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CRITICAL AERODYNAMIC LOADS ANALYSIS OF HORIZONTAL TAIL OF AN AIRCRAFT UNDER LONGITUDINAL DYNAMICS

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ABSTRACT

Aircrafts have multidisciplinary working systems and these complex structures are exposed to various type of loads because of the aircraft maneuvers. The loads may lead to structural failure and damage on aircraft horizontal tail. Therefore, tail loads must be calculated before flight testing processes. The aircraft equation of motions and equations of horizontal balanced tail loads are modelled on MATLAB/Simulink for defining critical situation about the limit structural tolerance. Calculated critical values may lead to designing optimization and structural analyses process. In this paper, calculating loads of horizontal tail which is existed from aircraft dynamics with determined parameters of RCAM (Research Civil Aircraft Model) are studied.

INTRODUCTION

The aircraft's horizontal tails encounter a lot of condition while they are flying. For instance, the aircraft maneuvers and gusts may lead to structural loads and the aerodynamics of horizontal tail also leads to aerodynamic loads. In addition, propeller and empennage interaction effects should be considered [Schroijen et al, 2010]. The uncalculated loads results in structural failure and it becomes a cause of economic and time problem. The total horizontal tail loads may classify as many parts. One of these loads is balanced tail loads which are required to stabilize an aircraft with trims of control surface (elevators) [Lomax, 1996]. The other one is abrupt maneuver tail loads which is the maneuvers becomes a critical value about the structural tolerance [Decker, 2015]. The total load factor of aircraft reaches a maximum point as well as the tails load. These loads on horizontal tail are considered to determine the critical structural analyses. If the determined loads above the critical structural tolerance, the designing optimization or selecting material process are required to safety flight condition. So that, loads should be computed before the complete the design and product processes. All of aircraft maneuvers generate a rotational acceleration and the most effective of them for horizontal tail is pitching acceleration which is generated by abrupt pull-down and pull-up maneuvers. The other maneuvers also affect the loads on horizontal tail. In some other cases, the total load on horizontal tail may disturb as unsymmetrically. It is also analyzed as epistemic uncertainty on empennage loads during

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dynamic maneuver [Ruxandra et al, 2018]. The other type of load is gust loads which is generated by the perpendicular component of abrupt gusts on aircraft surfaces. The gust loads may also become dangerous as well as the abrupt pitching maneuver. The prediction of the empennage gust loads is also important about designing and structural analyzes processes [Philippe, 2000]. The other load is an aerodynamic load which is affected from shape of tail airfoil. The buffet loads which are effect from airflow around of the empennage are also considered [Schmit and Levinski, 2007].

Traditionally, the parameters from the solution of aircraft equations of motion (EoM) are considered independently of structural approach and analyses. Fundamentally, the mass distribution, moment of inertia and basic aerodynamic parameters etc. were considered only about aircraft structure. This methodology is incapable to define how this load is existed and how the other working disciplines associate each other on aircraft systems. The modelling of aircraft equation of motion on Simulink produces a generated loads time history graphs on each maneuver and determines the relationship between control surfaces and EoM. Thus, the resulting data defines the relationship between aircraft controls, EoM, structural approach. The same approach is using Lockheed Martin's AMC-X project [Marietta, 2008].

Determining the critic loads and classifying the type of loads are defined about the aircraft classman. In this paper, Federal Aviation Regulation (FAR) criteria's may be guiding the calculations of load programs and crtical aerodynamic loads are considered [Miedlar, 1997].

METHOD

Mathematical Model of Aircraft

Table 1: State Variables in EoM

p	Rolling rate (Body fixed frame)
q	Pitching rate (Body fixed frame)
r	Yawing rate (Body fixed frame)
u	x direction linear velocity (Body fixed frame)
v	y direction linear velocity (Body fixed frame)
w	z direction linear velocity (Body fixed frame)
I	Inertia
$\phi(phi),$	Roll (Euler) angle
$\theta(theta)$	Pitch (Euler) angle
$\psi(psi)$	Yaw (Euler) angle
F_G	Gravity Force
F_A	Aerodynamic Force
F_T	Thrust Force
L	Rolling moment (Also using for Lift)
M	Pitching moment
N	Yawing Moment
m	Mass

The aircraft EoM is underlying the mathematical model of aircraft. Aircraft EoM has different forms depends on reference sources. However, there are basic parameters which exists the fundamental of total model's output. Aircraft EoM is defining as [Yechout et al, 2003]:

Applied Forces:

$$
\begin{aligned}
m(\dot u + qw - rv) &= F_{G_x} + F_{A_x} + F_{T_x} \\
m(\dot v + ru - pw) &= F_{G_y} + F_{A_y} + F_{T_y} \\
m(\dot w + pv - qu) &= F_{G_z} + F_{A_z} + F_{T_z}
\end{aligned}
\qquad (1)
$$

Left-hand side and right hand-side are represented in body axis system. The right hand-side represents the basic forces acting on aircraft on each three axes.

