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LATERAL-DIRECTIONAL CONTROL OF A HIGHLY MANEUVERABLE JET AIRCRAFT BASED ON COMBINATION OF MODAL AND OPTIMAL CONTROL THEORY

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ABSTRACT

A feedback matrix is calculated to assign closed loop eigenvalues of the lateral-directional stability augmentation system of the aircraft with low closed loop eigenvalue sensitivity at minimum cost which guarantees +inf and -6 dB gain margins and +60 and -60 degrees phase margins by combining modal and optimal control theory. The method based on combination of modal and optimal control reduces effort to satisfy gain and phase margins criteria while allowing eigenvalue assignment to achieve desired performance characteristics of the aircraft.

INTRODUCTION

Stability augmentation systems (SAS) are widely used in highly maneuverable jet aircrafts. Although SAS have similar architectures or block diagram representations, different methods have been used in control feedback matrix design to satisfy minimum requirements for the aircrafts.

In the design of a flight control law, one of the most significant requirements is to satisfy the minimum stability margins of the aerodynamic closed loops of the aircrafts. Consider the military specifications, flight control systems (FCS) of the aircraft must have certain gain margins to be acceptable. The stability margins are required for the FCS to tolerate gain and phase variations in its feedback loops. In the limit of minimum and maximum operational speeds of the aircraft, required minimum gain and phase margins are defined as (+6 dB, -6 dB) and (+45 deg, -45 deg) respectively for the mode frequency lower than the first aeroelastic mode.

In order to achieve required minimum gain margins and phase margins criteria, especially, optimal control design methods have been widely used in flight control problems [Gangsaas, Bruce, Blight and Ly, 1986] [Thompson, Coleman and Blight, 1987] [Amato, Mattei, Scala and Verde, 2000] [Boughari and Botez, 2012]. One of the optimal control solutions to such flight control problems is the linear quadratic regulator (LQR) which describes the cost as a quadratic function. A single input single output (SISO) LQR guarantees infinite upward gain margin with minimum 50 percent gain reduction corresponds to +infinite and -6 dB and minimum phase margin of +60 degree and -60 degree [Anderson and Moore, 1971]. For multi input multi output (MIMO) systems, the choosing of weighting matrix R diagonal provides same guaranteed margins [Lehtomaki, Sandell and Athans, 1981].

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Classical approach in LQR algorithm is to choose proper weighting matrices, Q and R, to find a symmetric positive definite matrix, P that is solution to the algebraic Riccati equation (ARE). When ARE solution matrix P is found, the feedback gain matrix, K can be calculated according to it. The resultant feedback gain matrix, K, determines the closed loop eigenvalues, which characterize the response and the performance parameters of the aircraft such as damping ratio and natural frequency.

There is a very crucial disadvantage of the classical LQR method when the performance requirements of the jet aircraft are concerned because the closed loop eigenvalues cannot be assigned arbitrarily to achieve desired damping ratio and natural frequency. On the other hand, modal control theory allows to assignment of closed loop eigenvalues.

Modal control theory not only assigns eigenvalues but also concerns about eigenvector assignment of the closed loop systems [Porter, Crossley, Tzafestas and Higgins, 1973]. For desired closed loop eigenvalues of the multi input systems, there is more than one set of corresponding closed loop eigenvectors of the solution [Moore, 1975]. Therefore, for each corresponding set of closed loop eigenvectors results different feedback matrices [Andry, Shapiro and Chung, 1983]. Since there are two inputs as aileron and rudder for conventional jet aircrafts, this method can be used for controller design on the lateral-directional axes [Sobel and Lallman, 1989].

In aerospace applications of the modal control theory, eigenstructure assignment is widely used to provide decoupling of lateral-directional dynamics of the aircraft [Faleiro and Pratt, 1996] [Sobel and Shapiro, 1985] [Harris and Black, 1996]. The method is based on the selection of the closed loop eigenvector structure for desired closed loop eigenvalues of the aircraft. There are some suggested eigenvector structures, which focus on decoupling of roll, and spiral mode but the selection of the eigenvector structures can be many [Albostan, 2018].

