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SCATTER AND SENSITIVITY ANALYSIS OF A PROJECTILE MODELED UNDER MAGNUS EFFECT

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ABSTRACT

The most common type of ammunition in military areas is unguided projectile. This is because these ammunitions are cheap and easy to use. However, these unguided ammunitions have several shortcomings. They are spin-stabilized which is prone to Magnus Effect. They are also sensitive to disturbances and launch errors. In this study, these shortcomings will be examined through Monte-Carlo simulations.

INTRODUCTION

The grenades and many artillery ammunitions are munitions without stabilizing wings. Therefore, spin stabilization is necessary. There are many studies aimed at understanding and developing spin stabilized systems. DeSpirito worked on this issue to understand the behavior of spin-stabilized projectile during flight, with high angle of attack caused by guided flight maneuvers [Despirito, 2017]. In recent years, Fresconi, Rogers and Celmins have published studies on similar topics. These studies focus on unguided large diameter systems such as 175mm mortar and unguided small diameter systems such as 40 mm grenade [Fresconi, Cooper, Celmins, Despirito, and Costello, 2010; Fresconi and Celmins, 2018; Fresconi and Rogers, 2014].

An initial spin provided by the grooves in the barrel. In order to achieve gyroscopic stabilization, the projectile needs to attain a sufficient roll rate depending on the flight conditions and its geometry [McCoy 2012];

$$S_g = \frac{P^2}{4M} = \frac{I_x^2 p^2}{2\rho I_y S d V^2 C_{m_\alpha}}$$

For the spin stabilization of a statically unstable projectile $S_g > 1$ criteria must be ensured. Therefore, the projectile must fly with an appropriate roll rate. The projectile that is used in this study and shown in Figure 1, has 76 m/s muzzle speed and about 63 Hz spin rate [Army ammunition data sheets, 1991], It is also seen that there is an equation between the velocity of the muzzle and the rotational velocity of the muzzle based on the gun's barrel features.

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Figure 1 – M433 Grenade

According to this equation, the muzzle rotation speed of the grenade depends on the speed of the bomb and the helix angle of the barrel. Barrel twist rate (ηd) is taken from the equation which is related to muzzle velocity and muzzle rotation speed in order to use in study.

$$P_{muzzle} = \frac{2\pi v_{muzzle}}{\eta d} \ Hz$$

Gyroscopic stabilization equation is examined with the help of a simulation model. Two different scenarios are evaluated. One of them is the launch of the grenade at the defined muzzle velocity and rotation rate specified in the literature, and the other scenario is the launch of grenade with the same muzzle velocity but nearly 40 percent reduced rotation speed. These simulations are presented in Figure 2 and Figure 3.

