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COVARIANCE-SCALING ROBUST ATTITUDE ESTIMATION USING INERTIAL MEASUREMENTS

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ABSTRACT

Considering the attitude (i.e., roll and pitch) estimation problem using an inertial measurement unit (IMU) composed of accelerometer and gyroscope triads, this paper proposes a covariance-scaling based robust Kalman filter (KF) algorithm. Attitude estimation problem can be solved by KF based and complementary filtering (CF) based methodologies. Appropriate tuning of the covariance matrices make KF based attitude filters efficient and optimal. Adaptive way for this tuning procedure is given by the algorithm we propose, and it can accurately estimate the attitude in two axes. The proposed methodology is split into two main methods, single-scale factor (SSF) and multiple-scale factor (MSF) methods. They are tested and compared with other existing filtering methodologies in the literature under different dynamical conditions and using real-world experimental dataset in order to validate their effectiveness.

INTRODUCTION

Attitude estimation has been a significant problem for decades in various navigation and localization applications [Kok, Hol and Schön, 2017]. The optimal algorithms for this problem have been developed for last fifty years and divided into two common groups, Kalman filters (KFs) and complimentary filters (CFs), respectively. The attitude is evaluated via combining the gyroscope and the accelerometer readings in a complementary way in [Fourati, 2015; Wu, Zhou, Chen, Fourati and Li, 2016; Madgwick, 2010; Mahony, Hamel and Pflimlin, 2008] whereas KF-based methods are used for solving the attitude estimation problem via focusing on accurate compensation of the external acceleration and appropriate representation of the noise covariances with different approaches such as fuzzy, cascaded, adaptive, and even manual tuning methods [Choukroun, Bar-itzhack and Oshman, 2006; Javed, Tahir and Ali, 2020; Kang, Park, 2009; Li, Chang and Hu, 2015; Park, Park and Park, 2017]. In this case, when the one looks into the recent works in the literature [Nazarahari, Rouhani, 2021], KF-based methodologies proposed for attitude estimation problem aim to minimize the negative effect of external acceleration and disturbances during the estimation process by correctly constructing the measurement noise covariance matrix since the accelerometer itself can be used for roll and pitch angle estimation. Some of these different approaches were compared in our previous work [Candan, Soken, 2020].

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Even though there are methods for compensating the external acceleration and disturbances, they lack for constructing an accurate measurement noise covariance matrix during the estimation process since these methods only include basic switching and tuning mechanisms [Chiella, Teixeira and Pereira, 2019; Sabatini, 2006; Suh, 2010; Hyyti, Visala, 2015; Wu, 2020].

This paper proposes a robust-adaptive KF algorithm for estimating the two-axis attitude (i.e., roll and pitch angles) using the measurements of an inertial measurement unit (IMU). The method deals with the covariance uncertainty due to the external accelerations via adaptively scaling the measurement noise covariance matrix. Two different approaches are proposed for scaling the matrix based on the comparison of the theoretical and real innovation covariances in line with the methods given by [Hajiyev, Soken, 2016; Soken, Hajiyev, 2013]. Both approaches are evaluated using the dataset for a micro aerial vehicle (MAV) provided by [Majdik, Till and Scaramuzza, 2017] and the results are compared with a bunch of benchmark algorithms.

THEORETICAL BACKGROUND

Attitude Representation

There are three common methods for representing the attitude of a system. These are Euler angles, direction cosine matrix (DCM) and quaternions. In this work, DCM representation is preferred and now let I and S represent the inertial and the sensor frame coordinates, respectively. ${}^{I}_{S}\mathbf{R}$ is the DCM of the sensor frame with respect to the inertial frame and to be denoted $\mathbf{R}$ for convenience. Using the conventional Z-Y-X Euler angles, $\mathbf{R}$ can be constructed as following.

$$\mathbf{R} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta\ c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & c\alpha s\beta s\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \tag{1}$$

