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System Identification of a Fighter Aircraft Using Frequency Domain Methods

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ABSTRACT

Longitudinal dynamics of F-16 aircraft in a level trim, unaccelerated flight are identified by using frequency domain system identification techniques. The nonlinear system is excited by sine sweep elevator input and angle of attack and pitch rate responses are collected. The data in time domain are converted to frequency domain by using fast Fourier transform (FFT) and corresponding transfer functions at appropriate order are fitted. Finally, dimensional derivatives are estimated based on these transfer functions and are non-dimensionalised to compare them with those of the nonlinear model. Entire work is performed in MATLAB and Simulink environment.

INTRODUCTION

System identification is a powerful process to obtain linear model of aircraft based on input-output relation extracted from flight data or simulated data and linear models are very useful to estimate the aircraft parameters directly. Aircraft parameters can be estimated either in time domain or frequency domain. Frequency domain analysis has certain advantages, namely 1) physical insight in terms of frequency content, 2) direct applicability to control system design, and 3) no risk of divergence because no numerical integration is involved. The basis for frequency domain identification method is the finite Fourier transform which transforms the time domain data into the frequency domain [Mohamed, 2014]. Once data is transformed into frequency domain, corresponding transfer function, whose order is known a priori, can be fitted by employing Sanathanan and Koerner (S-K) iteration [Sanathanan and Koerner, 1963] based algorithm given in [Ozdemir and Gumussoy , 2017]. The order of the transfer function to be fitted can be determined by considering the flight dynamics. Then, the parameters of aircraft are identified from the estimated transfer functions, which are also shown in state-space form. This study stemmed from the need of simpler approach to identify system dynamics and parameters in frequency domain without depending on any dedicated software.

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METHOD

Aircraft Model & Dynamics

Nonlinear F-16 Simulation model provided in [Russell , 2003] is used in this study. The aerodynamic database of the simulation is the wind tunnel aerodynamic data for a 16% scale model of the F-16 aircraft flying at relatively low Mach numbers (<0.6), out of ground effect, with landing gear retracted and no external stores. [Morelli , 1998] [Nguyen, L.T., et al., 1979]. The model includes elevator, thrust, aileron, rudder and leading edge flaps (LEFs) as control surfaces and actuator dynamics with position and rate limits. It is observed that the longitudinal and lateral dynamics of the aircraft are not coupled.

Procedure

The goal of this study is to identify the short-period mode of the aircraft and the short-period approximation given in [Stevens, Lewis and Johnson , 2015] is considered. The dynamics are simplified by neglecting the derivatives with respect to angle of attack rate ($\dot{\alpha}$), which are $M_{\dot{\alpha}}$ and $Z_{\dot{\alpha}}$ (1). The rationale behind the entire procedure and application is based on the reference [Tischler and Remple, 2012].

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix} = \begin{bmatrix} Z_\alpha/V_{T_e} & 1+Z_q/V_{T_e} \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix}\alpha \\ q\end{bmatrix} + \begin{bmatrix} Z_{\delta_e}/V_{T_e} \\ M_{\delta_e}\end{bmatrix} \begin{bmatrix}\delta_e\end{bmatrix} \tag{1}$$

In order to excite and identify short-period dynamics, sine sweep input is given to the elevator command (δ_{e_c}) to the aircraft which is in steady wings-level flight at 5000 ft altitude with 350 ft/s (∼0.3 M) trim airspeed (V_{T_e}). In addition, aircraft mass and dimension characteristics are given in Table-I in the reference [Nguyen, L.T., et al., 1979]. Then, actual elevator position (δ_e), angle of attack (α) and pitch rate (q) responses of the aircraft are collected. The sampling of the simulation is adjusted to 100 Hz as in real flight test and white noise is added to the (α) and (q) responses. The block diagram is presented in Figure 1. The elevator sweep command and the resultant elevator deflection from trim are given in Figure 2. The outputs are shown in Figure 3.

