

## THE THERMO-GAS-DYNAMIC DESIGN METHOD FOR THE LIQUID ROCKET ENGINE CHAMBER

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### ABSTRACT

*Analysis of the thermodynamic and thermophysical properties of combustion products in the liquid rocket engine (LRE) chamber shows that their dissociation degree depends on temperature  $T$ , gas expansion degree  $\epsilon$ , etc. Practically, combustion products are always chemically active working fluid, therefore the number of moles  $N$  of the products varies along the length of the LRE chamber in the entire reaction mixture. The local values of the parameters  $T$  and  $N$  depend on the specific physical conditions. Therefore, the distribution of local numbers of moles of the components of the gas mixture and its heat capacities can be represented as dependencies  $N \sim f(T)$  and  $c \sim g(T)$ . For this purpose on the basis of the numerical values of the moles and the heat capacities of the gas mixture components in the main sections of the LRE chamber are formed as corresponding empirical functions through interpolation. The system of equations for the thermodynamic calculation of LRE chamber is solved by taking into account new functions. Such approach allows forming the optimal contour of the LRE chamber at the preliminary stage of engine design and improving results of the gas-dynamic calculation and nozzle profiling by modified method of characteristics.*

### INTRODUCTION

As known, one of the main directions in rocket and space technologies development is design of highly efficient propulsion systems, which include liquid rocket engines (LRE). Design of LRE and its optimization scheme consists of choosing a combination of parameters of the workflow, which achieves the most advantageous combination of traction characteristics and weight of the structure. There accumulated a large scientific and practical experience in the development of various LRE. However, determining the design parameters of a new designed LRE camera is still a difficult process.

In LRE development their initial geometry, pneumatic-hydraulic scheme (PHS) of the engine and parameters of these energy relations are determined. Next, on the basis of this PHS is selected, at all characteristic point of PHS pressures, consumption of fuel components, required pump features and power consumed by them and components temperatures of working gases are determined. These engine parameters obtained are the

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initial data for the design of the LRE combustion chamber (CC), gas generator, pumps, turbines, regulators, etc.

The pressure and the ratio of fuel components in the CC is selected taking into account obtaining a maximum specific impulse of the engine, its dimensions and reliable cooling of the chamber. At this design stage many parameters of LRE and its aggregates are taken approximately based on the experience of previous developments. Therefore, great accuracy in determining of certain engine parameters at characteristic points of PHS and LRE chamber should not be expected.

For determination of the thermodynamic characteristics of combustion products (CP) have been done many researches and developed a number of different software (for example, CEA (NASA, USA), Astra.4/pc (MSTU named after N.E.Bauman, Russia), RPA (Alexander Ponomarenko, Germany), etc.

In these applications is assumed that (for CC exit, the nozzle inlet):

- fuel mixing is complete,
- physical incomplete combustion missing,
- the combustion process takes place at a constant pressure in the CC ( $p_c = const$ ),
- combustion products systems at the CC exit are in a thermodynamic equilibrium state,
- there is no heat exchange with CC walls,
- gas phase is described by the ideal gas state equation,
- solubility of gases in the liquid and solid phases is missing,
- condensed substances form one-component immiscible phases, etc.

For the expansion process calculating in the nozzle, the following assumptions are made:

- the expansion process is chemically and energetically extremely balanced,
- no fuel burnout in the nozzle,
- no heat transfer to the nozzle walls,
- there is no friction and gas-dynamic losses in the nozzle [Alemasov,1989; Babkin, 1990; Glushko,1976; Gurtovoy, 2016; Sutton, 2010; Vasiliev,1983].

In the known methods, these problems are mainly considered from general theoretical positions and thermo-gas-dynamic features of the working processes are not taken into account. The correct accounting of these features would allow creating the correct mass and geometric configuration of LRE camera. Consequently an improved technique for the preliminary design of LRE, taking into account certain features of the working processes in the engine, is considered in this paper.

## THE AIM OF RESEARCHES

As known, preliminary geometry of LRE chamber is formed both by thermodynamic models of combustion and outflow processes and by gas-dynamic models of the gas flow. In the engineering practice, thermodynamic calculation precedes the gas-dynamic design of LRE chamber. However, inaccuracies in thermochemical modeling (for example, incorrect modeling of the distribution of CP thermodynamic parameters over the chamber volume) of LRE's can lead to certain errors in the engine configuration at the next design stages, which lead to improvements in the basic thermodynamic models.

Therefore, the purpose of the paper is to improve the methodology for the correct geometry formation of LRE chamber (combustion chamber and nozzle) based on the existing method refinement for thermodynamic calculation.

