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Multi-Objective Controller Synthesis Using Hybrid Formulation

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ABSTRACT

This paper addresses the problem of finding optimum parameters for a given control law architecture with the given conflicting goals including stability robustness, handling qualities and agility. A multi-objective design approach named as compromise Decision Support Problem (cDSP) is proposed for an existing longitudinal flight control architecture of a fighter aircraft which is responsible for fulfillment of multiple objectives simultaneously. Problems related to decision support provide a means of modeling decisions found in the design. Desired values of the criteria are determined beforehand. Then, they are transferred to the optimization goals using the deviation variables in the cDSP formulation. The weakness of the cDSP formulation is that cost function is the weighted sum of deviation variables. So, cDSP formulation could get stuck on the local minimum solutions in non-convex optimization problems. In the scope of the study, criteria that are subjected to optimization are found to be non-convex. In order to find a feasible solution; once the problem is formulated with cDSP, a hybrid algorithm that combines genetic and sequential quadratic programming (SQP) will be used to solve it. Results of linear and non-linear analyses show the effectiveness of cDSP with hybrid optimization algorithm in the flight control design problem which takes multiple objectives into account. The aim of this paper is both to show that cDSP could optimize the flight control algorithm design problem and to show that genetic algorithm could overcome the weakness of cDSP.

INTRODUCTION

Most of the recent engineering problems is to satisfy more than one requirement. One of the problems to be addressed in the engineering field is the design of a control law architecture. It requires that different design objectives such as stability, time domain requirements, frequency response requirements to be taken into account simultaneously while designing a control law architecture. Objectives cannot be dealt with separately, as they sometimes conflict with each other. Finding an optimal controller involves an inevitable trade-off between design goals such as performance and stability robustness [1]. There are numerous studies using optimization for flight control algorithm design in the literature [2],[3] and [4].

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Even though there are numerous criteria while designing a flight controller, they could be clustered in three classes which are stability, flying and handling qualities and agility. In the scope of this paper, four design goals and a hard constraint are selected. Two of design goals belong to handling quality which are Neal-Smith criterion [7] and Gibson Average Phase Rate criterion [7], [8]. Handling quality criteria are selected through their success rate as it will be shown later. One of design goals is stability robustness using the Nichols Exclusion Zone [11] which is based on the industrial knowledge. The last design goal is agility using the STEM maneuver which was developed to increase the evaluation maneuvers currently used by flight qualities and flight testing communities [9], [10]. Conflictions between the design goals will be shown in the paper. Beside the design goals, there could be hard constraints such as actuator limits, load limits etc. which are strictly to be met. In the scope of this study, there is only one hard constraint. Hard constraint is that the linear model which is linearized at the relevant flight condition has no root on the right half of s-plane.

It is needed to use a multi-objective optimization formulation to find an optimal controller with conflicting design goals. While dealing with the multi-objective optimization, there are numerous methods to model the problem because of the complexity of the problem. In the scope of this study, the method which will be used is “Compromise Decision Support Problem” [5]. The weakness of the formulation will be shown with using only ‘sqp’ algorithm. After the problem is formulized, hybrid algorithm of ‘sqp’ and ‘genetic algorithm’ will be used to solve the problem.

cDSP is a powerful technique for defining an objective in mathematics. The main idea behind this is to define an objective with proper borders. It is based on the goal programming. The main difference between the goal programming and cDSP is that constraints can be modelled in the cDSP whereas it is not the case for goal programming. In the scope of this paper, there is one hard constraint to be modelled in the problem. The difference between cDSP and goal programming will be shown in later chapters.

In this study, the mathematical model of a generic fighter aircraft was used [6]. The mathematical model and the algorithms were all developed in the MATLAB. In order to design a controller, linear model is required. In order to find equilibrium points (trim points), Newton-Raphson algorithm was used [12].

METHOD

1. Multi Objective Optimization and cDSP Formulation

Multi-objective optimization is formulized as stated below.

Satisfying the constrained as shown in equation (1) and equation (2).

$$g_i(x) \leq 0 \,, i = 1,2, \dots, m \tag{1}$$

$$h_i(x) = 0 \,, i = 1,2, \dots, p \tag{2}$$

Minimizing the vector of cost functions as shown in equation (3).

$$\vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \tag{3}$$

Where;

$\vec{x}$: Vector of decision variables
$\vec{f}(\vec{x})$: Vector of cost functions
$g(\vec{x})$: Non-equilibrium constraints
$h(\vec{x})$: Equilibrium constraints

While minimizing the cost functions, there are numerous points which satisfy the design goals simultaneously. In this case, problem requires to find global minimum solution which provides the best decision over the design space with respect to pre-defined cost function. In multi-objective optimization problem, there are a few global minimum solutions depending on the weights of the design goals. In this case, cluster of global minimum solution is named as Pareto-Front and is defined as follows.

