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TRAJECTORY OPTIMIZATION OF A PURE PURSUIT GUIDED MUNITION IN TERMS OF CONTROL EFFORT AND CROSSTRACK ERROR BY MEANS OF VIRTUAL TARGET TRACKING APPROACH

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ABSTRACT

In this paper, the trajectory of the pursuer consisting of linear and circular segments is optimized by tracking a virtual target that follows the intended path. The trajectory is shaped by an objective function which aims to minimize the total control effort and the cross-track error while the virtual target is being pursued towards a destination point. The pursuer implements the "Pure Pursuit" guidance law wherein an upper limit is assigned as the maximum allowable acceleration command in accordance with the maneuver capability and structural strength of the pursuer. "Steepest Descent" optimization algorithm is conducted to find minima of the design variables, namely the speed of the virtual target and the radius of curvature regarding the circular path. In conclusion, the pursuer is shown to track the optimal trajectory satisfying the minimum cross-track error and control effort criteria, as determined by iterative solutions implemented in the optimization process.

INTRODUCTION

In general, three different types of approaches have been explored for trajectory- tracking guidance scheme that are vector-field-based approach, error-regulation-based approach, and virtual-target-based approach. The main idea of the virtual-target-based approach is that the pursuer is controlled to follow a moving virtual target point along reference trajectory. The virtual target approach being followed in this work is the pursuit of designed path that contains consecutive three segments, respectively linear, circular, and linear segments similar to the implementation in [1]. Trajectory following has been research content over the last two decades, thus many methods have been conducted to assure efficient and steady path tracking guidance systems [2]. Most of the methods have been developed for the trajectory shaping in planar motion, as the scenario studied in this paper. While the munition is following the trajectory, the approach being implemented in this work is based on moving a virtual target, which relies on the minimization of the cross-track error and control effort. The trajectory shaping is to be conducted and finalized prior to the launch of the pursuer. A moving virtual target approach is applicable on the problem of trajectory tracking with Pure Pursuit guidance law that generates acceleration command to directly steer the velocity vector to be aligned with the intended path. When applying Pure Pursuit guidance law, pursuer has limits in terms of structural strength and aerodynamic maneuverability. Therefore, while following the circular segment of the desirable path, the radius of curvature is restricted by the minimum value. In accordance with the constraints, the requirements are calculated with the use of steepest

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descent optimization technique. Steepest descent is among the most preferred optimization techniques in the engineering area because of the feasible unconstrained property of the method and it might be considered as one of the fastest methods in terms of computational time. The minimization process is for finding optimum trajectory criteria, which are cross-track error and control effort. The objective function comprises of both cross-track error and control effort terms together. At the end of the process, optimal trajectory tracking is obtained with the optimum design variables.

PURE PURSUIT GUIDANCE LAW BY VIRTUAL TARGET APPROACH

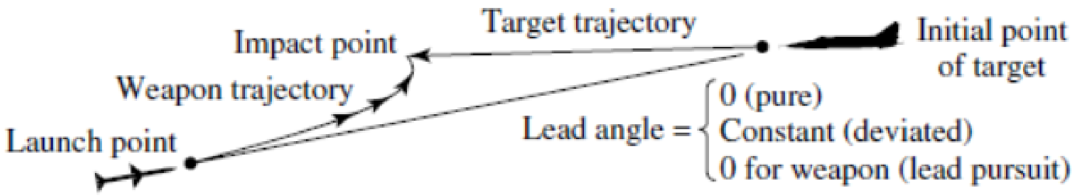

Figure 1 Pursuit: pure pursuit / deviated pursuit / lead pursuit [3]

The logic of the theory of the pure pursuit guidance is based on going in the direction of a target by forcing the angle between the line-of-sight to the target and the forward axis of the pursuer's velocity to zero as shown in Figure 1. This movement is succeeded by commanding the virtual target to travel along the predefined path. When the virtual target moves along the path, the pursuer moves to form a pursuit trajectory towards the target. As the target is moving, the LOS is changing direction and, the pursuer has to compensate for this change, simultaneously. The general representation of the pursuit guidance is depicted in Figure 2. As can be seen, the biggest advantage of the pursuit guidance is in its simplicity in implementation [3].

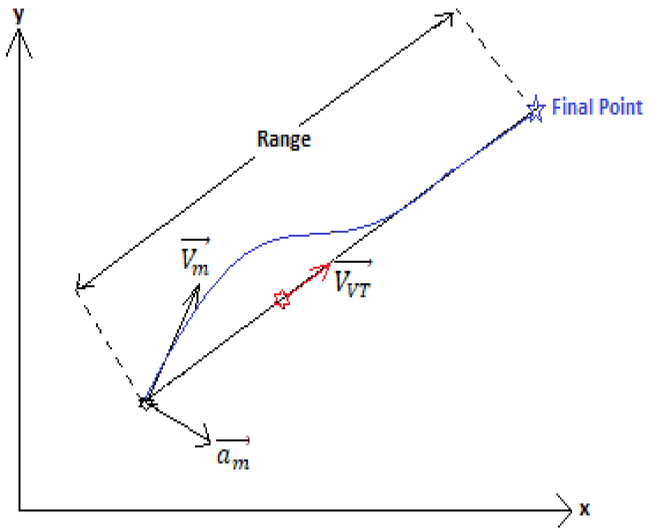

Figure 2 Pursuer-virtual target engagement scenario

The guidance algorithm computes the appropriate commands so that the velocity vector of the pursuer may point in the direction of the instantaneous LOS. According to Figure 3, γ (gamma) is flight path angle and λ (lambda) is azimuth LOS angle and they are used to decide to which direction the acceleration command is to be applied.