Aplied Moments:

$$
\begin{aligned}
\dot p I_{xx} + qr\left(I_{zz} - I_{yy}\right) - (\dot r + pq)I_{xz} &= L \\
\dot q I_{yy} - pr(I_{zz} - I_{xx}) - (p^2 - r^2)I_{xz} &= M \\
\underbrace{\dot r I_{zz}}_{\text{Angular acceleration terms}} + \underbrace{pq\left(I_{yy} - I_{xx}\right)}_{\text{Gyro precession terms}} + \underbrace{(qr - \dot p)I_{xz}}_{\text{Coupling terms}} &= N
\end{aligned}
\qquad (2)
$$

The important point is the letter L is used as also for lift. The reader should check carefully the letter to use for. The assumptions made in developing the equations of motion were: the mass of the aircraft is constant, the aircraft is a rigid airframe, the Earth axis system is an inertial reference frame, the mass distribution of the aircraft is constant, and the aircraft has a xz plane of symmetry.

Longitudinal Equation of Motion:

The aircrafts xz plane is assumed coincident with an xz plane fixed in space. The aircraft can translation in x and z directions, and rotation about y axis. Thus, the forces are existed on x, z and moment is existed on y axis. In these equations L refers to lift.

$$
\begin{gathered}
m(\dot u + qw - rv) = -mgsin\theta + (-Dcos\alpha + Lsin\alpha) + Tcos\phi_T \\
\dot q I_{yy} - pr(I_{zz} - I_{xx}) + (p^2 - r^2)I_{xz} = M_A + M_T \\
m(\dot w + pv - qu) = mgcos\phi cos\theta + (-Dsin\alpha - Lsin\alpha) - Tsin\phi_T
\end{gathered}
\qquad (3)
$$

Lateral-Directional Equations of Motion:

Aircraft can translation in the y direction, roll about the x axis, and yaw about the z axis. The lateral-directional EOM consist of the y force, x moment, and z moment equations.

$$
\begin{gathered}
\dot p I_{xx} + qr\left(I_{zz} - I_{yy}\right) - (\dot r + pq)I_{xz} = L_A + L_T \\
m(\dot v + ur - pw) = mgsin\phi cos\theta + F_{A_y} + F_{T_y} \\
\dot r I_{zz} + pq\left(I_{yy} - I_{xx}\right) + (qr - \dot p)I_{xz} = N_A + N_T
\end{gathered}
\qquad (4)
$$

Kinematic Equation:

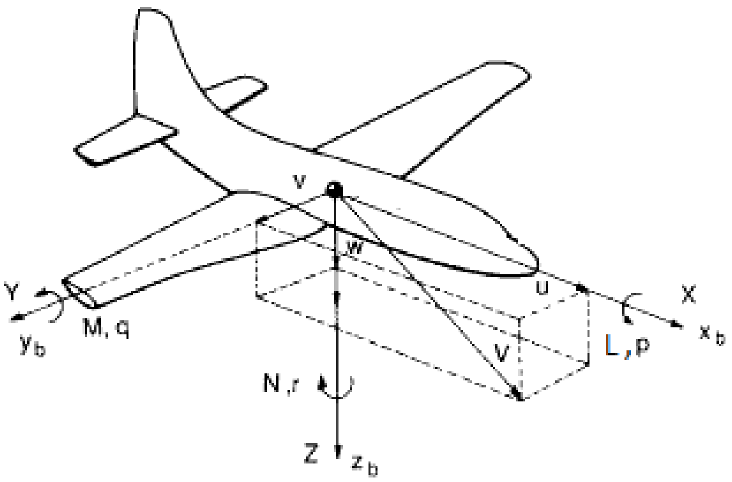

Figure 1: Definition of forces, moments and velocity components in a body fixed frame [Nelson, 1998].

The kinematic EoM are necessary because there are more than six unknowns due to the presence of the Euler angles in the force equations. These additional equations are helpful to explain the relationship between Earth axis system and Body axis system. In brief, the kinematic EoM:

$$
\begin{aligned}
p &= -sin\theta\dot{\psi} + \dot{\phi} \\
q &= sin\phi cos\theta\dot{\psi} + cos\phi\dot{\theta} \\
r &= cos\phi cos\theta\dot{\psi} - sin\phi\dot{\theta}
\end{aligned} \quad (5)
$$

The mathematical model is summarized above. The output parameters which are required to explain load equations will be determined. Figure 1 shows fundamental parameters on aircraft axis.