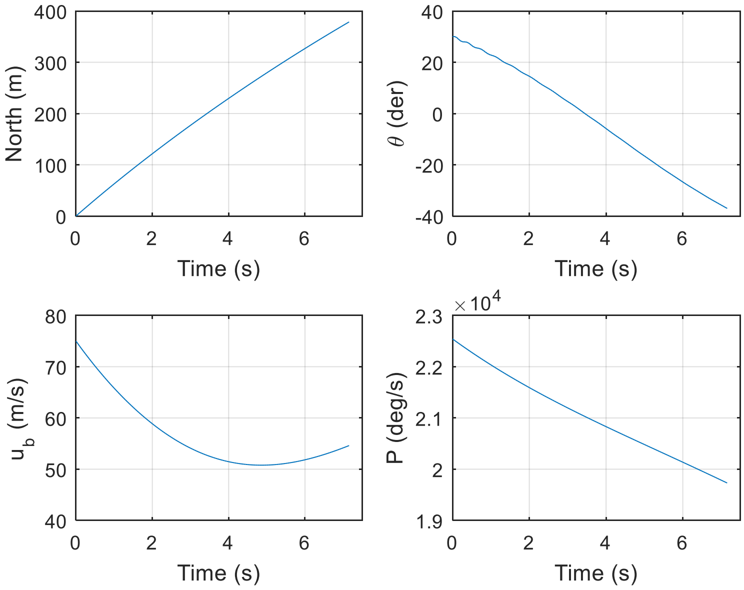

Figure 2 – 76 m/s Muzzle Velocity and 63 Hz Muzzle Spin Rate

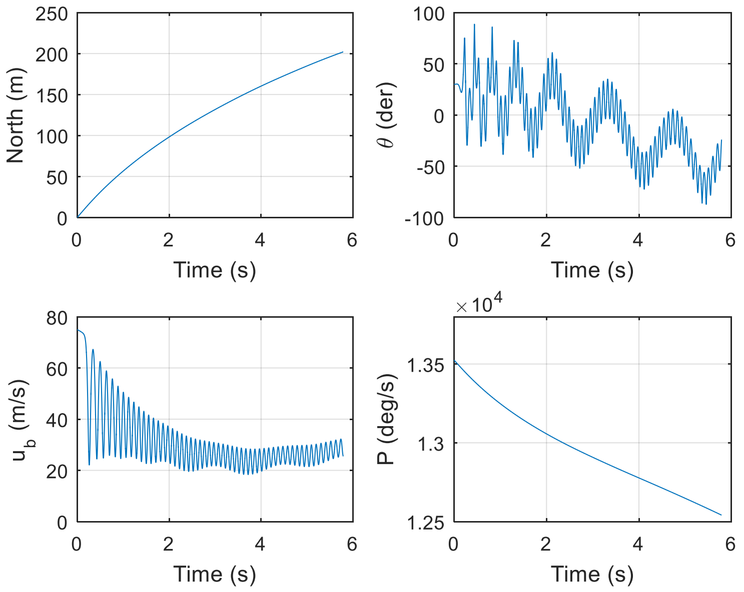

Figure 3 – 76 m/s Muzzle Velocity and 38 Hz Muzzle Spin Rate

From these figures, it may be observed that high spin rates are necessary for spin-stabilized systems. High rotational speeds and angle of attack causes the Magnus effect on the projectile. The Magnus effect is defined as the lifting force on a rotating body when it is exposed to a fluid that hits the body and is perpendicular to the axis of rotation [Seifert, 2012]. Magnus effect can be divided into two subparts called Magnus force and Magnus moment. The coefficient $C_{Y_{p\alpha}}$ is used for obtaining Magnus force. In addition, Magnus force is always on the axis that is perpendicular to the plane of yaw [McCoy 2012]. Magnus moment coefficient $C_{n_{p\alpha}}$ may be positive or negative depending on the geometric characteristics of the projectile.

Other distortions such as wind velocity, air density error, boresight retention error, muzzle velocity error, distance error, cant error, weapon-target altitude error, aim error and manufacturing error of aerodynamic shape of grenade shall also be considered.

Distortions

Wind velocity: Range wind and crosswind are the atmospheric disturbances acting on the grenade, affecting vertical and horizontal miss distances respectively [Strohm, 2013].

Air density error: High or low air density prolongs or shortens flight of grenade.

Boresight retention error: When the trigger is grasped and fired, the harmony between the leaf sight and the barrel hole may deteriorate in an unknown manner due to vibrations and the use of weapons [Strohm, 2013].

Muzzle velocity error: Muzzle velocity error is caused by the use of the barrel and the amount of gunpowder in the barrel. This cause a change in spin rate due to the helix angle.

Distance error: The distance which is between target and launcher plays important role in determining the elevation angle. The error in determination of distance of target causes vertical miss distance.

Cant error: Cant is the angle between the grenade launcher axial vector and the gravity vector, which leads to muzzle mistakes shift.

Weapon-target altitude error: The difference in altitude between the weapon and the target leads to errors for miss distance.

Aim error: Aiming error is a kind of human error which caused by misalignment of elevation and azimuth angle

Manufacturing error of aerodynamic shape of grenade: Grenades are not always productively identical. This causes the aerodynamic coefficients to be different.