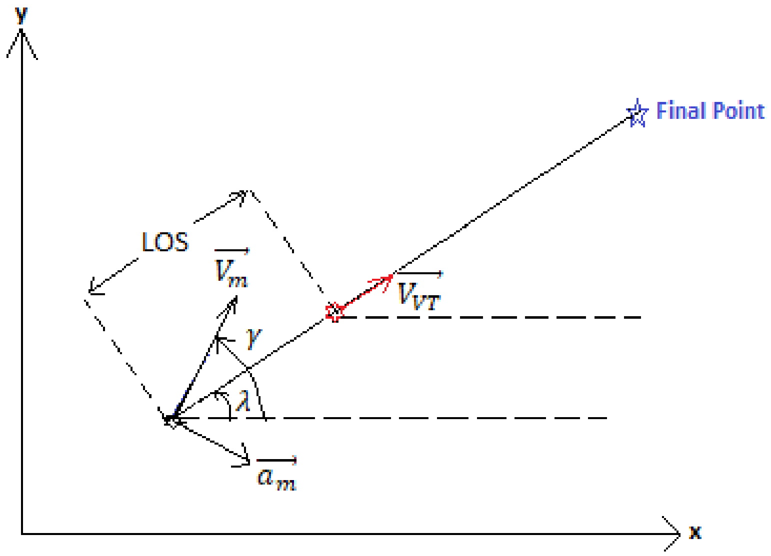

Figure 3 LOS angle and heading error ($\boldsymbol{\lambda}$ ***and*** $\boldsymbol{\gamma}$) with pursuer and virtual target velocity vectors

The purpose of the pure pursuit guidance is to coincide the velocity vector and LOS vector. 'K' is the design proportionality constant. Then, the desired flight path angle rate of change can be found as followed by the relation equation.

$$\gamma = tan^{-1}(\frac{Vm_y}{Vm_x})$$

$$R = \sqrt{(y_{vt} - y_m)^2 - (x_{vt} - x_m)^2}$$

$$\lambda = tan^{-1}(\frac{R_y}{R_x})$$

$$\frac{d\gamma_{desired}}{dt} = K(\lambda - \gamma)$$

CROSS TRACK ERROR AND CONTROL EFFORT

Path-following error convergence is a significant point of concern when addressing path-following algorithms. In other words, when implementing a path-following guidance rule, it is crucial to scrutinize the pursuer's dynamic response to positional changes and angular deviations from the desired path.

Cross-Track Error of Straight Line Segment

When the pursuer tracks the virtual target along the straight path, the cross-track error can be calculated with an analytical formulation. The coordinates of two points on a linear line on a plane $(\boldsymbol{x_1}, \boldsymbol{y_1})$ and $(\boldsymbol{x_2}, \boldsymbol{y_2})$, and the position coordinates of the follower $(\boldsymbol{x_0}, \boldsymbol{y_0})$ that aims to follow this line can be found with the equation below, wherein the perpendicular distance of the follower to the straight line is $(\boldsymbol{d_0})$.

$$\boldsymbol{d_0} = \frac{|(\boldsymbol{y_2} - \boldsymbol{y_1})\boldsymbol{x_0} - (\boldsymbol{x_2} - \boldsymbol{x_1})\boldsymbol{y_0} + \boldsymbol{x_2}\boldsymbol{y_1} - \boldsymbol{y_2}\boldsymbol{x_1}|}{\sqrt{(\boldsymbol{y_2} - \boldsymbol{y_1})^2 + (\boldsymbol{x_2} - \boldsymbol{x_1})^2}}$$

Cross-Track Error of Circular Line Segment

The calculation of the cross-track error in the circular path is different from the straight path. The central(x_c, y_c), coordinates can be found by solving the following equations in which the distance $(d_0)$is the cross-track error while tracking a circular trajectory of radiusρ.

$$d_0 = |P - r|$$

$$\text{where } P = \left|\sqrt{(x_0 - x_c)^2 + (y_0 - y_c)^2}\right|$$

Control Effort

In guidance terminology, the time integral of the squared magnitude of the acceleration command equals the control effort of the pursuer as represented by the equation below:

$$\kappa = \int_0^{t_{final}} |a_{command}|^2 dt$$

STEEPEST DESCENT OPTIMIZATION METHOD

The steepest descent optimization algorithm is one of the most preferable and easy to be applied compared to other gradient search methods. The steepest descent is a gradient method, wherein the following vector notation can find the gradient of a function:

$$\underset{n \times 1}{\nabla f} = \begin{Bmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \\ . \\ . \\ . \\ \partial f/\partial x_n \end{Bmatrix}$$

The gradient function has the crucial role for the optimization process, if the process moves along the gradient direction from the starting point, the function results in a change with the highest rate [5]. Therefore, the gradient direction is called 'the direction of the steepest 'ascent'. It is shown in Figure 4.

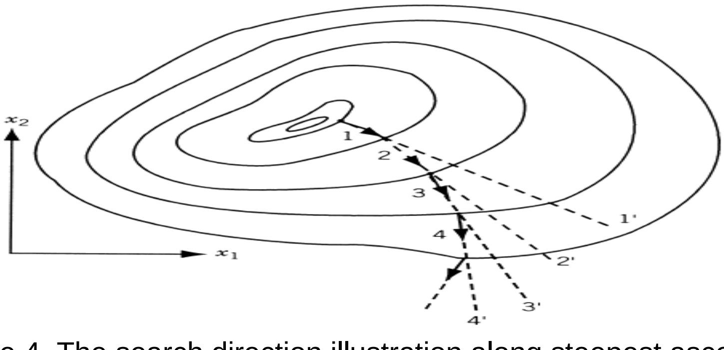

Figure 4 The search direction illustration along steepest ascent [4]

The steepest descent direction is the direction along which the gradient is descending with the highest rate as opposed to the depiction above. According to that, when the minimization process is applied, gradient vector direction is expected to converge to the minimum point as expected.

