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DISCUSSION OF ANGLES ONLY METHODS FOR THE CASE OF MULTI-OBSERVER

Mehmet Fatih Erturk*
University of Turkish Aeronautical Association
Ankara, Turkey

ABSTRACT

This work is a discussion about the application of angles only preliminary orbit determination methods for multi-observers. The methods discussed in this work are the Gauss and Gooding's methods. The application of Gauss method is easier, but it needs adjustments for the elevation and the angle between observations. On the other hand, using Gooding's method is a bit involved; however, this method works with various conditions and gives high accuracy results. Revealing these points and making a discussion about the results of these methods are the main targets of this paper. The study presents a nice perspective for angles only methods for multi-observer case.

INTRODUCTION

As the number of satellites are increased, determining and following them are getting more important. There are several way to determine the orbit of a satellite or space debris, which are based on optical, radar and lidar systems. Radar and lidar systems are costlier than optical systems. That makes optical systems more practicable. Also, any curious person can achieve a basic initial orbit determination with an optical system on a tight budget. Accuracy of calculations depends on both quality of optical system and application of initial orbit determination methods. In this paper, main discussion will be about these methods.

Below discussion is about two angles only methods, which are Gauss and Gooding's method. Application of these methods main references are Curtis H. D. [2013], Gooding R. H. [1993], and Gooding R. H. [1997]. Gauss method is applied and results are obtained clearly. On the other hand, Gooding's method algorithm is written but, it does not work good. So that, there will be a the theoretically discussion part about its solution algorithm.

METHOD

Optic measurements give just two angles data of target. They are form of declination (δ) and right ascension (α). At least three observations needed to obtain initial orbit of a satellite. State vectors are findable with only angles data. To obtain them, a few methods can be applicable as Gauss and Gooding's methods.

Gauss Method

In this section, there will be a short recall for Gauss method.

*Student in Astronautical Department, Email: mfatiherturk@outlook.com

Gauss method of preliminary orbit determination method requires at least three observations with their times and position vectors of observer. There are some rules for these observations. Basically, Gauss method tries to determine geocentric position vector of satellite with the help of (1).

$$\vec{r} = \vec{R} + \vec{\rho} = \vec{R} + \rho\hat{\rho} \tag{1}$$

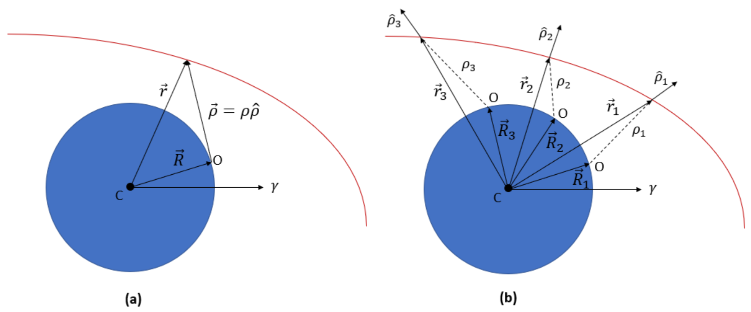

Figure 1: Earth and Orbiting Satellite Representation

Firstly, position vector of observer should be calculated for observation site. Actually, calculation of ρ and $\hat{\rho}$ is the main part of Gauss method. (2), which comes from figure 1-a, and (1) are guides of solution. Also, $\vec{r}_1$ and $\vec{r}_2$ can be written in terms of $\vec{r}_2$, $\vec{v}_2$ and Lagrange coefficients with the help of figure 1-b.

$$\vec{r}_2 = c_1\vec{r}_1 + c_3\vec{r}_3 \quad (2) \qquad \vec{r}_1 = f_1\vec{r}_2 + g_1\vec{v}_2 \quad (3) \qquad \vec{r}_3 = f_3\vec{r}_2 + g_3\vec{v}_2 \quad (4)$$

Equalization of (1) and (2) writes ρ and $\hat{\rho}$ in terms of c_1 and c_3. Then, solving these equations for c_1 and $c_3$with the help of (2), (3) and (4) makes a relation between c, f, g and $\vec{r}$ terms. Also Lagrangian terms are common known elements. In this paper, Gauss method has small time intervals to decrease complexity of these elements. $\tau_1 = t_1 - t_2$, $\tau_3 = t_3 - t_2$, and $\tau = \tau_3 - \tau_1$ are representations of time intervals. With these small time intervals, ignoring of high order terms in Lagrange coefficients is meaningful.

After the knowledge of f and g terms, writing c_1 and c_3 approximately is possible. End of the problem is getting closer. However, still there are some missing parts in (1). Equalization of (1) and (2) with (3) and (4) gives a nice chance to isolate ρ_2 and make a contact between ρ_2 and r_2.

Expanding and rearranging all the equations leads to an eight-order polynomial. After solving this equation results come as slant ranges ρ_1, ρ_2, and ρ_3. Replacing them and other values into (1) and solving this equation for $\vec{v}_2$ yields

$$\vec{r}_2 = \frac{1}{f_1}\vec{r}_1 - \frac{g_1}{f_1}\vec{v}_2 \quad (5) \qquad \vec{v}_2 = \frac{1}{f_1 g_3 - f_3 g_1}\left(-f_3\vec{r}_1 + f_1\vec{r}_3\right) \quad (6)$$

There are some iteration techniques for increasing accuracy of results. They can easily effect results. However, there will be no discussion about them in this paper.

Gooding's Method

In this section, there will be a discussion of Gooding's method with respect to its algorithm, which is written in Algorithm (1). This algorithm includes its main logic, not a detailed algorithm.

R. H. Gooding published his method about initial orbit determination in 1993. Gooding's method roots in estimations of two ranges. Also orbit type, retrograde or prograde, should be estimated. If all of these estimations are so far away from the correct situation, Gooding's method cannot converge Henderson, T. A., Mortari, D. and Davis, J. [2010]. Being applicable for several revolutions case, independent from observer's position and observation time makes the method a nice option for preliminary orbit determination.

Everything starts with an important estimation of ρ_1 and ρ_3 at times t_1 and t_3. $\hat{\rho}_1$ and $\hat{\rho}_3$ have already known from observation data. Then, calculation of $\vec{r}_1$ and $\vec{r}_3$ with (1) is the key point for the method. Now, a Lambert solution is the way for computing the estimation of an initial orbit. As a guideline Lancaster, E. R., Blanchard, R. C. and Devaney, R. A. [1966] and Lancaster, E.R. and Blanchard, R.C. [1969] are chosen from Gooding R. H. [1990] point of view. However, as stated in Zuehlke D. [2019], any Lambert solution could be suitable to use, and Curtis H. D. [2013] is used as the Lambert solver of this paper. On the other hand, after gaining a true algorithm, Gooding's Lambert solution should be use to increase accuracy of results.

The Lambert's problem is a describing process for the velocity determination of an object from its known position vectors. So that, the Lambert's problem is important for finding velocity vector from distance vector. To attain velocity components of an object, Lambert uses known quantities as $\vec{r}_1$, $\vec{r}_3$, t_1, t_3 and angles between two points. Equating these relations with a basic geometric logic, gives the key for velocity vector. However, reorganizing and solving these geometric relations is complicated. There are several different cases in the geometry and they directly affects results. So, the solution of the Lambert's problem contains various cases. Complexity of the cases causes different solution ways. As stated in Sangra, D. T. and Fantino, E. [2015], there are five main approach for the Lambert's problem. There are universal variables, semi-major axis, semi-latus rectum, eccentricity vector, Kustaanheimo-Stiefel regularized coordinates as a guide. In this paper, a universal variable approach is followed like Gooding, Lancaster and Curtis.

After having $\vec{v}_1$ from Lambert's problem, $\vec{v}_1$ and $\vec{r}_1$ give orbital elements with a Keplerian propagator. Rearranging orbital elements for $\tau_{12} = t_2 - t_1$ and using Keplerian propagator again, gives calculated $\vec{r}_2$ and $\vec{v}_2$ as $\vec{r}_{2c}$ and $\vec{v}_{2c}$. Then, $\hat{\rho}_{2c}$ and ρ_{2c} can be written from (1).

Now on, we can start to find elements of Newton-Raphson process which is written in(7). ρ_1 and ρ_3 are represented as x and y. Also essential symbolization explained in (8), (9), and (10).

