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# CFD BASED QUASI-STEADY AIRCRAFT LOADS PREDICTION USING RADIAL BASIS FUNCTIONS

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## ABSTRACT

*In this study, a surrogate modeling approach is used to predict aircraft surface pressure distributions in the transonic regime. This model is using high-fidelity computational fluid dynamics (CFD), but it is computationally much cheaper than the original CFD model. In order to make predictions, radial basis function (RBF) interpolation is used. Which is an effective way for interpolating the scattered data. In order to measure the performance of these radial basis functions, they are compared with the high-fidelity computational fluid dynamics solutions. The optimal radial basis function is selected based on its accuracy and stability. Finally, the effect of a number of sample points in the dataset on the prediction quality of RBF will be investigated.*

## INTRODUCTION

Load assessment on aircraft and its components is an important aspect of aircraft development process. At the preliminary design stage of aircraft development steady and unsteady aerodynamic loads have to be computed on the most current iteration of the external shape in order to size the aircraft structure. The newly sized structure is then assessed for its impact on the loads aero elastics and aerodynamic performance. Therefore, loads computations is central in the overall flight physics process and it is important to achieve a good level of accuracy to ensure optimum weight, performance, dimensions and safety. The accuracy of loads computations is dependent on the fidelity of aerodynamics methods used. Ideally wind tunnel results or flight test data could be used. However, in preliminary stages it is not feasible to use those since the external shape and structure are not mature enough. Other option is to use high fidelity RANS computations but this requires long times and high computational resources for the number of computations needed for the loads process. The less accurate but feasible option is to use low fidelity panel methods, however these methods are used with limitations i.e. linear, incompressible and inviscid flow and they are unable to model aerodynamic non linearities like shocks and separations. To overcome the constraint of high computational cost and achieve more accurate results than panel methods, it is proposed to use a CFD surrogate model. For aircraft loads [Lillian et al.,

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2011] uses system identification to predict surface pressure distribution, whereas [Juretschke et al., 2019] uses constrained optimization on the loads model and also tries kriging to build the surrogate. Also, [Ripepi et al., 2018] uses thin plate spline which is a form of radial basis function to predict aircraft pressure distribution. This study will show the usage of several types of radial basis functions to build a CFD surrogate model and comparison of prediction against original surface pressure distribution.

## METHOD

### Radial Basis Functions

Radial basis function is a real valued function whose value depends on the Euclidean distance from the center [ Buhmann, M. D, 2003]. RBF can be show in following form:

$$y(x) = \sum_{i=1}^{n} \omega_i \, \phi(||x - x_i||)$$

Where $\omega_i$ are called weights and $x_i$ is the center. In matrix form it can be shown as

$$\begin{bmatrix} \phi(||x_1 - x_1||) & \cdots & \phi(||x_1 - x_n||) \\ \vdots & \ddots & \vdots \\ \phi(||x_n - x_1||) & \cdots & \phi(||x_n - x_n||) \end{bmatrix} \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

Since there are many types of radial basis functions, it is important to select the radial basis function suitable for problem. However, the best radial basis function for specific application is unclear [Toit, 2008]. Therefore, multiple radial basis functions will be considered. Radial basis functions used for this study given in Table 1.

Table 1: Radial basis functions formulas

| **Function Name** | **Formula** |
|---|---|
| Multiquadric | $\sqrt{(\frac{r}{\varepsilon})^2 + 1}$ |
| Inverse | $\frac{1}{\sqrt{(\frac{r}{\varepsilon})^2 + 1}}$ |
| Gaussian | $e^{(-\frac{r}{\varepsilon})^2}$ |
| Linear | $r$ |
| Cubic | $r^3$ |
| Quintic | $r^5$ |
| Thin Plate | $r^2 \log(r)$ |

Where ε is the shape parameter and r is the radius. Shape parameter adjusts the steepness of a function. [Toit, 2008].

## NASA Common Research Model

To test the performance of the suggested methodology NASA common research model (CRM) will be used as the test case. The NASA Common Research Model (CRM) consists of a contemporary supercritical transonic wing and a fuselage that is representative of a wide body commercial transport aircraft. The CRM is designed for a cruise Mach number of 0.85 and a corresponding design lift coefficient of 0.5 [Vassberg et al., 2008].
The wing-body of the CRM model is discretized 2202 and flow field around it has 42201 elements (Figure 1). Total grid density is kept especially low due to lack of computational resources; however, the aim of this study is not to obtain the most accurate CFD solutions but also to validate the methodology under development. Once model provides sufficiently accurate comparison, it can be applied to high-fidelity CFD solutions.