In this representation, α (yaw), β (pitch), and γ (roll) are the rotation angles about the Z, Y, and X axes, respectively and c and s stand for cosine and sine trigonometric functions. It is clear that the two-axis attitude estimation can be done using only the last row of matrix $\mathbf{R}$ including only the pitch and roll angles, which are desired to be estimated. These angles can be evaluated from basic trigonometric identities as,

$$\gamma = tan^{-1}(\frac{R_{32}}{R_{33}}) \tag{2}$$

$$\beta = tan^{-1}(\frac{-R_{31}}{\sqrt{R_{32}^2 + R_{33}^2}}) \tag{3}$$

where R_{ij} represents (i, j) entry of the matrix $\mathbf{R}$. Therefore, this last row ($\mathbf{R}_l$) can also be used as the state vector for this work shown as following.

$$\mathbf{x} = \mathbf{R}_l^T \mathbf{e} \tag{4}$$

and the vector $\mathbf{e}$ is defined as $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

Sensor Models

Measurement signals from the gyroscope ($\mathbf{y}_G$), and the accelerometer ($\mathbf{y}_A$) are modelled respectively as following.

$$\mathbf{y}_G = {}^{S}\omega + \mathbf{n}_G \tag{5}$$

$$\mathbf{y}_A = {}^{S}\mathbf{a} + {}^{S}\mathbf{g} + \mathbf{n}_A \tag{6}$$

Note that, $\mathbf{a}$ and ω are the ideal external acceleration and angular rates sensed by the accelerometer and the gyroscope, respectively. $\mathbf{g}$ is the gravity vector, n_A and n_G are the sensor noises assumed to be uncorrelated, zero-mean white Gaussian noise. It is important to emphasize that in practice each sensor exhibits not only constant bias offset, but also varying bias errors, which are not accounted in the given models. Moreover, in [Lee, Park and Robinovitch, 2012], external acceleration (${}^{S}\mathbf{a}$) was modeled as a first-order low-pass filtered white noise process as following,

$$ {}^{S}\mathbf{a}_t = c_a {}^{S}\mathbf{a}_{t-1} + \boldsymbol{\epsilon}_t \tag{7} $$

where c_a is a dimensionless, determinator constant specified for the cutoff frequency and the value of this constant is varying between 0 and 1. Time-varying error during the acceleration process is represented by $\boldsymbol{\epsilon}_t$.

Filter Process Model (Prediction)

In the proposed method, the process model is governed by following equation as,

$$ \mathbf{x}_t^- = \boldsymbol{\Phi}_{t-1}\mathbf{x}_{t-1} + \mathbf{w}_{t-1} \tag{8} $$

where $\boldsymbol{\Phi}$ is the state transition matrix that propagates the system states from previous time and $\mathbf{w}$ is the noise vector for the process model, assumed to be zero mean white Gaussian. In order to derive expressions for the state transition matrix and the process noise covariance matrix, time propagation of the rotation matrix, $\mathbf{R}$, is to be investigated. First order approximation for this propagation with gyro measurements can be given as,

$$ \mathbf{R}_t = \mathbf{R}_{t-1}\left(\mathbf{I_3} + \Delta t\widetilde{\boldsymbol{\omega}}_{t-1}\right) \tag{9} $$

where Δt is the sampling time and $\widetilde{\boldsymbol{\omega}}_{t-1}$, a skew-symmetric matrix, includes ideal gyro rates of the body at time $t-1$. The symbol "$\sim$" represents the cross-product operator which is transforming a vector to a matrix form. From (9), the propagation of the state vector can be given as following

$$ \mathbf{x}_t^- = \left(\mathbf{I_3} + \Delta t\widetilde{\boldsymbol{\omega}}_{t-1}\right)^T \mathbf{x}_{t-1} \tag{10} $$