## SOME NOTES ON THE GAS-DYNAMIC CALCULATION OF THE LRE CHAMBER

As known, the task of LRE gas-dynamic calculation is to determine the main geometric dimensions in various sections of the combustion chamber (CC), nozzle and the calculation of the expected characteristics of the engine. The calculation is based on the assumption of chemical inertness of the fuel combustion products (CP) during their flow

through the nozzle. Along with proposals for adiabaticity and one-dimensional flow, this assumption allows to use the gas-dynamic functions, which describe the motion of a gas flow with constant composition in an energy-insulated channel, i.e. in the frozen expansion [Alemasov, 1989; Babkin, 1990; Glushko, 1976; Gurtovoy, 2016; Vasiliev, 1983].

However, at high temperatures ( $T > 2000\text{ K}$ ), the combustion products are chemically active working fluids (medium) in which dissociation and recombination reactions take place. During the expansion of such working fluid in the nozzle, due to temperature decreasing the dissociation decreases. In this process there happens an increase in the recombination phenomena that occurs with the heat releasing. As a result, the chemical equilibrium state is not realized due to the short residence time of PC in the nozzle and the final rates of chemical reactions. Consequently, the changing composition of the working fluids in LRE chamber does not allow the correct determination of engine parameters [Belov, 2013; Babkin, 1990; Brykov, 2017; Gurtovoy, 2016].

Thus, in the classical formulation of this problem the gas-dynamic calculation in separately or in the private combination with some results of thermodynamic calculation does not allow the formation of the correct geometry of LRE chamber. This circumstance leads to improvements in the methodology of thermodynamic calculation for LRE, taking into account the gas-dynamic design problems.

## FEATURES OF THERMODYNAMIC CALCULATION OF LRE

As known, at high temperatures ( $T > 2000\text{ K}$ ), a thermal dissociation of the working fluids occurs in LRE. Dissociation processes lead to a decrease in the total conversion of the fuel chemical energy  $U_{chem}$  into heat ( $U_{chem} \rightarrow Q_{chem}$ , in the ideal case  $U_{chem} = Q_{chem}$ ), which should be taken into account during preliminary design of the engine. In addition, the temperature and pressure of the gas flow also decrease, which have different effects on the dissociation degree. Existing studies have shown that the temperature effect on the gas dissociation degree is greater and at gas temperatures  $T < 2000\text{ K}$  the degree of dissociation is smaller [Babkin, 1990; Gurtovoy, 2016].

Therefore, when considering the gas flow in the nozzles, changes in the chemical composition and chemical energy  $U_{chem}$  due to recombination processes of CP are considered as small and are not taken into account in the calculations. Analysis of thermodynamic and thermophysical properties of CP [Brykov, 2017; Glushko, 1976] shows that the degree of dissociation of CP also depends on the expansion degree  $\varepsilon = p_c / p_e$  of PC and the oxidizer excess ratio  $\alpha = K_m / K_m^0$  in the LRE chamber. For example, for a kerosene-oxygen fuel pair at  $\varepsilon \geq 20 \dots 30$ ,  $\alpha \leq 0.7$  and temperatures  $T < 2000\text{ K}$ , the combustion products are practically a chemically active working fluids. In this case, it would be correct to carry out a thermodynamic calculation of the LRE, taking into account the dependence  $Q_{chem} \sim f(p, T, \varepsilon, \alpha, \bar{x}, \bar{y}, \dots)$ , where  $\bar{x} = x / y_{cr}$  or  $\bar{x} = x / x_{min}$  - the relative length,  $x$  - coordinates of the point considered on the LRE chamber axis,  $y_{cr}$  - radius of the nozzle throat (critical section),  $\bar{y} = y / y_{cr}$  - the relative radius of the considered engine section.

Thus, if changes due to recombination reactions are not taken into account, then the error of calculation of thermo-gas-dynamic parameters can be several percent. In addition, despite the change in the composition of the CP along the LRE chamber length the ratio of specific heats  $\gamma = c_p / c_v$  in the calculations is considered only in the main sections of the engine. Consequently, the thermodynamic calculation with some average value of the isentropic index  $\gamma$  leads to an incorrect configuration of the LRE.