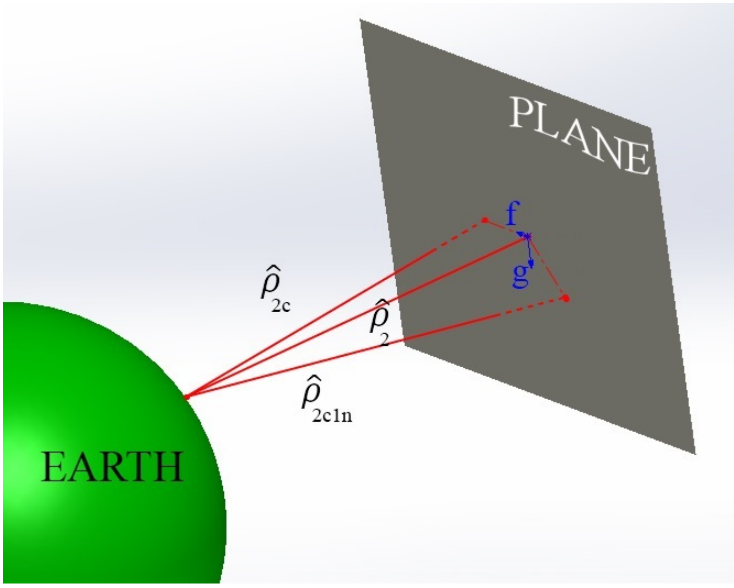

Figure 2: Gooding's Method

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = - \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix}^{-1} \begin{pmatrix} f \\ g \end{pmatrix} \tag{7}$$

$$\delta x = -D^{-1} f g_y \quad (8) \qquad \delta y = D^{-1} f g_x \quad (9) \qquad D = f_x g_y - f_y g_x \quad (10)$$

Firstly, we should determine our target functions as f and g. f lies on the plane, which is perpendicular both $\hat{\rho}_2$ and $\hat{\rho}_{2c}$.Taking cross product of $\vec{\rho}_{2c}$ and $\hat{\rho}_2$ gives us that plane. Then, taking another cross product with $\hat{\rho}_2$ gives the direction of that f function. Similarly, taking cross product of $\vec{P}$ and $\hat{\rho}_2$ gives g function. So, g lies on the plane and is perpendicular to the f. Now, finding their values is possible with equations (13) and (14). By definition, g comes 0 or so close to 0. So, equations (8), (9) and (10) are usable directly. If f comes 0 in (13), assumptions are true and process should be stop. However, it cannot be 0 in the beginning, if your assumptions are not true values. Actually, f may not be exactly 0. As an error 10^{-6} or 10^{-10} are acceptable. Lower errors give higher accuracy and more iteration. To gain computation time, 10^{-6} accepted as error margin for this paper. Henderson, T. A., Mortari, D. and Davis, J. [2010] reveals that situation clearly.

$$\vec{P} = (\hat{\rho}_2 \times \vec{\rho}_{2c}) \times \hat{\rho}_2 \quad (11) \qquad \vec{N} = \hat{\rho}_2 \times \vec{\rho}_{2c} \quad (12)$$

$$f(x,y) = \frac{\vec{P} \cdot \vec{\rho}_{2c}}{\left\|\vec{P}\right\|} \quad (13) \qquad g(x,y) = \frac{\vec{N} \cdot \vec{\rho}_{2c}}{\left\|\vec{N}\right\|} \quad (14)$$

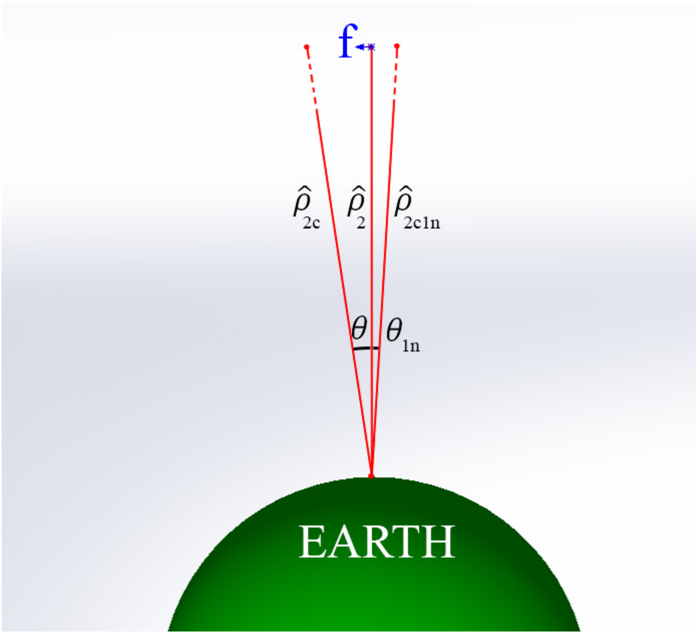

Figure 3: Upper View for Triangles

Then, first range estimations can be changed to find derivatives. Estimations multiplied by 0.001 as stated in Gooding R. H. [1997]. That process should be followed for third assumption too, but following steps will be written for first range assumption.

After that, new range assumption is $\rho_{1n} = 1.001\rho_1$. Then calculate $\hat{\rho}_{2c1n}$. Now, there is a new unit vector, and new f and g should be calculated for it.Using equation (11) to (14) is enough. These new f and g values can be written as $f_{\rho 1n}$ and $g_{\rho 1n}$. Finally, taking derivative of f and g is possible.

$$f_x = \frac{f_{\rho 1n} - f}{\rho_{1n} - \rho_1}, \quad g_x = \frac{g_{\rho 1n} - g}{\rho_{1n} - \rho_1}, \quad f_y = \frac{f_{\rho 3n} - f}{\rho_{3n} - \rho_3}, \quad g_y = \frac{g_{\rho 3n} - g}{\rho_{3n} - \rho_3} \tag{15}$$

Finally, (7) gives rearrangement factor of range assumptions. Then, modified range assumptions come like (16). Later on all of these steps repeated until f comes in the error margin. Also, there might be a shooting algorithm of range estimation to reduce time of coincidence or just use a multiple of Earth's radius.

$$\rho_{1NR} = \rho_1 + \delta x, \quad \rho_{3NR} = \rho_3 + \delta y \tag{16}$$

RESULTS AND DISCUSSION

In this section, Gauss initial orbit determination method's results for multi-observer case is given. During application of Gauss process, Curtis H. D. [2013] is strongly followed. Gooding's method is basically application of ρ and$\hat{\rho}$ into the Lambert's problem as assumed $\vec{r}_1$, and $\vec{r}_3$. Then, recalculating of estimated ρ_1, and ρ_3 with Newton-Raphson iterations for better results. In the applications MatLab is used for calculation software and all of the codes are written by the author. Also, detailed results are published in appendix. Discussion of this section will be through summary of the results.

The code for Gooding's method can easily be written with the help of algorithm (1). However, constricts of the iterations is missing. As a result of correction factors, method's accuracy is lower than expected. Also, Gooding's method takes its estimations from Gauss results to be sure that, it increases accuracy. However, Gauss sometimes have really bad answers and Gooding's method cannot correct them. So, Gauss's error make Gooding's results inaccurate too. As a nice example of that situation can be seen with the observations which have equal time separation. We know that, times would always be at uniform intervals Gooding R. H. [1993]. On the other hand, Gauss method needs less than $60°$ angular separation between observations Vallado D. A. [2001]. That condition is provided just for same angular separation observation cases. If the time intervals between observations are equal, angel between observations might be bigger than $60°$ as a result of geographical change on the observer. So, accuracy of result dramatically decreases and these bad results turn Gooding's inputs. Gooding cannot improve these really bad results and its results comes bad too. Finally, we can observe thar, Gooding should work better with equal time intervals, but bad inputs which come from Gauss method, decrease Gooding's results without any guilty.

At least 3 clear images needed as input data to make initial orbit determination. Any poor quality of these images can disturb results. So that, angles information obtained from Stellarium, which is a planetarium software that shows the celestial objects and satellites. Also, to check the results Keplerian elements are needed. Some other programs could be used for that. However, a TLE propagator is used like Erturk, M. F., Koprucu, S. U., Tugcular, U., Arda, I., Erkan, Y. B., Sisman, T. C. [2018]. A rearranged version of that propagator is used as a controller for this paper.