The Aerodynamic Equations of Horizontal Tail

Table 2: Required parameters

L_H	Horizontal tail lift
D_H	Horizontal tail drag
M_H	Horizontal tail moment
a	slope of lift curve for horizontal tail, per radian
a_g	geometric angle of attack of tail, measured from zero lift, ($a_W + i_H - i_W$)
a_H	Horizontal tail angle of attack
ϵ	downwash angle at tail caused by wing (or lifting system), positive downward
q	Dynamic pressure
q_H	Horizontal tail dynamic pressure
S_H	Horizontal tail referance area
$C_{M,0,H}$	pitching-moment coefficient of horizontal tail lift, positive nose up
$C_{L,H}$	Lift coefficient of horizontal tail

$C_{D,H}$	Drag coefficient of horizontal tail
$C_{D,0}$	Profile-drag coefficient of horizontal tail
$C_{M,H}$	Moment coefficient of horizontal tail
c_{mac}	Main aerodynamic chord of horizontal tail
η	Dynamic pressure ratio
M_{cg}	Total pitching moment about the cg
M_W	The pitching moment of wing
L_W	Lift of wing
M_F	The pitching moment of fuselage
M_E	The pitching moment of engines
M_N	The pitching moment of nacelles
x_{cg-ac}	Distance between cg-ac
l_H	Distance between wing ac and horizontal tail ac
i_W	angle between fuselage reference line and chord line of wing
i_H	angle between fuselage reference line and zero-lift line of horizontal tail
ρ	Density of air
V	Velocity of aircraft

The aerodynamic forces are determined as Lift and Drag create the moment because of the shape of airfoils. The following equations explain these concepts [Heyson, 1966]:

$$\begin{gathered} L_H = a\left(a_g - \epsilon\right)q\left(\frac{q_H}{q}\right)S_H = C_{L,H}q_H S_H \\ D_H = \left[C_{D,0} + \frac{a^2}{\pi A}\left(a_g - \epsilon\right)^2\right]q\left(\frac{q_H}{q}\right)S_H = C_{D,H}q_H S_H \\ M_H = \left[C_{M,0,H} + \frac{dC_{M,H}}{d_a}\left(a_g - \epsilon\right)\right]q\left(\frac{q_H}{q}\right)S_H c_{mac} = C_{M,H}q_H S_H c_{mac} \end{gathered} \tag{6}$$

The dynamic pressure q_H is less than the dynamic pressure of wing. The reason for this is a delay in the flow caused by the wing drag. The reduction in dynamic is expressed by the dynamic pressure ratio.

$$\eta = \frac{q_H}{q} \tag{7}$$

Tail cannot be considered independently from total aircraft. The aerodynamic forces also affect the total stability of aircraft system. Aircraft's total forces, moments and angels are showed in Figure 2 and Figure 3. Total pitching moment of aircraft about the center of gravity is [10]:

$$M_{cg} = M_W + L_W x_{cg-ac} + M_F + M_E + M_N - L_H\left(l_H - x_{cg-ac}\right) + M_H \tag{8}$$

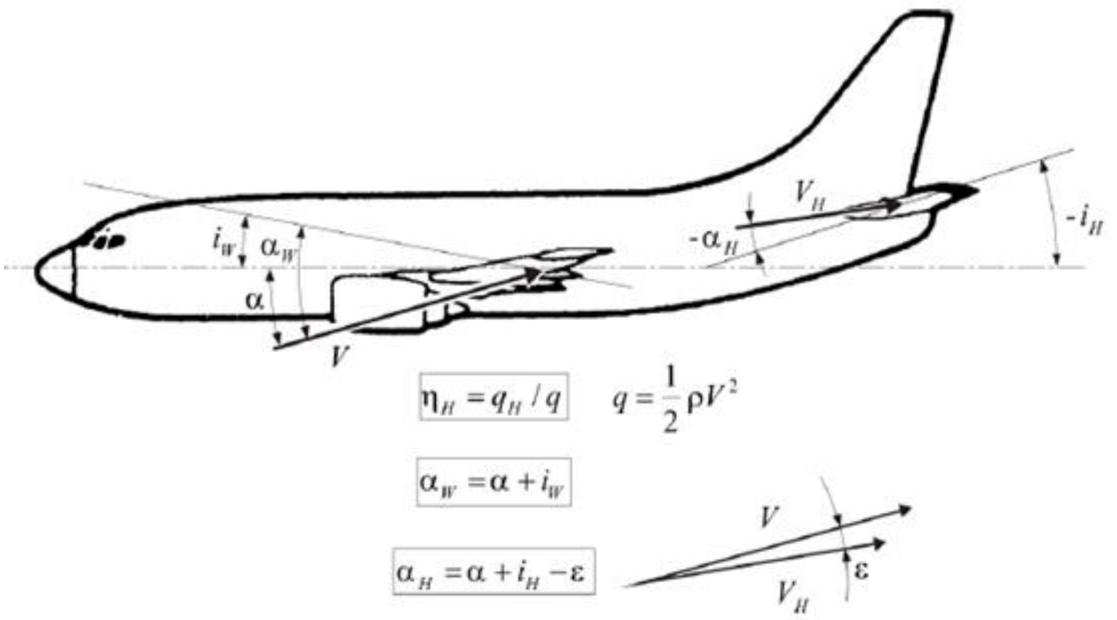

Figure 2: Angles and flow velocities to calculate pitching moment [Scholz, 2013].

