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PARAMETER ESTIMATION OF A MAGNETORQUER BY USING THE DIFFERENTIAL CORRECTOR

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ABSTRACT

A method to define the maximum current and the time constant parameters which can not exactly be known for a Magnetic Torque Rod (Magnetorquer) that is used for angular momentum management, detumbling and reaction wheel desaturation on Low Earth Orbit (LEO) satellites was proposed. In order to apply an improved modulation technique efficiently to the magnetorquer, it is crucial to know these uncertain parameters. Because taking these parameters constant can avoid the magnetorquer to produce the required torque to control the attitude of the satellite. The differential corrector method was used to estimate these parameters. Results show that the proposed algorithm can estimate the maximum current and the time constant parameters with high accuracy.

INTRODUCTION

Magnetic Torque Rods as a part of Attitude Determination and Control System (ADCS) [Amin, J., Lightsey, E., G., 2019] are used in Low Earth Orbit (LEO) Satellites for angular momentum management, detumbling and reaction wheel desaturation [Desouky, M., A., A., Abdelkhalik, O., Gauchia, L., 2020]. Besides this, as advanced level usage, they can be used also for attitude control in certain cases [Desouky, M., A., A., Abdelkhalik, O., 2019].

Basically, a Magnetic Torque Rod consists of a serial resistance-inductance (RL) circuit with a time constant [Mehrjardi, M., F., Mirshams, M., 2010]. In order to apply the required torque to the satellite, the magnetic torque rod is supplied with the input voltage and the magnetic dipole moment produced by the current flowing through the magnetorquer coils interacts with the Earth's magnetic field [Jan, Y., W., Tsai, J., R., 2012] which is highly variable for typical orbits [Gulmammadov, F., 2015].

Magnetic Torque Rods are driven by an H-Bridge circuit in the interface board and commonly by the Pulse Width Modulation (PWM) technique. A modulation algorithm is used to calculate the firing duration of the magnetorquer in order to produce the magnetic dipole moment demanded by the controller. Because of the fact that classical PWM techniques neglects the exponentially convergent behavior of the RL circuit, improved modulation techniques such as shark fin modulation which takes into account the exponentially convergent behavior can be

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used [Gulmammadov, F., Sozen, N., 2019]. In these improved modulation techniques, in order to understand the RL characteristics better, it is important to know some uncertain magnetorquer parameters such as the maximum current and the time constant more accurately [Psiaki, M., L., 2005]. For that reason, an estimation process can be designed and applied.

In this paper, the differential corrector method was used to estimate the maximum current and the time constant parameters. The motivation for this work comes from the previous satellite projects of TUBITAK Space Technologies Research Institute that used classic modulation technique.

In first section, the parameter estimation method is explained and the following results section gives the estimation results.

METHOD

A magnetorquer is an inductive load which can be modelled as a serial RL circuit. Assuming the inductance (L) and resistance (R) do not change or change slowly with time, the current flowing through the magnetorquer is given by the solution of the following first order differential equation.

$$V(t) = I(t)R + L\frac{d_i(t)}{dt} \tag{1}$$

The solution of (1) for the current is given as:

$$I(t) = I_{\max}\left(1 - e^{-t/\tau}\right) \tag{2}$$

Where $I_{\max} = V/R$ and $\tau = L/R$ which is the time constant.

The magnetic dipole moment generated by the magnetorquer is given by the following formula:

$$m(t) = nI(t)A \tag{3}$$

Where n is the number of the turns of wire, $I(t)$ is the current provided and A is the vector area of the coil. Putting (2) into (3) we get the following equation for the magnetic dipole moment.

$$m(t) = I_{\max}\left(1 - e^{-t/\tau}\right) \tag{4}$$

Where $m_{\max} = nI_{\max}A$.

In a satellite control or momentum management problem, the magnetorquer is driven by Pulse Width Modulation (PWM) and the controller output is an effective magnetic dipole moment m_f requested from the magnetorquer. The m_f produced by PWM corresponds to the duty cycle of the PWM multiplied by the maximum magnetic dipole moment that magnetorquer can achieve.