In total, there are 7 observers with 4 different conditions. These conditions are; all observers are the same, all of them have same longitude or latitude and each latitude and longitude elements are different. Table 1 gives their coordinates and altitudes. Figure 4 is a visualization of the observations.

	Observer-1	**Observer-2**	**Observer-3**	**Observer-4**	**Observer-5**	**Observer-6**	**Observer-7**
Latitude ($°$)	39.9455	39.9455	39.9455	41.6337	38.2573	39.7806	36.8241
Longitude ($°$)	32.6871	38.3928	26.9814	32.6871	32.6871	41.2265	30.3355
Altitude (**km**)	0.811	2.185	0.210	0.475	0.985	3.170	2.500

Table 1: Observer's Locations

Figure 4: Observer Locations on The Map

Observations searched in mainly 2 different groups. They have same time difference or same angular separation. First group has 15 seconds, 2 minutes or 3 minutes time differences. Second group has 20, 30 or 40 degree separation between each observation. To determine the angle, (17) is used for each observation. So, the angle between lines is obvious. Lastly, table 2 gives the order of observations with their observers.

$$\hat{\vec{p}} = \cos\delta\cos\alpha\hat{\boldsymbol{I}} + \cos\delta\sin\alpha\hat{\boldsymbol{J}} + \sin\delta\hat{\boldsymbol{K}} \tag{17}$$

	One Observer	**Same Latitude**	**Same Longitude**	**3 Different Observer**
1. Observation	OBS-1	OBS-2	OBS-4	OBS-6
2. Observation	OBS-1	OBS-1	OBS-1	OBS-1
3. Observation	OBS-1	OBS-3	OBS-5	OBS-7

Table 2: Observations with Observers

5 satellites selected to for their different eccentricity or inclination values. These satellites are ISS, Meteor M2-2, Glonass K-1, Molniya 1-86, Molniya 3-50. Their inclinations and eccentricities are in the table 3.

	ISS	**Meteor M2-2**	**Glonass K-1**	**Molniya 1-86**	**Molniya 3-50**
NORAD	255444	44387	37372	22671	25847
i $(°)$	51.6419	98.5713	65.7543	63.0930	62.1353
e	0.0007356	0.0002181	0.0007568	0.4843974	0.7238609

Table 3: Selected Satellites

There are some extraordinary situations for Gauss method. They become noticeable when eccentricity, inclination or both of them closing to zero. Main comparison points with these conditions should be respectively argument of latitude $(\omega + \theta)$, longitude of periapsis $(\Omega + \omega)$, and true longitude $(\Omega + \omega + \theta)$. Neglecting this information and comparing them one by one causes wrong judgments. Considering this situation provides an understanding about in which conditions Gauss method works properly. Also, there should be maximum $60°$ separation between observations, otherwise accuracy of results will be low as stated before. Some of the observations with equal time differences break the rule, because of multi-observer cases. As a natural result of geographical change on the observer, the angle between observations increases directly. That is another unforgettable criterion.

Gooding's method is weak for near polar inclinations. The method should be run twice, once assuming prograde and another assuming retrograde Schaeperkoetter A. V. [2011]. Then, picking the true results is a right way to have accurate results.

Results searched with three relative error tolerance such as 0.1, 0.01 and 0.001. Detailed results can be searched in Appendix. In general, one observer case has more accurate results. However, if you look more close for each satellite or method, having multi observers might be an advantage. Especially, Gooding's method has nearly same accuracy for same latitude and same observer cases. Gooding's results with the observers, which have same longitude, might be better as others, if inputs of it were nice. Results basically told us that, Gooding's method gives more accurate results for most of the cases, but Gauss method has a little bit more accurate results. That shows, if Gooding's method applied completely without Gauss' bad inputs, it would be perfect.

With the light of table results, having only one observer seems enough to make accurate orbit determination. However, orbit of satellite may affect that decision. Especially, one observer may not be enough, while satellite's inclination increasing. The best thing is considering both orbit and observation details. So, multi observers are better, when true satellite and true observation technique are selected.

CONCLUSIONS

This study aims to compare angles only methods. For this comparison, both Gauss and Gooding's method are chosen and results for 5 different satellites presented. These satellites selected because of their various inclination and eccentricity values. These results can be extended with different satellites for a better generalization.

Gauss method is applied for many years, but just works well with suitable conditions. Results show that obviously, multi observer case affects accuracy of initial orbit determination. Orbital characteristics and observation technique, such as having same time or angle separation, directly affect accuracy too.

On the other hand, Gooding's method is relatively new method. It claims to work with nearly any conditions. Exact method could not applied but, preliminary results support that claim. If the inputs are good enough, it can work with any observer.

Having a knowledge of satellites, which are orbiting around us, is an important data especially for ground segment of any mission. Suchlike, the accuracy of this data is another important feature. As discussed above, this accuracy can be achieved by true methods and true applications.

ACKNOWLEDGEMENTS

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APPENDIX

There are some notes about appendixes to make them clear. Firstly, Appendix A is the general algorithm for Gooding's method. There are more steps in real application. However, in this work, main purpose is giving an idea about application of Gooding's method. Secondly, for the Appendix B and C, given times are 2. observation's time. All of listed results valid for OBS-1 location. Also given TLE is the closest one to the calculation time. Thirdly, the errors in Appendix B and C are compared with respect to 7 orbital elements. As a result of special orbit cases, which are stated in results and discussion part, sometimes argument of latitude chosen as a comparison element apart from just ω and θ.

Appendix A

Algorithm 1 Gooding's Method Algorithm

1. Define observation values as δ_{1-2-3}, α_{1-2-3}, t_{1-2-3}
2. Specify orbit type "retrograde" or "prograde"
3. Find observer position vectors ($\vec{R}$) with respect to t_{1-2-3}
4. Calculate $\hat{\rho}_1$, $\hat{\rho}_2$ and $\hat{\rho}_3$
5. Find $\Delta t_{13} = t_3 - t_1$, $t_3 > t_1$
6. Make range assumptions as ρ_1 and ρ_3
7. Calculate $\vec{r}_{1-3}$ vectors with $\vec{r} = \vec{R} - \hat{\rho}\rho$
8. Go to a Lambert Solver.
   (a) Inputs: $\vec{r}_1$, $\vec{r}_3$, Δt_{13}, "retrograde" or "prograde"
   (b) Outputs: $\vec{v}_1$, $\vec{v}_3$
9. Use Keplerian function
   (a) Find E and M.
   (b) Rearrange them for $(t_1{+}\Delta t_{12})$.
   (c) Find orbital elements for $(t_1 + \Delta t_{12})$.
   (d) Go to $\vec{r}_{2c}$ and $\vec{v}_{2c}$ from these elements .
10. Find $\vec{P}$, $\vec{N}$, f and g.
11.$\rho_{1n} = 1.01\rho_1$ or $\rho_{1n} = 1.001\rho_1$. Find $\hat{\rho}_{2c1n}$ and $\vec{\rho}_{2c1n}$.
12. Find $\vec{P}_{\rho 1n}$, $\vec{N}_{\rho 1n}$, $f_{\rho 1n}$ and $g_{\rho 1n}$.
13. Calculate $g_x = \frac{g_{\rho 1n} - g}{\rho_{1n} - \rho_1}$, $f_x = \frac{f_{\rho 1n} - f}{\rho_{1n} - \rho_1}$.
14. Repeat step 12, and 13 for $\rho_{3n} = 1.01\rho_3$ or $\rho_{1n} = 1.001\rho_1$. Find $\hat{\rho}_{2c3n}$ and $\vec{\rho}_{2c3n}$.
15. Calculate $g_y = \frac{g_{\rho 3n} - g}{\rho_{3n} - \rho_3}$, $f_y = \frac{f_{\rho 3n} - f}{\rho_{3n} - \rho_3}$.
17. Find determinant of $\begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix}$ as $D = (f_x g_y - f_y g_x)$.
18. Find $\delta x = -\frac{1}{D} f g_y$ and $\delta y = \frac{1}{D} f g_x$.
19. Calculate new range assumptions $\rho_{1NR} = \rho_1 + \delta x$ and $\rho_{3NR} = \rho_3 + \delta y$.
20. Repeat the process until $f = 0$ or close enough to zero.
21. Calculate orbital elements from $\vec{r}_{2True}$ and $\vec{v}_{2True}$.

