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A SUB-OPTIMAL STAGING TOOL FOR MULTI-STAGE LAUNCH VEHICLES

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ABSTRACT

In order to start to design a launch vehicle, mission requirements, such as the payload mass, required ΔV budget, target orbit, path constraints, acceleration thresholds, and dynamic pressure, must be known. However, in preliminary design phase, these requirements can be reduced to only two: the payload mass and the required ΔV. It is known that the design of a multi-stage launch vehicle can optimally be determined with these two parameters. Nevertheless, in most cases, current technological level, mission requirements or time constraints limit the design and the optimality deteriorate. In order to include such constraints, a sub-optimal staging tool is developed in this study. Having developed the tool, a commercial and active launch vehicle's optimality is analyzed. Comparing the staging tool output and the Vega launch vehicle, the tool resulted 14.3% *greater performance.*

INTRODUCTION

Design, manufacturing and trajectory optimization of a launch vehicle (LV) requires great effort in terms of both technology and economy due to complex nature of the LVs. Designs should be kept as simple as possible, however, it is not always viable to choose the simpler design due to technological constraints. For instance, multi-staging which uses the idea of disposal of unnecessary structural masses in order to accelerate the vehicle faster with the same amount of propellant, are generally employed for space launches, even though the simpler single-stage-to-orbit (SSTO) configurations are being argued for decades and considered as feasible [Freeman, Talay, & Austin, 1996].

In order to manufacture a feasible, affordable and competitive LV, the design process is essential. Once a LV is not designed very well at the early phases of design, extensions on schedules of design and manufacturing processes are possible. Even if a LV with a poor design is manufactured somehow, much larger scaled troubles are likely to appear when the time comes to launch phase. It is probable to have failed launch attempts with such a design whether the trajectory is optimized at the state-of-the-art level. What's worse, the whole space program may be canceled with fatalities like happened in the Brazilian case [Brazilian Rocket Explodes On Pad: Many Dead, 2003].

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METHOD

A single stage vehicle's ΔV capacity can be determined easily from the famous equation of Konstantin Tsiolkowsky [Brown, 2002], i.e. ideal rocket equation, which is derived from Newton's second law of motion.

$$\Delta V = V_{exh} \ln\left(\frac{m_0}{m_f}\right) \tag{1}$$

In equation (1), m_0 and m_f are the initial and final masses (kg), respectively, while V_{exh} stands for the nozzle exhaust velocity (m/s) of exhaust products with respect to (wrt) the vehicle. V_{exh} can be determined by:

$$V_{exh} = I_{sp} g_0 \tag{2}$$

Where, I_{sp} is the specific impulse (s) and g_0 is the gravitational acceleration on earth which is equal to $9.80665\ m/s^2$. It is possible to write m_0 and m_f in equation (1) as

$$m_0 = m_s + m_p + m_{pl} \tag{3}$$

$$m_f = m_s + m_{pl} \tag{4}$$

In equation (4), m_s, m_p and m_{pl} are structural, propellant and payload masses (kg), respectively. Since dealing with dimensionless parameters instead of mass terms is advantageous in such problems, mass ratios are defined. The most convenient and useful ratios are the payload ratio,(π_*) and the structural factor (ϵ).

$$\pi_* = \frac{m_{pl}}{m_0} \tag{5}$$

The first, payload ratio, which is an indicator of overall performance of LVs, shows the ratio of essential mass of the vehicle to overall mass. For low earth orbit (LEO) missions, this ratio is around 1% [Payload Fraction, 2021].

$$\epsilon = \frac{m_s}{m_s + m_p} \tag{6}$$

The second, structural factor, is the ratio of structural mass to overall mass excluding the payload mass. It is also a design criterion and an indicator of technological level. The final mass to overall mass can be rewritten as

$$\frac{m_f}{m_0} = \frac{m_s + m_{pl}}{m_s + m_p + m_{pl}} + \frac{m_p - m_p}{m_s + m_p + m_{pl}} = 1 - \frac{m_p}{m_0} \tag{7}$$

Multiplying the nominator and the denominator with $(m_s + m_p)$, one can get

$$\frac{m_f}{m_0} = 1 - \frac{m_s + m_p}{m_0}\frac{m_p}{m_s + m_p} \tag{8}$$

Now it is easier to substitute the dimensionless parameters into equation (8):

$$\frac{m_f}{m_0} = 1 - (1 - \pi_*)(1 - \epsilon) \tag{9}$$

$$\frac{m_f}{m_0} = \epsilon + (1 - \epsilon)\pi_* \tag{10}$$

Finally, substituting equation (10) into equation (1), noting that the term in equation (10) is m_f/m_o while in equation (1) is m_0/m_f, the ideal rocket equation can finally be written with mass ratios instead of mass parameters.

$$\Delta V = -V_{exh} ln(\epsilon + (1-\epsilon)\pi_*) \tag{11}$$

Equation (11) shows that the ΔV of a rocket is strictly dependent to exhaust velocity, structural factor and payload ratio.