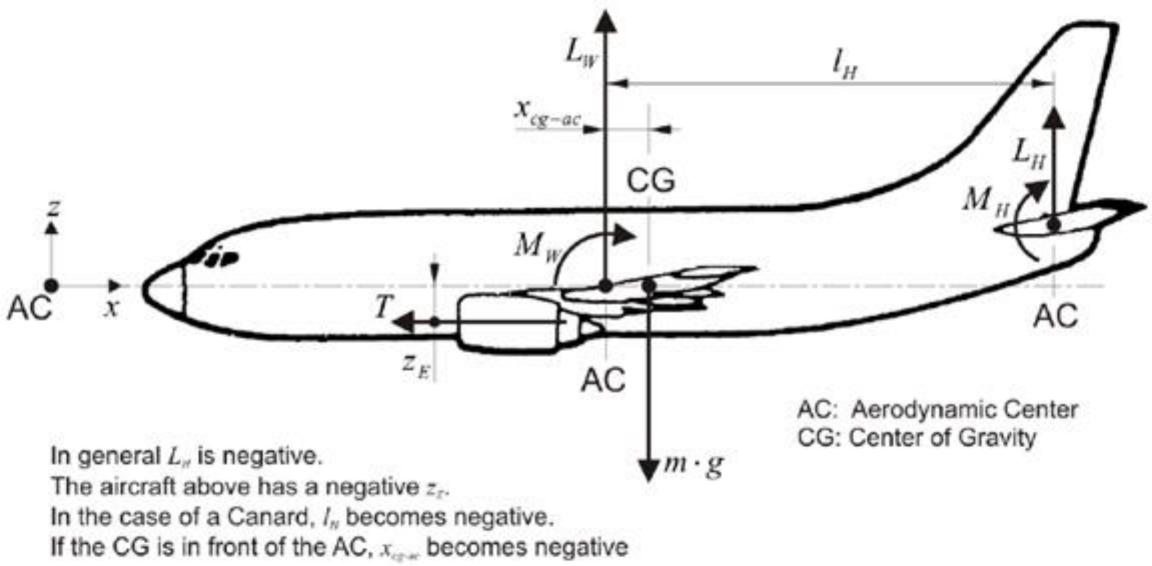

Figure 3: Forces, moments and lever arms to calculate pitching moment [Scholz, 2013].

Horizontal Tail Loads:

Horizontal tail loads should be considered for designing significant part of aircraft. Generally, the horizontal tail stabilizer and elevator, the body structure aft of the pressure bulkhead and horizontal tail support structure, the aft fuselage monocoque structure, the fuselage center section (overwing) structure, and the stabilizer actuator (jackscrew mechanism) are affected by horizontal tail loads. In this page the horizontal tail load types are determined with respect to aircraft longitudinal axis maneuvers.

<u>Horizontal Tail Load in Longitudinal Axis Maneuvers:</u>

1. Steady-State Symmetrical Maneuver:

This type of load has a relationship with pitching moment, stabilizer angle of attack, and elevator angle. In steady-state maneuver, the pitching acceleration is assumed zero. The summation of the forces in the x and z axes is:

$$N_F = n_z W = L cos a_w + D sin a_w$$
$$C_F = n_z w + T_{eng} = D cos a_w - L sin a_w \tag{9}$$

The balance equations on pitching condition:

$$L + L_H = n_z W$$
$$M_{0.25} + n_z W x_{cg-ac} + n_z W z_a = L_H l_h - M_H - T_{eng} z_e \tag{10}$$

The horizontal drag terms are neglected, because small with respect to the effect on aircraft pitching moment. Different approach for L_H is:

$$L_H = \frac{[(cg - 0.25)C_{La} + C_M] q S_w c_w}{l_h} + \frac{M_H}{l_h} + \frac{T_{eng} z_e}{l_h} \tag{11}$$

So that the balancing tail load (BTL) is determining as a traditional equation:

$$BTL = \frac{[(cg - 0.25)C_{La} + C_M] q S_w c_w}{l_h} + \frac{M_H}{l_h} + \frac{T_{eng} z_e}{l_h} = L_H - \frac{M_H}{l_h} \tag{12}$$