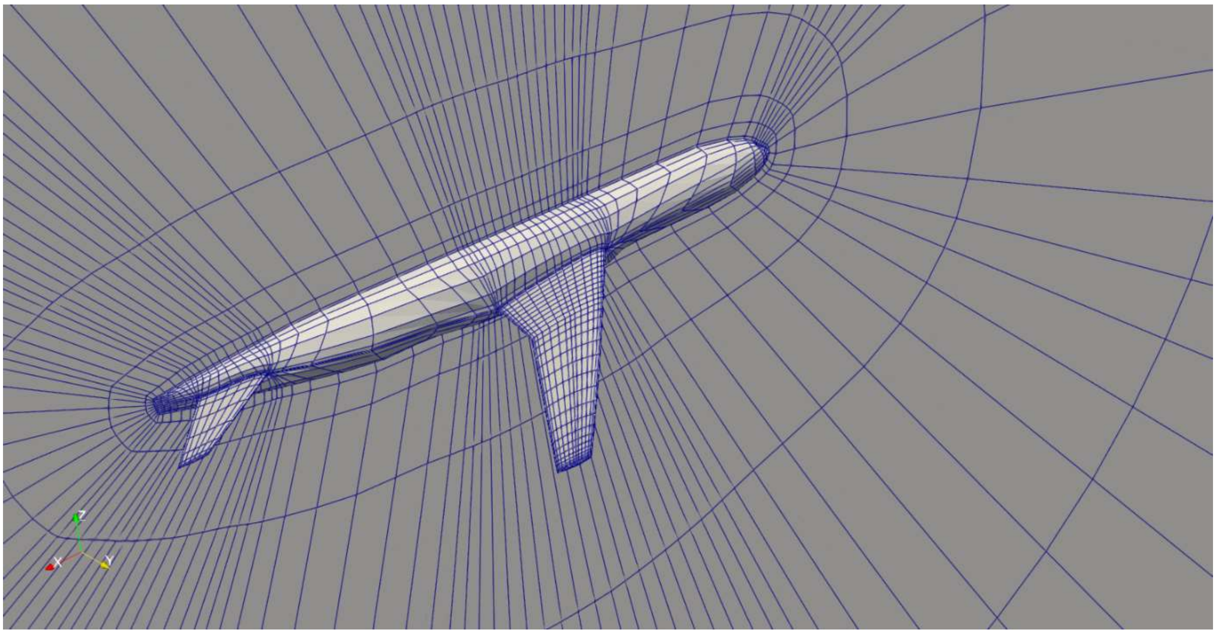

Figure 1 Aircraft surface and symmetry plane grid representation

## Dataset

In order to create a surrogate model, required dataset is obtained from open source CFD solver SU2. Flow field and aircraft are discretized as mentioned above, and Euler equations are used to get flow solutions. In total 209 different data points are solved for Mach between 0.8-0.9 with 0.01 increments and angle of attack between 0-9 degrees with an increment of 0.5 degrees (Figure 2). Each CFD case requires approximately 6 minutes physically and the creation of whole dataset took almost a day with a 3.7 GHz processor. With using whole dataset as training matrix, reconstruction of additional prediction using RBF is 1.8 seconds. Typical RANS case having 30 million volume elements with more than 500000 surface cells takes approximately 1 hour with 1000 cores. Solving these kinds of high fidelity datasets with personal computers is almost impossible, even with powerful HPC systems required resource is enormous. However, RBF methodology calculations are simple algebraic calculations, which means that increasing number of elements in the computational domain will not change reconstruction time significantly; therefore, prediction time will be in order of seconds once sample training dataset is ready. Figure 3 shows surface pressure distributions of random points in dataset. Red tones in color map represents compressions, white are neutral and blue to green colors show suction regions. Since CRM aircraft is optimized for transonic regimes, typical aft shock is observed near trailing edge of the wing.