Appendix B

ISS									
Closest TLE	1 25544U 98067A 19238.13982639 .00004845 00000-0 91557-4 0 9996 2 25544 51.6437 14.2544 0007486 327.1999 9.5168 15.50394402 186101								
Time	26.8.2019 − 06 : 57 : 10 (UTC0)								
20Degree Separation	$\alpha\,(^\circ)$	330.32076 296.24621 256.93171		332.81258 296.24621 256.33339		332.04311 296.24621 258.47092		330.07904 296.24621 258.85055	
	$\delta\,(^\circ)$	52.9433 61.6224 57.3969		54.9767 61.6224 57.8444		54.5148 61.6224 56.3198		52.7367 61.6224 55.7474	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\,\left(\frac{km^2}{s}\right)$	52038.16	51743.84	51752.82	55588.39	44313.79	51590.71	51601.45	475.50	81928.83
e	0.000749	0.010660	0.010371	0.170408	0.291223	0.015967	0.015643	0.992822	1.6728083
$\Omega\,(^\circ)$	13.51	13.05	13.054	9.17	11.81	12.77	12.81	109.54	289.54
$i\,(^\circ)$	51.64	51.58	51.59	51.99	51.67	51.57	51.57	88.78	91.22
$w+\theta\,(^\circ)$	93.84	89.91	94.60	96.99	95.38	90.97	94.76	227.36	133.65
a (km)	6793.7	6717.8	6720.1	7984.1	5383.1	6679.1	6681.8	39.65	−9364.3
30Degree Separation	$\alpha\,(^\circ)$	340.66464 296.24621 236.82812		343.90338 296.24621 242.04374		342.60624 296.24621 244.76029		340.50223 296.24621 245.26901	
	$\delta\,(^\circ)$	45.6776 61.6224 46.6748		47.7178 61.6224 52.1765		49.9140 61.6224 50.2336		45.4306 61.6224 50.0111	
$h\,\left(\frac{km^2}{s}\right)$	52038.16	51726.04	49449.90	58424.93	58358.51	51678.85	51612.33	40154.63	40148.79
e	0.000749	0.011643	0.098302	0.256507	0.253678	0.013243	0.041226	0.404575	0.405014
$\Omega\,(^\circ)$	13.51	13.21	2.40	12.56	12.60	13.09	18.67	306.95	306.96
$i\,(^\circ)$	51.64	51.58	50.98	52.19	52.19	51.57	51.90	75.65	75.67
$w+\theta\,(^\circ)$	93.84	86.87	101.68	101.58	94.09	93.02	91.06	91.99	130.45
a (km)	6793.7	6713.4	6194.6	9166.8	9131.8	6701.4	6694.3	4836.8	4837.5
40Degree Separation	$\alpha\,(^\circ)$	348.01461 296.24621 233.5195		351.49110 296.24621 231.78941		349.86460 296.24621 235.12557		347.77102 296.24621 235.34077	
	$\delta\,(^\circ)$	37.1090 61.6224 43.6517		39.2087 61.6224 44.9085		38.1995 61.6224 42.7698		36.9860 61.6224 42.6170	
$h\,\left(\frac{km^2}{s}\right)$	52038.16	51478.30	51462.29	50518.34	50547.02	51468.46	51466.95	34318.74	34306.45
e	0.000749	0.020559	0.020935	0.056191	0.055143	0.020912	0.020783	0.558697	0.558924
$\Omega\,(^\circ)$	13.51	13.24	13.25	13.14	13.15	13.13	13.12	335.01	335.00
$i\,(^\circ)$	51.64	51.52	51.52	51.46	51.47	51.52	51.53	58.62	58.62
$w+\theta\,(^\circ)$	93.84	83.27	94.49	88.55	94.56	84.84	94.57	110.91	118.03
a (km)	6793.7	6651.1	6647.1	6422.9	6429.5	6648.7	6648.2	4295.6	4294.1

Table 4: ISS with Equal Angular Separations

MeteorM2 2									
Closest TLE	1 44387U 19038A 19233.46746596 -.00000044 00000-0 -72411-6 0 9993 2 44387 98.5734 195.3015 0001194 353.7973 6.3191 14.23333501 6719								
Time	21.8.2019 − 13 : 05 : 10 (UTC0)								
20Degree Separation	$\alpha\,(^\circ)$	154.66174 143.37805 129.20397		145.41041 143.37805 157.83410		154.41026 143.37805 130.16763		154.56609 143.37805 138.53426	
	$\delta\,(^\circ)$	−0.4476 16.3085 31.7964		−3.5660 16.3085 31.2986		−0.3622 16.3085 32.2813		−0.4484 16.3085 35.9316	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gooding's Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$	53542.74	53631.42	53559.73	52794.40	52750.96	54160.10	53474.36	175671.67	280379.19
e	0.000119	0.004154	0.002133	0.028599	0.030243	0.087554	0.432699	12.013580	33.543346
$\Omega\,(^\circ)$	195.38	195.41	195.29	195.92	195.96	198.36	14.59	302.64	318.82
$i\,(^\circ)$	98.57	98.67	98.52	99.23	99.28	102.12	94.79	139.46	134.99
$w+\theta\,(^\circ)$	38.49	34.92	38.48	34.13	38.59	34.82	139.99	106.64	117.86
a (km)	7192.2	7216.2	7196.8	6998.3	6987.5	7415.9	4497.5	−540.2	−175.4
30Degree Separation	$\alpha\,(^\circ)$	159.85144 143.37805 120.27730		151.34072 143.37805 143.69952		159.87651 143.37805 121.05878		159.77377 143.37805 128.84109	
	$\delta\,(^\circ)$	−8.9983 16.3085 38.4723		−12.7214 16.3085 46.5221		−9.0183 16.3085 39.3282		−9.0463 16.3085 43.8650	
$h\left(\frac{km^2}{s}\right)$	53542.74	53456.05	53467.35	54719.55	55437.55	53281.83	18031.41	140755.69	140674.62
e	0.000119	0.003479	0.003108	0.046275	0.070322	0.008900	0.872718	5.652608	7.641073
$\Omega\,(^\circ)$	195.38	195.27	195.27	195.34	194.26	195.26	17.43	343.29	343.291
$i\,(^\circ)$	98.57	98.46	98.75	98.76	94.50	98.43	91.14	128.34	128.34
$w+\theta\,(^\circ)$	38.49	309.55	38.48	31.53	38.32	14.45	140.27	111.44	124.16
a (km)	7192.2	7169.0	7172.1	7528.0	7748.6	7122.9	3422.0	−1605.8	−865.1
40Degree Separation	$\alpha\,(^\circ)$	164.96541 143.37805 109.22409		156.89509 143.37805 129.57315		165.16827 143.37805 110.07288		164.91077 143.37805 116.81724	
	$\delta\,(^\circ)$	−17.5382 16.3085 44.1969		−21.4728 16.3085 59.9004		−17.5368 16.3085 45.1601		−17.6380 16.3085 50.2803	
$h\left(\frac{km^2}{s}\right)$	53542.74	53303.88	53149.51	53488.05	53580.92	53277.15	12040.71	159260.58	159724.45
e	0.000119	0.008180	0.012831	0.001557	0.002077	0.009230	0.943183	9.299980	11.353621
$\Omega\,(^\circ)$	195.38	195.28	195.41	195.27	195.23	195.25	26.47	19.76	19.74
$i\,(^\circ)$	98.57	98.45	98.56	98.49	98.46	98.41	81.12	122.51	122.51
$w+\theta\,(^\circ)$	38.49	10.84	38.51	13.38	38.47	8.89	139.52	140.33	131.24
a (km)	7192.2	7128.7	7088.1	7177.6	7202.5	7121.7	3294.4	−744.3	−500.4