$$m_f = m_{\max}\text{duty\_cycle}_{pwm} \tag{5}$$

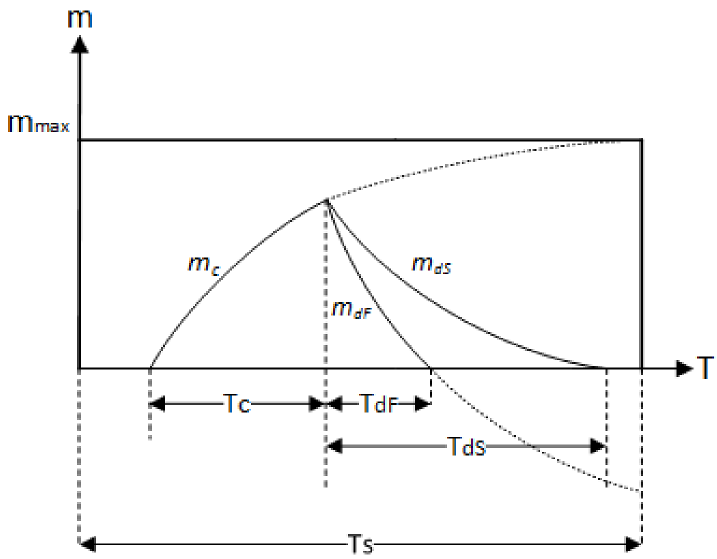

Figure 1: Charge-discharge curve of a Magnetorquer

In Figure 1, T_c is the charge duration, T_{ds} is discharge duration and the T_s is the sampling period. $m_c(t)$ and $m_{ds}(t)$ are given by the following equations:

$$m_c(t) = m_{\max}\left(1 - e^{-t/\tau}\right) \tag{6}$$

$$m_{ds}(t) = m_{\max}\left(1 - e^{-T_c/\tau}\right)e^{-t/\tau} \tag{7}$$

The duty cycle is the ratio between the area created during $T_c + T_{ds}$ and the total area during T_s. So the effective magnetic dipole moment m_f can be given as:

$$m_f = m_{\max}\frac{\int_0^{T_c} m_c(t)dt + \int_0^{T_{ds}} m_{ds}(t)dt}{T_s m_{\max}} \tag{8}$$

Here we can define the firing duration T_f as:

$$T_f = \frac{\int_0^{T_c} m_c(t)dt + \int_0^{T_{ds}} m_{ds}(t)dt}{m_{\max}} \tag{9}$$

And using the firing duration T_f, the effective magnetic dipole moment m_f can be calculated as:

$$m_f = m_{\max}\frac{T_f}{T_s} \tag{10}$$

When the controller requests an effective magnetic dipole moment m_f from the magnetorquer which is driven by PWM one needs to calculate the required charge T_c and discharge T_{ds} durations. Charge and discharge durations can be calculated using the following formulas where their derivations can be found in [Gulmammadov, F., Sozen, N., 2019].

$$T_c = \frac{m_f T_s}{m_{\max}} + \tau\frac{m_{zero}}{m_{\max}} \tag{11}$$

$$T_{ds} = \tau \ln\left[\frac{m_{zero}}{m_{\max}}\left(1-e^{-T_c/\tau}\right)\right] \tag{12}$$

Here m_{zero} is a design parameter which can be determined at the beginning. As it can be seen from equations (11) and (12) above, in order to find the required charge and discharge durations one needs to know $I_{\max}$ (where $m_{\max} = nI_{\max}A$) and τ values which are the parameters of the RL circuit that is used to model the magnetorquer.

In order to find the characteristics of the magnetorquer by means of $I_{\max}$ and τ, samples from the charge equation $m_c(t)$ can be used. During a randomly determined charge duration which is greater than τ $(T_c > \tau)$ 2 or more magnetic dipole moment values sampled from $m_c(t)$ can be used to estimate the $I_{\max}$ and τ.

In order to estimate the parameters differential corrector method can be used. The differential corrector is a numerical algorithm that solves problems in the form;

$$y = f(x) \tag{13}$$

In this problem y is m_c values sampled from $m_c(t)$ at different sampling times (t) and $x = [I_{\max}, \tau]$ are the parameters that needs to be estimated.

$$y = f(x,t) = m_c(t) = m_{\max}\left(1-e^{-t/\tau}\right) \tag{14}$$

In order to find a solution for the equation above using samples, a Taylor series expansion of $f(x,t)$ is taken to first order about the initial values x_0;

$$y = f(x,t) = f(x_0,t) + J_0(x - x_0) \tag{15}$$

$$x = x_0 + J_0^{-1}\left(y - f(x_0,t)\right) \tag{16}$$

Where J is the Jacobian matrix, t is the sampling time and y is the sample.

Using equation above the Differential Corrector algorithm can be given iteratively as;

$$x_{k+1} = x_k + J_k^{-1}\left(y - f(x_k,t)\right) \tag{17}$$

and repeated until the error $y - f(x_{k+1},t)$ is under tolerance.

RESULTS

In the simulation we assume the resistance (R) circuit is changed slowly by time. After every $t = 7\tau n$ sampling period $(where, n = 1.....N)$ a R value is changed using the formula $210 + 60 * \sin(2\pi t / 6000)$. $V = 28$ V, $L = 25$ H and $R = 210$ Ohm are given initially yielding $I_{\max} = V/R = 0.1333$ A and $\tau = L/R = 0.119$ (sec). After every 7τ period, a new R value is calculated using the formula so RL circuit have a new characteristic (having a new $I_{\max}$ and τ value).