TRAJECTORY OPTIMIZATION

The purpose of the study is the minimization of the cross-track errors and the control effort resulting from trajectory tracking between moving virtual target and pursuer. The chosen method is steepest descent optimization algorithm. There is a reference path which is modified iteratively depending on the design variables which are velocity of the virtual target ($\boldsymbol{v_{VT}}$) and radius of curvature ($\boldsymbol{\rho}$) that pursuer follows. The optimization process may be regarded as a mission-planning phase and is completed before the pursuit. Optimum design variables are determined in advance so that a reference trajectory is set for the pursuer to be tracked. The cost function ($\boldsymbol{Q}$) includes with control effort and cross-track error terms.

$$\boldsymbol{Q} = \int_{0}^{t_{final}} [|a_{command}|^2 + |d_0|]dt$$

SIMULATION STUDIES

The minimization process of the cost function is implemented. The pursuer is launched with $\mathbf{25}°$ heading error at the starting point. Moreover, the virtual target is started in front of the pursuer. The reference path includes 4 points and 3 segments namely the starting point of A in the straight-line segment, non-fixed point of B which is the end of the straight-line segment and the starting of the circular line segment, fixed point of C which is the end of the circular-line segment and the starting of the final straight segment, and D is the final destination point is shown in Figure 5.

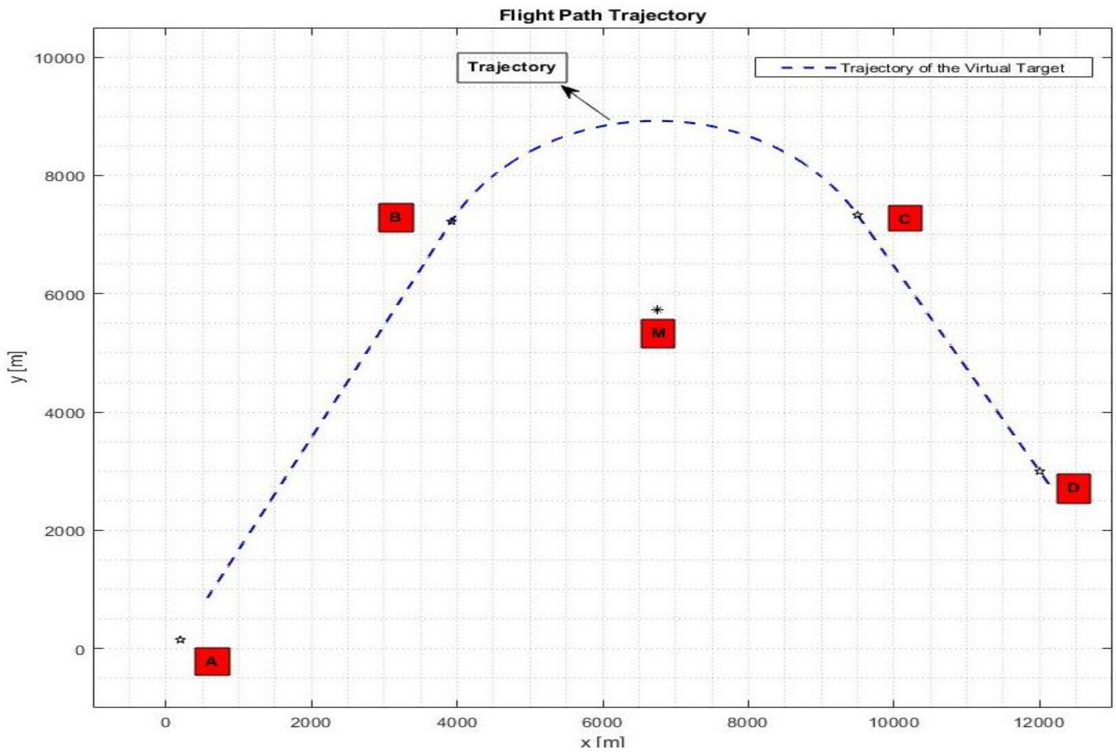

Figure 5 Trajectory Segments

First of all, the cost function is classified into two function which are cross-track error and control effort, as known. Regarding the design variables, the objective cost function needs initial guess points, referred as$(\boldsymbol{v}_{VT,0}, \boldsymbol{\rho}_0)$.

To begin with, manual search is implemented to locate the minima regions around which an optimization can be conducted. The search interval for the velocity of the virtual target is between 250 and 300 meters per second, and for the radius of curvature it is in between 3185 and 5000 meters. Those values are selected according to pursuer's structural strength and maneuver capability while tracking the intended path. As a result of scan process at evenly spaced design variable combinations, the surface graph is plotted for the total cross-track error values with respect to the velocity of virtual target and the radius of curvature as shown in Figure 6.

According to the results, the minimum point of cross-track error is 255.47 m at the 260 meter per second of $\boldsymbol{v}_{VT}$ and 4835 meters of $\boldsymbol{\rho}$.