Therefore, LQR problem solutions have lack of achievement of the desired performance parameters such as natural frequency and damping ratio while guaranteeing gain and phase margin criteria for the aircrafts. On the other hand, modal control design methods such as eigenstructure assignment does not guarantee the gain and stability margins. These problems lead to trade-off between stability and performance of the aircrafts. In both modal and optimal control theory, there must be extra effort to meet with minimum requirements.

In order to overcome this trade-off problem and to eliminate the extra effort on the design of a stability augmentation system for the aircraft, many researches have been developed. Generally, inverse LQ methods and direct search on the feedback matrix have been used. Wilson and Cloutier have developed a procedure which fix the eigenvalues and attain the constraints to the set of possible eigenstructure to minimize a cost function [Wilson and Cloutier, 1990]. Broussard has presented an algorithm which determines the weighting matrices by attempting to place closed-loop eigenvalues near desired locations [Broussard, 1982]. These methods guarantee the gain and phase margins, but exact pole assignment is impossible. Choi and Seo have proposed an algorithm which find weighting matrices according to the desired pole locations [Choi and Seo, 1999]. Although their method achieves exact eigenvalue assignment, gain and phase margins are not guaranteed.

In this paper, combination of modal and optimal control theory is introduced to design a stability augmentation system for the jet aircraft. This algorithm is more efficient than direct search methods on the elements of the feedback matrix which satisfy both closed loop eigenvalues and minimum cost [Moore and Klein, 1976]. Also, pole placement method has been evaluated to compare the stability margins. To prove that stability margins are guaranteed by combining modal and optimal control, SAS design based on the algorithm has been performed for trim points on flight envelope of the aircraft.

METHOD

The block diagram of the lateral-directional stability augmentation system is given in Figure 1.

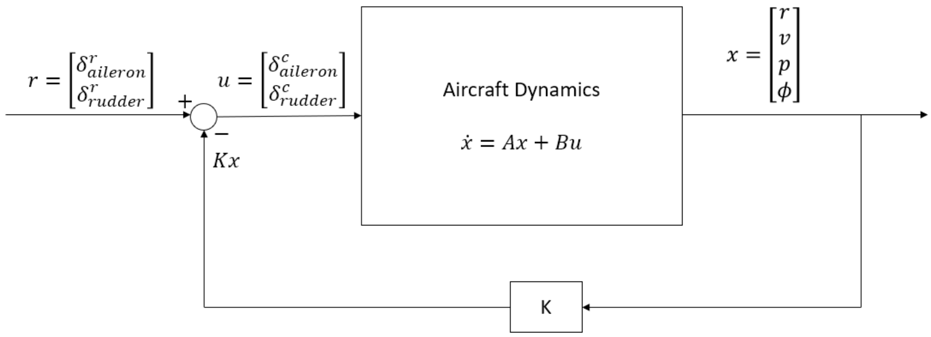

Figure 1: Stability augmentation system on lateral-directional axes

Modal optimal control is based on linear time invariant system model. The linear model of the aircraft is derived by the small perturbations in the body axis reference frame for each trim point. The inputs and outputs of the linear system are given in Figure 1.

The feedback matrix is found from the minimization of the quadratic cost function,

$$J = \int_0^\infty (x^T Q x + u^T R u) dt$$

where Q and R are positive semidefinite weighting matrices. In order to assign the desired eigenvalues to desired position, closed loop eigenvectors should be used. Sensitivity robustness of the eigenvalues also considered.