Table 5: Meteor M2-2 with Equal Angular Separations

Glonass K 1									
Closest TLE	1 37372U 11009A 19232.68367858 -.00000023 00000-0 00000+0 0 9996 2 37372 65.7635 37.8716 0007902 219.0910 100.0289 2.13107500 65974								
Time	21.8.2019 − 06 : 41 : 17 (UTC0)								
20Degree Separation	$\alpha\,(^\circ)$	45.65467 56.39346 78.02362		45.01616 56.39346 78.83101		46.09970 56.39346 77.27910		45.60219 56.39346 76.96491	
	$\delta\,(^\circ)$	30.8924 49.1721 65.6979		31.0969 49.1721 65.3800		30.74150 49.1721 66.0031		30.8964 49.1721 66.1322	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\,\left(\frac{km^2}{s}\right)$	100833.10	99243.39	99106.92	99415.16	99302.95	99265.01	42335.55	161867.57	40837.74
e	0.000790	0.024963	0.0261920.024173	0.022481	0.023501	0.024649	0.841166	1	0.773952
$\Omega\,(^\circ)$	37.85	37.67	37	37.67	37.67	37.66	310.19	37.87	314.72
$i\,(^\circ)$	65.76	65.74	65.74	65.75	65.75	65.73	38.63	67.83	41.00
$w+\theta\,(^\circ)$	55.57	38.88	55.5	39.55	55.51	39.18	12.92	62.19	138.43
a (km)	25507.6	24725.0	24658.6	24807.8	24752.9	24735.4	15375.7	76495538200.5	10433.8
30Degree Separation	$\alpha\,(^\circ)$	41.65685 56.39346 100.49210		41.00029 56.39346 101.04527		42.06326 56.39346 99.84048		41.61375 56.39346 99.57468	
	$\delta\,(^\circ)$	21.5341 49.1721 71.6956		21.7464 49.1721 71.3478		21.4140 49.1721 72.0790		21.5459 49.1721 72.2767	
$h\,\left(\frac{km^2}{s}\right)$	100833.10	97374.23	96776.96	97751.60	97227.23	97441.80	68926.32	135713.03	141313.26
e	0.000790	0.054819	0.059724	0.049075	0.053451	0.053787	0.471287	0.595421	2.544314
$\Omega\,(^\circ)$	37.85	37.69	37.69	37.70	37.70	37.67	200.07	38.14	201.28
$i\,(^\circ)$	65.76	65.65	65.65	65.68	65.68	65.64	130.80	67.10	122.59
$w+\theta\,(^\circ)$	55.57	30.3	55.56	30.84	55.55	30.62	23.78	79.51	69.49
a (km)	25507.6	23859.3	23580.8	24031.7	23783.8	23889.7	15322.0	71585.9	−9152.9
40Degree Separation	$\alpha\,(^\circ)$	38.05404 56.39346 133.88594		37.37911 56.39346 133.69173		38.42245 56.39346 134.06806		38.0213 56.39346 134.09424	
	$\delta\,(^\circ)$	12.1402 49.1721 73.5696		12.3837 49.1721 73.2566		12.0222 49.1721 74.0014		12.1683 49.1721 74.2463	
$h\,\left(\frac{km^2}{s}\right)$	100833.10	95450.9	93938.42	95948.55	94571.00	99505.16	29160.40	119572.92	123494.7
e	0.000790	0.087241	0.097634	0.079470	0.089008	0.086386	0.876848	0.317094	2.367713
$\Omega\,(^\circ)$	37.85	37.73	37.73	37.75	37.75	37.71	199.77	38.29	237.86
$i\,(^\circ)$	65.76	65.52	65.52	65.56	65.56	65.51	127.38	66.66	73.52
$w+\theta\,(^\circ)$	55.57	21.24	55.62	21.72	55.60	21.57	87.66	68.64	253.88
a (km)	25507.6	23032.5	22351.6	23242.9	22616.9	23055.2	9229.5	39879.6	−8306.7

Table 6: Glonass K-1 with Equal Angular Separations

Molniya 1-86									
Closest TLE	1 22671U 93035A 19233.45122225 -.00000030 00000-0 10326-4 0 9999 2 22671 63.1020 112.4662 4841687 352.7072 2.2904 5.62587110 294241								
Time	$21.8.2019 - 11:05:12$ (UTC0)								
20Degree Separation	$\alpha\,(^\circ)$	86.31635 88.36616 90.57380		85.73913 88.36616 98.48477		89.50012 88.36616 82.76617		85.95806 88.36616 78.72076	
	$\delta\,(^\circ)$	36.3333 56.2373 76.1438		36.2362 56.2373 76.00500		36.3113 56.2373 76.1108		36.2422 56.2373 75.9422	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$	63836.40	4246.76	64034.98	50480.87	50638.85	64336.32	64318.09	74239.81	80280.18
e	0.484169	0.846099	0.49	0.405710	0.404995	0.497747	0.499763	0.517268	0.815111
$\Omega\,(^\circ)$	112.46	199.76	112.28	116.27	116.20	122.31	112.31	97.77	91.98
$i\,(^\circ)$	63.10	57.49	63.13	63.55	63.53	63.19	63.18	59.13	57.26
$w\,(^\circ)$	352.71	59.90	353.32	293.76	294.48	354.35	354.17	67.98	89.12
$\theta\,(^\circ)$	67.37	177.74	66.71	110.08	124.05	55.45	65.84	22.13	340
a (km)	13353.9	159.2	13589.5	7652.8	7695.5	13804.3	13833.4	18878.5	48179.9
30Degree Separation	$\alpha\,(^\circ)$	82.20205 88.36616 92.22337		84.80871 88.36616 118.39830		88.05659 88.36616 67.90563		84.89788 88.36616 58.49726	
	$\delta\,(^\circ)$	26.25540 56.2373 86.3161		26.3047 56.2373 85.6165		26.2063 56.2373 85.9639		26.3261 56.2373 85.6417	
$h\left(\frac{km^2}{s}\right)$	63836.40	64010.44	64018.55	62837.15	62854.20	66434.80	64451.00	68957.8	68958.29
e	0.484169	0.493406	0.501368	0.464727	0.469228	0.499453	0.506710	0.430905	0.433206
$\Omega\,(^\circ)$	112.46	112.41	112.42	112.42	112.41	112.26	112.26	105.09	105.09
$i\,(^\circ)$	63.10	63.17	63.17	63.04	63.04	63.18	63.18	61.16	61.16
$w\,(^\circ)$	352.71	353.21	352.87	349.43	349.33	354.70	354.38	25.27	24.91
$\theta\,(^\circ)$	67.37	53.28	67.10	56.36	70.67	53.06	65.64	34.76	38.13
a (km)	13353.9	13587.1	13734.3	12634.7	12709.7	13878.0	14021.3	14649.9	14685.94
40Degree Separation	$\alpha\,(^\circ)$	83.98285 88.36616 272.75831		83.68281 88.36616 255.15063		86.57203 88.36616 285.56273		83.69089 88.36616 289.97361	
	$\delta\,(^\circ)$	16.3911 56.2373 83.76740		16.3794 56.2373 83.5868		16.1981 56.2373 83.4796		16.3329 56.2373 83.3147	
$h\left(\frac{km^2}{s}\right)$	63836.40	64081.42	64165.79	63183.27	63175.35	64398.21	64502.48	65478.69	64779.18
e	0.484169	0.489133	0.507309	0.470380	0.480915	0.499853	0.5182245684	0.449843	0.436535
$\Omega\,(^\circ)$	112.46	112.208614	112.20	112.33	112.33	112.30	112.29	109.65	109.56
$i\,(^\circ)$	63.10	63.10	63.10	63.03	63.02	53.18	63.18	62.52	62.31
$w\,(^\circ)$	352.71	353.74	353.23	350.73	350.27	354.57	354.09	1.87	359.73
$\theta\,(^\circ)$	67.37	50.50	66.83	52.68	69.76	50.81	65.93	45.57	61.44
a (km)	13353.9	13542.1	13908.9	12860.9	13025.3	13869.6	−515.7	13485.1	13006.2