The load factor acting along the x axis is:

$$n_x = \frac{\left(D cos a_w - L sin a_w - T_{eng}\right)}{W} \tag{13}$$

If the aerodynamic coefficient using as linearized, BTL becomes:

$$C_L = C_{L0} + C_{Law} a_w$$
$$C_{M0.25} = C_{M0} + \frac{dC_M}{dC_L} C_L \tag{14}$$

$$BTL = \frac{\left[n_z W \left(CG - 0.25 + \frac{dC_M}{dC_L}\right) + C_{m0} q S_w\right]}{\frac{l_h}{c_w} + \frac{dC_M}{dC_L}} \tag{15}$$

$$a_w = \frac{\left[\frac{n_z W - BTL}{q S_w} - C_{L0}\right]}{C_{Law}} \tag{16}$$

Using the graphical solution is more useful when the aerodynamic coefficients are nonlinear BTL becomes:

$$C_{La} = \frac{n_z W}{q S_W} \tag{17}$$

$$C_L = C_{La} - \frac{BTL}{qS_W}$$

$$BTL = \frac{[(cg - 0.25)C_{La} + C_{M0.25}]qS_W\, c_w}{l_h} \tag{18}$$

Calculating $C_{M0.25}$ process:

1) Determine axis system representing the center of gravity for aircraft, which for this example is noted as a forward center of gravity$(cg - 0.25)C_L$.

2) Calculate the airplane lift coefficient.

3) Calculate the sloping line representing the tail-off lift coefficient from related graphs.

4) The resulting intersection of this slope with the tail-off pitching moment curve will give the desired pitching moment coefficient $C_{M0.25}$ and the tail-off lift coefficient C_L .

5) The reference wing angle of attack is determined from the tail-off lift coefficients graphs.

6) Balancing tail loads are then calculated.

If there is power on condition:

$$\Delta C_M = \frac{T_{eng} z_e}{qS_W\, c_w} \tag{19}$$

The graphical method of solving for balancing tail loads and wing angle of attack for nonlinear coefficients may also be solved by using the nonlinear representation of the lift and pitching moment curves in a table.

2. Abrupt Pitching Maneuver:

Horizontal tail loads due to the abrupt elevator input:

$$\begin{aligned} \Delta L_{H\theta} &= L_{a_h}\Delta a_h + L_{\delta_e}\Delta\delta_{emax} \\ \Delta M_{H\theta} &= M_{a_h}\Delta a_h + M_{\delta_e}\Delta\delta_{emax} \end{aligned} \tag{20}$$

Lift coefficient due to the elevator deflection [Philip et al, 2015]:

$$C_{L_{\delta_e}} = \frac{S_H \times \eta \times C_{L,a,H} \times \tau}{S_W} \tag{21}$$

The aircraft response factor for abrupt unchecked maneuver:

$$k_r = \frac{\Delta L_{H\theta}}{L\Delta\delta_{emax}} \tag{22}$$

Change in stabilizer angle of attack:

$$\Delta a_H = (k_r - 1)\left(\frac{L_{\delta_e}}{L_{a_h}}\right)\Delta\delta_{emax} \tag{23}$$

Horizontal tail load due to abrupt unchecked elevator in terms of k_r:

$$L_H = L_{hnz=1} + k_r L_{\delta_e} \Delta\delta_{emax} \tag{24}$$

Relationship between horizontal tail load due to abrupt checked elevator and pitching acceleration is:

$$I_y \ddot{\theta} = M_{H\ddot{\theta}} - l_h L_{H\ddot{\theta}} \tag{25}$$

Neglect the horizontal tail pitching moment term as small, net horizontal tail load in checked elevator is:

$$L_H = L_{hnz} - \frac{I_y \ddot{\theta}}{l_h} \tag{26}$$

Simulink Models:

Established models are showed in Figure 4 below.

Eq. (20) and Eq. (21) are combined in Abrupt Pitching Tail Load Model block. To reach the solution, required equations are provided from different sources and became available in this process. Thus, aerodynamic coefficients are used with dynamic equations. Considering aerodynamics and mechanics of aircraft as related systems, the method obtained above is developed.