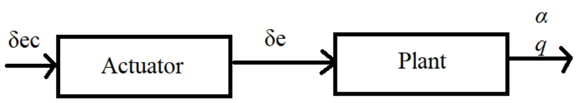

Figure 1: Block Diagram

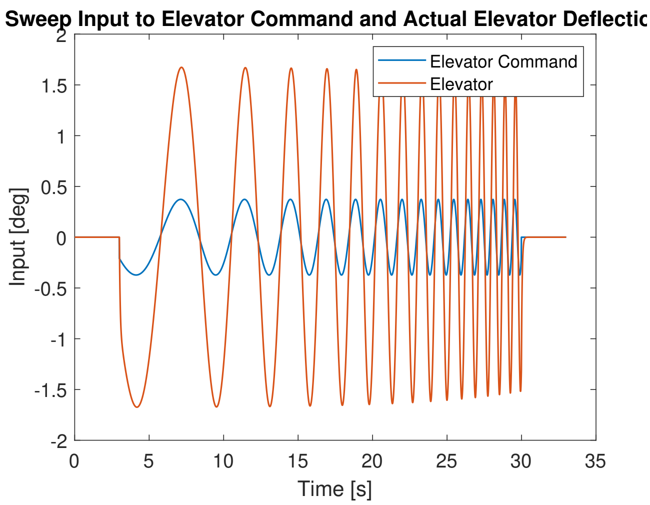

Figure 2: Sweep Input

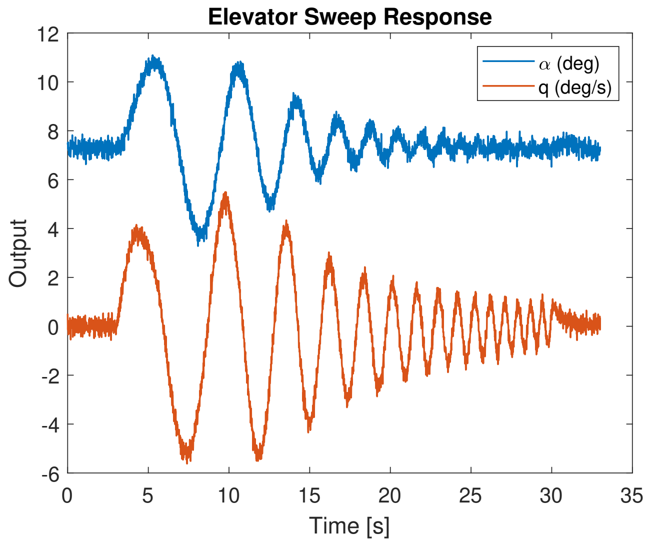

Figure 3: Outputs

The time domain input and outputs are converted into frequency domain response data by using FFT techniques.

Moving to the estimation of transfer functions based on frequency domain data, the aircraft dynamics should be considered in order to determine the order of these transfer functions, before applying any fitting process. The short-period mode can be approximated by second-order transfer functions which are in the form of:

$$\frac{\alpha}{\delta_e} = \frac{Z_{\delta_e}/V_{T_e}s + (1 + Z_q/V_{T_e})M_{\delta_e} - (M_q Z_{\delta_e}/V_{T_e})}{s^2 - (Z_\alpha/V_{T_e} + M_q)s + (M_q Z_\alpha/V_{T_e}) - (1 + Z_q/V_{T_e})M_\alpha} \tag{2}$$

$$\frac{q}{\delta_e} = \frac{M_{\delta_e}s + (M_\alpha Z_{\delta_e} - Z_\alpha M_{\delta e})/V_{T_e}}{s^2 - (Z_\alpha/V_{T_e} + M_q)s + (M_q Z_\alpha/V_{T_e}) - (1 + Z_q/V_{T_e})M_\alpha} \tag{3}$$