The effects of various distortions on accuracy and precisions are investigated in this study. Accuracy is described as the closeness to nominal value, while precision is described as the closeness to each other [Montgomery, Runger and Huberle, 2006]

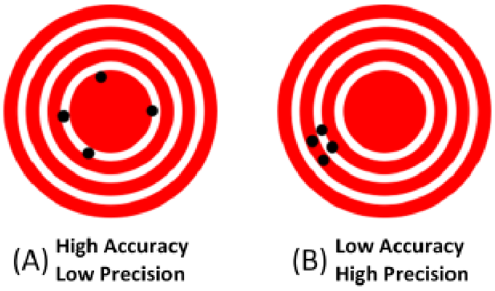

Figure 4 – Accuracy and Precision Demonstration Source: Courtesy of Army Research Laboratory

METHOD AND SIMULATION

M433 grenade that is shown in Figure 1 is selected for the analysis carried out below. The study is performed using MATLAB/Simulink and PRODAS. By using MATLAB/Simulink, six degrees of freedom model "6DOF Quaternion Model" is implemented using non-rotating and flat earth assumptions [Stevens and Lewis, 2003; Zipfel, 2007]. Wind model effects angle of attack and sideslip angle. The aerodynamic model required for the analysis was prepared in PRODAS (Projectile Rocket Ordanance Design & Analysis Systems).

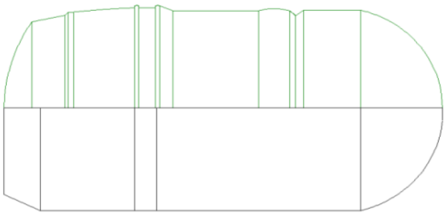

Figure 5 –Modeled M433 Grenade

Aerodynamic Model

The related aerodynamic coefficients with their meanings are listed in the Table 1 below.

Table 1 – Aerodynamic Model Coefficients

C_{X_0}	Zero Yaw Axial Force Coefficient
$C_{X_{\alpha^2}}$	Yaw Axial Force Coefficient Derivative
C_{N_α}	Normal Force Coefficient Derivative
$C_{Y_{p\alpha}}$	Magnus Force Coefficient Derivative
$C_{Y_{\gamma\alpha}}$	Induced Side Force Coefficient Derivative
C_{l_p}	Roll Damping Coefficient
C_{l_δ}	Roll Moment Fin Deflection Derivative
$C_{l_{\gamma\alpha}}$	Induced Roll Moment Coefficient
C_{m_α}	Pitching Moment Coefficient Derivative
C_{m_q}	Pitch Damping Coefficient
$C_{n_{p\alpha}}$	Magnus Moment Coefficient Derivative
$C_{n_{\gamma\alpha}}$	Induced Side Moment Coefficient Derivative
$C_{n_{sm}}$	Out of Plane Side Moment Coefficient

Additional parameters used in the aerodynamic model are listed in Table 2.

Table 2 – Other Parameters Used in the Simulation Model

α_T	Total Angle of Attack
N	Number of Fin Blades
ϕ'	Aerodynamic Roll Angle
δ	Fin Cant
u, v, w	Body Frame Velocity Components
p	Roll Rate
q	Pitch Rate
r	Yaw Rate
V	Total Velocity
d	Reference Length
Q	Dynamic Pressure
A	Reference Area

With the help of these parameters, three aerodynamic moments and three aerodynamic forces may be calculated. The related equations are given below:

$$F_x = -QA\left[C_{X_0} + C_{X_{\alpha^2}} \sin^2 \alpha_T\right]$$

$$F_y = QA\left[-C_{N_\alpha} sin\alpha_T\ sin\phi' + C_{Y_{p\alpha}} sin\alpha_T\ cos\phi' \left(\frac{pd}{2V}\right) + C_{Y_{\gamma\alpha}} \sin^3 \alpha_T\ \ cos\phi' \sin(N\phi')\right]$$

$$F_z = QA\left[-C_{N_\alpha} sin\alpha_T\ cos\phi' - C_{Y_{p\alpha}} sin\alpha_T\ sin\phi' \left(\frac{pd}{2V}\right) - C_{Y_{\gamma\alpha}} \sin^3 \alpha_T\ \ sin\phi' \sin(N\phi')\right]$$