CALCULATIONS

The state-space representation of aircraft linear model is

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$x = \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix}, \qquad u = \begin{bmatrix} \delta_{ail} \\ \delta_{rud} \end{bmatrix} \qquad \text{and} \qquad C = I^{4*4}$$

Let K_Λ is the set of feedback gain matrices that closed loop system has set of eigenvalues,

$$\Sigma = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$$

Also, class of all possible sets of closed loop eigenvectors corresponding to the eigenvalues in Λ is E such as

$$\mathrm{E} = \{\mathrm{E}_1, \mathrm{E}_2, \dots, E_i, \dots\} \quad \text{where} \quad i = 1,2,3, \dots$$

The aim is to find an eigenvector set E_i, which minimize the quadratic cost function. Hence, let

$$\Lambda = diag(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

If following conditions are met then, there is a $K \epsilon (A - BK)V = V\lambda$ [Moore, 1976].

1) V is nonsingular
2) v_i and v_j, the i^{th} and j^{th} columns of V respectively are conjugate pairs of $\lambda_i = \lambda_j^*$

3) A vector w_i exist such that $(\lambda_i I - A)v_i = Bw_i$ for $i = 1,2,3,4$

Therefore,

$$K = -WV^{-1}$$

where

$$W = [w_1 \quad w_2 \quad w_3 \quad w_4]$$

In addition, eigenvalue sensitivity can be defined with inverse of the sensitivity parameter s_i for each closed loop eigenvalue λ_i [Wilkinson, 1965].

$$s_i = e_i^T v_i$$

where e_i and v_i are unit left and right eigenvectors corresponding λ_i. In order to minimize sensitivity, general approach is minimization of

$$\sum (s_i^* s_i)^{-1} = \sum \|s_i\|_2^{-1}$$

In LQR problems minimum cost solution is [Kirk, 2004]

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt = x_0^T P x_0$$

where P is the symmetric solution of Algebraic Ricatti Equation (ARE).

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$

Also,

$$u = -Kx = -R^{-1}B^T Px$$

and

$$K = R^{-1}B^T P$$

If we substitute K with the solution comes from eigenvector solution then,

$$-WV^{-1} = R^{-1}B^T P$$

Hence,

$$B^T P = -RWV^{-1}$$

And

$$PB = -V^{*-1}W^* R^T$$

Substitute into ARE,

$$PA + A^T P + V^{*-1}W^* RWV^{-1} + Q = 0$$

Multiply the ARE by V^* from left and by V from right.

$$V^* PAV + V^* A^T PV + W^* RW + V^* QV = 0$$

It can be also written as

$$\Lambda V^* PV + \Lambda V^* PV + W^* RW + V^* QV = 0$$

Let $P = EP_0E^*$ where P_0 is symmetric also and $E = [e_1, e_2, e_3, e_4]$. Then,

$$\Lambda V^* EP_0E^* V + \Lambda V^* EP_0E^* V + W^* RW + V^* QV = 0$$

Multiply the equation by S^{*-1} from left and by S^{-1} from right where $S = diag(s_1, s_2, s_3, s_4) = E^* V$.

$$\Lambda P_0 + \Lambda P_0 + S^{*-1}(W^* RW + V^* QV)S^{-1} = 0$$

Then minimization of P_0, also minimize the quadratic cost. Therefore,

$$P_0 = -\frac{1}{2\Lambda} S^{*-1}(W^* RW + V^* QV)S^{-1}$$

For simplification, minimize the trace of P_0 [Moore and Klein, 1976],

$$J \triangleq trP_V = -\sum_{i=1}^{n} (\frac{1}{2 * s_i^* s_i Re(\lambda_i)})(v_i^* Q v_i + w_i^* R w_\text{i})$$

Where s_i is the sensitivity parameter, v_i is the right eigenvector of the closed loop system and w_i is a vector determined by v_i. Interior-Point Algorithm [Byrd, Gilbert and Nocedal, 2000] has been used to evaluate the cost. The purpose is to find the closed loop eigenvectors corresponding desired eigenvalues from orthonormal null space of the closed loop system to minimize the cost. Once the eigenvectors were found, the feedback matrix can be easily calculated.