The transfer functions (2) and (3) are obtained from the short-period approximation given in (1). Determining the order, transfer functions that will represent the dynamics embedded in the collected frequency response data can be estimated at this stage. In addition, the frequency range where the transfer functions will be fitted is another vital aspect. Dynamics of the aircraft to be identified (short-period, phugoid, dutch-roll, ...) is the guide to determine this frequency range initially and comparison of identified model and linearised model on Bode plots is found to be helpful for final tuning of the range. Finally, the transfer functions are fitted by using the S-K iteration based algorithm explained in [Ozdemir and Gumussoy , 2017]. Original S-K iterations are performed to solve the nonlinear least squares problem, which is the minimisation of the following cost function J [MathWorks , 2012]:

$$J = \sum_{i=1}^{n_f} \left| W(\omega_i) \left(y(\omega_i) - \frac{N(\omega_i)}{D(\omega_i)} u \right) \right|^2 \tag{4}$$

In (4), n_f is the number of frequencies, W is the frequency-dependent weight function, ω is the frequency, y and u are measured output and input while N and D are the numerator and denominator of the transfer function to be estimated. This original S-K algorithm is enhanced in [Ozdemir and Gumussoy , 2017] by applying a second set of iterations for refinement and reduction of numerical errors. The enhanced version presented in [Ozdemir and Gumussoy , 2017] is employed in this study as transfer function fitting algorithm.

After completing the work in the frequency domain and fitting the transfer functions, the identified model should be verified in time domain by using a dissimilar input which is not used in identification. For this study, since response data from frequency-sweep inputs were used for the identification, data from step or multistep inputs can be used for verification as suggested in [Tischler and Remple, 2012]. Hence, for instance, doublet input can be given to the system as elevator input.

Eventually, aerodynamic derivatives are identified by matching them with the coefficients of estimated transfer functions' and are converted to nondimensional forms.

Finally, by using the estimated transfer functions, state-space representation of the system can also be written and can be directly compared with the linearised form of the nonlinear system.

RESULTS AND DISCUSSION

Sweep input is given as elevator command input and the outputs are collected as described in previous section. Converting the collected time input-output relation into frequency domain by using FFT, transfer functions are estimated by employing the algorithm given in [Ozdemir and Gumussoy , 2017]. While setting the frequency interval for the estimation, the short-period dynamics and the Bode plots in Figure 4 are considered together. The linear model in Figure 4 is obtained by linearisation of nonlinear Simulink model and includes both longitudinal and lateral dynamics. In addition, identified transfer functions represent the dynamics from δ_e to α and q, not from δ_{e_c}. Therefore, the identified transfer functions are multiplied with actuator transfer functions and then the Bode diagrams given in Figure 4 are plotted. Finally, frequencies from 0.5 rad/s to 8 rad/s are found to be suitable for this study and the estimation is performed in this frequency interval. The estimated transfer functions, whose Bode plots with actuator dynamics are shown in Figure 4, are found as:

$$\frac{\alpha}{\delta_e} = \frac{-0.1106s - 4.026}{s^2 + 1.537s + 1.836} \tag{5}$$