$$M_x = QAd\left[C_{l_p}\left(\frac{pd}{2V}\right) + C_{l_\delta}\delta + C_{l_{\gamma\alpha}} \sin^2 \alpha_T\ \ \sin(N\phi')\right]$$

$$M_y = QAd\left[C_{m_\alpha} sin\alpha_T\ cos\phi' + C_{m_q}\left(\frac{qd}{2V}\right) + C_{n_{p\alpha}} sin\alpha_T\ sin\phi'\ \left(\frac{pd}{2V}\right) + \left(C_{n_{\gamma\alpha}} + C_{n_{sm}}\right) \sin^3 \alpha_T\ \ sin\phi' \sin(N\phi')\right]$$

$$M_z = QAd\left[-C_{m_\alpha} sin\alpha_T\ sin\phi' + C_{m_q}\left(\frac{rd}{2V}\right) + C_{n_{p\alpha}} sin\alpha_T\ cos\phi'\ \left(\frac{pd}{2V}\right)\right.$$
$$\left. + \left(C_{n_{\gamma\alpha}} + C_{n_{sm}}\right) \sin^3 \alpha_T\ \ cos\phi' \sin(N\phi')\right]$$

Simulation

In order to analyze the scatter and sensitivity of projectile, Monte Carlo analysis is performed. The variable parameters of analysis have Gaussian distribution with mean and standard deviation values, as shown at Table 3.

Table 3 – Monte Carlo Parameters for Scatter Analysis

Parameter Error	Mean	Standard Deviation	Unit
Range wind velocity	0	2	Knot
Crosswind velocity	0	2	Knot
Air density error	0	1	Percent
Boresight retention error $(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})_{\boldsymbol{t=0}}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{0\ 5\ 5}]$	Degree/Second
Muzzle velocity error $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})_{\boldsymbol{t=0}}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{2\ 0\ 0}]$	Meter/Second
Distance error $\mathbf{NED}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})_{\boldsymbol{t=0}}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{2\ 0.5\ 0}]$	Meter
Cant error $(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi})_{\boldsymbol{t=0}}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{5\ 0\ 0}]$	Degree

Weapon-target altitude error $\mathbf{NED}(x, y, z)_{t=0}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{0\ 0\ 2.5}]$	Meter
Aim error $(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi})_{t=0}$	$[\mathbf{0\ 0\ 0}]$	$[\mathbf{0\ 3\ 3}]$	Degree
Manufacturing error of aerodynamic shape of grenade	0	1	Percent

Initial Conditions:

Unguided projectile's initial conditions are taken from literature sources. These are muzzle exit velocity ($\boldsymbol{u_{muzzle}}$), roll rate at the muzzle exit ($\boldsymbol{p}$), and the optimum elevation angle ($\boldsymbol{\theta}$) of grenade launcher. The other initial conditions are taken with engineering intuitive approach, which makes analysis easier.

Table 4 – Initial Conditions

Parameter	Value	Unit
Muzzle position defined in NED Frame$(\boldsymbol{X_N}, \boldsymbol{X_E}, \boldsymbol{X_D})_{t=0}$	$[\mathbf{0\ 0\ 0}]$	Meter
Muzzle exit velocity in Body Frame$(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})_{t=0}$	$[\mathbf{76\ 0\ 0}]$	Meter/Second
Euler angles at muzzle exit $(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi})_{t=0}$	$[\mathbf{0\ 35\ 0}]$	Degree
Angular speeds at muzzle exit $(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r})_{t=0}$	$\left[\frac{\boldsymbol{u_{muzzle}}}{\mathbf{1.1978}}, \mathbf{0}, \mathbf{0}\right]$	Hz

RESULTS

Scatter Analysis Results:

After carrying out the 5000 simulations using the statistical properties of various parameters listed in Table 3, the results are analyzed and listed in tables 5-7.