It should be noted that the change in specific heats and the isentropic index along the LRE nozzle length were considered in some previous studies [Colonno, 2008; Fu, 2016; Kestin, 1950; Kyprianidis, 2009; Rizkalla, 1990; Zebbiche, 2011]. However, as a rule,

mathematical modeling of these changes is narrow and does not allow revealing the entire energy potential of the gas flow. In these studies, changes in the isentropic index are considered as a separate problem in order to justify the use of the improved method of characteristics. It is well known that the energy formation of a gas stream along the LRE chamber is extremely complex and dynamic. Consequently, it becomes necessary to solve the problem of correctly applying the method of characteristics, taking into account the features of thermochemical models of combustion processes in the LRE chamber.

Therefore, preliminary design of LRE requires additional researches for improving the thermodynamic calculation, which is considered in the next paragraph of this paper.

## SOLUTION OF SOME PROBLEMS OF THERMODYNAMIC CALCULATION OF THE LRE CHAMBER

In the general case, the geometric profile of the LRE chamber and its thrust characteristics are determined by the distribution of the moles of the gas mixture and its components, heat capacities, isentropic index, chemical or internal energy over the engine chamber length (or volume):

$$N_i = f_1(\bar{x}, \bar{y}), c_i = f_2(\bar{x}, \bar{y}), \gamma = f_3(\bar{x}, \bar{y}), U_{chem} = f_4(\bar{x}, \bar{y}) \text{ or } U = f_5(\bar{x}, \bar{y}) \quad (1)$$

Depending on the nature of these distributions, certain fields and isosurfaces of parameters ( $p, T, w$  etc.) are formed in the LRE chamber, which affect the engine thrust characteristics.

It should be noted that taking into account the distribution of these parameters in the LRE chamber determines the improvement of the method of characteristics for supersonic nozzle profiling [Anderson, 1982].

As known, one of the main gas flow parameters affecting the thrust characteristics of LRE is heat capacity. The specific heat values  $c_p$  and  $c_v$  (respectively, other parameters) for the considered LRE chamber section depend on the properties of individual substances (gases) and their moles in the PC mixture. Theoretically, the specific heat capacities are determined by the following formulas [Alemasov, 1989; Glushko, 1976; Vasiliev, 1983]

$$c_p = \sum c_{p_i} N_i + \sum J_i \left( \frac{\partial N_i}{\partial T} \right)_{p=const} \quad (2)$$

$$c_v = \sum c_{v_i} N_i + \sum U_i \left( \frac{\partial N_i}{\partial T} \right)_{v=const} \quad (3)$$

where  $c_{p_i}$  and  $c_{v_i}$  - are the specific heat capacities of the  $i$ -th component of CP (individual substance of the gas mixture) for the considered temperature,  $N_i$  - the number of moles of the  $i$ -th component for the considered conditions (pressure  $p_i$  and temperature  $T_i$ ),  $J_i$  and  $U_i$  - the enthalpy and the internal energy of the  $i$ -th component for the considered temperature [Gurvich, 1982]. In the calculations for gaseous components of the mixture instead of  $N_i$  the partial pressure of the components  $p_i$  is used ( $N_i = p_i$ ). In addition, for the considered conditions  $c_{p_i} - c_{v_i} = R_i$  can be used, where  $R_i = R/\mu_i$  - the gas constant and  $\mu_i$  - the molecular mass of the  $i$ -th component.

As a rule, in existing studies it is accepted that the heat capacity depends on temperature in the form

$$c = c_0 + aT + bT^2 + dT^3 \dots \text{ or } c \approx c_0 + aT \quad (4)$$

where  $c_0$  - heat capacity at 298.15K,  $a, b, d, \dots$  - constant coefficients. Usually, coefficients  $b$  and  $d$  are not taken into account because of their smallness. However, analysis shows that for different temperature ranges (1500-2000K, 2000-3000K, >3000K) and conditions

$(\alpha, p_c, K_m, \varepsilon)$  changes of heat capacities of individual substances in CP  $\Delta c(\Delta T)$  have different effects on engine performance [Bulygin, Rachuk, 1997]. As an example, table 1 shows changes in the heat capacity of the CP of the kerosene-oxygen fuel [Glushko, 1976]. Consequently, for different design conditions  $(\alpha, p_c, K_m, \varepsilon)$  of LRE chamber the nature of the changes of parameters  $c_p$  and  $c_v$  must be taken into account.

Thus, resulting heat capacity of the gas at the considered point of the flow is formed by the variety and number of different substances, which is almost impossible to simulate mathematically. Therefore, on the LRE chamber calculation the heat capacities are not considered in the engine chamber cross sections between “c”, “cr” and “e” (**first approximation**), which leads to a distortion of the nozzle geometry (Fig. 1).