$$\frac{q}{\delta_e} = \frac{-4.418s - 2.823}{s^2 + 1.153s + 1.836} \tag{6}$$

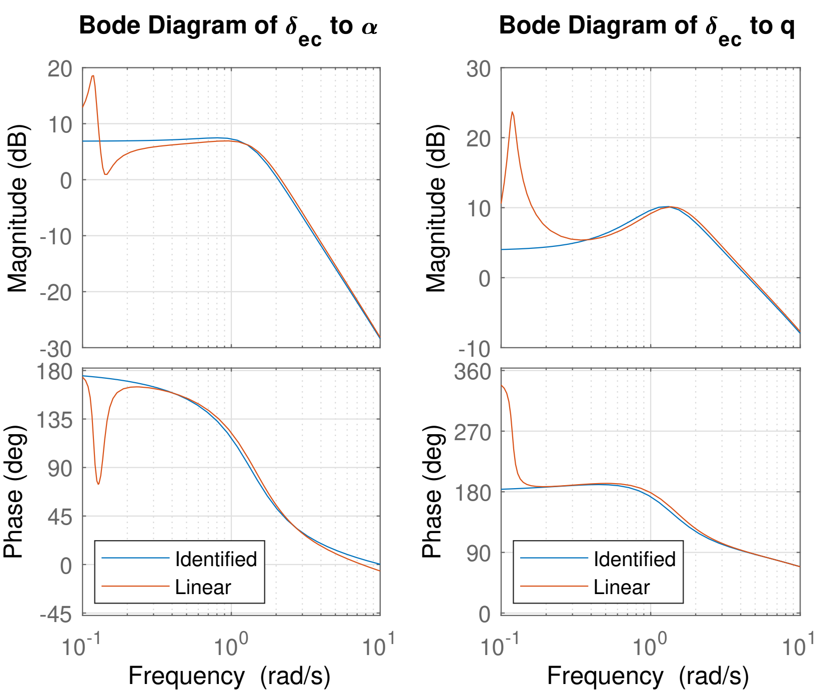

Figure 4: Bode Plots

Then, the identified model is tested in time domain with a doublet input of 3 degrees and the outputs are compared with nonlinear model and linearised model in Figure 5.

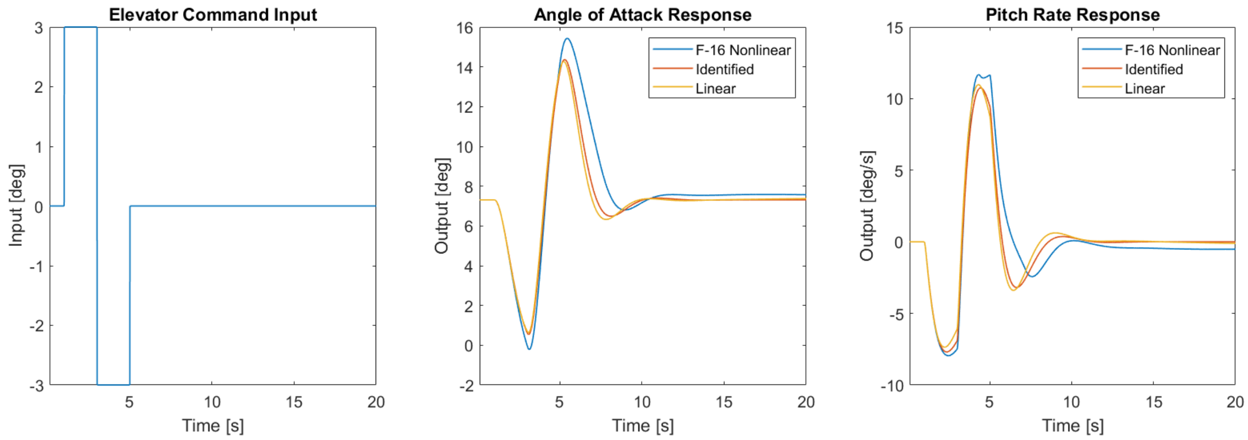

Figure 5: Doublet Input and Time Responses

Having matching responses, aerodynamic coefficients are estimated as next step. The transfer functions are already in such a form that the aerodynamic coefficients can be found directly. Equating the numbers in identified transfer functions (5) and (6) to related parameters in (2) and (3), the identified aircraft parameters are found as:

$$\begin{aligned} Z_{\delta_e} &= -11.7982 \;\; M_{\delta_e} = -4.4185 \;\; M_q = -0.8628 \\ Z_\alpha &= -235.9334 \;\; M_\alpha = -1.4096 \;\; Z_q = -11.7754 \end{aligned} \tag{7}$$

In state-space form, similar to the representation in (1):

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.6741 & 0.8896 \\ -1.4096 & -0.8628 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.1106 \\ -4.4185 \end{bmatrix} [\delta e] \tag{8}$$