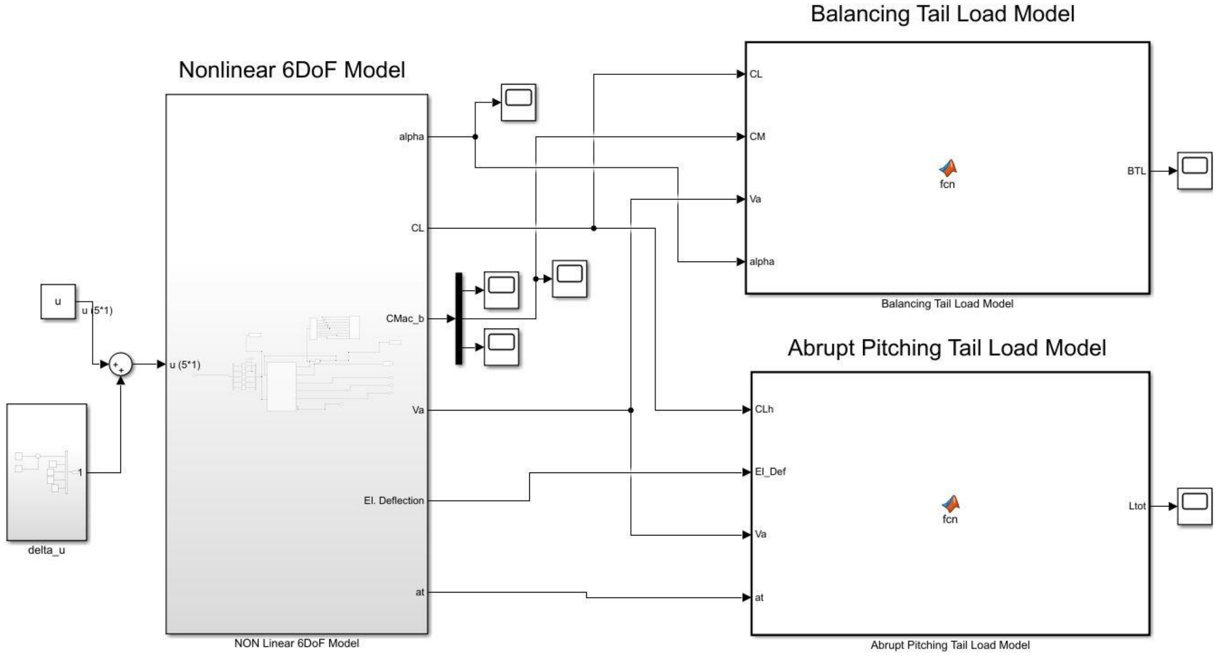

Figure 4: Simulink loads model of BTL and APTL

Results and Discussion:

Balancing tail load in steady-state maneuver is related with all parameters above. However, the elevator deflection is most effective variable in this process. Figure 5, shows the balancing tail load in steady-state maneuver ($\delta_e = 0$) in terms of newton (N). In the designing, selecting the composite material and optimization processes, maximum BTL may be considered. However, BTL may not be critical load value.

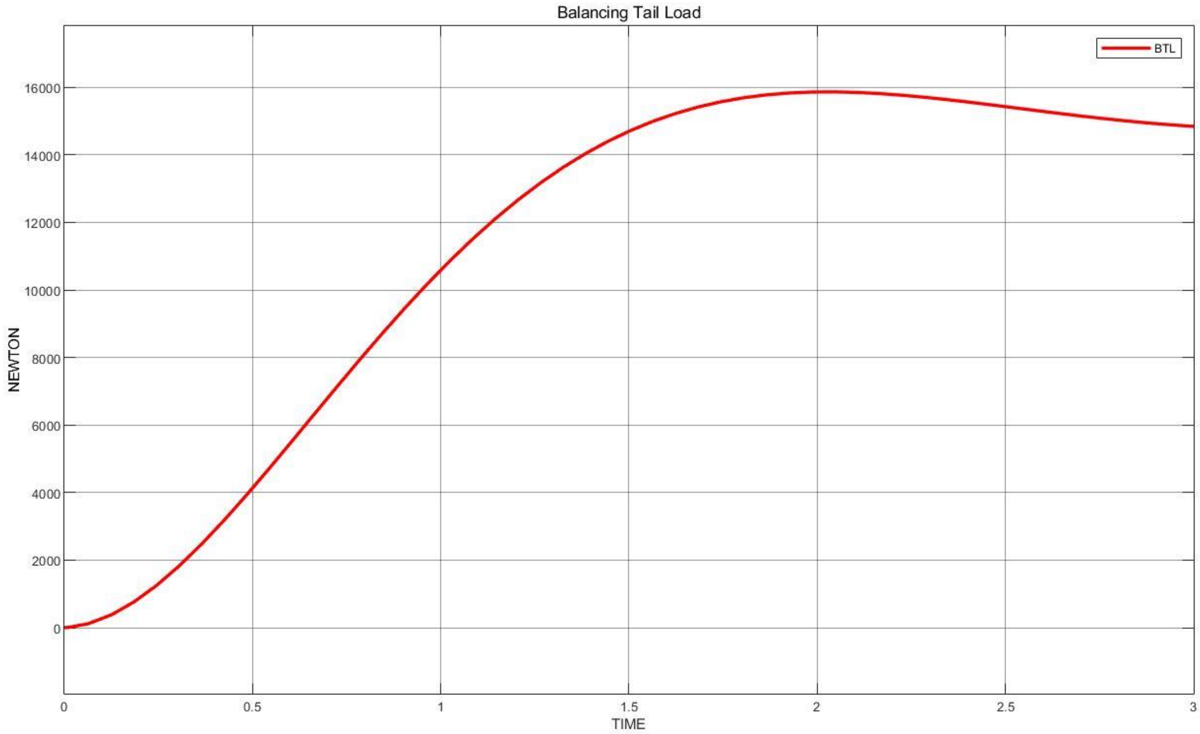

Figure 5: Balancing Tail Load

Horizontal tail load in abrupt elevator trim condition is calculated in Figure 6. If the peak points in two graphs are compared, abrupt pitching horizontal tail load is bigger than balancing tail load. Besides, the load lines which is shown as red in both of graphs are different. Load line in abrupt pitching condition has high value. This condition may cause in fatigue effect on material. Thus, it becomes critical condition. Abrupt pitching maneuver may cause of critical load situation. Therefore, this type of load should be considered.