Multiple Staging

Staging uses the idea of the disposal of unnecessary mass during the ascent. Once the propellant of a stage is consumed, it separates and the next stage is ignited. Having ejected the structural mass of the previous stage, the vehicle accelerates much faster [Coşkun, 2013]. A serial stage configuration together with mass definitions are illustrated in Figure 2.1 :. Owing to staging phenomena which is suggested first by Tsiolkowsky [Wiesel, 1989], accelerating a vehicle to orbital velocities is much feasible than it would be for SSTO designs with current technology.

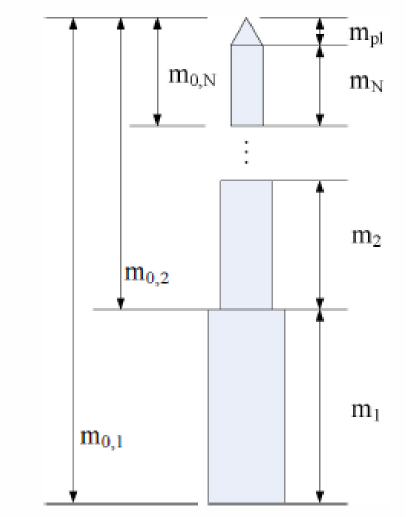

Figure 2.1 : Serial staging mass definitions.

In order to determine the total ΔV of multiple stage, first, the dimensionless parameters should be redefined in accordance with the stage numbers. Before starting the definitions, some perspectives should be highlighted. The payload of a stage is defined as the final mass of the rocket excluding the stage's structural mass. Correspondingly, overall mass of (n+1)th stage completely is the payload of nth stage. The payload ratio of a stage for multistage vehicles can be calculated via:

$$\pi_i = \frac{m_{pl,i}}{m_{0,i}} = \frac{m_{0,i+1}}{m_{0,i}} \tag{12}$$

The subscript i stands for the stage number where the preceding subscript of 0 means the gross or sub-gross mass. It is also helpful to define the overall payload ratio, π_*, which is the product of stage payload ratios.

$$\pi_* = \frac{m_{pl}}{m_{0,1}} = \frac{m_{pl}}{m_{0,n}} \frac{m_{0,n}}{m_{0,n-1}} \dots \frac{m_{0,2}}{m_{0,1}} \tag{13}$$

Since the structural factor excludes payload mass and deals only with the propellant and structural masses of the stage of interest, no big differences in the definition occur.

$$\epsilon_i = \frac{m_{s,i}}{m_{s,i} + m_{p,i}} \tag{14}$$

Having redefined the dimensionless parameters, now it is possible to proceed the calculation of the total ΔV (ΔV_*) of the rocket. It can simply be calculated summing ΔV of each stage successively.

$$\Delta V_* = \sum_{i=1}^{n} -V_{exh,i} ln(\epsilon_i + (1-\epsilon_i)\pi_i) \tag{15}$$

Using the same exhaust velocities and structural ratios, as 0.1, change of ΔV_* with payload ratio and number of stages is shown in Figure 2.2 :. It is apparent that the number of stages improve the ΔV_*, however, advantage of increasing the number of stages dwindle rapidly as there exist a limiting value for ΔV_*. Also, costs and reliability worsen with increasing number of stages, hence, having the least number of stages that provide the required ΔV_* comes out to be the best configuration.