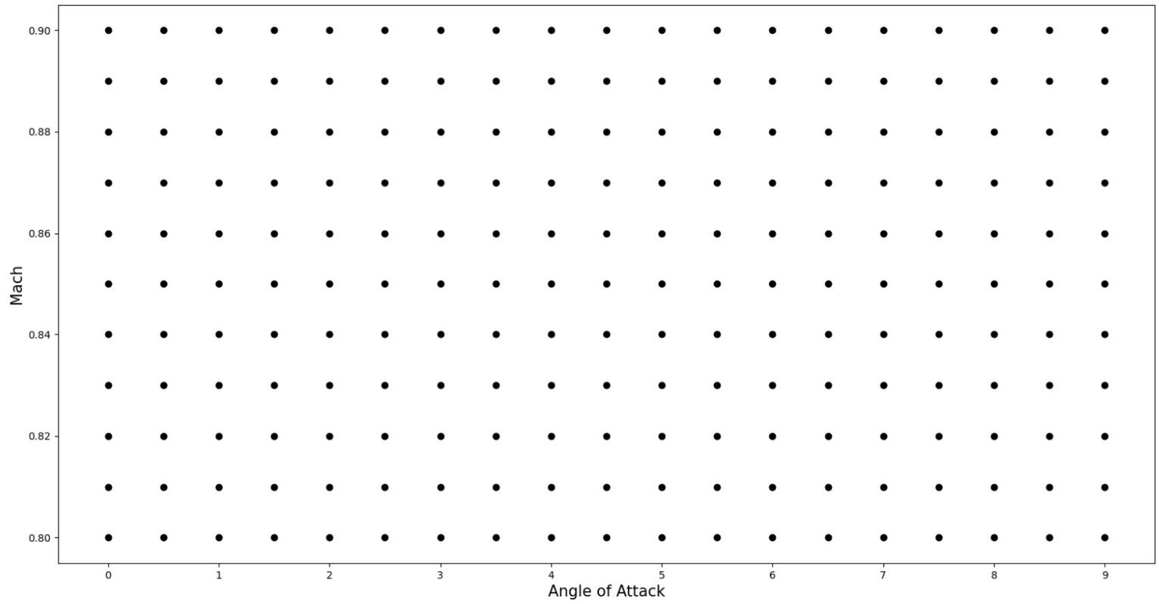


Figure 2 Mach number and angle of attack distribution of total dataset

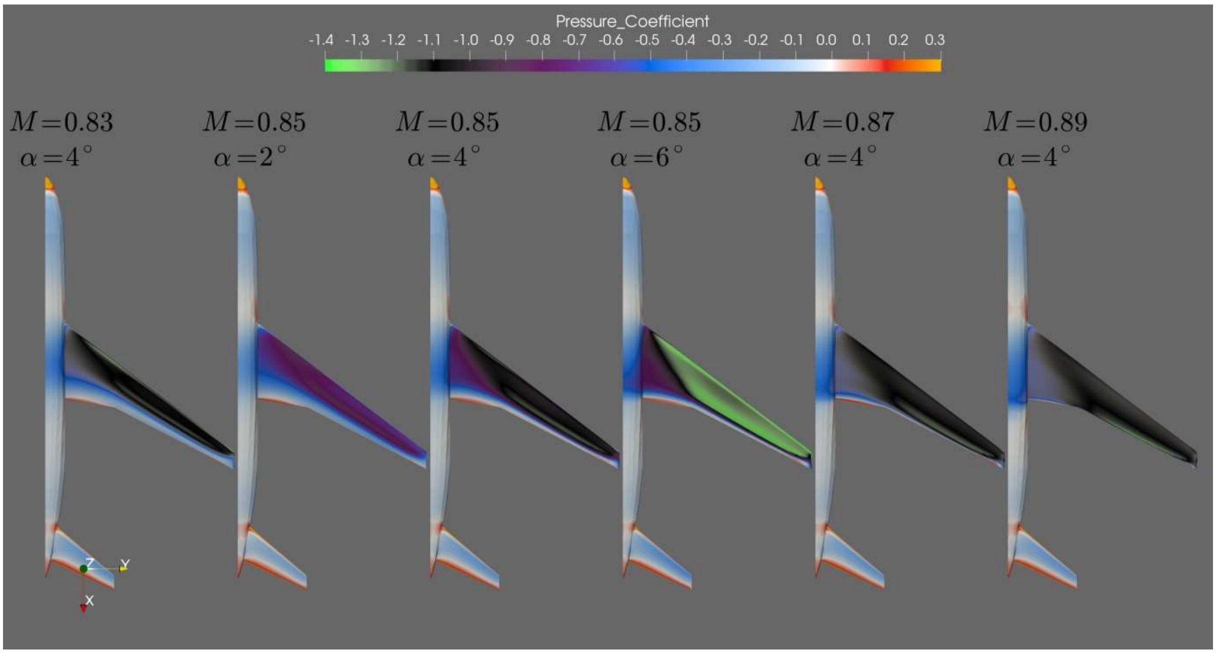


Figure 3 Surface pressure coefficient distributions of random points from dataset

**Interpolation**

In order to find the prediction points, radial basis function interpolation is used which is using Mach and angle of attack as inputs. First, a matrix is constructed which has all of the information necessary to interpolation method requires.

$$[A] = \begin{bmatrix} Cp1,1 & \cdots & Cp1,n \\ \vdots & \ddots & \vdots \\ Cpm,1 & \cdots & Cpm,n \end{bmatrix}_{mxn}$$

Matrix A contains the coefficient of pressure distributions. Each column represents a sample point for a specific Mach and angle of attack combination, whereas each row represents a surface grid point in a CFD solution.Predictions for an unknown prediction point is made by finding the coefficient of pressure value for each grid point. After obtaining for all of the points the final pressure distribution result is obtained.