Table 1: Changes in the heat capacity of the PC of the kerosene-oxygen fuel

Case	Parameters of LRE, fuels and combustion process				
A)	$\varepsilon \leq 50$	$\alpha = 0.5$	$p_c = 0.1 \text{ MPa} - 50 \text{ MPa}$	$K_m = 1.704$	$c_p \downarrow$
B)	$\varepsilon > 50$	$\alpha = 0.5$	$p_c = 0.1 \text{ MPa} - 50 \text{ MPa}$	$K_m = 1.704$	$c_p \uparrow$
C)	$\varepsilon \leq 10$	$\alpha = 1.0$	$p_c = 0.1 \text{ MPa} - 50 \text{ MPa}$	$K_m = 3.409$	$c_p \uparrow$
D)	$\varepsilon > 10$	$\alpha = 1.0$	$p_c = 0.1 \text{ MPa} - 50 \text{ MPa}$	$K_m = 3.409$	$c_p \downarrow$
E)	$\varepsilon \leq 5000$	$\alpha = 2.0$	$p_c = 0.1 \text{ MPa} - 50 \text{ MPa}$	$K_m = 6.815$	$c_p \downarrow$

Here, the heat capacity average value is considered unchanged due to the recombination reactions between the indicated cross sections. Accordingly, the heat can be approximately taken constant (i.e.  $\bar{Q}_{chem} \approx const$ ). In this case, the PC enthalpy to be calculated by the formula [Alemasov, 1989; Glushko, 1976; Vasiliev, 1983]

$$J = i + \bar{Q}_{chem} = \int_{T_0}^T c_p dT + \bar{Q}_{chem}$$

where  $T_0$  - the reference (or initial) temperature (298.15K),  $T$  - the temperature of considered CP.

It is known that due to chemical reactions along the LRE chamber length in the entire reacting gas mixture the number of moles  $N_i$  of components changes. At the same time, the local value of this parameter is determined by the thermophysical conditions ( $p_i, T_i$ , etc.) at the point in question. Therefore, from the point of view of energy conversion, the local number of moles can be represented as a function

$$N_i = f_6(T_i), \text{ where } T_i = f_7(\bar{x}, \bar{y}) \quad (5)$$

Analysis of numerical studies shows that, based on  $N_i$  values in the main sections of the LRE chamber using interpolation it is possible to define a function  $N_i = f_6(T_i)$  in the form

$$N_i = aT_i + b, N_i = aT_i^b + c, N_i = a \ln(T_i) + b, N_i = aT_i^2 + bT_i + c. \quad (6)$$

Depending on the specific tasks, one of these functions can be taken into account in formulas (2) and (3). In this case, for the formation of the LRE chamber geometry, the thermo-gas-dynamic calculation is repeated taking into account new dependencies (6). This approach allows us to obtain more refined values of heat capacities for the considered point on the LRE chamber axis taking into account the specific nature of the change  $T_i$  along the engine cross section.

Thus, in the **second approximation**, between the indicated sections of the LRE

chamber, changes in heat capacities values  $c'_p$  ( $c'_p \downarrow$  or  $c'_p \uparrow$ ) will be taken into account. Then taking into account the condition  $U'_{chem} \rightarrow Q'_{chem} \neq const$  ( $U'_{chem} \downarrow$  or  $U'_{chem} \uparrow$ ) enthalpy of the combustion products should be calculated taking into account the changing internal thermal energy

$$J' = i' + U'_{chem} = i' + Q'_{chem} = \int_{T_0}^T c_p dT + Q'_{chem} \quad (7)$$

Taking into account the above, based on the values of the total enthalpy for two "c" and "e" sections of the LRE chamber, we find the velocity of the gas flow in the section "e"

$$w'_e = \sqrt{2c'_{p0}(T_e - T_0) + a(T_e^2 - T_0^2)}$$

Thus, if for any two "n-1" and "n" sections of the LRE chamber the initial heat capacities are taken as  $c'_{p0} = c'_{p0_{n-1}}$  or  $c'_{p0} = c'_{p0_n}$ , then

$$w'_n = \sqrt{2c'_{p0}(T_n - T_{n-1}) + a(T_n^2 - T_{n-1}^2)}, \quad w'_n = \sqrt{2(i'_n - i'_{n-1}) + a(T_n^2 - T_{n-1}^2)} \quad (8)$$

In order to optimize the nozzle, the value  $w'_n(\bar{x})$  obtained by the formula (8) is compared with the value of a predetermined sigmoidal function as [Abdullayev, 2017]

$$w'_n(\bar{x}) - w_{sigmoid}(\bar{x}) \leq \Delta w_n(\bar{x})$$

The best case is  $\Delta w_n(\bar{x}) \approx 0$ , i.e.