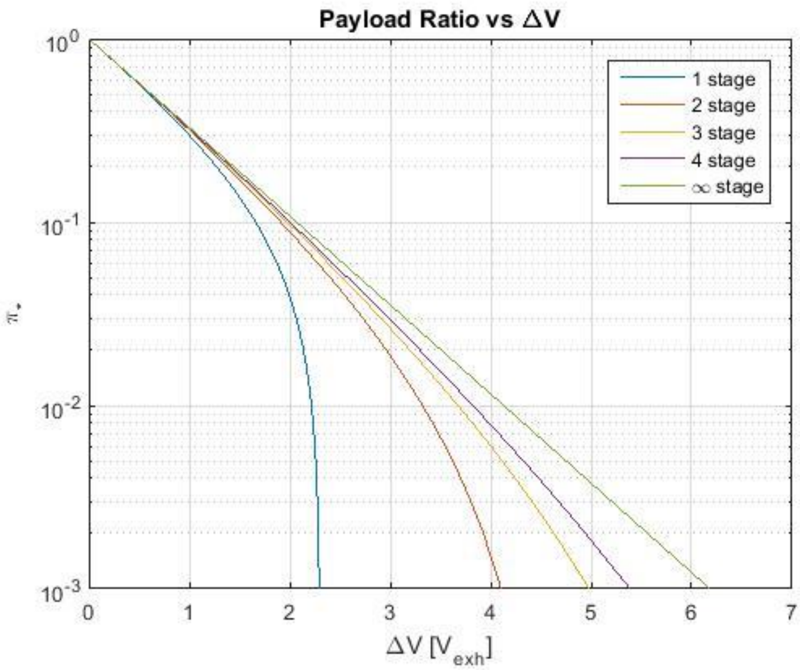

Figure 2.2 : ΔV_* versus payload ratio by number of stages.

Optimal Staging

Multistage rockets have certain advantages over single stage rockets as explained in the previous sections. On the other hand, staging brings the optimality problem together with it, since it is possible to accelerate a payload to target velocity with various designs. Remembering the ΔV_* equation, equation (15), V_{exh}, ϵ and π can have various values that satisfy the required ΔV_*. Freezing V_{exh} and ϵ values to available technological limits, only parameter left to be determined is the π for each stage. Optimization problem here can be treated as maximizing the overall payload ratio satisfying the required ΔV_*.

$$\pi_* = \prod_{i=1}^{n} \pi_i \tag{16}$$

Taking natural logarithm of both sides in equation (16) equation (17) can be obtained.

$$\ln \pi_* = \sum_{i=1}^{n} \ln \pi_i \tag{17}$$

Now the optimization problem can be written in more convenient way

$$maximize \ \ln \pi_* = \sum_{i=1}^{n} \ln \pi_i \tag{18}$$

$$subject\ to\ \Delta V_* = \sum_{i=1}^{n} -V_{exh,i} ln(\epsilon_i + (1-\epsilon_i)\pi_i) \tag{19}$$

The maximum can simply be determined by calculating n partial derivatives of equation (18) and setting them to zero if there was no constraint. Having the constraint equation (19), one of the π_i is dependent to $V_{exh,i}$, ϵ_i and V_*. Solving the equation (19) for dependent π_i, substituting it into equation (18) and calculating $n-1$ partial derivatives for independent π_i would give the result. However, since this type of problems can also be solved using Lagrange multiplier method, no complex mathematical procedure is required. The method suggests to multiply the constraint equation with an arbitrary number, called Lagrange multiplier, and to add the multiplication to objective function.

$$\ln \pi_* = \sum_{i=1}^{n} \left[\ln \pi_i + \lambda \left(\frac{\Delta V_*}{n} + V_{exh,i} ln(\epsilon_i + (1-\epsilon_i)\pi_i) \right) \right] \tag{20}$$

Applying the method, the expression in equation (20) is obtained. In equation (20), λ is the Lagrangian multiplier. The multiplied term in parenthesis is in fact a complex form of zero. Taking partial derivative of equation (20) as if each of the π_i is independent, the expression given in equation (21) is evaluated.

$$\frac{\partial \ln \pi_i}{\partial \pi_i} = \frac{1}{\pi_i} + \frac{\lambda V_{exh}(1-\epsilon_i)}{\epsilon_i + (1-\epsilon_i)\pi_i} \tag{21}$$

In order to solve equation (21), λ is chosen so that the equation is zero for dependent π_i. Having the rest of the π_i independent, $n-1$ equations can now be set to zero.

$$\pi_i = -\frac{\epsilon_i}{(1-\epsilon_i)(1+\lambda V_{exh,i})} \tag{22}$$

The specific λ value setting equation (21) zero for dependent λ can be determined by substituting equation (22) into constraint equation (19).

$$\Delta V_* = -\sum_{i=1}^{n} V_{exh,i} \ln \left(\epsilon_k - \frac{\epsilon_k}{1+\lambda V_{exh,i}} \right) \tag{23}$$

Equation (23) is quite challenging to solve for λ analytically but can easily be solved numerically. Once determined, it is substituted into equation (22) to obtain each π_i for optimal mass distribution. In some special cases in which all $V_{exh,i}$ and ϵ_i are equal, each π_i becomes equal to each other for an optimal design, however, that is not the case for most of the designs.