RESULTS

The highly maneuverable jet aircraft has been trimmed at [400, 600, 800] ft/sec, [10000, 30000] ft altitude and center of gravity position at 0.35 cruise condition. The weighting matrices Q and R are chosen as identity matrices.

The closed loop eigenvalues of the aircraft have been chosen as follows:

i. Dutch-roll mode natural frequency is same with open loop dutch-roll mode natural frequency.
ii. Dutch-roll damping ratio is increased 100%.
iii. Roll mode root remains same with airframe root.
iv. Spiral mode root remains same with airframe root.

In form,

$$\text{eig}(\text{A}_{\text{cl}}) = \begin{bmatrix} -\omega_{ol}(2\zeta_{ol}) + \omega_{ol}\sqrt{1-(2\zeta)^2}i \\ -\omega_{ol}(2\zeta_{ol}) - \omega_{ol}\sqrt{1-(2\zeta)^2}i \\ \lambda_{roll} \\ \lambda_{spiral} \end{bmatrix}$$

In order to make comparison between gain and phase margins, MATLAB ***place()*** command is used to assign the closed loop eigenvalues based on [Kautsky, Nichols and Van Dooren, 1985].

Frequency response of the aileron loop gain based on combination of modal and optimal control is shown on the Nichols plot for frequency range [0.01:0.01:100] rad/s (Figure 2).

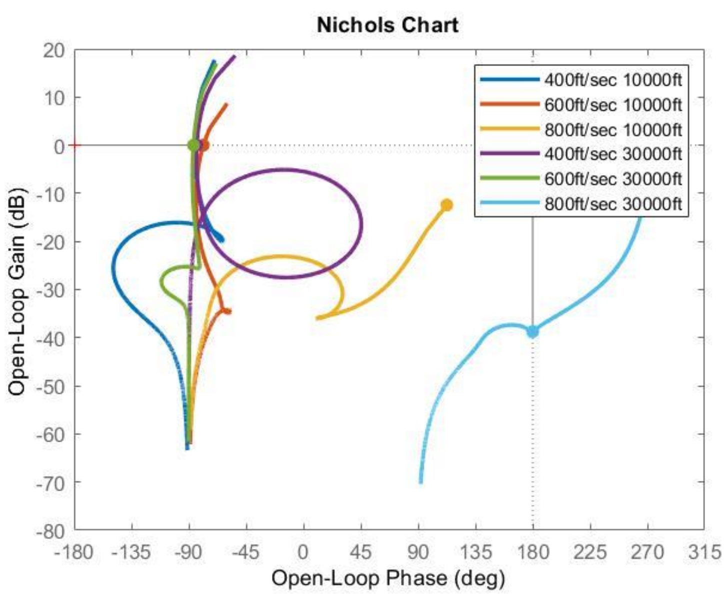

Figure 2: Nichols plot for the loop gain from aileron input

The comparison table of minimum gain and phase margins are given in Table 1.

	Modal – Optimal Control		**Poles Placement**	
Trim Point	**Gain Margin [dB]**	**Phase Margin [degree]**	**Gain Margin [dB]**	**Phase margin [degree]**
V = 400 ft/sec h = 10000 ft	[-inf, +inf]	[-inf, 94.9]	[-inf, 12.9]	[-inf, 44.9]
V = 600 ft/sec h = 10000 ft	[-inf, +inf]	[-inf, 101.0]	[-inf, 7.6]	[-inf, 26.0]
V = 800 ft/sec h = 10000 ft	[-inf, +inf]	[-inf, +inf]	[-inf, 1.6]	[-inf, 12.6]
V = 400 ft/sec h = 30000 ft	[-inf, +inf]	[-inf, 96.3]	[-inf, 18.4]	[-120, 72.1]
V = 600 ft/sec h = 30000 ft	[-inf, +inf]	[-inf, 93.7]	[-inf, +inf]	[-inf, 68.5]
V = 800 ft/sec h = 10000 ft	[-inf, +38.7]	[-inf, 99.4]	[-inf, 1.49]	[-inf, 8.07]