Therefore, the state transition matrix, process noise and the process noise covariance matrix can be defined from (10) as,

$$ \boldsymbol{\Phi}_{t-1} = \mathbf{I_3} - \Delta t\widetilde{\mathbf{y}}_{G,t-1} \tag{11} $$

$$ \mathbf{w}_{t-1} = \Delta t(-\widetilde{\mathbf{x}}_{t-1})\mathbf{n}_G \tag{12} $$

$$ \mathbf{Q}_{t-1} = E[\mathbf{w}_{t-1}\mathbf{w}_{t-1}^T] = -\Delta t^2\widetilde{\mathbf{x}}_{t-1}\boldsymbol{\Sigma}_G\widetilde{\mathbf{x}}_{t-1} \tag{13} $$

where $\boldsymbol{\Sigma}_G$ is the noise covariance matrix of the gyroscope which is given as $\sigma_G^2\mathbf{I}_3$ assuming that the variance of gyro noise, σ_G^2, is distributed equal to all axes for the same gyro.

Filter Measurement Model (Correction)

In the proposed method, the measurement model is governed by following equation as,

$$ \mathbf{z}_t = \mathbf{H}\mathbf{x}_t + \mathbf{v}_t \tag{14} $$

In order to derive terms for the measurement model components,(6) can be divided into two equations given below as,

$$ \mathbf{a}_t^- = c_a\mathbf{a}_{t-1}^+ \tag{15} $$

$$ \boldsymbol{\epsilon}_t = \mathbf{a}_t - \mathbf{a}_t^- \tag{16} $$

where $\mathbf{a}_t^-$ is the predicted (a priori) acceleration at time t, and $\mathbf{a}_{t-1}^+$ is the estimated (a posteriori) external acceleration at time $t-1$. Now, it is possible to use (6) via inserting (15) and (16) into the equation and extract the measurement model expressions.

$$ \mathbf{z}_t = \boldsymbol{y}_{A_t} - c_a\mathbf{a}_{t-1}^+ \tag{17} $$

$$ \mathbf{H} = g\mathbf{I_3} \tag{18} $$

$$ \mathbf{v}_t = \boldsymbol{\epsilon}_t + \boldsymbol{n}_A \tag{19} $$

where $\boldsymbol{\epsilon}_t$ cannot be related to $\boldsymbol{n}_A$, resulting the appearance of following measurement noise covariance matrix as following.

$$ \mathbf{M}_t = E[\mathbf{v}_t\mathbf{v}_t^T] = \boldsymbol{\Sigma}_{acc} + \boldsymbol{\Sigma}_A \tag{20} $$

It is crucial to focus that when $\boldsymbol{\Sigma}_A$ is set as $\sigma_A^2\mathbf{I}_3$, assuming that the variance of accelerometer noise, σ_A^2, is distributed equal to all axes for the same accelerometer.

However, time varying component of (20), $\mathbf{\Sigma}_{acc}$, cannot be analytically obtained since the external acceleration during the measurements is unknown. Following chapter is to introduce the proposed method in order to overcome from this problem.

METHOD

Adaptive measurement noise covariance scaling methodology is basically to change KF gain via tuning the measurement noise covariance matrix autonomously. Therefore, the filter can adapt itself to this new environment via comparing theoretical and real values of the innovation covariance [Soken, Hajiyev, 2013; Hajiyev, Soken, 2020] when there exist external acceleration/disturbances on the measurement system so that these effects can be compensated. Definiton of innovation in KF structure is,

$$\mathbf{e}_t = \mathbf{z}_t - \mathbf{H}\mathbf{x}_t^- \tag{21}$$

where $\mathbf{x}_t^-$ is the predicted state vector and $\mathbf{e}_t$ is the innovation sequence. Kalman filter gain changes with varying innovation covariance if there exist mismatches between the process and measurement models, therefore the innovation covariance after filter adaptation can be defined as,

$$\hat{\mathbf{C}}_{e_t} = \mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \mathbf{S}_t\mathbf{M}_t \tag{22}$$

and KF gain becomes,

$$\mathbf{K}_t = \mathbf{P}_t^-\mathbf{H}^T(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \mathbf{S}_t\mathbf{M}_t)^{-1} \tag{23}$$