$$w'_n(\bar{x}) = \sqrt{2[i'_n(\bar{x}) - i'_{n-1}(\bar{x})] + a[T_n^2(\bar{x}) - T_{n-1}^2(\bar{x})]} = w_{sigmoid}(\bar{x}) \quad (9)$$

For any two "n-1" and "n" sections of the LRE chamber, the temperature can be found as

$$T_n = T_{n-1} + \left( \frac{w_{n-1}^2}{2(c'_{p0(n-1)} + aT_{n-1})} - \frac{w_n^2}{2(c'_{p0(n)} + aT_n)} \right) \quad (10)$$

As can be seen, unlike the traditional thermodynamic calculation scheme of the engine, the temperature and velocity of the CP can be determined sequentially along the axis of the LRE chamber.

In view of the foregoing, we will consider a modified thermodynamic calculation of the LRE chamber, which takes into account the average gas dynamics of the engine.

## MODIFIED TECHNIQUE FOR THERMODYNAMIC CALCULATION OF THE LRE CHAMBER

In general, for this technique is considered fuel with source elements C, H, O and N. For determining of the composition and temperature of the combustion products in each LRE chamber section a system of equations is composed using [Alemasov, 1989; Bonnie, 2002; Cantwell, 2019; Glushko, 1976; Gordon, 1994; Gurtovoy, 2016; Hill, 1992; Pashayev, 2018; Vasiliev, 1983]

- The chemical equilibrium law,
- The equation of material balance (law of conservation of matter),
- The Dalton's law (partial pressure balance equation),
- The law of masses action.

The system of equations is solved accurately using the Newton-Raphson method. Taking into account the main provisions of previous paragraphs sequence of calculation will consist of the following steps (Fig.1).

### A. Combustion chamber (“c”, “c0” sections)

By the solution of equations system for a given pressure  $p_c$  in the CC are determined the composition of the CP (mass or mole fraction  $N_{c,i}$  for every  $i$ -th component), the partial pressure of CP components  $p_{c,i}$ . Further, using the condition  $I_F = I_c$  ( $I_F$  and  $I_c$  are the enthalpy of the fuel and combustion products), are determined the temperature  $T_c$  in the CC, entropy  $S_c$ , molecular weight  $\mu_c$ , gas constant  $R_c$ , density  $\rho_c$ , heat capacities  $c_{p,c}$  and  $c_{v,c}$ , the isentropic index  $\gamma_c$  and speed of sound  $a_c$  in the initial section of the CC.

### B. Nozzle exit (“e” section)

For given pressure  $p_e$  by the solution of the equations system are determined the composition ( $N_{e,i}$ ) and the partial pressure  $p_{e,i}$  of CP components. Next, using the condition  $S_c = S_e$  ( $S_c$  and  $S_e$  are the entropy of the combustion products in the relevant sections «c» and «e» of the LRE chamber) are determined temperature  $T_e$ , molecular mass  $\mu_e$ , gas constant  $R_e$ , density  $\rho_e$ , heat capacities  $c_{p,e}$  and  $c_{v,e}$ , isentropic index  $\gamma_e$ , specific area  $F_{e,sp}$  and speed of sound  $a_e$  at the nozzle exit.

### C. Nozzle throat section (“th” section)

Based on the solution of the equations system of for each pressure value  $p_{th,j}$  (from the range  $[p_{th,min}, p_{th,max}]$ ,  $j=1,2,..$ ) is set one value temperature  $T_{th,k}$  (from the range  $[T_{th,min}, T_{th,max}]$ ,  $k=1,2,..$ ) of the gas mixture and are determined the composition and entropy  $S_{th}$  of the combustion products. The temperature  $T_{th} \in [T_{th,min}, T_{th,max}]$  for which the condition is met  $S_{th} = S_c$  is taken as final. Besides at each solution cycle for each value  $p_{th,j}$  from the range  $[p_{th,min}, p_{th,max}]$  also the composition  $N_{th,i}$ , molecular mass  $\mu_{th}$ , gas constant  $R_{th}$ , density  $\rho_{th}$ , heat capacities  $c_{p,th}$  and  $c_{v,th}$ , isentropic index  $\gamma_{th}$ , specific area  $F_{th,sp}$  and speed of sound  $a_{th}$  of the gas mixture are determined. The true nozzle throat section will then when a concretely value of the pressure  $p_{th}$  and other parameters provide the minimum specific area  $(F_{th,sp})_{min}$ . In this section will also be satisfied the equality of velocities  $a_{th} = w_{th}$ .