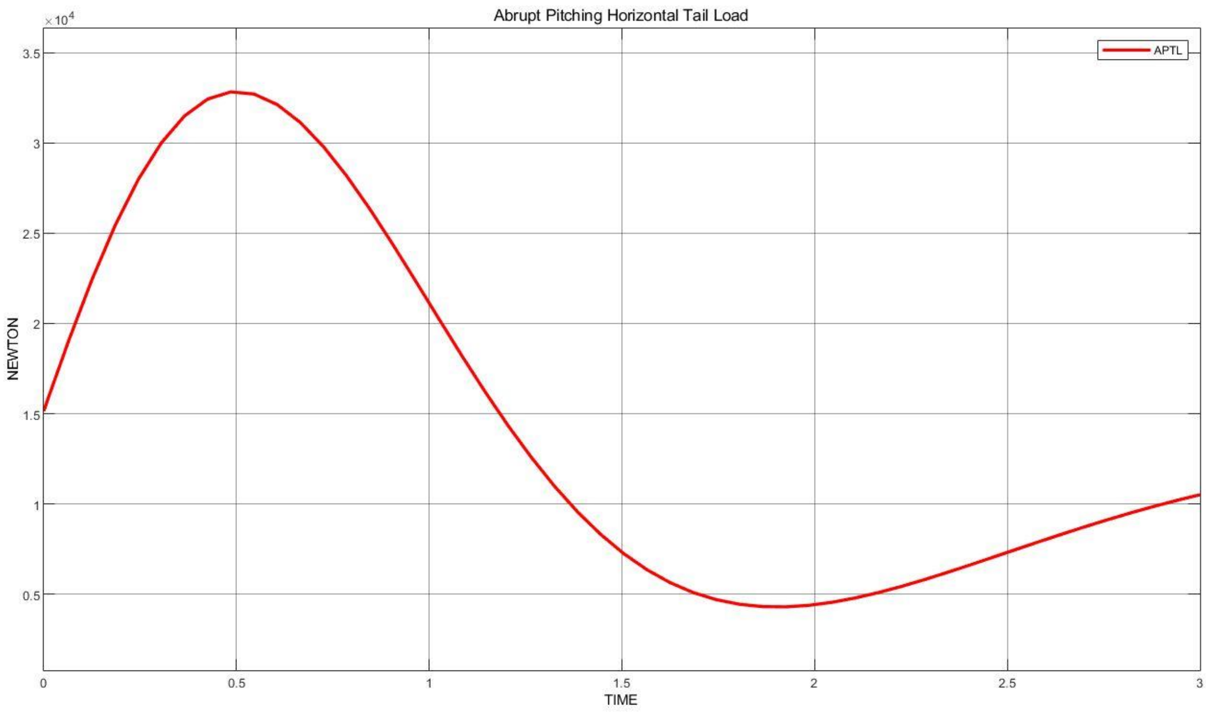

Figure 6: Abrupt Pitching Tail Load

Aircraft mathematical model makes clear to understand relationship between aircraft dynamics and load programs. Pitching acceleration which is one of the mathematical model outputs may be critical parameter to consider in longitudinal axis. All of the outputs have a mathematical relationship each other. Figures below (Figure 7 and Figure 8), show state variable. Deflection on the aircraft control surfaces is considered as input. Instead of close loop method, open loop method is used in this model because it can provide to reach critical values with less calculation.

Table 3: Basic Initial Values

δ_{emax}	$0.46\ rad$
I_y	$64\ m^4$
l_h	$24.8\ m$
S_W	$260\ m^2$
S_H	$64\ m^2$
c_w	$6.6\ m$
m	$120\ 000\ kg$