where $\mathbf{P}_t^-$ is the predicted covariance matrix during the KF process and $\mathbf{S}_t$ is the measurement noise covariance matrix scaling factor (SF). It is the fact that if the real value of KF error exceeds the theoretical error, which can be shown as,

$$tr(\mathbf{e}_t\mathbf{e}_t^T) \geq tr(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \mathbf{M}_t) \tag{24}$$

filtering process must be done adaptively. In (24), $tr(.)$ denotes that the trace of the related matrix. There are two possible options for scaling the measurement noise covariance matrix, single-scale factor (SSF) option and multiple-scale factor (MSF) option, respectively.

Single-Scale Factor (SSF) Method

In the first approach, in the SSF is introduced directly to modify the $\boldsymbol{M}_t$ matrix as in (23). In order to obtain the SSF, first, let us insert $\mathbf{S}_t$ into the inequality in (24) and take the condition where two innovations are equal as the basis as,

$$tr(\mathbf{e}_t\mathbf{e}_t^T) = tr(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T) + S_t tr(\mathbf{M}_t) \tag{25}$$

therefore, S_t can be expressed as,

$$S_t = \frac{\mathbf{e}_t^T\mathbf{e}_t - tr\left\{\mathbf{H}_t\mathbf{P}_t^-\mathbf{H}_t^T\right\}}{tr\left\{\mathbf{M}_t\right\}} \tag{26}$$

If there is no external acceleration or disturbance detected i.e. condition given in (24) is not met, $\mathbf{S}_t$ simply becomes $\mathbf{S}_t = 1$.

Multiple-Scale Factor (MSF) Method

In the second approach, the MSF methodology is introduced. Rather than to adaptively tune the Kalman gain via scalar factor, it becomes appropriate to use a matrix structure with multiple factors since SSF approach rejects the measurements from all channels, even if the external acceleration is sensed in one direction. Therefore, using the same method for derivation of the SSF, but instead estimating a matrix composed of multiple factors $\boldsymbol{S}_t$ can be written as,

$$\boldsymbol{S}_t = (\mathbf{e}_t\mathbf{e}_t^T - \mathbf{H}\mathbf{P}_t^-\mathbf{H}^T)\mathbf{M}_t^{-1} \tag{27}$$

Same as mentioned, if there is no external acceleration or disturbance detected i.e. condition given in (24) is not met again, now, $\mathbf{S}_t$ simply becomes as following.

$$\mathbf{S}_t = diag(s_1, s_2, s_3), \tag{28}$$

$$s_i = max\left\{1, \mathbf{S}_{ii}\right\}, i = 1, 2, 3. \tag{29}$$

RESULTS AND DISCUSSION

Benchmark Methods

As mentioned earlier, there are two common strategies in filtering methods for attitude estimation problems, Kalman filtering and complementary filtering, respectively. In order to compare the proposed method with the literature of CFs and KFs in the sense of performance, different methods are selected. Following, the reader can find brief explanations about each method chosen for the performance comparison task.

Accelerometer-Only Attitude Estimation: The attitude estimation problem is tried to be solved using only the accelerometer measurements without compensating the external acceleration.

Gyroscope-Only Attitude Estimation: In this method, the attitude estimation problem is to be solved only relying to the gyroscope measurements (by propagating the initially obtained attitude angles without any update).

Madgwick's Filter [Madgwick, 2010]: Madgwick treats the attitude estimation as minimization problem and proposes a CF method, which is basically depending on the gradient decent strategy that uses the steepest decent algorithm to solve the problem recursively.

Lee's Filter [Lee, Park and Robinovitch, 2012]: Lee proposed a KF structure, which is able to adapt itself by automatically tuning the unknown component of the measurement noise covariance matrix. This method assumes that the external acceleration is distributed same along all axes.