Table 7: Molniya 1-86 with Equal Angular Separations

Molniya 3-50										
Closest TLE	1 25847U 99036A 19233.76210190 -.00000163 +00000-0 -62306-2 0 9998 2 25847 062.1495 136.6226 7244562 272.6202 014.1955 02.00619306147433									
Time	21.8.2019 − 06 : 55 : 00 (UTC0)									
20Degree Separation	$\alpha\,(^\circ)$		162.8847 169.49017 39.9455		161.77970 169.49017 181.80450		162.60967 169.49017 180.94822		162.92396 169.49017 181.36461	
	$\delta\,(^\circ)$		6.2136 25.2018 42.8668		6.6172 25.2018 42.4349		6.3125 25.2018 42.8572		6.2107 25.2018 42.6672	
			One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
		Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$		70919.66	71224.24	72008.76	71691.09	72577.42	71453.70	72198.26	45642.16	46522.26
e		0.724456	0.711774	0.756869	0.712951	0.760627	0.713392	0.756140	0.773743	0.790739
$\Omega\,(^\circ)$		136.55	136.71	136.71	136.79	136.79	136.71	136.71	134.02	133.91
$i\,(^\circ)$		62.15	62.51	62.51	62.53	62.53	62.50	62.50	63.50	63.18
$w\,(^\circ)$		272.63	271.96	275.38	272.59	276.19	272.37	275.57	240.23	244.71
$\theta\,(^\circ)$		124.18	91.05	121.21	90.60	120.37	91.16	121.02	128.62	152.82
a (km)		26555.4	25795.2	30454.6	26223.6	31356.2	26083.6	30536.2	13022.7	14489.8
30Degree Separation	$\alpha\,(^\circ)$		160.51178 169.49017 190.13322		159.32243 169.49017 191.00769		160.16037 169.49017 190.10380		160.55613 169.49017 190.47373	
	$\delta\,(^\circ)$		−3.5090 25.2018 50.6667		−3.1217 25.2018 50.1794		−3.4027 25.2018 50.6859		−3.5276 25.2018 50.4954	
$h\left(\frac{km^2}{s}\right)$		70919.66	69902.91	74365.63	70649.58	75409.08	70223.24	74516.95	54307.02	57876.98
e		0.724456	0.695012	0.856782	0.695486	0.866249	0.696345	0.852118	0.734368	0.818987
$\Omega\,(^\circ)$		136.55	137.02	137.02	137.15	137.15	137.03	137.03	135.87	135.87
$i\,(^\circ)$		62.15	62.76	62.76	62.81	62.81	62.74	62.74	63.43	63.43
$w\,(^\circ)$		272.63	268.26	281.97	269.27	283.27	268.84	281.94	247.84	262.18
$\theta\,(^\circ)$		124.18	88.18	114.49	87.57	113.13	88.14	114.51	109.70	134.64
a (km)		26555.4	23713.62	52173.5	24253.8	57153.5	24017.6	50861.5	16060.3	25523.1
40Degree Separation	$\alpha\,(^\circ)$		158.51319 169.49017 203.16695		157.20377 169.49017 203.97230		158.07646 169.49017 203.09448		158.56361 169.49017 203.40853	
	$\delta\,(^\circ)$		−13.3430 25.2018 57.1672		−12.9757 25.2018 56.6251		−13.2295 25.2018 57.2240		−13.3819 25.2018 57.0445	
$h\left(\frac{km^2}{s}\right)$		70919.66	66358.49	91730.58	67292.05	93003.95	66628.29	90458.21	56153.66	82725.13
e		0.724456	0.689069	1.710733	0.687049	1.719906	0.689093	1.626383	0.735021	1.764494
$\Omega\,(^\circ)$		136.55	137.11	137.10	137.32	137.32	137.12	137.12	136.46	136.46
$i\,(^\circ)$		62.15	62.85	62.85	62.95	62.95	62.82	62.82	63.22	63.22
$w\,(^\circ)$		272.63	260.36	305.76	261.57	306.36	260.88	304.71	248.14	301.30
$\theta\,(^\circ)$		124.18	88.87	90.66	88.22	89.97	88.87	91.71	101.73	95.33
a (km)		26555.4	21035.1	−10957.1	21517.2	−11082.5	21207.8	−12478.4	17206.9	−8123.6

Table 8: Molniya 3-50 with Equal Angular Separations

Appendix C

ISS									
Closest TLE	1 25544U 98067A 19235.90586947 -.00000030 00000-0 74576-5 0 9996 2 25544 51.6419 25.3268 0007356 318.4296 141.4573 15.50391036185763								
Time	24.8.2019 − 03 : 42 : 00 (UTC0)								
15Seconds Separation	$\alpha\,(^\circ)$	43.88388 52.52196 63.20203		10.99760 52.52196 214.13694		36.98771 52.52196 214.14559		33.03665 52.52196 63.18270	
	$\delta\,(^\circ)$	1.4167 9.2638 18.1069		−13.1807 9.2638 42.8394		−23.9627 9.2638 42.8411		−17.5740 9.2638 18.0921	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$	52038.20	54336.27	54322.62	136.54	207765.50	77009.30	77659.29	1241392.254093	1252186.52
e	0.000736	0.098642	0.106520	0.986176	16.050865	0.665972	1.375568	966.82	982.597251
$\Omega\,(^\circ)$	24.10	23.68	23.38	44.67	152.77	242.05	241.76	68.36	68.22
$i\,(^\circ)$	51.64	52.78	52.57	77.28	140.12	87.79	88.28	64.01	64.01
$w+\theta\,(^\circ)$	46.74	50.17	50.34	202.69	91.85	141.78	140.22	103.34	127.96
a (km)	6793.7	7479.8	7488.2	1.7	−422.0	26736.1	−16958.8	−4.1	−4.1
2Minutes Separation	$\alpha\,(^\circ)$	18.84364 52.52196 141.13795		3.70393 52.52196 129.67540		18.38704 52.52196 124.37464		14.43201 52.52196 141.14175	
	$\delta\,(^\circ)$	−24.0779 9.2638 41.0163		−23.0954 9.2638 15.5783		−31.7037 9.2638 42.7982		−28.6585 9.2638 41.0159	
$h\left(\frac{km^2}{s}\right)$	52038.20	53907.12	53900.90	363.66	15814.4	9571.92	9570.31	7328.62	196934.49
e	0.000736	0.095254	0.095462	0.999013	0.903101	0.964696	0.964704	0.624117	19.530497
$\Omega\,(^\circ)$	24.10	23.30	23.30	308.59	128.59	300.23	300.22	17.50	197.50
$i\,(^\circ)$	51.64	52.54	52.54	41.27	138.73	43.10	43.10	58.31	121.69
$w+\theta\,(^\circ)$	46.74	42.49	50.41	286.55	73.47	113.02	112.75	241.25	116.08
a (km)	6793.7	7357.2	7355.8	168.1	3402.4	3314.0	3313.5	220.7	−255.8
3Minutes Separation	$\alpha\,(^\circ)$	7.68064 52.52196 157.45115		233.79930 52.52196 144.83445		12.15945 52.52196 151.99017		8.58791 52.52196 157.44580	
	$\delta\,(^\circ)$	−29.3472 9.2638 37.8201		0.9967 9.2638 20.7490		−34.2447 9.2638 39.8306		−31.9100 9.2638 37.8217	
$h\left(\frac{km^2}{s}\right)$	52038.20	53524.81	53519.64	839.73	17288.40	46942.29	14880.02	8800.93	148158.64
e	0.000736	0.083977	0.084927	0.997875	0.891492	0.552613	0.915931	0.730234	8.751337
$\Omega\,(^\circ)$	24.10	23.28	23.28	289.49	109.48	5.01	359.15	20.97	200.97
$i\,(^\circ)$	51.64	52.55	52.55	45.50	134.50	47.04	42.43	55.52	124.48
$w+\theta\,(^\circ)$	46.74	38.34	50.45	300.53	59.36	48.37	67.56	232.95	123.29
a (km)	6793.7	7238.5	7238.2	416.6	3653.5	7958.7	3448.7	416.3	−728.6