### Sample Calculation

One dimensional example will be solved to show how radial basis function interpolation works. Suppose 3 datapoints as: $x_1 = 2$, $x_2 = 5$, $x_3 = 6$ and their values are:

y ($x_1$) = 0.5, y ($x_2$) = 0.6, y ($x_3$) = 1. For simplicity choose radial basis function as cubic function. Which is given as $\phi(r) = r^3$

By constructing the matrix as given in the radial basis function section:

$$\begin{bmatrix} \phi(0) & \phi(3) & \phi(4) \\ \phi(3) & \phi(0) & \phi(1) \\ \phi(4) & \phi(1) & \phi(0) \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0 & 27 & 64 \\ 27 & 0 & 1 \\ 64 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_n \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.6 \\ 1 \end{bmatrix}$$

Solving these equations yield $\omega_1 = 0.019,\ \omega_2 = -0.2,\ \omega_3 = 0.09$

Hence, the resultant function can be written as:

f(x) = 0.019 $\phi(|x-2|)$ – 0.2 $\phi(|x-5|)$ + 0.09 $\phi(|x-6|)$

For any x corresponding value can be found via the above equation.

## RESULTS AND DISCUSSION

First step of this study is to identify the best radial basis function for this problem. This will be decided by their accuracy and stability performance. To measure the accuracy performance of functions a statistical parameter called root mean square error will be used.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Predicted_i - Actual_i)^2}{N}}$$

Because it's taking the square of the difference between the prediction and actual value, the error is monotonically increasing which ensures that there is no error canceling.
For the prediction points shown in different radial basis functions predictions made. In total there are 95 prediction points and their results are compared with a box plot shown in Figure 3.