Even though an optimal design is chosen for a LV initially, its optimality deteriorate as the design and production processes advance. In most cases, initial design parameters cannot be achieved and the gross mass of the vehicle increases [Brown, 2002]. Still, it provides an elegant starting point. Even if the exact optimality cannot be achieved, it assists the design to be close to optimal which lighten the gross mass for any specified payload mass. Beyond that, with better assumptions for ϵ and V_{exh}, the optimality can be conserved almost completely even at the final phases of design.

Design of a Launch Vehicle

At this point, with given ϵ and V_{exh} for each stage, an optimal staging can be determined for any $\Delta V_* < \sim 6V_{exh}$ and m_{pl}. As described in the previous section, an optimal staging tool (OST) can be used to design any vehicle of interest following the procedure. In order to have a reasonable comparison, a LV with similar performance parameters to a commercial and active LV, the Vega LV, will be designed.

It is designer's choice to pick any mission type of the Vega LV to start design since a vehicle designed for one of the missions, will be capable of achieving any mission of the Vega. Among the missions, 750 km Sun synchronous orbit (SSO) mission of which the payload capacity is 1300 kg, is chosen in this study [Ariane Space, 2014]. Referencing the data for mass and performance parameters for Vega, i.e. ϵ and I_{sp} values, an optimal design is obtained from OST for a m_{pl} of

$1352\ kg$ and a ΔV_* of $9282.7\ m/s$ which is the ΔV performance of the Vega LV without fairing ejection.

Table 2.1 : shows a comparison on designs for the Vega and the OST. ϵ for the 4th stage is marginally high, though, it is not rare for upper stages to have a bad ϵ due to sensitivity requirements and relatively low amount of propellant capacity. As a result, the OST gave trivial results such as $\pi > 1$ for the 4th stage. To have non-trivial designs with the OST in such cases, there are some tricks to deceive the OST. In the first, some of necessary mass of avionics, actuators, payload adaptor etc., all included here as structural mass, can be considered as payload mass. In this manner, the OST will perceive the upper stage with a higher performance, and distribute the rolls of each stage in a way obeying the design constraints. Another method to deceive the OST is taking all upper stage and m_{pl} as payload. Subtracting the upper stage ΔV capacity, the design can be made for 3 stage rocket. However, this method can be applied only if the upper stage design is an advanced level considering its mass and ΔV capability is required.

Table 2.1 : Vega vs OST design comparison.

	Stages	1	2	3	4
Common	ϵ	0.0887	0.0945	0.1190	0.5440
	V_{exh}	2745.9	2819.4	2901.8	3085.2
Vega	π	0.3011	0.3656	0.2083	0.5993*
OST	π	0.2076	0.2054	0.2446	1.8346*

* including fairing

The 1st method is followed in this study since the upper stage also is in design process and less information required for the upper stage. It is possible to find a “sub-optimal” design via this method with an iterative process. Remembering the OST results payload ratios and the final payload mass, general mass distribution of the vehicle including each stage’s propellant and structural masses can be obtained.

As before, the method suggests manipulating the payload and the structural masses. Starting from the real payload mass, the manipulated payload mass ($m_{pl,m}$) is increased until the sum of this manipulated payload mass and manipulated OST output of 4th stage structural mass ($m_{s,4m}$) is equal to sum of m_{pl} and real (estimated) $m_{s,4}$.

$$m_{pl,m} = m_{pl} + m_m \tag{24}$$

Accordingly, $m_{s4,m}$ should be defined as

$$m_{s4,m} = m_{s4} - m_m \tag{25}$$

Where, m_m is the manipulated mass and will be increased until the equation (26) is satisfied:

$$m_{pl} + m_{s4} = m_{pl,m} + m_{s4,m} \tag{26}$$

It is designer’s task to estimate the $m_{s,4}$ with the available technology.