Table 9: ISS with Equal Time Separations

MeteorM2-2									
Closest TLE	1 44387U 19038A 19214.90892900 -.00000044 00000-0 -48019-6 0 9999 2 44387 98.5713 177.2208 0002181 34.4318 325.7002 14.23333281 4073								
Time	03.8.2019 − 01 : 48 : 00 (UTC0)								
15Seconds Separation	$\alpha\,(^\circ)$	241.61163 255.66209 269.65243		265.70568 255.66209 119.72696		259.95626 255.66209 252.84943		239.93663 255.66209 217.83760	
	$\delta\,(^\circ)$	75.3429 75.0867 73.8486		60.1425 75.0867 83.5220		76.4092 75.0867 73.9301		75.1199 75.0867 78.2015	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\,\left(\frac{km^2}{s}\right)$	53542.74	53671.49	53595.92	54192.16	54317.34	53526.77	53629.26	624.55	2875860.32
e	0.000218	0.005126	0.002711	0.024559	0.028901	0.003653	0.003014	1.12308	3637.779973
$\Omega\,(^\circ)$	177.38	177.23	177.24	181.15	181.18	177.25	177.22	12.57	192.57
$i\,(^\circ)$	98.57	98.70	98.71	95.24	95.21	98.60	98.62	95.08	84.92
$w+\theta\,(^\circ)$	130.65	165.8	130.67	130.22	131.00	36.38	130.53	243.34	129.74
a (km)	7192.2	7227.1	7206.6	7322.2	7408.0	7188.0	7215.6	−3.7	−1.6
2Minutes Separation	$\alpha\,(^\circ)$	192.77617 255.66209 317.64007		223.50040 255.66209 357.13396		193.20399 255.66209 309.70497		192.73306 255.66209 311.84403	
	$\delta\,(^\circ)$	66.0478 75.0867 42.9468		62.6967 75.0867 55.3021		67.7185 75.0867 50.8421		65.8261 75.0867 64.6666	
$h\,\left(\frac{km^2}{s}\right)$	53542.74	53441.23	53433.23	53721.92	53719.25	53368.34	53361.74	2169.54	146818.88
e	0.000218	0.003394	0.003609	0.006944	0.006912	0.005768	0.005943	0.946183	8.110550
$\Omega\,(^\circ)$	177.38	177.25	177.25	177.30	177.30	177.29	177.29	350.83	170.83
$i\,(^\circ)$	98.57	98.71	98.71	98.64	98.64	98.68	98.68	133.15	46.84
$w+\theta\,(^\circ)$	130.65	104.25	130.68	123.69	130.66	123.74	130.70	126.51	229.72
a (km)	7192.2	7165.1	7162.9	7240.8	7240.1	7145.7	7143.9	112.7	−834.8
3Minutes Separation	$\alpha\,(^\circ)$	185.15300 255.66209 324.99597		208.43927 255.66209 351.85291		184.59332 255.66209 312.85310		185.21264 255.66209 324.24663	
	$\delta\,(^\circ)$	60.3150 75.0867 19.1749		59.5686 75.06867 24.9701		61.2320 75.0867 28.0368		60.1725 75.0867 40.7896	
$h\,\left(\frac{km^2}{s}\right)$	53542.74	52836.39	52834.14	53135.15	53133.81	34872.51	34860.96	459.89	32861595.70
e	0.000218	0.024381	0.024259	0.013987	0.013835	0.557979	0.558101	0.766226	31402.008242
$\Omega\,(^\circ)$	177.38	177.33	177.32	177.17	177.17	179.48	179.47	338.41	158.41
$i\,(^\circ)$	98.57	98.71	98.71	98.82	98.81	98.66	98.66	102.56	77.44
$w+\theta\,(^\circ)$	130.65	120.69	130.78	120.27	130.72	130.40	133.41	260.45	94.35
a (km)	7192.2	7007.9	7007.2	7084.5	7084.1	4430.2	4428.2	1.3	−2.7

Table 10: Meteor M2-2 with Equal Time Separations

Glonass K-1									
Closest TLE	1 37372U 11009A 19214.73481554 -.00000028 +00000-0 +00000-0 0 9998 2 37372 065.7543 038.4734 0007568 221.6059 007.4938 02.13107584065591								
Time	3.8.2019 − 12 : 40 : 00 (UTC0)								
15Seconds Separation	$\alpha\,(^\circ)$	182.20258 182.39991 182.59630		179.42153 182.39991 185.27948		182.10563 182.39991 182.69417		182.21334 182.39991 183.97101	
	$\delta\,(^\circ)$	58.8691 58.7418 58.6140		58.9525 58.7418 58.3492		58.3446 58.7418 59.1309		58.9230 58.7418 59.4703	
		One Observer		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$	100833.09	55046.65	55047.04	109968.98	111109.20	112024.34	112468.27	406.50	16686046.21
e	0.000757	0.618845	0.618522	0.185867	0.221313	0.717342	0.735675	6.40	259478.73
$\Omega\,(^\circ)$	38.45	37.40	37.39	39.63	40.12	21.71	21.48	162.20	342.20
$i\,(^\circ)$	65.75	64.72	64.72	65.20	64.98	74.38	74.51	78.02	101.99
$w+\theta\,(^\circ)$	117.36	81.39	118.42	120.66	116.52	210.83	123.49	240.67	120.59
a (km)	25507.6	12320.2	12312.4	31424.7	32566.6	64858.8	69169.8	−0.01	−0.1
2Minutes Separation	$\alpha\,(^\circ)$	180.78189 182.39991 183.93052		177.92922 182.39991 186.53956		180.69492 182.39991 184.03625		180.79185 182.39991 185.28681	
	$\delta\,(^\circ)$	58.7505 58.7418 57.7123		59.8125 58.7418 57.4325		59.2284 58.7418 58.2323		59.8041 58.7418 58.5669	
$h\left(\frac{km^2}{s}\right)$	100833.09	26370.76	26182.41	104148.24	104162.2	117781.55	117785.68	7713.83	282272.10
e	0.000757	0.923246	0.942936	0.057320	0.057579	0.258727	0.258840	0.479352	80.585869
$\Omega\,(^\circ)$	38.45	157.41	157.41	38.53	38.53	39.82	39.82	162.05	342.05
$i\,(^\circ)$	65.75	80.71	80.71	65.72	65.72	65.45	65.45	76.79	103.21
$w+\theta\,(^\circ)$	117.36	108.11	271.38	117.53	117.17	123.82	116.48	170.80	130.17
a (km)	25507.6	11818.7	15511.8	27302.1	27310.2	37299.9	37304.8	193.8	−30.8
3Minutes Separation	$\alpha\,(^\circ)$	179.93794 182.39991 184.66451		177.04385 182.39991 187.23269		179.85768 182.39991 184.77424		179.94773 182.39991 186.00997	
	$\delta\,(^\circ)$	60.2467 58.7418 57.1913		60.2952 58.7418 56.9032		59.7257 58.7418 57.7124		60.2998 58.7418 58.0446	
$h\left(\frac{km^2}{s}\right)$	100833.09	99560.37	99541.04	104330.35	104298.18	92820.94	92814.94	9146.56	130215.99
e	0.000757	0.019694	0.020052	0.058714	0.058119	0.116656	0.116807	0.446507	16.891524
$\Omega\,(^\circ)$	38.45	38.26	38.26	38.48	38.48	37.75	37.76	161.69	341.69
$i\,(^\circ)$	65.75	65.77	65.77	65.75	65.75	65.84	65.84	76.03	103.97
$w+\theta\,(^\circ)$	117.36	116.1	117.32	117.26	117.19	111.2	117.62	190.15	133.17
a (km)	25507.6	24877.3	24868.0	27402.1	27383.3	21913.2	21911.1	262.1	−149.6