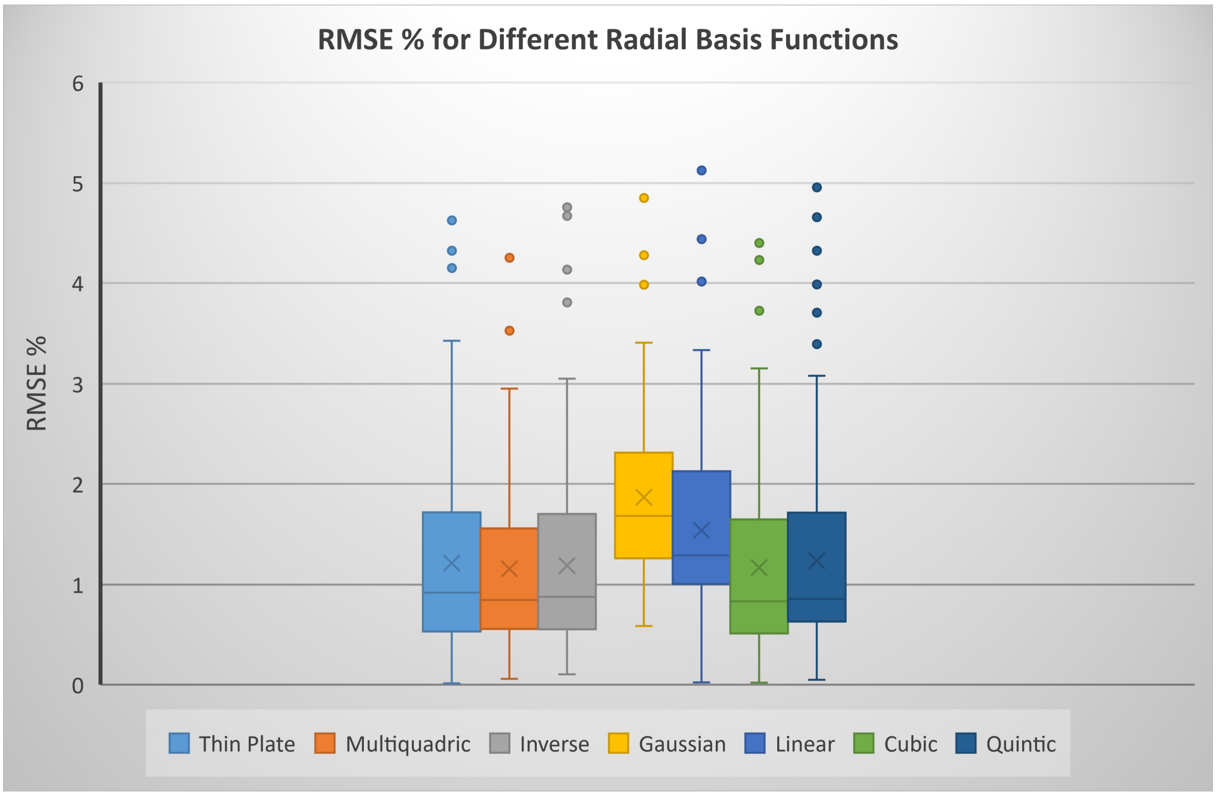


Figure 4 Box plot for different radial basis functions. Mean errors indicated with “x” symbol.

Box plot is a good way for showing the distribution of the data. It’s showing how tightly the data is grouped as well as the mean error. Also, it shows the outlier points which has the larger error compare to majority of the points. For the selection of the optimal radial basis function, one should select the function which has lowest RMSE percentage and its data is tight rather than widely spread. According to these two criteria multiquadric function is selected as the optimal radial basis function for this problem.

It is widely known that accuracy of this kind of statistic-based model highly dependent on number of samples in training matrix. To check sensitivity of the number of samples two different training matrices are used for predictions. Figure  shows the first training matrix which has 114 of the solutions are selected as sample points (blue squares) and 95 of the solutions will be predicted with radial basis functions (red triangles). The second training matrix is represented in the right hand side of the figure, which covers 40 sample points for training and same 95 points are kept constant as predictions. All predictions are generated using selected RBF function and root mean square error distributions calculated between original CFD and prediction values. RMS values of each prediction point are represented in Figure  for 114 samples and 40 samples respectively. It is seen that highest error is observed in lower boundary. Since dataset is created in Mach range between 0.8 and 0.9, predictions of 0.89 is close to upper Mach boundary which affects accuracy of estimations. In addition, transonic flow nonlinearities are much more significant in 0.89 that can reduce order of accuracy around those regions. It is also seen that effect of sample points is an important parameter. Average error distribution of cumulative pressure coefficient is 1.15 for 114 sample dataset and 2.85 for 40 sample dataset respectively. Also, highest error is reduced from 7 to 4 at upper Mach boundary. Error distribution of 40 sample dataset indicates that predictions are highly dependent on angle of attack value provided in training set. Since half degree angle of attack values are not provided (i.e. 1.5°, 2.5°…), errors at those regions are higher. For example, have a look at Mach 0.85 angle of attack 4, 4.5 and 5 degrees error values: 1.49, 2.24 and 1.53 respectively. 4 degree and 5 degree AoA values are provided in training set for different Mach values and error values are low for them. Similar, characteristics are observed for most of the predictions and for future work weights of the angle of attack should be carefully considered.

Heatmap style error distributions are pretty good for observing general behavior of the problems. However, in order to find what causes that error requires additional data analysis. 2 different cases are chosen to investigate error root causes deeper. Table 1 shows specifications of these cases:

Table 1 RMS error values of cases of detailed analysis

| Dataset | Error of Mach 0.85, AoA 5 | Error of Mach 0.89, AoA 2 |
|---|---|---|
| 40 sample | 1.07 | 4.30 |
| 114 sample | 1.52 | 7.66 |

Figure 8 and Figure 9 shows pressure coefficient contours and four chordwise pressure distributions of CRM wing-body aircraft for Mach 0.85, AoA 5 degrees and Mach 0.89, AoA 2 degrees respectively. It can be said that overall pressure distribution are quite well captured for first case by 40 sample and 114 sample dataset. Leading edge suctions are on top of each other for each span station. However, shock predictions causes rise in error values. For case 2, similar comment can be done, yet 114 sample predictions are superior to 40 sample ones in terms of upper surface flow field after mid-chord.