Results

The performance of the proposed method is verified via using The EuRoC Micro Aerial Vehicle (MAV) Dataset, which is including time-synchronized images, IMU and ground truth data collected on-board MAV, provided by [Majdik, Till and Scaramuzza, 2017]. If it is desired to obtain more details, the readers can refer to this work. Table 1 presents the attitude estimation results of the benchmark methods and the proposed method with its two approaches, SSF and MSF methods, in terms of root mean square error (RMSE).

Table 1: Attitude Estimation Results in terms of RMSE (°)

Methods	**Roll**	**Pitch**
Gyroscope-Only	136.9165	35.4860
Accelerometer-Only	13.3190	4.3743
Madgwick's Filter	7.1589	2.4608
Lee's Filter	1.6999	1.3085
SSF Method	1.2915	1.1948
MSF Method	1.1217	1.1819

It is clear that the SSF method improves the estimation quality of KF-based Lee's filter and CF-based Madgwick's filter. On the other hand, it is observed that the performance of the proposed method with its second approach is superior against all of the comparison methods even Lee's filter which is one of the widely accepted and reliable KF-based solution method for attitude estimation problem. The reason behind this improved performance in the terms of the attitude estimation quality is basically the robust-adaptive covariance-scaling strategy implemented into the KF structure.

This provides better measurement covariance matrix construction and hence, attitude estimation performance during the filtering process which Lee's filter cannot provide. Moreover, Fig. 1. demonstrates that the proposed algorithm with its second approach, MSF method, compared with Lee's filter, achieves better accuracy in terms of attitude (i.e., roll and pitch) estimation error during the flight course of MAV. Especially, after a short warm-up period approximately until 40th second when highly disturbed/accelerated motion begins, the proposed method is able to compensate the deteriorative effects of disturbances mainly caused by external accelerations along the flight course.

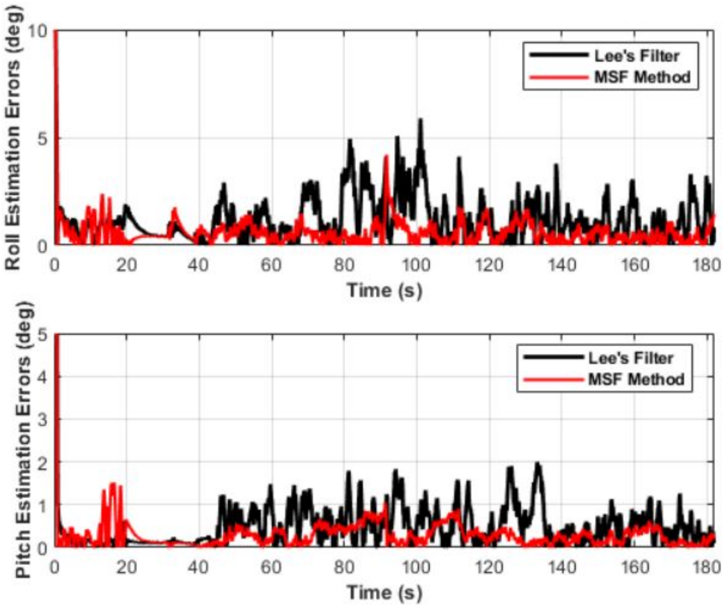

Figure 1: Attitude Estimation Errors of MSF Method for the EuRoC MAV (MH-01) Dataset

CONCLUSIONS

In this paper, a novel covariance-scaling robust Kalman filter with two different methods is proposed for the attitude estimation problem. The performance of the proposed method in terms of the estimation quality is evaluated using a real-world dataset and compared with the methods selected from the current literature. Main contribution of the proposed method is to introduce two robust-adaptive approaches for scaling the measurement noise covariance matrix in Kalman filter (KF) structure. The performance of these approaches is evaluated using a real-world dataset and it is observed the first approach slightly improves the estimation quality of previous KF-based and complementary filter (CF) based filters while the second approach shows superior performance against the selected literature including both KF and complementary filters. Furthermore, future studies to investigate rather than tuning the measurement covariance matrix via scaling, the use of estimation for obtaining the specific component of measurement noise covariance that cannot be obtained analytically as mentioned previously.

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