In addition, derivatives in non-dimensional form can be also found based on [Stevens, Lewis and Johnson , 2015] and they can be compared to the aerodynamic data embedded in the nonlinear model which are given as look-up tables in [Nguyen, L.T., et al., 1979]. Since α is in small region, it is assumed that $C_{L\alpha} = -C_{Z_\alpha}$, $C_{L_q} = -C_{Z_q}$ and $C_{L_{\delta_e}} = -C_{Z_{\delta_e}}$. The non-dimensional derivatives are given in (9):

$$\begin{aligned} C_{Z_{\delta_e}} = \frac{Z_{\delta_e} m}{\bar{q}S} \;\; C_{M_{\delta_e}} = \frac{M_{\delta_e} J_y}{\bar{q}S\bar{c}} \;\; C_{M_q} = \frac{2M_q J_y V_{T_e}}{\bar{q}S\bar{c}^2} \\ C_{Z_\alpha} = \frac{Z_\alpha m}{\bar{q}S} \;\; C_{M_\alpha} = \frac{M_\alpha J_y}{\bar{q}S\bar{c}} \;\; C_{Z_q} = \frac{2Z_q m V_{T_e}}{\bar{q}S\bar{c}} \end{aligned} \tag{9}$$

The identified non-dimensional derivatives at 5000 ft altitude and 350 ft/s trim airspeed are found as in (10):

$$\begin{aligned} C_{Z_{\delta_e}} &= -0.6542 \;\; C_{M_{\delta_e}} = -0.5782 \;\; C_{M_q} = -6.9817 \\ C_{Z_\alpha} &= -3.9877 \;\; C_{M_\alpha} = -0.1844 \;\; C_{Z_q} = -40.3823 \end{aligned} \tag{10}$$

The non-dimensional derivatives for F-16 nonlinear model at the same conditions are (11):

$$C_{M_q} = -5.7131 \;\; C_{Z_q} = -30.8692 \tag{11}$$

By repeating the same procedure, identification can be performed at various trim points to cover the entire flight envelope. For instance, a small application is presented in Table 1. The procedure is repeated and the short-period approximation is obtained for various Mach numbers at constant altitude (5000 ft). It is seen that the identification procedure provides consistent results for a range of Mach numbers.

Conclusion

A practical approach for short-period approximation by using frequency-domain system identification techniques is proposed in this study. It is observed that the method predicts the time-domain response successfully and the parameter estimation is open to enhancement by improving the fitting algorithm used in the study. Still, the proposed method provided acceptable results in the different points of the flight envelope. Since the results are acceptable and can be improved, the identified parameters found by this technique can be considered as good initial inputs to more sophisticated system identification tools.

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Airspeed (ft/s)	**Mach**	$C_{M_{\delta_e}}$	$C_{Z_{\delta_e}}$	C_{M_q}	C_{Z_q}	C_{M_α}	C_{Z_α}	α_{trim} **(deg)**	$\delta_{e\ trim}$ **(deg)**	$LEFs_{trim}$ **(deg)**	**Freq. fit range rad/s**	C_{M_q} **F-16**	C_{Z_q} **F-16**
230	0.209	-0.6123	-0.3555	-6.5737	-27.0567	-0.1926	-4.7751	17.53	-5.4018	25	0.6-10	-6.1883	-28.8841
350	0.317	-0.5782	-0.6542	-6.9817	-40.3823	-0.1844	-3.9877	7.0377	-3.2458	10.8892	0.5-8	-5.7131	-30.8692
450	0.410	-0.5817	-0.1269	-6.4234	-29.6030	-0.1407	-4.1621	3.9757	-2.1453	5.8695	1.2-10	-5.4561	-29.7049
495	0.451	-0.6172	-0.3154	-6.8071	-28.7120	-0.1079	-4.2974	3.0309	-1.8815	4.3416	0.85-8	-5.4618	-29.8938
555	0.506	-0.6613	-0.6345	-7.4264	-30.7967	-0.0834	-4.1434	2.0962	-1.6204	2.7197	0.75-9	-5.4674	-30.0808
605	0.551	-0.6650	-0.4178	-7.8374	-28.3190	-0.0887	-3.6950	1.51	-1.4523	1.6053	0.7-8.5	-5.4709	-30.1980

Table 1: Non-dimensional Derivatives at 5000 ft