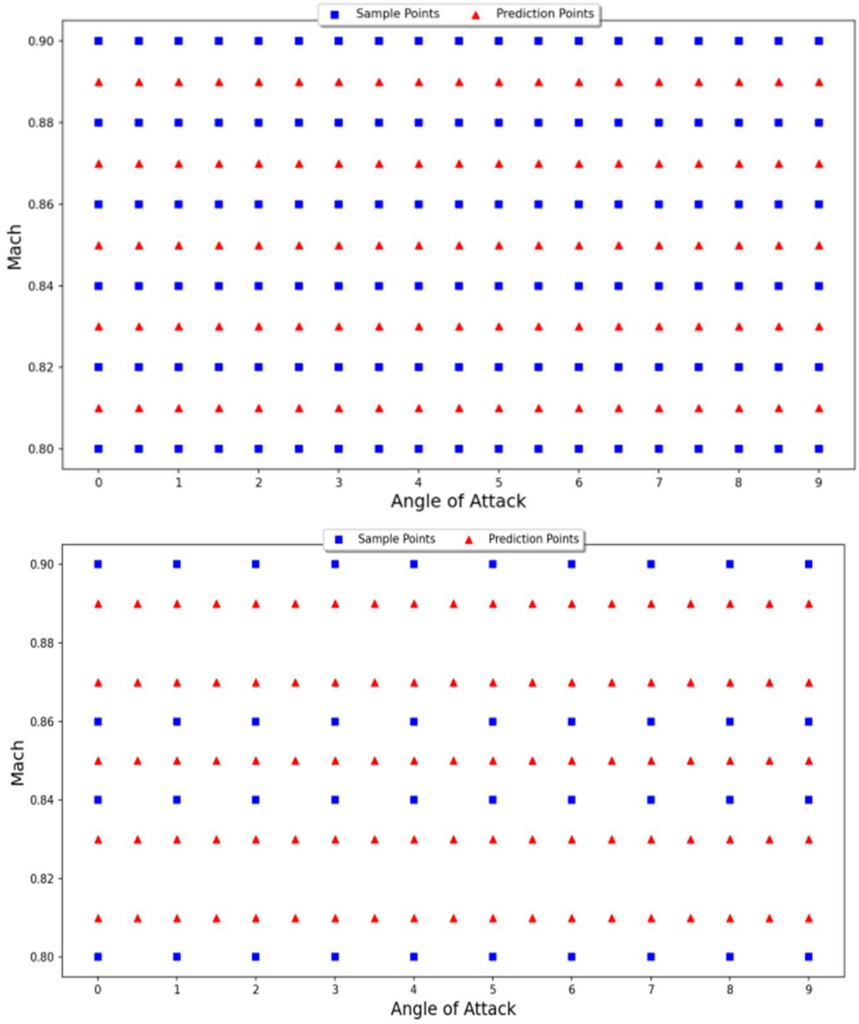


Figure 5 Training dataset (blue squares) with 114 samples (above), 40 samples (below) for 95 predictions (red triangles)