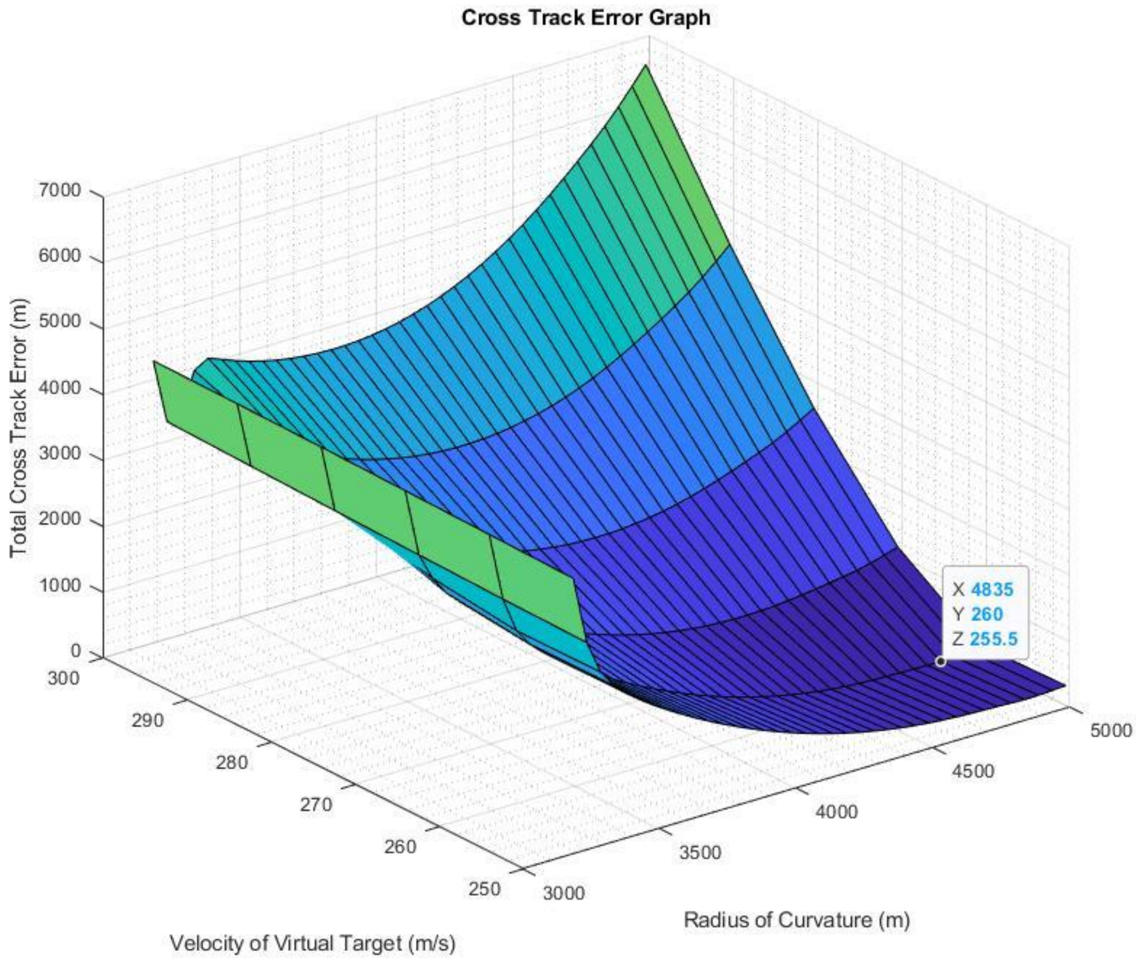

Figure 6 Surface graph of cross-track error

The same initial large specific interval is applied for the control effort function. After that, the surface graph is plotted for the control effort values with respect to velocity of virtual target and radius of curvature as shown in Figure 7.