Definition 1: Decision vector of $(\vec{x})$ is dominant over decision vector of $(\vec{y})$, if and only if the cost of the decision vector of $(\vec{x})$ is less than the cost of the decision vector of $(\vec{y})$ in at least one design goal.

$$\vec{x} \; is \; dominant \; over \; \vec{y}$$
$$\forall i \in \{1, \dots, k\}: f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1, \dots, k\}: f_i(\vec{x}) < f_i(\vec{y})$$

Definition 2: If there is no such a decision vector which is dominant over $(\vec{x})$, decision vector of $(\vec{x})$ is said to be non-dominant in design space. Thus, decision vector of $(\vec{x})$ is Pareto-Front of the optimization problem.

In the scope of this study, “Compromise Decision Support Problem (cDSP)” formulation will be used to solve the problem. Formulation of cDSP is given in Figure 1. cDSP formulation is based on goal programming. A demonstration of basic differences between goal programming, cDSP and traditional programming is shown in Figure 2. cDSP formulation puts all weighted deviation variables sum into one cost function. So, it does not guarantee to find global minimum solution. Instead, cDSP provides the feasible solution which satisfy all the design goals. In control algorithm design problem, finding a best controller with respect to given cost function is a challenge. So, genetic algorithm will be integrated with gradient-based optimization in order to overcome the drawback of the cDSP.While minimizing the cost functions, there are numerous points which satisfy the design goals simultaneously. In this case, problem requires to find global minimum solution which provides the best decision over the design space with respect to pre-defined cost function. In multi-objective optimization problem, there are a few global minimum solutions depending on the weights of the design goals. In this case, cluster of global minimum solution is named as Pareto-Front and is defined as follows.

Given

Assumptions that come from physics of the problem.

n Number of decision variables

p Number of equilibrium constraints

q Number of non-equilibrium constraints

m Number of design goals

$g_i(x)$ Functions of equilibrium constraints

$$\boldsymbol{g_i(x) = C_i(x) - D_i(x)}$$

$f_k(d_i)$ Deviation variables which are subject to optimization

W_i Weights of deviation variables to be used in cost function

Find ;

Values of system decision variables; X_j j = 1,…,n

Values of deviation variables; d_i^+, d_i^- ; i = 1,…,m

Satisfy ;

System constraints;

$g_i(X) = 0$; i = 1,…,p

$g_i(X) \geq 0$; i = 1,…,q

Design goals;

$A_i(X) + d_i^- - d_i^+ = G_i$, i = 1,…, m

Lower and upper bounds of decision variables ;

$X_j^{min} \leq X_j \leq X_j^{max}$, j = 1,…,n

Lower and upper bounds of deviation variables;

$d_i^-, d_i^+ \geq 0$, $d_i^- * d_i^+ = 0$

Minimize ;

Function of deviation variables ;

$$Z = \sum_{i=1}^{m} W_i(d_i^- + d_i^+); \sum W_i = 1 ; W_i \geq 0$$

Figure 1 cDSP formulation

OPTIMIZATION PROBLEM		GOAL PROGRAMMING		cDSP		TRADITIONAL PROGRAMMING	
Find ;		Find ;		Find ;		Find ;	
	Design Variables (x)		Design Variables (x)		Design Variables (x)		Design Variables (x)
			Deviation Variables		Deviation Variables		
Satisfy ;			(d_i^+,d_i^-)		(d_i^+,d_i^-)		
	Constraints	Satisfy ;				Satisfy	
	$A_u(x) \leq G_u$		Goals ;	Satisfy ;			System Constraints
	$A_u(x) = G_u$		$A_i(x) + d_i^- - d_i^+ = G_i$		System Constraints		$g_i(x) = 0, i = 1, \dots, p$
	$A_u(x) \geq G_u$				$g_i(x) = 0, i = 1, \dots, p$		$g_i(x) \geq G_i$
	Goals				$g_i(x) \geq 0, i = p+1, \dots, u$	Minimize ;	
	$A_t(x) \leq G_t$	Minimize ;			Goals ;		$A(x)$
	$A_t(x) = G_t$		a)Preemptive		$A_i(x) + d_i^- - d_i^+ = G_i$		
	$A_t(x) \geq G_t$		$Z = [f_i(d_i^+,d_i^-), \dots, f_k(d_i^+,d_i^-)]$		System Borders ;		
					$X_j^{min} \leq X_j \leq X_j^{max}$		
Maximize ;			b)Archimedean		$d_i^+, d_i^- \geq 0$		
	$A_r(x)$				$d_i^+ * d_i^- = 0$		
Minimize ;			$Z = \sum Wi(d_i^+ + d_i^-)$	Minimize ;			
	$A_s(x)$				a)Preemptive		
			$\sum Wi = 1 ; W_i \geq 0$		$Z = [f_i(d_i^+,d_i^-), \dots, f_k(d_i^+,d_i^-)]$		
					b)Archimedean		
					$Z = \sum Wi(d_i^+ + d_i^-)$		
					$\sum Wi = 1 ; W_i \geq 0$		
where ;		where ;		where ;		where ;	
	t : Design goals		G = Design goals		g : System constraints		G: Design goals
	u : Design constraints		A : System achievements		X : System variables		A : Cost value
	r : Goals to be maximized		W : Weights of deviation variables				
	s : Goals to be minimized						