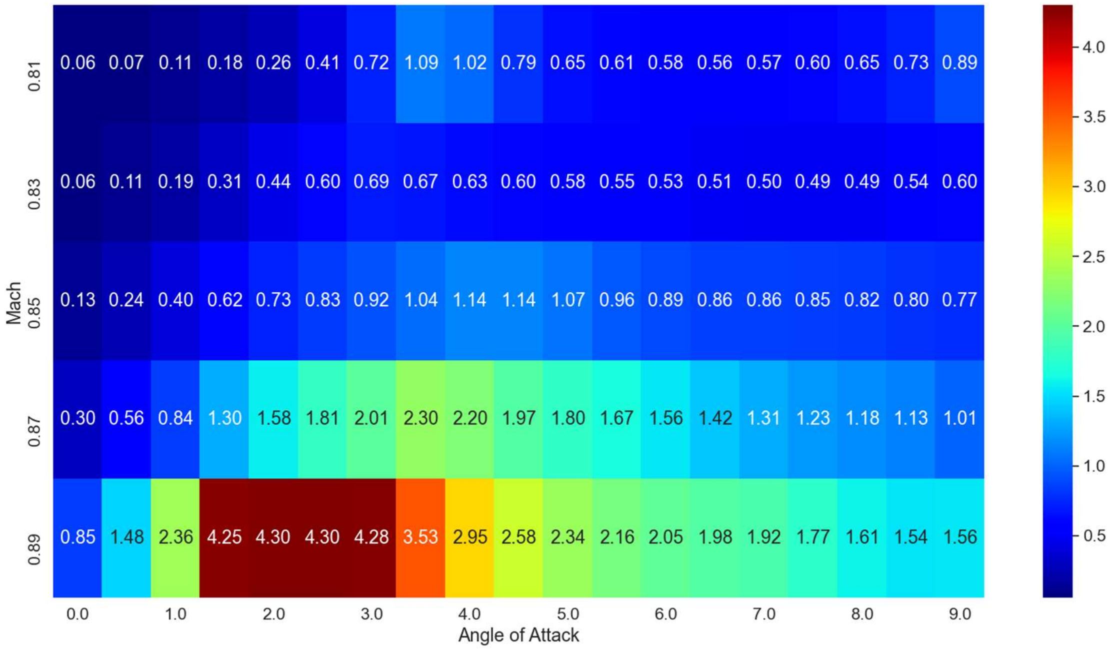


Figure 6 Error distribution of predictions based on 114 samples

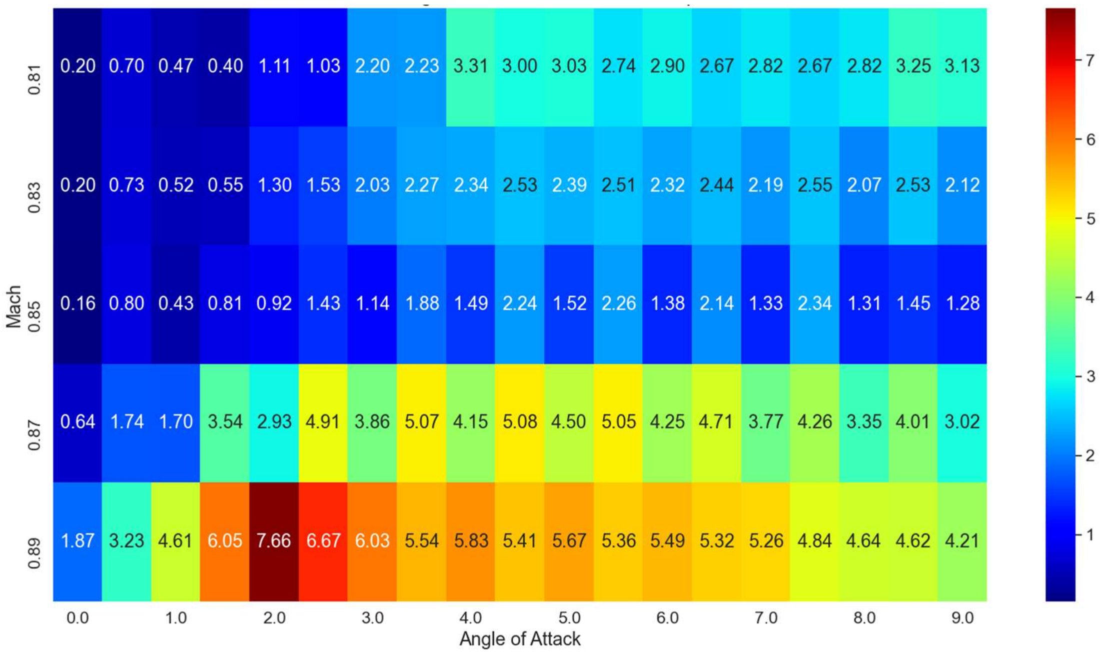


Figure 7 Error distribution of predictions based on 40 samples

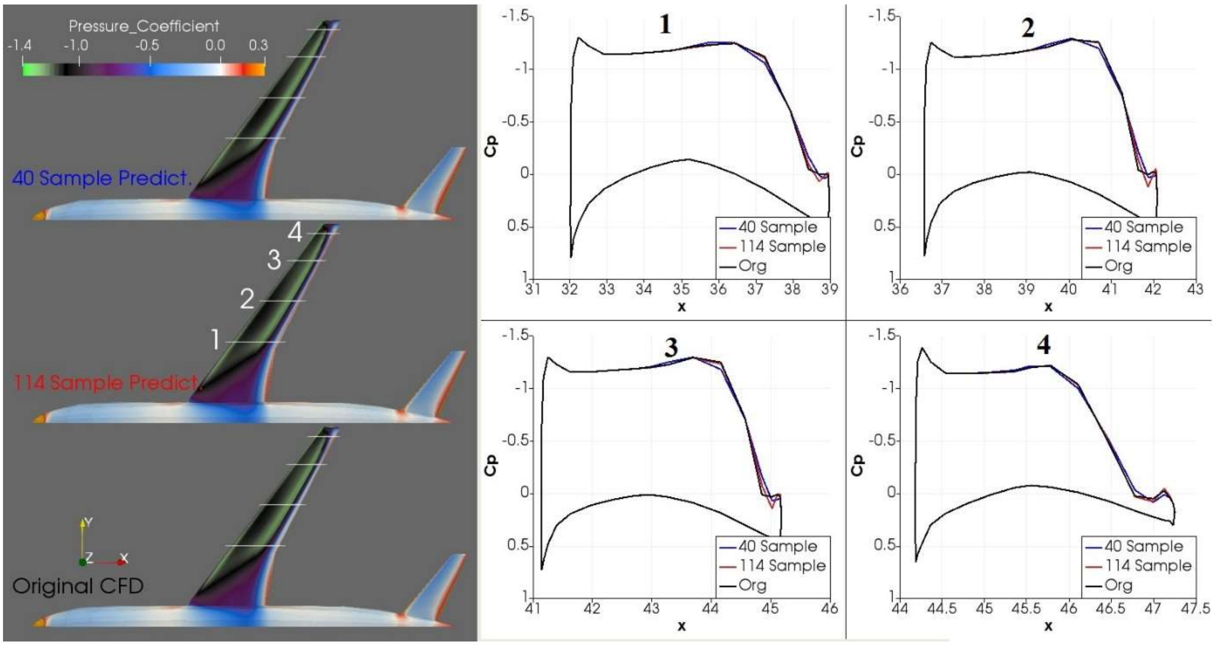


Figure 8 Wing pressure distributions of CRM wing-body for M 0.85, AoA 5°

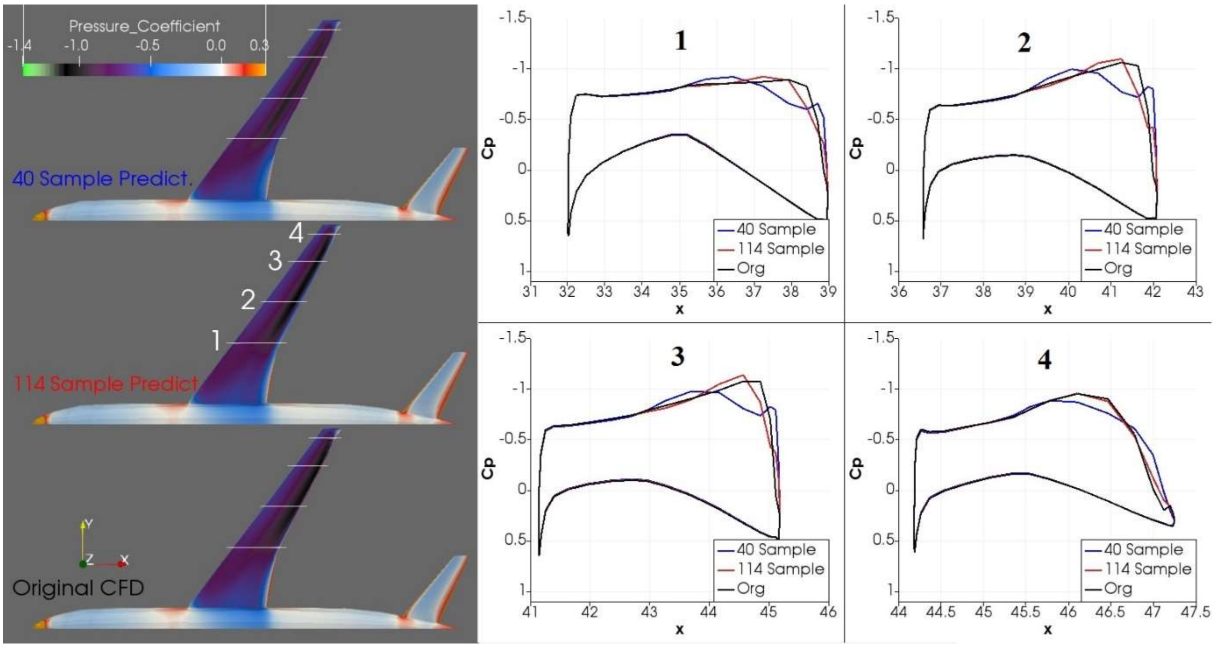


Figure 9 Wing pressure distributions of CRM wing-body for M 0.89, AoA 2°

## CONCLUSION

In this study, a method for determining aerodynamic loads via a CFD-based surrogate model is presented. Which aims to reduce the computational cost of obtaining aerodynamic loads and still provide similar accuracy levels with high-fidelity CFD solutions. To obtain the new predictions an interpolation method called radial basis function interpolation is used. Several types of radial basis functions are investigated by their prediction performance on a test aircraft CRM in a transonic flow regime. It has been observed that the 'Multiquadric' function performs better amongst the functions used. Also, the sensitivity of the number of sample points used for this model is studied by comparing two different datasets with their prediction quality on two different points. This comparison suggests that using a good sampling strategy can achieve good accuracy levels with fewer sample points. Finally, by using a surrogate model it's shown that obtaining new solutions is much faster.

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