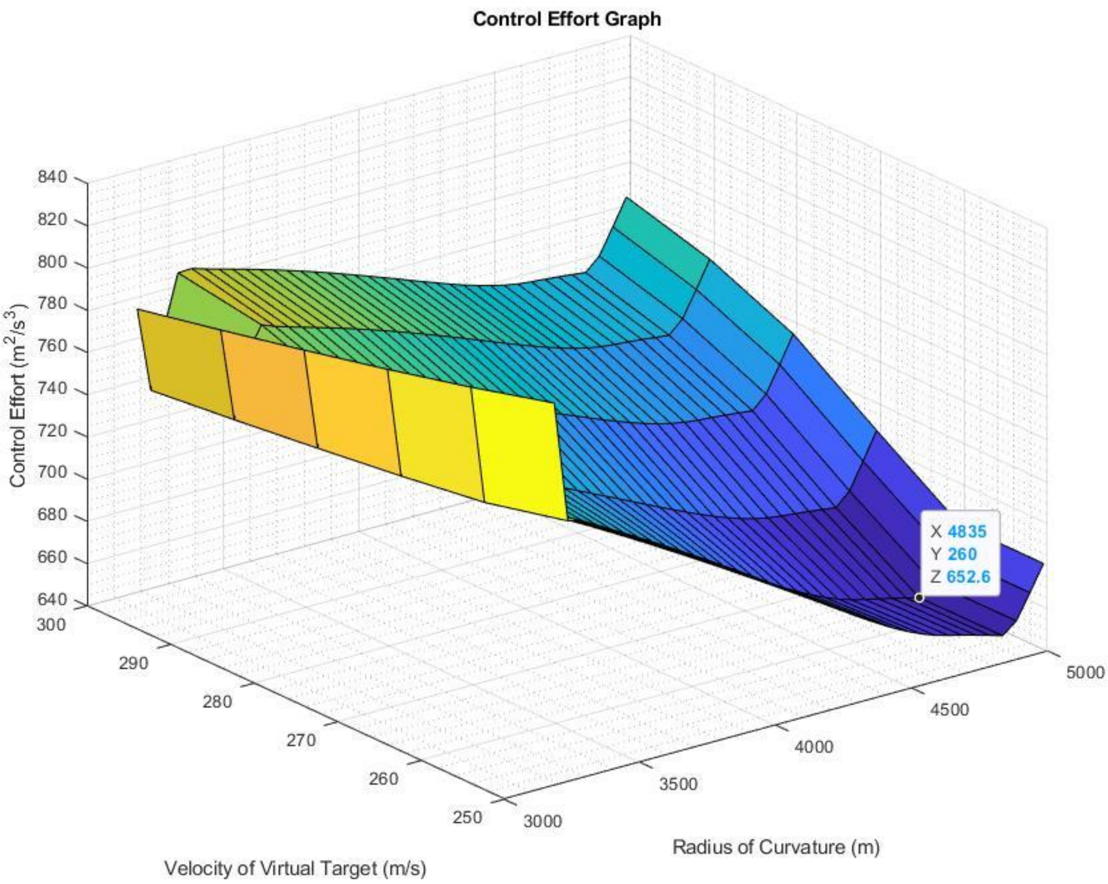

Figure 7 Surface graph of control effort

According to the results, the minimum point of control effort is 652.6 meters square per second cube at the 260 meters per second $\boldsymbol{v_{VT}}$ and 4835 meters of $\boldsymbol{\rho}$.

Then, the steepest descent algorithm is applied to determine the minimum values more precisely for each function as shown in Table 1.

Table 1 the steepest descent optimization results in terms of cross-track error and control effort

$\boldsymbol{d_{0,min} = 206.7868\ \mathrm{m}}$	$\boldsymbol{v_{VT} = 257.5353\ m/s}$	$\boldsymbol{\rho = 4858.1\ m}$
$\boldsymbol{\kappa_{min} = 648.6996\ m^2/s^3}$	$\boldsymbol{v_{VT} = 255.9462\ m/s}$	$\boldsymbol{\rho = 4855.3\ m}$

Optimized cross-track error and control effort values are used as normalization factors to acquire dimensionless terms since cross-track error and control effort have different units, yielding a merged cost function as presented with the equation below.

$$\boldsymbol{Q = \int_0^{t_{final}} \left[|a_{command}|^2 + |d_0|\right] dt}$$

$$Q_{normalized} = \int_0^{t_{final}} \left[\frac{\kappa}{\kappa_{min}} + \frac{d_0}{d_{0,min}}\right] dt$$

Afterwards, normalized cost function is scanned in the same search interval and the result is plotted as surface graph as shown in Figure 8. The minimum point of the cost function is 2.2419 where the velocity of the virtual target is 260 meters per second and the radius of curvature is 4835 meters.

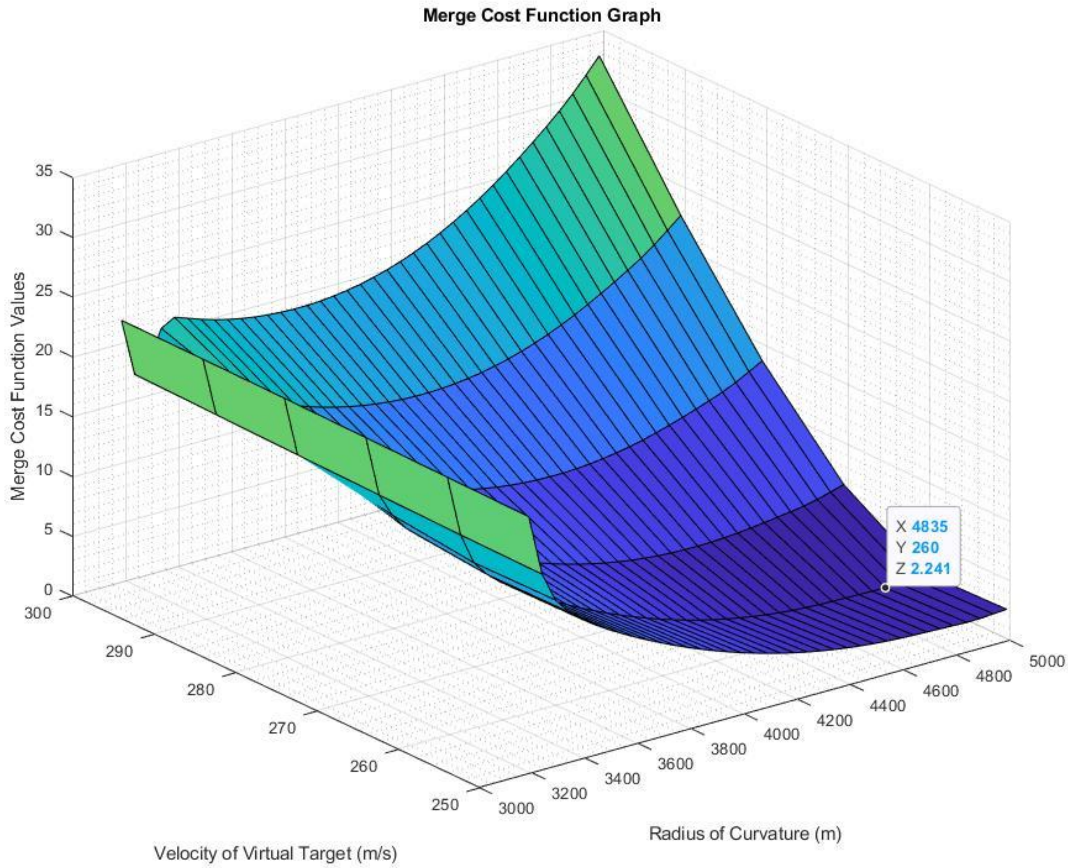

Figure 8 Surface graph of merged cost function

According to those results, the initial points for design variables in the steepest descent algorithm are decided. After 341 iterations and 14574.33 seconds of computation, the minimum point of cost function is found to be 2.0028 where the velocity of virtual target is 257.4642 m/s and the radius of curvature is 4855.583 m and the absolute error is 9.22 e-07. Some iteration values are specified in the Table 2.