During every 7τ period noisy samples are taken from charge equation ($m_c(t)$) at randomly determined sampling moments which is greater than τ. 2 and 3 samples are taken at sampling moments $T_c/2, T_c/4$ and $T_c, T_c/2, T_c/4$ where $T_c > \tau$. Using these samples, $I_{\max}$ and τ values are estimated for every 7τ period. The estimated $I_{\max}$ and τ values for the current sampling period are used as the initial conditions for the next sampling period.

In the first sampling period, estimation is initialized with $I_{max\,initial} = 1.1 * I_{max}$ and $\tau_{initial} = 1.1 * \tau$ values. If there are no initial guess about the I_{max} and τ below approximation can be used to initialize the parameters.

$$\frac{m_c\left(T_c/2\right)}{m_c\left(T_c/4\right)} = \frac{m_{max}\left(1-e^{-(T_c/2)/\tau}\right)}{m_{max}\left(1-e^{-(T_c/4)/\tau}\right)} \cong \frac{\left[1-\left(1-\frac{T_c}{2\tau}+\frac{T_c^2}{8\tau^2}-\frac{T_c^3}{48\tau^3}\right)\right]}{\left[1-\left(1-\frac{T_c}{4\tau}+\frac{T_c^2}{32\tau^2}-\frac{T_c^3}{384\tau^3}\right)\right]} \tag{18}$$

$$F(\tau)=\tau^3\left[6m_c\left(\frac{T_c}{2}\right)\frac{T_c}{4}-6m_c\left(\frac{T_c}{4}\right)\frac{T_c}{2}\right]-\tau^2\left[3m_c\left(\frac{T_c}{2}\right)\frac{T_c^2}{16}-3m_c\left(\frac{T_c}{4}\right)\frac{T_c^2}{4}\right]+\tau\left[m_c\left(\frac{T_c}{2}\right)\frac{T_c^3}{64}-m_c\left(\frac{T_c}{4}\right)\frac{T_c^3}{8}\right]=0 \tag{19}$$

$$\tau_{initial} = abs\left(\text{max\_root}\left(F(\tau)\right)\right) \tag{20}$$

$$I_{max\,initial} = \frac{1}{N}\sum\nolimits_{i=1}^{N}\frac{m_c\left(t_i\right)}{n*A*\left(1-e^{-t_i/\tau_{initial}}\right)}$$, N = 2 or 3 and t_i is the i^th sampling time.

For Jacobian simplicity, I_{max} and $1/\tau$ are estimated using the Differential Corrector. Zero mean Gaussian white noise having a standard deviation as $I_{max}/2048$ are added to samples.

At every sampling period (totally N = 5000 sampling periods), estimations are done. After a sample Monte Carlo run, estimation errors for I_{max} and τ values are given in Figure 2 and Figure 3. Change of the true parameter values can be seen from Figure 4. As can be seen from Figure 2 and Figure 3, the estimation accuracy increases as the number of samples increases.

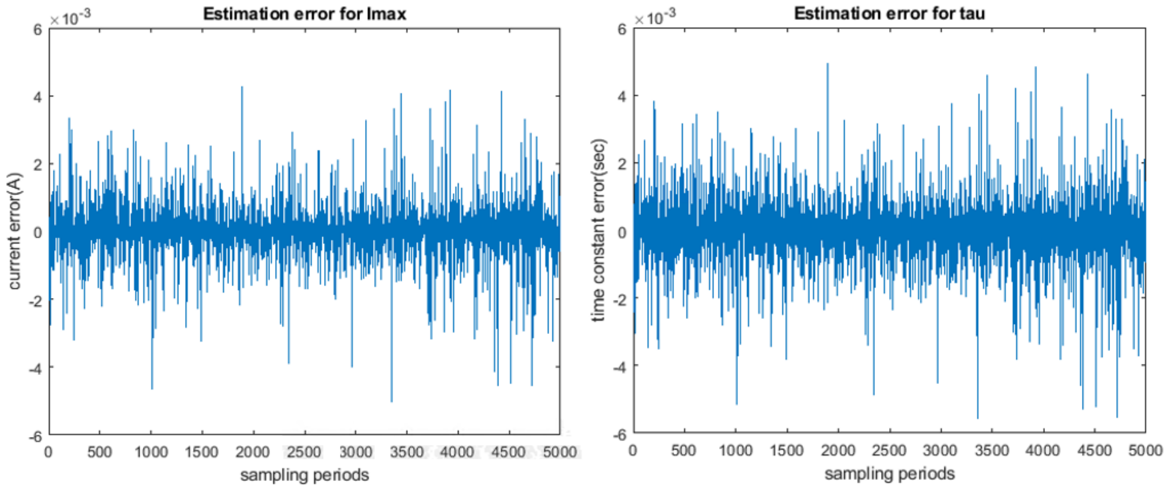

Figure 2: Magnetorquer parameter estimation error for I_{max} (left) and for τ (right) for 2 samples case