Starting a new design, it is necessary to make another assumption on the fairing since it is ejected at some point during the powered flight. In this case, such an assumption is not required as the Vega LV is imitated. However, it is sure better to show the calculations as a guide. Continuing with the Vega LV, it is reasonable to assume the fairing as the 3rd stage structural mass considering it is ejected during the 3rd stage flight [Ariane Space, 2014], yet, this would deteriorate the ϵ_3 and the OST will decrease the roll, therefore the mass, of the 3rd stage. Instead, it is more appropriate to take the fairing as payload with a coefficient smaller than 1. At the initial phases of the ascent, the vehicle is much less sensitive to mass variations. For $1\ m/s$ velocity change of Vega requires a final mass variation of roughly 19 kg, 6.5 kg, 1.7 kg and 0.7 kg for 1st, 2nd, 3rd and upper stages, respectively. Using a course weighting, noting the fairing ejection is held in the middle of 3rd stage boost:

$$(m_{fair} - m_{fair,m}) * \left(\frac{1}{19} + \frac{1}{6.5} + \frac{0.5}{1.7}\right) = m_{fair,m}\left(\frac{1}{0.7} + \frac{0.5}{1.7}\right) \quad (27)$$

$$(m_{fair} - m_{fair,m}) * 0.5006 = m_{fair,m} * 1.7227 \quad (28)$$

Where m_{fair} and $m_{fair,m}$ are the real and the manipulated fairing masses. Gathering all manipulated fairing mass terms to left hand side:

$$\frac{m_{fair}}{m_{fair,m}} - 1 = 3.4413 \quad (29)$$

$$m_{fair,m} = \frac{m_{fair}}{4.4413} \quad (30)$$

Remembering the fairing mass is $540\ kg$

$$m_{fair,m} = 121.59\ kg \quad (31)$$

Manipulating the fairing mass as $121.59\ kg$, ΔV gained with the decrement of $418.41\ kg$ in the first 2.5 stage will be compensated by 121.59 kg extra mass in the upper 1.5 stage. This assumption sure has deficiencies, though, it is sufficient at initial phases of design.

Returning to manipulated payload mass, with $m_{fair,m} = m_{fair}$, it can be redefined as

$$m_{pl,m} = m_{pl} + m_m + m_{pla} + m_{fair,m} \quad (32)$$

Where m_{pla} is the payload adapter mass. The only unknown parameter in equation (32) is m_m. Starting from 0, value of the m_m will be increased until equation (25) is satisfied together with the OST output m_{p4} is equal to AVUM propellant mass. The parameter ϵ_4 automatically gets its manipulated form $\epsilon_{4,m}$, in order to manage the mass distribution of the stages and upper stage propellant and structural masses.

$$\epsilon_{4,m} = \frac{m_4\epsilon_4 - m_m}{m_4 - m_m} \quad (33)$$

It is an easy task to determine the value of the m_m manually considering high sensitivity is not a concern at this point. After couple of trials, an m_m of $470\ kg$ and an $\epsilon_{4,m}$ of 0.2744 are found to satisfy the conditions.

RESULTS and CONCLUSION

Table 2.2 : shows the Vega and the SOST mass distributions in detail. With the same upper stage performance and payload mass, the SOST designed a 12.5% lighter LV which makes the SOST design 14.3% greater performanced vehicle in terms of payload ratios. The Vega's performance still should be considered as good compared to the SOST since along the design and production processes, unexpected problems probably appear and the optimality deteriorate. Comparing the stage masses, the most optimality-spoiling stage is the 3rd one as seen from Table 2.2 :. Though the actual reason is not known, reasons why the 3rd stage designed much heavier than it optimally should be are possibly technological inability to keep same ϵ for a lighter stage, shortening the design process benefiting from a more ready-to-launch stage, and production constraints.

Table 2.2 : Vega vs SOST design parameters.

	Stages	1	2	3	4
Common	V_{exh}	2745.9	2819.4	2901.8	3085.2
Vega	ϵ	0.0887	0.0945	0.1190	0.5440
	m (kg)	96243	26300	12000	1265
	m_p	87710	23814	10567	577
	m_s	8533	2486	1433	688
	π	0.3011	0.3656	0.2083	0.5993*
	m_{pl} & π_*	1352 & 0.0098			
SOST	ϵ	0.0887	0.0945	0.1190	0.2744#
	m (kg)	86674	24548	6253	772#
	m_p	78986	22228	5509	560
	m_s	7688	2320	744	682**
	π	0.2801	0.2721	0.3186	0.7358#
	m_{pl} & π_*	1352 & 0.0112			

* including fairing
\# manipulated parameters
** $m_{s4} = m_{s4,m} + m_m$

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