Table 2 the steepest descent optimization results in terms of merged cost function iterations

Iterations Number	$\boldsymbol{v_{VT}}$	$\boldsymbol{\rho}$	**Q**
1	256.2664	4854.25	2.086821
25	256.5647	4854.229	2.049733
50	256.7861	4854.224	2.030053
75	256.9515	4854.207	2.01904
100	257.0756	4854.193	2.012801
125	257.1726	4854.406	2.008811
150	257.2455	4854.487	2.007013
175	257.2915	4854.476	2.005846

200	257.3171	4854.512	2.004991
225	257.3598	4854.728	2.004135
250	257.3931	4854.831	2.004
275	257.4192	4855.047	2.003165
300	257.4397	4855.263	2.00278
325	257.4558	4855.367	2.002957
341	**257.4642**	**4855.583**	**2.002384**

The pursuer follows the virtual target along a referenced trajectory. The aim of the study is to find the minimum cross-track error and control effort values for the determination of the optimum trajectory. After all of the minimization process, the steepest descent optimization method gives successful results. According to the steepest descent algorithm, trajectory iterations are shown in Figure 9.

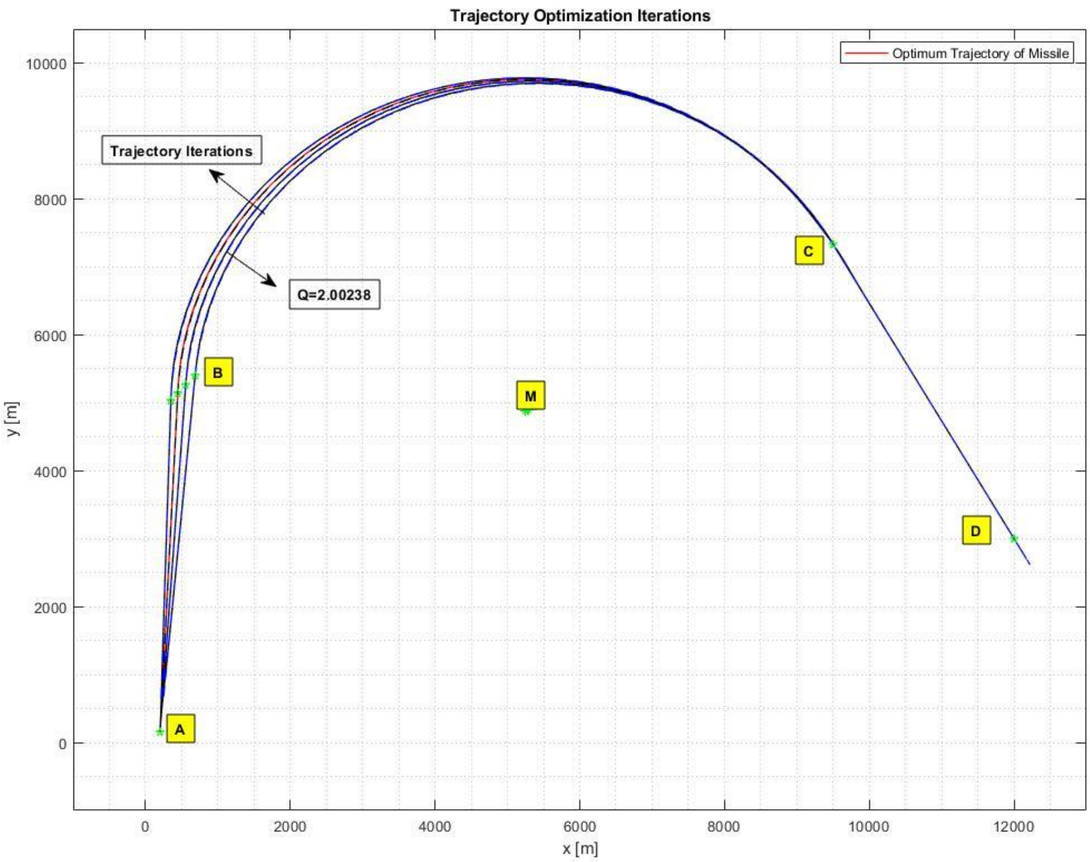

Figure 9 Trajectory iterations because of steepest descent optimization

In order to decrease the cross-track error and control effort, the trajectory segments are shaped. Since the pursuer is initially launched with a certain heading error, the maximum cross-track error is introduced initially and then while following the path in a circular motion huge control effort is expended. In minimization, our design variable, the radius of curvature, increases to reduce the initial cross-track error so that the pursuer can start following the straight line immediately. At the same time, the velocity of the virtual target increases so that the pursuer does not have to follow the virtual target with high control effort. The difference of the initial trajectory and the final optimum trajectory is shown in Figure 10.