Figure 2 Comparison of formulations

2. Formulation of Cost Function

In the scope of this study, there are four design goals and one hard constraint.

<u>Hard Constraint</u>;

• There must be no root of the linear system on the right half of s-plane. If there exists, value of cost function must be very large number.

Design Goals;

• Evaluation of the Neal-Smith criterion must be within Level-1 borders.

• Evaluation of the Gibson Average Phase Rate criterion must be within Level-1 borders.

• The closest distance between Nichols Exclusion Zone and frequency response of the system must be 1.

• Agility evaluations with the optimized controller must be better than the existing controller.

Design goals with given explanations as above are shown in equations of (4) and (5). Maneuver time depends on the flight condition. So, there is no standard design goal for this criterion. Instead, design goal is linked to the maneuver time with the existing controller. Cost function are obtained using the cDSP formulation as shown in equations of (6) and (7). Here, each goal is equal weighted. Different design variables could be found by using different weights.

$$G_1 = 1\,; G_2 = 1\,; G_3 = 1\,; \tag{4}$$

$$G_4 = (Existing\ maneuver\ time) - 1 \tag{5}$$

$$d_1 = 1 - \frac{A_1}{G_1}\,; d2 = 1 - \frac{A_2}{G_2}\,; d_3 = \left(\frac{A_3}{G_3}\right) - 1; d_4 = 1 - \left(\frac{A_4}{G_4}\right) \tag{6}$$

$$Z = 0.25 * d_1 + 0.25 * d_2 + 0.25 * d_3 + 0.25 * d_4 \tag{7}$$

3. Hybrid Optimization Algorithm

As a local search algorithm, “sqp” has a fast convergence to local or global optimum according to its initial condition but it is generally not possible to feed “sqp” with an appropriate initial condition that leads to global optimum in opposite, GA is insensitive to initial parameter set in means of global convergence but it takes long time to converge to global optimum for GA. In this research, a hybrid genetic algorithm is presented as a better way for solving the optimization problem. Hybrid GA aims to combine strong sides of “sqp” and GA. It is expected to be insensitive to initial condition and provide fast convergence to global optimum.

Hybrid GA starts with a fixed initial population. Secondly, objective function-based evaluation is performed by using fitness function. This evaluation assigns probability of usage to each individual for selection process. In next step, selection, crossover and mutation functions are performed to generate offspring individuals. Then the new population’s objective function values are obtained and the parameters of individual which has best fitness among the offspring individuals, are set as initial condition of sqp. For first generation, feeding the sqp with the best individual is a default action but for next generations, hybrid GA has Rule 1 to make decisions of action. Rule 1 states that if best individual of new population produced in present generation, has a better fitness than the best individual of previous generation, sqp will use the best individual of new population. If fitness is not better, new population remains as it Is. In former case, sqp makes its iterations and if it minimizes the objective function below a predetermined value, the hybrid algorithm will stop. But if sqp does not find a solution below that value, again hybrid GA has Rule 2 to make decision of action. First action is replacing the worst individual with sqp output in the case of improvement on the best individual of new population. Second action is remaining the new population as it is. In both cases, hybrid GA continues to next generation without checking whether new population is changed by sqp.

RESULTS

Results of the existing controller are given in Figure 3, Figure 4, Table 1 and Table 2. If the results are reviewed, there are evaluations of Level-2 both in Neal Smith criterion and Gibson APR criterion. There exists no violating result in the stability criterion if the existing controller is used. Maneuver time for each flight condition is shown in Table 1.