### D. Intermediate sections of the chamber with a given length (“n” section)

The purpose of this calculation stage is adjusting to the gas-dynamic calculation of the LRE chamber as close as possible. In the first approximation (parameters calculation in the intermediate sections of the LRE chamber) at first, based on the values  $c_{p,c}$ ,  $c_{p,th}$  and  $c_{p,e}$  using interpolation are formed functions  $c_p = f(T)$  that allow us to determine  $c_{p0,n}$  for the considered  $n$ -th section. With the linear form of this function  $c_p = c_{p0} + aT$  for the  $n$ -th section will be  $c_{p0,n} = c_{p,n-1}$ . The function  $c_p = f(T)$  without using of gas-dynamic functions allows providing minimal difference thermodynamic and gas-dynamic profiles of the LRE chamber using (8) and (10).

In the second approximation values of thermodynamic parameters in intermediate sections of the LRE chamber are refined using functions for moles  $N = f_6(T)$  and new values of  $c'_p$  and  $c'_v$ .

As noted in paragraph 3 gas-dynamic functions are applied with an average value of the isentropic index  $\gamma$ , which leads to certain errors in the formation of the LRE chamber geometry. On the other hand the determination of the local isentropic index  $\gamma$  by main

sections (cross sections “c”, “cr” and “e” of the LRE chamber) using thermodynamic calculation also doesn't allow correctly forming the gas-dynamic structure and take into account all the properties of the gas flow in gas-dynamic functions. Consequently, the use of functions  $c_p = f(T)$  and  $N = f_6(T)$  allows without use of gas-dynamic functions, maximally match the thermodynamic and gas-dynamic profiles of the LRE chamber using (9) and (10).

Further design is carried out using the method of characteristics, taking into account the changing values of the isentropic index of combustion products  $\gamma_n$  along the LRE chamber length. It should be noted that such approximations allow to be improved of the nozzle profiling contour results using the method characteristics. Thus, using the formulas (9) and (10) for the  $n$ -th section of the LRE chamber are determined parameters  $S_n$ ,  $I_n$ ,  $\mu_n$ ,  $R_n$ ,  $\rho_n$ ,  $c_{p,n}$ ,  $c_{v,n}$ ,  $\gamma_n$ ,  $a_n$  etc., which allow to provide the compatibility of the thermodynamic and gas-dynamic profiles of the LRE chamber. Consequently, the use of formulas (6)–(10) allows to combine these features and to form thermo-gas-dynamic calculation technique of the rocket engine chamber, which scheme is shown in Fig.1.