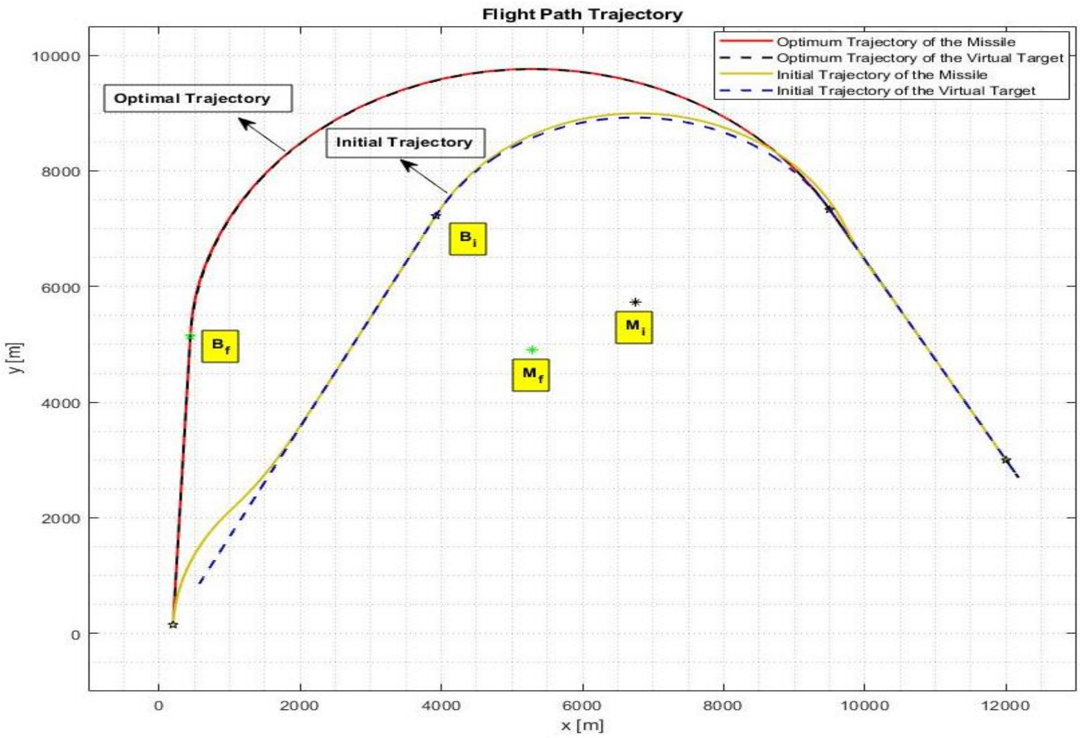

Figure 10 Initial and optimum trajectories of pursuer and virtual target

The change in between the initial cross-track error and the cross track error after minimization is plotted in the following Figure 11. As a result of the optimization, the pursuer follows minimum heading error due to increased radius of curvature. This is indicated as an enormous decrease in the total cross-track error.

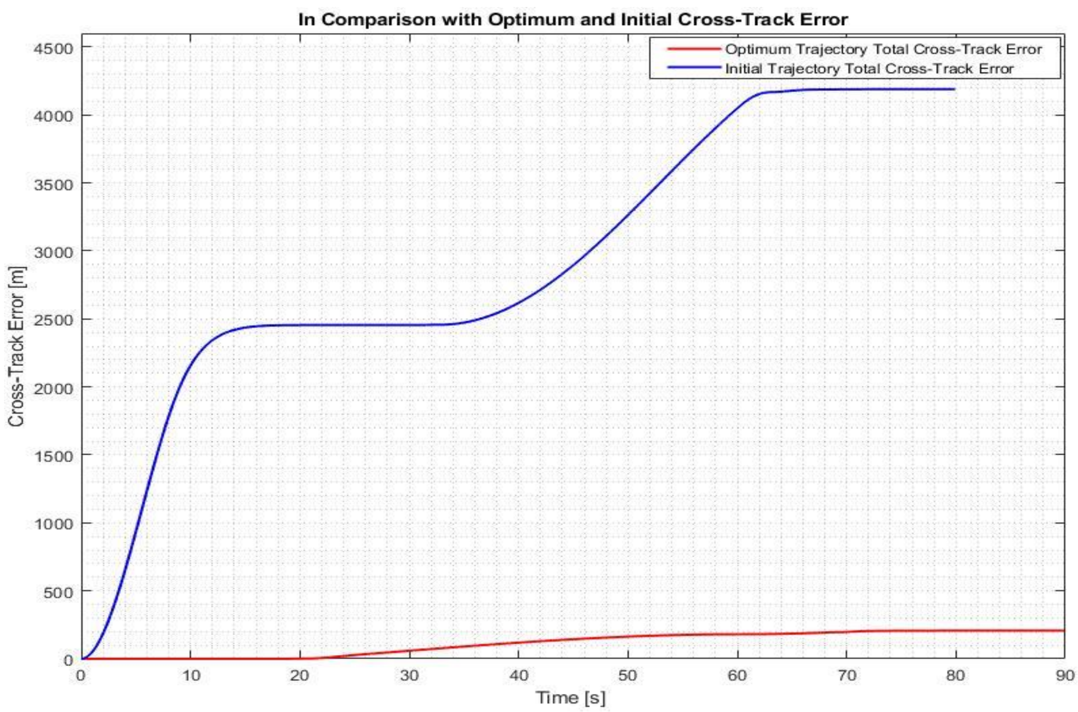

Figure 11 Initial cross-track error and optimized cross-track error

Objective function of control effort is changed after optimization process, and plotted in the following Figure 12. As mentioned before, increasing of the radius of curvature affected the trajectory shaping so that the pursuer consumed less control effort while tracking virtual target.