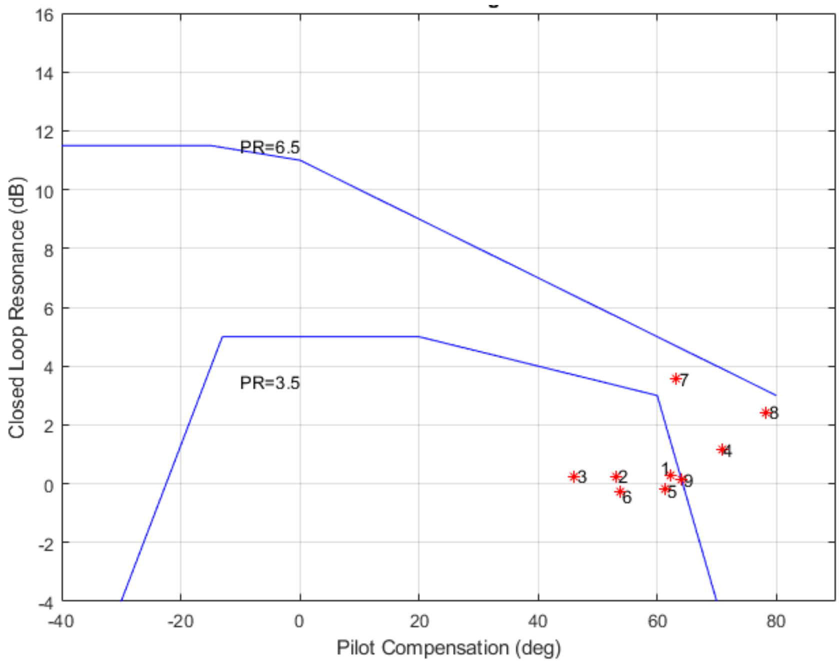

Figure 3 Evaluation of Neal Smith criterion with existing controller

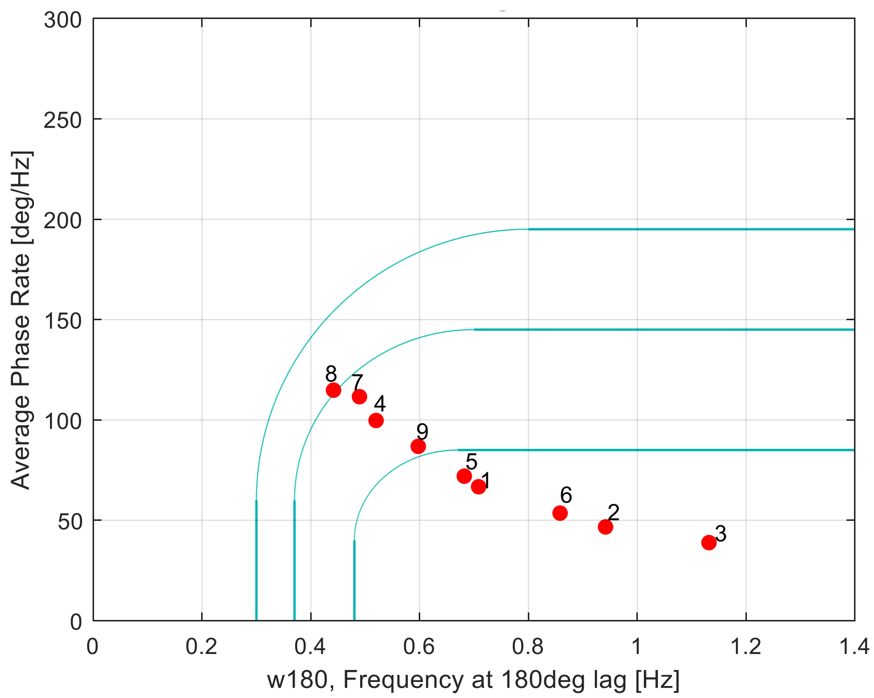

Figure 4 Evaluation of Gibson APR ecriterion with existing controller

Table 1 Evaluation of agility criterion with existing controller

Flight Condition	1	2	3	4	5	6	7	8	9

Maneuver Time (sec)	10.35	10.58	11.94	13.65	12.80	12.13	23.36	21.49	20.05

Table 2 Evaluation of stability criterion with existing controller

Flight Condition	1	2	3	4	5	6	7	8	9
Closest Distance to Nichols Exclusion Zone	2.6935	2.1342	1.8638	3.3728	2.7680	2.2898	3.0770	2.2663	3.0468

Optimization problem which is formulized using cDSP was solved and results are shown in Figure 5, Figure 6, Table 3 and Table 4. All design goals which are defined in method were met by using the optimization.