Table 1: Gain and phase margins for aileron loop gain

The design based on modal and optimal control at only one point had not guaranteed that positive gain margin is infinite. The reason of that, at low stability or at unstable points, it is hard to find a set of closed loop eigenvectors corresponding closed loop eigenvalues. The low stability can be seen by inspecting gain and phase margins comes from pole placement method. Depend on search algorithm used to minimize the cost, performance of the algorithm can be increased.

Frequency response of the rudder loop gain based on combination of modal and optimal control is shown on the Nichols plot for frequency range [0.01:0.01:100] rad/s (Figure 3).

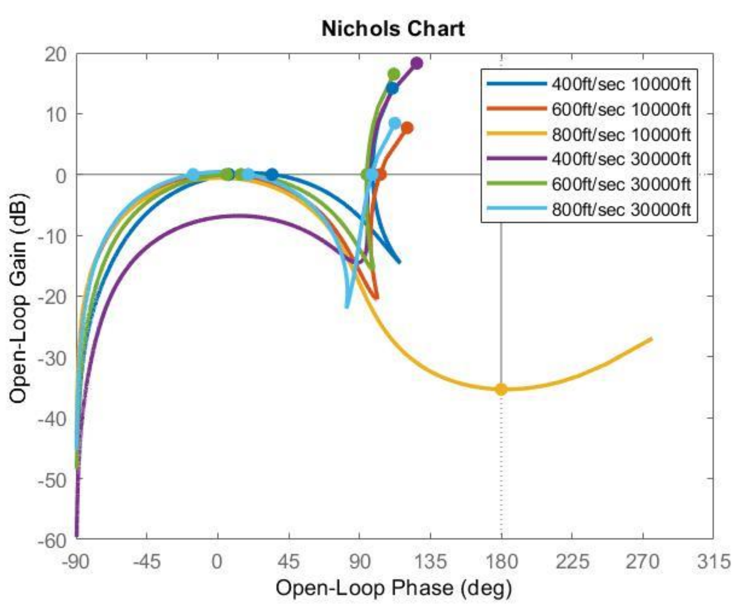

Figure 3: Nichols plot for the loop gain from rudder input

The comparison table of minimum gain and phase margins are given in Table 2.

	Modal – Optimal Control		Poles Placement	
Trim Point	**Gain Margin [dB]**	**Phase Margin [degree]**	**Gain Margin [dB]**	**Phase margin [degree]**
V = 400 ft/sec h = 10000 ft	[-inf, +inf]	[-81.0, +inf]	[-inf, +inf]	[-25.2, 163.0]
V = 600 ft/sec h = 10000 ft	[-inf, +inf]	[-77.0, +inf]	[-inf, +inf]	[-9.7, 145.0]
V = 800 ft/sec h = 10000 ft	[-inf, 35.3]	[-90, +inf]	[-inf, 0.1]	[-4.2, 141]
V = 400 ft/sec h = 30000 ft	[-inf, +inf]	[-84.2, +inf]	[-19.4, +inf]	[-90.1, +inf]
V = 600 ft/sec h = 30000 ft	[-inf, +inf]	[-85.3, +inf]	[-12.3, +inf]	[-113.0, +inf]
V = 800 ft/sec h = 10000 ft	[-inf, +inf]	[-82.3, 164.0]	[-inf, +inf]	[-inf, 138]

Table 2: Gain and phase margins for rudder loop gain

The design based on modal and optimal control at only one point had not guaranteed that positive gain margin is infinite. The reason of that is same with discussion at aileron loop gain.