Table 5 – Monte Carlo Analysis Result Nominal and Mean Points

Name of Point	Location of Point wrt North East Down Frame (m)
Nominal Point (Accuracy Point)	[399.4 10.2 0.0]
Mean Point (Precision Point)	[398.5 10.8 0.0]

Table 6 – 5000 Monte Carlo Analysis Result (Accuracy)

Radius From Nominal Point	Percentage of Ammunition Falling into the Radius
1	0.04 %
5	2.42 %
10	9.10 %

Table 7 – 5000 Monte Carlo Analysis Result (Precision)

Radius From Mean Point	Percentage of Ammunition Falling into the Radius
1	0.04 %
5	2.30 %
10	8.96 %

According to these results, accuracy is slightly better than precision by a small margin. However, both of them have less than %10 success to fall into a 10 m radius for related points.

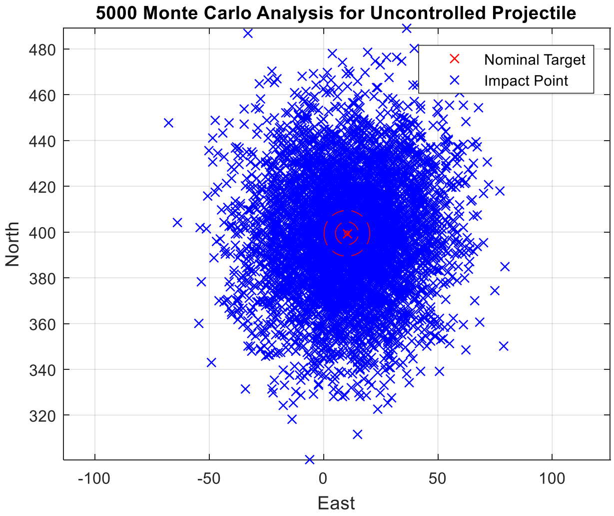

Figure 6 – Grenade Distribution

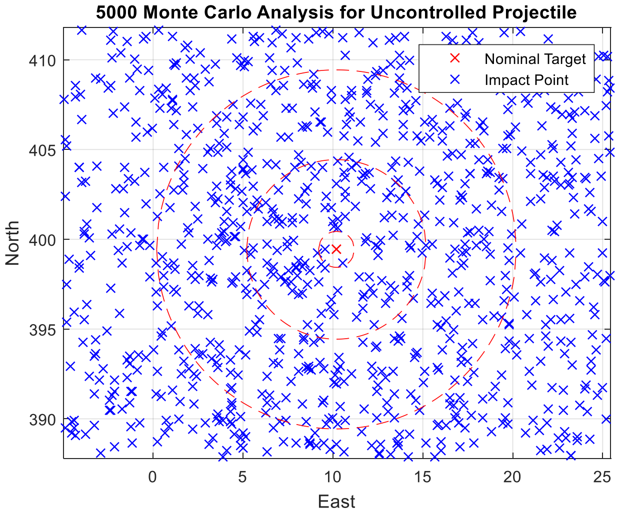

Figure 7 – Zoomed Grenade Distribution

Sensitivity Results:

Correlation coefficients are calculated for sensitivity analysis. Dependence or independence between two or more variables is examined by correlation [Kendall 1979].

$$\rho_{xy} = \frac{V_{xy}}{\sigma_x \sigma_y}$$

V_{xy} represents the covariance of *x* and *y* random variables, while σ_x and σ_y represent the standard deviation values of *x* and *y* variables [Cowan, 1998].

In order to examine sensitivity analysis, range wind, crosswind, air density error, muzzle velocity error, altitude error, aim error and aerodynamic shape error are considered.

Monte Carlo analysis is performed by considering the defined distortion parameters. Each set of random disturbances that are used as inputs for Monte Carlo analysis are recorded. In addition, the miss distances in *x* axis and *y* axis obtained for each Monte Carlo simulation are recorded. By using recorded data, correlation coefficients for each Monte Carlo parameter are calculated by performing above equation. The negative or positive sign of correlation coefficient represent the relation between Monte Carlo parameters and correlation coefficients, which means that Monte Carlo parameters are directly proportional to or inversely proportional to correlation coefficients. The obtained correlation coefficients are tabulated in Table 8. When the absolute value of the correlation coefficients indicated in the table is close to one, it shows that sensitivity level of the Monte Carlo parameter is high. On the contrary, when the absolute value of the correlation coefficients is close to zero, it shows that sensitivity level of the Monte Carlo parameter is low.

