber path optimization utilizing other NURBS variables; knot vector and weight is planned. Finally, knot vector, weight and control point optimization can be combined to achieve better results. Moreover, optimization can be performed for the prepreg tow placement by handling the gap and overlap issue.

References

Arora, J. (2012). Introduction to Optimum Design. In *Introduction to Optimum Design*. https://doi.org/10.1016/C2009-0-61700-1

Cooper, A. A. G. (1972). *Trajectorial Fiber Reinforcement of Composite Structures*.

GÜLDÜ, S. (2014). *Analysis and Optimization Of Cylindrical Structures Manufactured By Automated Fiber Placement Technique*.

GURDAL, Z., & OLMEDO, R. (1993). In-plane response of laminates with spatially varying fiber orientations - Variable stiffness concept. *AIAA Journal, 31*(4), 751–758. https://doi.org/10.2514/3.11613

Hoffmann, M., & Juhasz, I. (2001). Shape control of cubic B-spline and NURBS curves by knot modifications. *Proceedings Fifth International Conference on Information Visualisation*, 63–68. https://doi.org/10.1109/IV.2001.942040

Hyer, M. W., & Charette, R. F. (1989). *INNOVATIVE DESIGN OF THE COMPOSITE USE OF DESIGN FORMAT STRUCTURES : CURVILINEAR IN COMPOSITE STRUCTURE DESIGN*.

Hyer, M. W., & Lee, H. H. (1991). The use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes. *Composite Structures, 18*(3), 239–261. https://doi.org/10.1016/0263-8223(91)90035-W

Lukaszewicz, D. H. J. A., Ward, C., & Potter, K. D. (2012). The engineering aspects of automated prepreg layup: History, present and future. *Composites Part B: Engineering, 43*(3), 997–1009. https://doi.org/10.1016/j.compositesb.2011.12.003

Matsuzaki, R., Mitsui, K., Hirano, Y., Todoroki, A., & Suzuki, Y. (2021). Optimization of curvilinear fiber orientation of composite plates and its experimental validation. *Composite Structures, 255*(July 2020), 112956. https://doi.org/10.1016/j.compstruct.2020.112956

Nagendra, S., Kodiyalam, S., Davis, J., & Parthasarathy, V. (1995, April 10). Optimization of tow fiber paths for composite design. *36th Structures, Structural Dynamics and Materials Conference*. https://doi.org/10.2514/6.1995-1275

Nguyen, T. N. (2011). *Isogeometric Finite Element Analysis based on Bézier Extraction of NURBS and T-Splines*.

Parsopoulos, K. E. (2016). Handbook of Heuristics. In *Handbook of Heuristics*. https://doi.org/10.1007/978-3-319-07153-4

Piegl, L., & Tiller, W. (n.d.). *The NURBS Book*. Springer.

Rhinoceros (No. 6). (2020). Robert McNeel & Associates.

Talukder, S. (2011). *Mathematical Modelling and Applications of Particle Swarm Optimization*.

Waldhart, C. (1996). *Analysis of Tow-Placed, Variable-Stiffness Laminates* [Virginia Tech]. https://vtechworks.lib.vt.edu/handle/10919/36781

Wu, K. C., & Gurdalf, Z. (2001). Thermal testing of Tow-Placed, variable stiffness panels. *19th AIAA Applied Aerodynamics Conference, April*. https://doi.org/10.2514/6.2001-1190

Yamanaka, Y., Todoroki, A., Ueda, M., Hirano, Y., & Matsuzaki, R. (2016). Fiber Line Optimization in Single Ply for 3D Printed Composites. *Open Journal of Composite Materials, 06*(04), 121–131. https://doi.org/10.4236/ojcm.2016.64012

Zhao, W. (2017). *Optimal Design and Analysis of Bio-inspired, Curvilinearly Stiffened Composite Flexible Wings*.