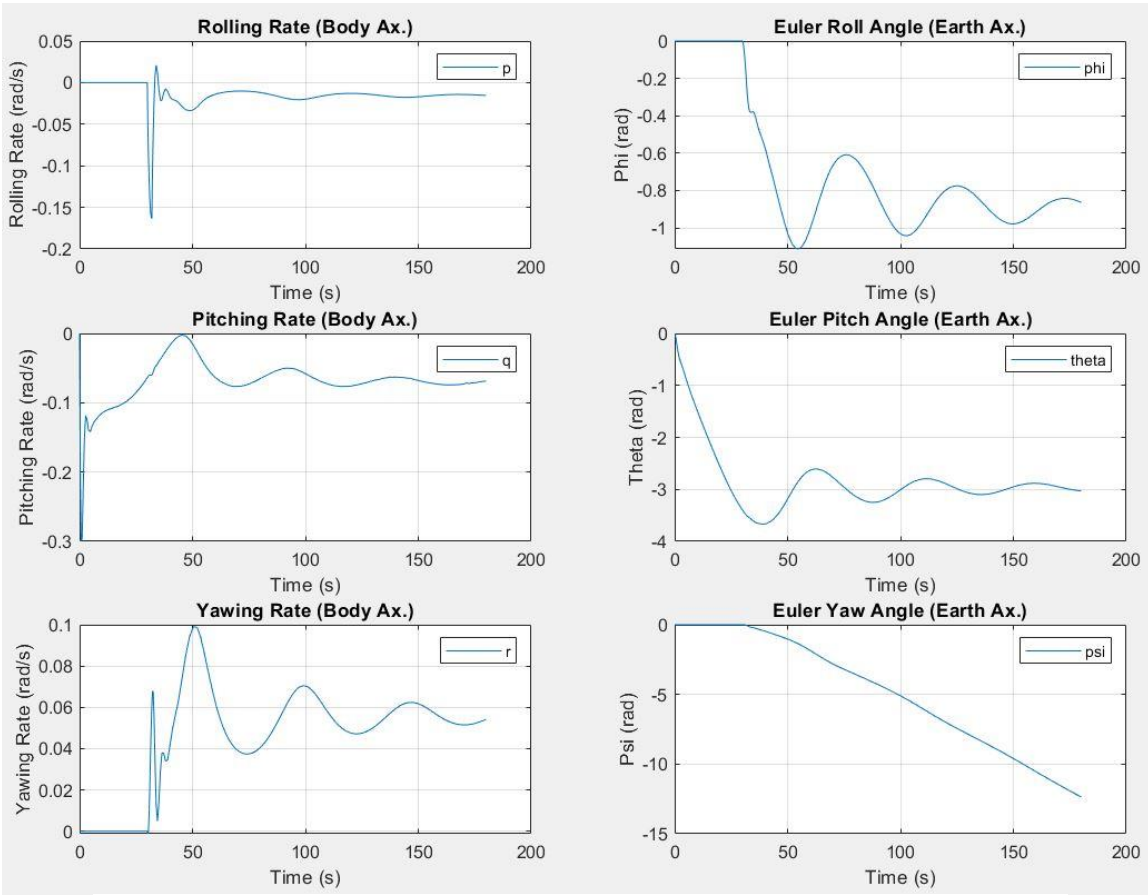

Figure 7: Rotational Velocities and Euler Angles

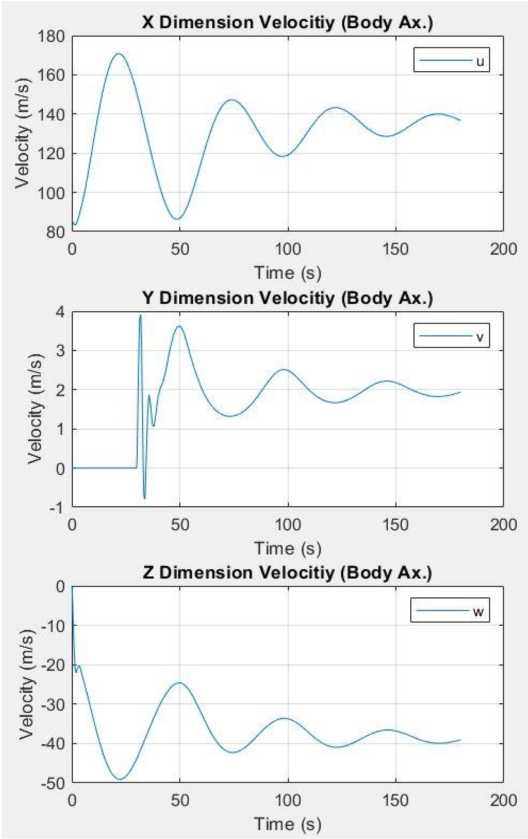

Figure 8: Translational Velocities

Conclusions and Future Works

Determining critical horizontal tail loads on aircraft are important points to consider in design, selecting material etc. processes. For this determination, in this paper aircraft mathematical model is established on MATLAB/Simulink. Using with aerodynamic and flight mechanics, horizontal tail loads were calculated. Output parameters of mathematical model are used as input parameters of load models of horizontal tail. In this paper, aircraft maneuvers in longitudinal axis are considered. Thus, established model on Simulink may be used for designing of aircraft significant parts. To be able to determine designing of horizontal tail, optimization for minimum effects from calculated loads and aeroelasticity analyses on horizontal tail planned as future work.

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