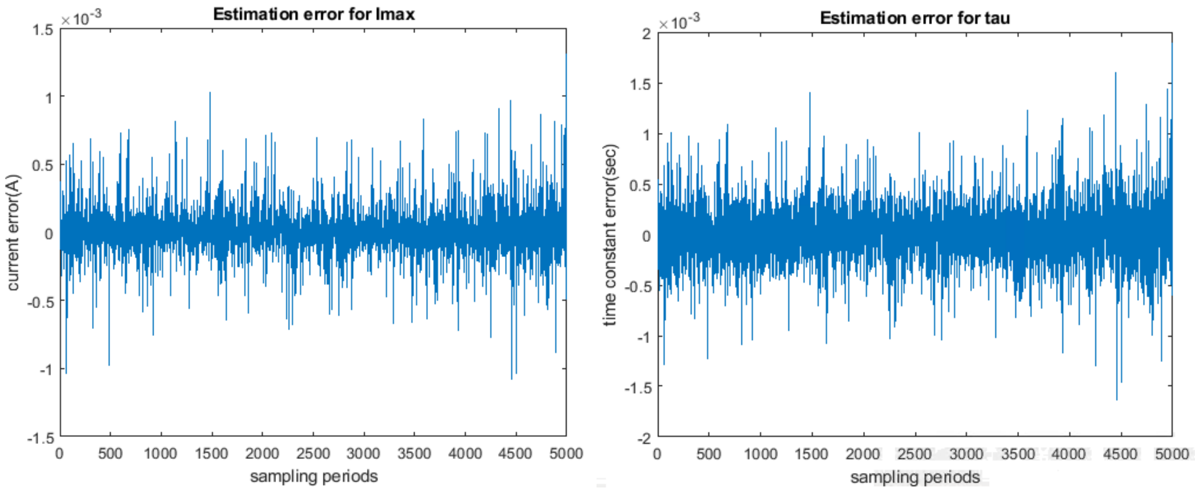

Figure 3: Magnetorquer parameter estimation error for I_{max} (left) and for τ (right) for 3 samples case

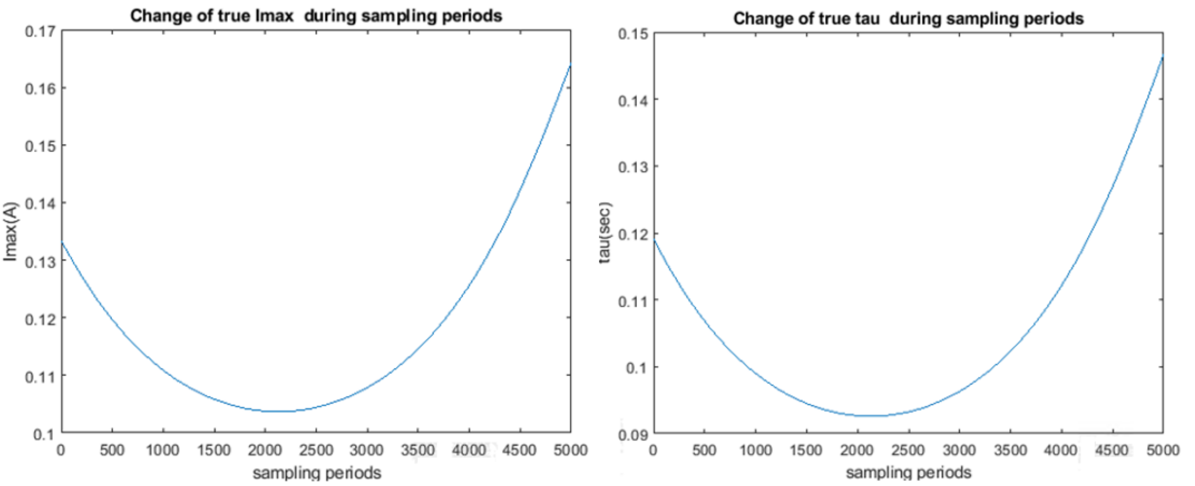

Figure 4: Change of true parameter values during sampling periods

CONCLUSION

A simple algorithm for magnetorquer parameter estimation is proposed. Results showed that the algorithm is capable of accurately estimating the parameters even when we can sample only two magnetic dipole moment values.

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