In order to observe dutch roll behavior of the augmented aircraft, the initial time response of the aircraft to 5 degrees sideslip angle is given as shown in Figure 4.

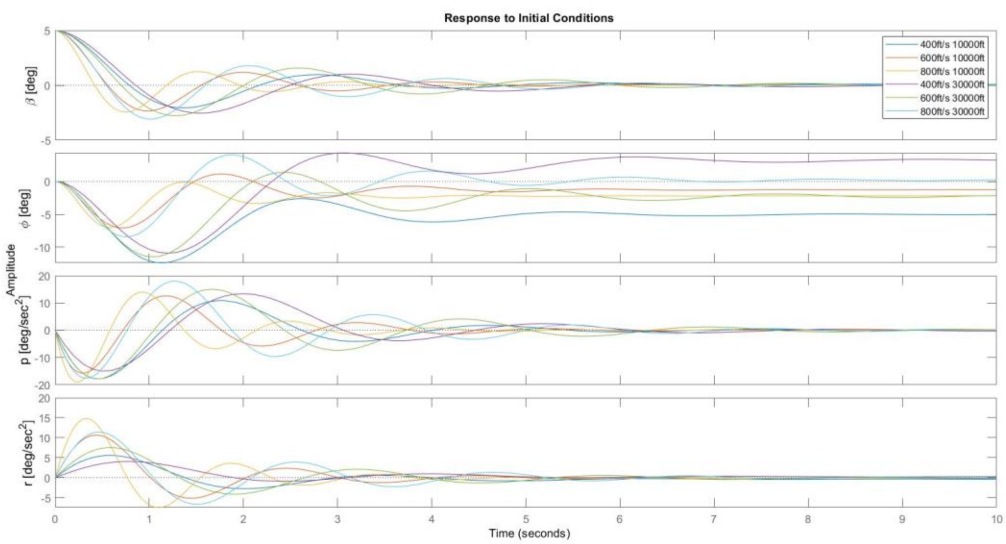

Figure 4: Initial time response of augmented aircraft

The oscillations on sideslip and yaw rate are applicable because dutch-roll mode has not been damped too much. The time response difference between augmented and unaugmented is given in Figure 5.

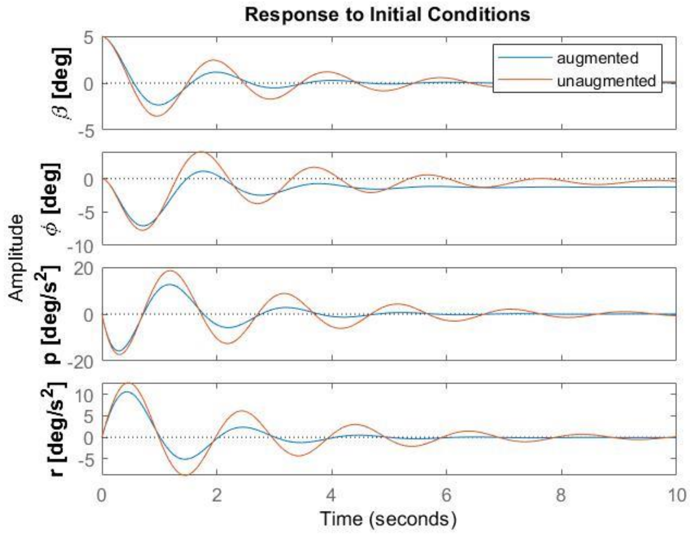

Figure 5: Initial time response of augmented aircraft

CONCLUSION

In conclusion, combination of modal and optimal control has two advantages in aerospace applications. First of all is that performance requirements can be easily met by assigning closed loop eigenvalues of the flight control system while satisfying stability margins as a result of optimal control theory. It provides minimization of the effort in trade-off between performance and stability. Second advantage of the method is that the cost function can be manipulated according to needs of the designer by using eigenvalues, left and right eigenvectors as used in this paper to minimize sensitivity parameter.

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