Table 11: Glonass K-1 with Equal Time Separations

Molniya 1-86									
Closest TLE	1 22671U 93035A 19215.49847584 -.00000023 00000-0 -21375-4 0 9994 2 22671 63.0930 122.8629 4843974 352.4523 2.3367 5.62587008 293235								
Time	3.8.2019 − 12 : 11 : 20 (UTC0)								
15Seconds Separation	$\alpha\,(^\circ)$	105.90415 106.73804 107.63104		99.57652 106.73804 123.39928		109.83052 106.73804 103.12812		105.43373 106.73804 103.50846	
	$\delta\,(^\circ)$	50.5810 53.3314 55.9977		43.1198 53.3314 63.8697		47.1362 53.3314 58.9050		50.8919 53.3314 64.8584	
		One Observer		*Same Latitude*		Same *Longitude*		3 Different Observers	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\left(\frac{km^2}{s}\right)$	63827.17	64121.54	63840.09	66648.13	66087.57	71050.40	98778.90	2582986.10	2578775.79
e	0.484397	0.494325	0.544198	0.662972	0.577773	0.637226	2.028928	3767.204206	3756.472090
$\Omega\,(^\circ)$	122.86	122.73	124.31	122.75	121.33	122.40	130.73	38.32	38.38
$i\,(^\circ)$	63.09	63.14	64.19	63.24	62.31	63.33	70.99	52.68	52.69
$w\,(^\circ)$	352.45	353.36	348.45	354.78	358.02	17.30	23.20	166.58	166.56
$\theta\,(^\circ)$	61.63	55.71	64.96	60.25	56.72	49.96	27.64	18.72	284.47
a (km)	13353.9	13650.7	14526.8	19883.2	16447.9	21323.1	−7854.5	−1.2	−1.2
2Minutes Separation	$\alpha\,(^\circ)$	101.22820 106.73804 117.30378		97.00264 106.73804 152.43567		104.31118 106.73804 110.39400		100.86462 106.73804 114.10634	
	$\delta\,(^\circ)$	30.2787 53.3314 72.3781		25.4178 53.3314 77.5914		26.5780 53.3314 74.6051		30.6066 53.3314 79.5360	
$h\left(\frac{km^2}{s}\right)$	63827.17	63828.93	63847.04	61242.35	60945.13	71050.40	63733.78	173534.76	173561.53
e	0.484397	0.485513	0.487911	0.427492	0.423434	0.637226	0.484644	4.17	6.176360
$\Omega\,(^\circ)$	122.86	122.74	122.73	123.26	123.33	122.40	122.72	111.34	111.34
$i\,(^\circ)$	63.09	63.10	63.10	63.18	63.2	63.33	63.08	64.67	64.68
$w\,(^\circ)$	352.45	352.57	352.51	343.10	341.81	17.30	352.21	64.22	64.40
$\theta\,(^\circ)$	61.63	52.49	61.61	58.25	72.08	49.96	61.91	0.51	353.02
a (km)	13353.9	13373.6	13422.1	11513.6	11354.1	21323.1	13319.0	−4600.02	−2034.4
3Minutes Separation	$\alpha\,(^\circ)$	99.06225 106.73804 130.04876		95.59229 106.73804 187.13477		101.82093 106.73804 120.76367		98.73976 106.73804 138.41261	
	$\delta\,(^\circ)$	18.8371 53.3314 79.5836		15.8338 53.3314 80.7530		15.3099 53.3314 91.6624		19.1534 53.314 85.6417	
$h\left(\frac{km^2}{s}\right)$	63827.17	63563.70	63568.93	62714.28	62746.01	34891.13	34870.13	128428.85	128426.86
e	0.484397	0.478197	0.482662	0.458655	0.463773	0.584885	0.585284	1.66	3.321602
$\Omega\,(^\circ)$	122.86	122.76	122.75	122.89	122.88	114.31	114.31	117.55	117.55
$i\,(^\circ)$	63.09	63.08	63.08	63.08	63.08	53.09	53.09	65.81	65.81
$w\,(^\circ)$	352.45	351.79	351.55	348.81	348.69	242.77	242.82	15.90	43.30
$\theta\,(^\circ)$	61.63	50.73	62.56	52.29	65.38	173.42	177.04	50.01	12.09
a (km)	13353.9	13141.4	13217.1	12495.9	12583.8	4642.2	4639.9	−23524.2	−4124.2

Table 12: Molniya 1-86 with Equal Time Separations

Molniya 3-50									
Closest TLE	1 25847U 99036A 19214.82186374 -.00000166 +00000-0 -72333-2 0 9994 2 25847 062.1353 139.3201 7238609 272.4611 014.2757 02.00632547147068								
Time	3.8.2019 − 08 : 47 : 00 (UTC0)								
15Seconds Separation	$\alpha\,(^\circ)$	179.06708 179.13349 179.19963		178.24044 179.13349 179.84569		178.69941 179.13349 179.55614		179.10922 179.13349 180.15033	
	$\delta\,(^\circ)$	37.4799 37.5716 37.6626		38.1746 37.5716 36.9486		37.0874 37.5716 38.0547		37.5157 37.5716 38.0758	
		One Observer		*Same Latitude*		*Same Longitude*		**3 Different Observers**	
	Real	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*
$h\,\left(\frac{km^2}{s}\right)$	70982.41	86078.12	92610.45	67793.95	66813.93	77452.85	81166.98	134566517.84	134547392.79
e	0.723861	0.622980	1.356886	0.656870	0.627657	0.680411	0.812302	8673134.630452	8664979.991399
$\Omega\,(^\circ)$	139.24	143.06	134.93	140.15	143.75	140.16	138.23	63.42	63.48
$i\,(^\circ)$	62.14	64.30	57.96	63.01	66.15	62.95	61.35	40.41	40.39
$w\,(^\circ)$	272.47	281.80	309.66	258.62	250.05	278.24	290.63	20.04	19.99
$\theta\,(^\circ)$	135.50	108.22	100.73	139.56	155.81	111.90	117.73	10.36	270.74
a (km)	26554.2	30378.8	−25580.9	20281.4	18479.5	28089.2	48588.2	0.00	−0.1
2Minutes Separation	$\alpha\,(^\circ)$	178.60121 179.13349 179.66321		177.76979 179.13349 180.30571		178.23194 179.13349 180.01815		178.64366 179.13349 180.60939	
	$\delta\,(^\circ)$	36.8259 37.5716 38.2902		37.5197 37.5716 37.5752		36.4268 37.5716 38.6756		36.8624 37.5716 38.6910	
$h\,\left(\frac{km^2}{s}\right)$	70982.41	71288.71	71411.42	67902.10	55463.39	70861.57	89197.44	1929784.82	1929338.89
e	0.723861	0.721370	0.728925	0.689878	0.702412	0.469471	0.775205	1399.33	1397.985045
$\Omega\,(^\circ)$	139.24	139.11	138.92	141.01	200.63	183.17	140.94	92.47	92.51
$i\,(^\circ)$	62.14	62.24	62.09	64.03	131.44	62.27	62.72	36.01	36.02
$w\,(^\circ)$	272.47	272.57	273.44	264.05	178.44	107.82	294.91	355.51	355.47
$\theta\,(^\circ)$	135.50	117.52	134.53	129.06	247.66	88.54	112.43	41.5	275.24
a (km)	26554.2	26582.9	27298.1	22072.0	15233.4	16159.0	50018.8	−4.8	−4.8
3Minutes Separation	$\alpha\,(^\circ)$	178.33421 179.13349 179.92766		177.50004 179.13349 180.56821		177.96408 179.13349 180.28173		178.37694 179.13349 180.87137	
	$\delta\,(^\circ)$	36.4425 37.5716 38.6405		37.1357 37.5716 37.9251		36.0397 37.5716 39.0222		36.4796 37.5716 39.0344	
$h\,\left(\frac{km^2}{s}\right)$	70982.41	71165.43	71163.70	63916.28	63906.06	67735.96	67744.88	234767.54	237403.34
e	0.723861	0.723383	0.723517	0.701743	0.701870	0.725780	0.725885	28.718049	29.336421
$\Omega\,(^\circ)$	139.24	139.03	139.03	141.17	141.16	138.55	138.55	53.59	53.58
$i\,(^\circ)$	62.14	62.20	62.20	64.58	64.58	62.06	62.06	43.07	43.06
$w\,(^\circ)$	272.47	272.63	272.64	259.61	259.61	268.61	268.63	25.23	24.80
$\theta\,(^\circ)$	135.50	116.93	135.27	134.36	147.17	122.46	139.43	27.57	28.45
a (km)	26554.2	26652.7	26662.1	20193.0	20193.7	24323.0	24337.2	−167.9	−164.5

Table 13: Molniya 3-50 with Equal Time Separations

Appendix D

Relative Error < 0.1	*One Observer*		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**		**Total**		
	Gooding's Method	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Total Number of Elements*
ANGLE	68	77	67	68	73	60	30	23	238	228	384
TIME	67	69	63	55	40	46	8	11	178	181	384
TOTAL	135	146	130	123	113	106	38	34	416	409	768

Relative Error < 0.01	*One Observer*		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**		**Total**		
	Gooding's Method	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Total Number of Elements*
ANGLE	38	48	29	35	36	23	6	2	109	108	384
TIME	38	43	24	21	22	27	0	1	84	92	384
TOTAL	76	91	53	56	58	50	6	3	193	200	768

Relative Error < 0.001	*One Observer*		*Same Latitude*		**Same** *Longitude*		**3 Different Observers**		**Total**		
	Gooding's Method	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Gooding's Method*	*Gauss Method*	*Total Number of Elements*
ANGLE	6	16	7	9	4	0	2	1	19	26	384
TIME	16	20	8	8	6	9	0	0	30	37	384
TOTAL	22	36	15	17	10	9	2	1	49	63	768

Table 14: Number of Accurate Results

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