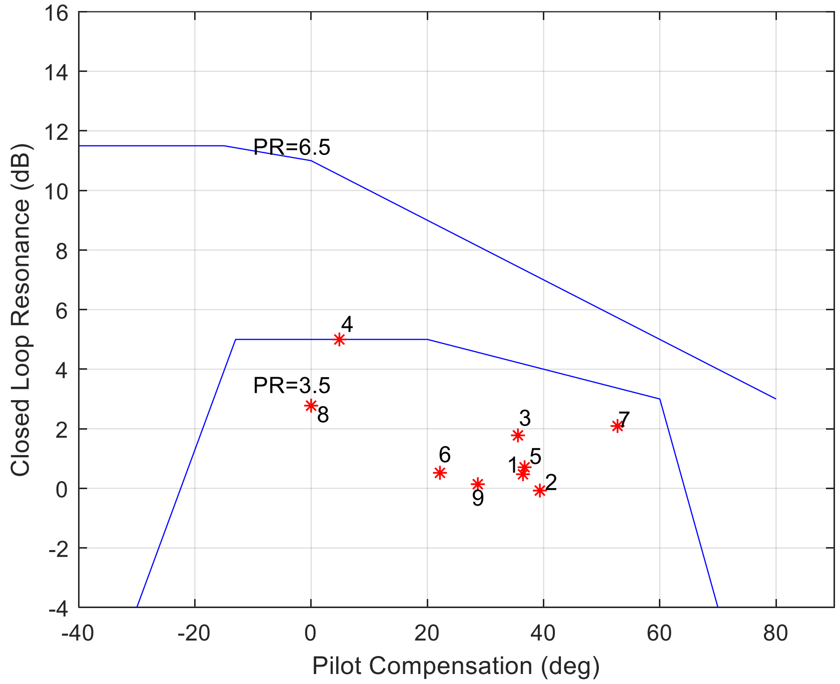

Figure 5 Evaluation of Neal-Smith criterion with optimized controller

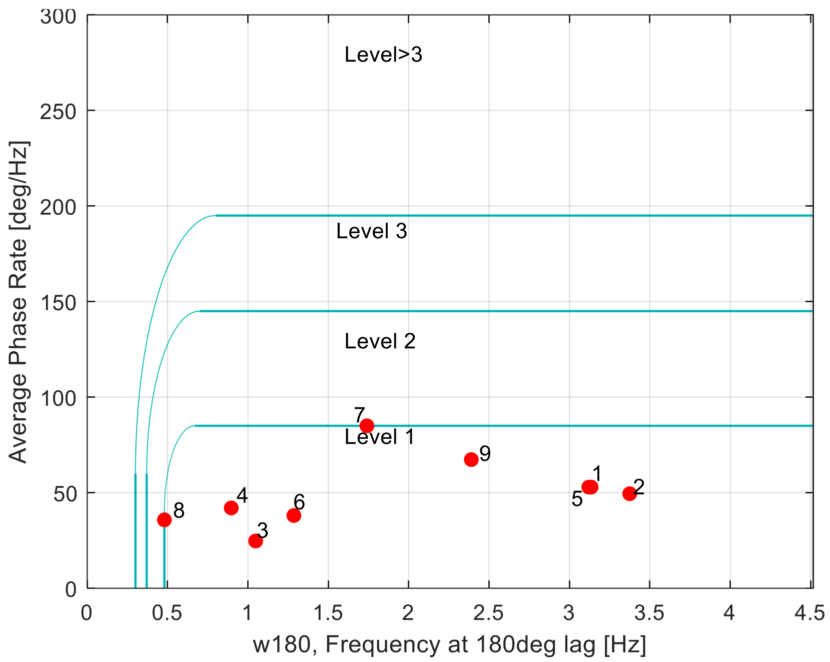

Figure 6 Evaluation of Gibson APR with optimized controller

Table 3 Evaluation of agility criterion with optimized controller

FC	1	2	3	4	5	6	7	8	9
Maneuver Time	9.46	8.75	8.11	12.82	11.52	10.9	23.03	20.71	18.96

Table 4 Evaluation of stability criterion with optimized controller

FC	1	2	3	4	5	6	7	8	9
Closest Distance to Nichols Exclusion Zone	1.003	1	1	1	1.31	1	1.61	1.01	1

In this paper, a multi-objective design and optimization method for controller design has been proposed. It is a process with 2 stages. First, the mathematical model is obtained by using the dynamics of the flight. Second, the cDSP multi-objective optimization formulation is used to solve the optimization problem. The results of the optimization show that the objectives of optimization are met with the optimized values of gains. Moreover, the gains can further be tuned using the different values of the weights used in cDSP formulation.

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