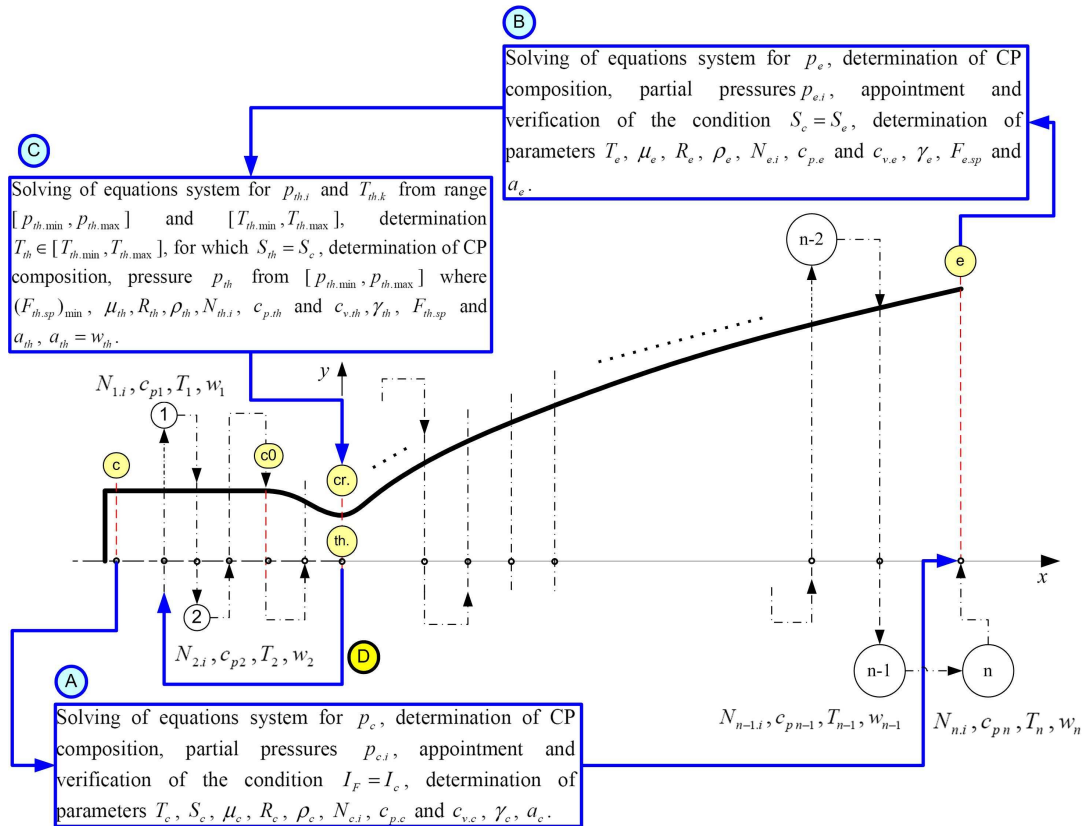


Figure 1: The design scheme of the LRE chamber geometry based on the modified thermo-gas-dynamic calculation technique.

As already mentioned, the distribution of parameters in the chamber of the LRE  $N_i = f_1(\bar{x}, \bar{y})$ ,  $c_i = f_2(\bar{x}, \bar{y})$ ,  $\gamma = f_3(\bar{x}, \bar{y})$  determines the correct application of the method of characteristics. Taking into account these distributions, in the next paragraph were considered applying results of the corrected method of characteristics in supersonic nozzle profiling [Abdulla, 2019]:

$$\nu = \nu(\gamma, M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \arctan\left(\sqrt{(M^2-1)\frac{\gamma-1}{\gamma+1}}\right) - \arctan(\sqrt{M^2-1})$$

where  $\gamma = f_3(T)$  -the variable heat capacity ratio ( $T = f_7(\bar{x})$ , can be take as  $T = T_{sigmoid}(\bar{x})$ ),  $M$  -the Mach number of the gas flow at the mentioned nozzle point (or at the beginning of uniform flow region).



## RESULTS AND DISCUSSION

From the above theoretical foundations of the thermodynamic calculation of LRE, it can be concluded that the engine nozzle must be designed with the isentropic index  $\gamma$  values changes. As a result of the variable  $\gamma$  application, as mentioned, the nozzle of a rocket engine becomes more accurate. Results of different inputs can be considered in order to discuss the effect of variable specific heat ratio implementation to the nozzle design. In order to demonstrate the variation, two different cases are analyzed for the project. Fig. 2-3 represent the outputs for different inputs. Two different nozzles are analyzed using the data given in Table 2. Figures clearly show that the nozzle contours obtained from constant and variable specific heat ratios are not the same. As it is observed from the figure legends, one of the contour is constructed based on constant  $\gamma$ , whereas another one is constructed using variable  $\gamma$  approach. Results yields, that the contours are different for two different cases. The contour built based on constant  $\gamma$  is inaccurate because, as mentioned previously, thermodynamic computations of the combustion process yield that  $\gamma$  varies along the nozzle length. On the other hand, a more accurate contour that is build based on varying  $\gamma$  is given in red color. As a result, it is observed that if  $\gamma$  increases starting from the nozzle throat until the nozzle exit, then the nozzle contour narrows.

Table 2: Experimental Cases with Properties

Case №	Performance Parameters	Specific Heat Ratio
Case 1	$w_e = 2250 \text{ m/s}$ , $T_e = 1500 \text{ K}$ , $R_e = 320 \text{ J/(kgK)}$ , $A_e / A^* = 25$	$\gamma = 1.18 = \text{const}$
		$\gamma_i = 1.16$ , $\gamma_e = 1.20$
Case 2	$w_e = 2250 \text{ m/s}$ , $T_e = 1500 \text{ K}$ , $R_e = 320 \text{ J/(kgK)}$ , $A_e / A^* = 25$	$\gamma = 1.18 = \text{const}$
		$\gamma_i = 1.20$ , $\gamma_e = 1.16$