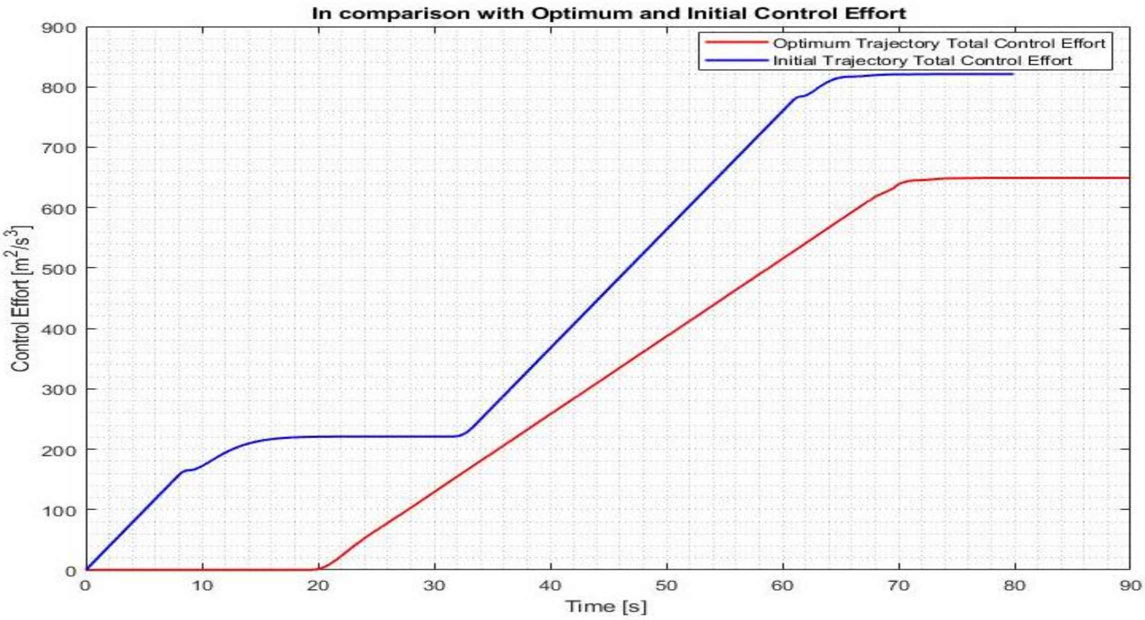

Figure 12 Initial total control effort and optimized total control effort

The pursuer acceleration is also of concern regarding the generated trajectory to be tracked. Due to the structural capabilities of pursuer its acceleration command is limited throughout motion. According to the scenario studied, initial acceleration demand is saturated by the limitations applied. Optimized trajectory is comprised of a straight line segment obtained by nullifying heading error of pursuer instead of acceleration's saturation, and a motion along circular segment wherein the necessity of acceleration is decreased.

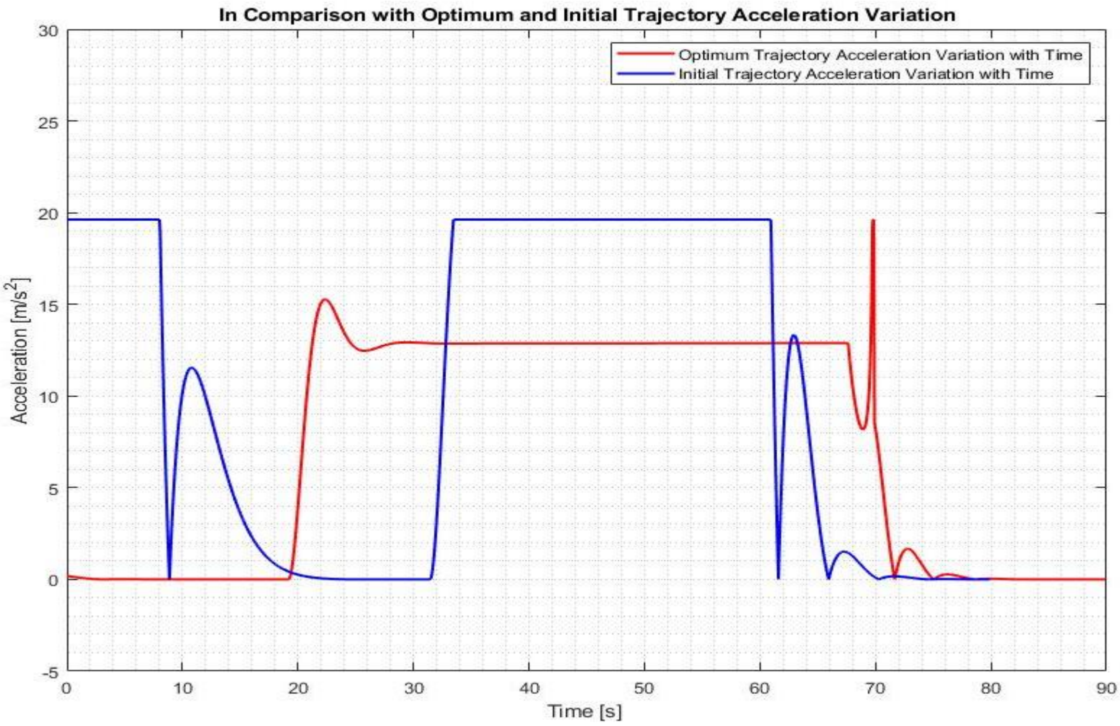

Figure 13 Initial and optimized pursuer acceleration variation with time

DISCUSSION AND CONCLUSION

In this study, virtual target approach is chosen as the trajectory optimization method. The Pure Pursuit guidance law is applied for generating acceleration command for the pursuer to track a moving virtual target along the intended path. According to the Pure Pursuit guidance, the acceleration command of the pursuer is always applied to match the velocity vector of the pursuer with the LOS vector. Thus, the pursuer tracks the virtual target in the desired trajectory with minimum control effort and cross-track error. However, there are some limits enforced by the pursuer's structural strength and maneuver capability while following the circular path. Thus, the initial search limits for the design variables are determined accordingly. The minimization process being applied is one of the most feasible methods to find optimum trajectory with optimum design variables. Consequently, cross-track errors caused by the heading error at the beginning of the engagement and then accumulating in the last phase of the circular path are examined in this study. In addition to that, when tracking of the virtual target along the referenced path, the total squared acceleration command defined as the control effort is also investigated and tried to be minimized.
The simulation results show the differences between the initial trajectory and optimized trajectory. According to the results, the optimized tracking is achieved successfully with minimized objective function values.

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