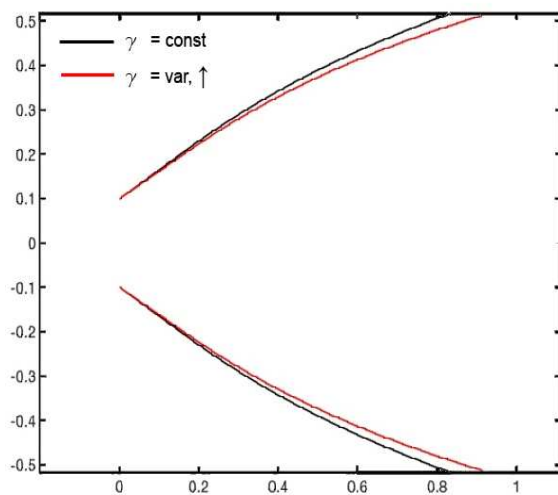


Figure 2: Contour of the LRE nozzle

Case 1:  $\gamma = \text{const}$ , and  $\gamma = \text{var}$  ( $\gamma \uparrow$ )

the abscissa axis -  $\bar{x}$ , the ordinate axis -  $\bar{y}$

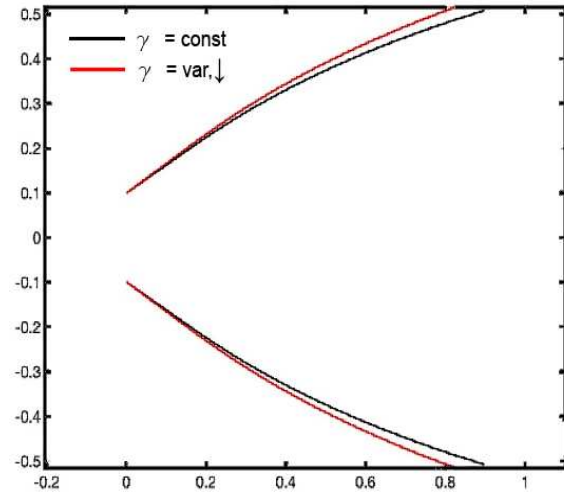


Figure 3: Contour of the LRE nozzle

Case 2:  $\gamma = \text{const}$  and  $\gamma = \text{var}$  ( $\gamma \downarrow$ )

On the other hand, Fig. 3 represents the nozzle contour for Case 2, in which all the properties remain the same as in Case 1. What differs Case 2 from Case 1 is the  $\gamma$  variation. In the first case  $\gamma$  increases from the nozzle throat until the nozzle exit, whereas in the second case  $\gamma$  decreases in the mentioned direction.

Thus, the LRE chamber geometry can be easily adapted to real conditions depending on the specific task (customer requirements for engine size and weight, type of flight vehicle, fuel and the main parameters of the engine work, etc.).

Based on the analysis of the results of a numerical experiment, it can be concluded that correctly taking into account changes in thermodynamic parameters of combustion products along the nozzle length allows us to solve the following problems:

- organize control of the LRE chamber function by changing the thermophysical properties of combustion products along the nozzle length
- organize, in flight, the correct gas-dynamic control of changes in the degree of expansion of gases in the LRE chamber
- control the influence of the initial expansion zone of gases on the distribution of the velocity field at the nozzle exit
- reduce the surface area of the cooling walls of the LRE chamber
- to form the optimal geometry of the entire LRE chamber

As we can see, taking into account changes in the properties of combustion products allows us to control the gas flow expansion in all flight conditions. This circumstance leads to the improvement of the pneumo-hydraulic scheme of the LRE. Generally, solutions of these problems require additional researches. As can be seen, accounting and control of thermophysical properties of combustion products along the LRE chamber length allows an application of new principles for the organization of the working processes of such engines and the improvement of their design schemes.

## CONCLUSION

The conducted studies show that the thermo-gas-dynamic calculation of the LRE, taking into account the distribution of energy parameters over the chamber volume, allows obtaining more accurate engine geometry. This circumstance determines the improvement of the constructive schemes of the LRE with the use of elements of the formation of local values of thermodynamic parameters in the chamber volume. Application of these elements can be implemented in the form of injection and afterburning of pre-burner gases in certain sections of the LRE chamber in order to change the local values of the main parameters.

Thus, a modified method for determining the optimal thermo-gas-dynamic profile of the LRE chamber using the results of thermodynamic calculation has been proposed. The technique is based on the distribution of the gas compositions and moles of its components, heat capacities, temperatures and the gas flow velocities along the length of the LRE chamber. The proposed modified method allows to carry out thermo-gas-dynamic calculations of LRE with maximum consideration of the gas-dynamic features of the PC in the engine chamber and to increase the efficiency of thermodynamic calculation. This approach allows forming the appropriate geometry of the LRE chamber at the preliminary stage of engine design and improving the nozzle profiling results by the modified method of characteristics.

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