Table 8 – Summary of Sensitivity Analysis Result

	Miss distance x-axis	Miss distance y-axis	Range Wind	Crosswind	Air density error	Muzzle velocity error	Altitude error	Elevation error	Azimuth error	Aero coefficient error
Miss distance x-axis	1.000	0.036	-0.032	-0.023	0.015	-0.926	0.015	-0.350	0.024	0.058
Miss distance y-axis		1.000	0.088	0.092	-0.002	-0.045	0.006	-0.041	-0.995	-0.009

The findings indicate that the velocity of the muzzle dominates the miss distance on the x-axis. Additionally, the elevation error is the second most significant consideration for the vertical axis. Also, aerodynamic coefficient error and wind factor also seem to be more essential than the remainder.

It is seen that the sensitivity of the error made in azimuth angle for the miss distance of y-axis is high. The effect of the crosswind at the level of 2 knots is perceivable but not at the expected level.

Magnus Effect Analysis Results:

In this analysis, the Magnus Effect for short range spin stabilized projectile is examined. To perform the analysis, the values for $\boldsymbol{C_{Y_{p\alpha}}}$ and $\boldsymbol{C_{n_{p\alpha}}}$ are included in order to model Magnus Effect in the simulation for one shot. First north position, east position and flight time are obtained. Then, they are compared with the results of another simulation that ignores the Magnus Effect. The obtained results are tabulated in Table 9.

Table 9 – Magnus Effect Comparison Table

	Final North Position (m)	Final East Position (m)	Flight Time (s)
Magnus Effect Coefficients Considered	399.7	10.74	8.134
Magnus Effect Coefficients Ignored	399.4	10.73	8.126

After the examination, it is seen that the difference between the results of the simulations is quite small. Therefore, the Magnus Effect may be neglected for short range small diameter projectiles.

For this flying system with an angle of attack, the Magnus components have an effect that tries to approach the nominal orbit rather than the scattering effect. The reason for this can be explained by equations of motion.

$$\frac{\mathrm{d}u}{\mathrm{d}t} = rv - qw + \frac{f_x}{m}$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = pw - ru + \frac{f_y}{m}$$
$$\frac{\mathrm{d}w}{\mathrm{d}t} = qu - pv + \frac{f_z}{m}$$

Gravity causes velocity vector deviates from x-axis of the body axis. This leads to an angle of attack in the grenade. A grenade having a positive rotation speed and angle of attack will have acceleration in the y-axis direction of the body axis. Therefore, it is experienced that the system deviates from the nominal ballistic orbit.

The Magnus Effect has distinct effects regardless of equations. The pressure difference around the grenade formed under a certain angle of attack and positive rotation speed creates the Magnus force and moment. This force is in the opposite direction to the y-axis of the body axis assembly. The effect of this condition is not seen much since the diameter of the grenade and its flight speed are low.

CONCLUSION

As a result of this study, the scattering values of the system are examined, the sensitivity analysis table is prepared and the Magnus effect of on the grenade considered is examined. In order to carry out the simulations, the aerodynamic model is obtained using PRODAS. Monte Carlo simulations are carried out to examine the effect of various errors on the miss distance. In addition, the effect of Magnus force on the nominal flight trajectory is also examined and it is concluded that this force does not have much effect on miss distance.

References

[1] Army ammunition data sheets: Artillery ammunition: Guns, howitzers, mortars, recoilless rifles, grenade launchers, and artillery fuzes (federal supply class 1310, 1315, 1320, 1390). (1991). Washington, DC: Headquarters, Dept. of the Army.

[2] Cowan, G. (1998). Statistical Data Analysis. Oxford University Press.

[3] Despirito, J. (2017). CFD Aerodynamic Characterization of 155-mm Projectile at High Angles-of-Attack. 35th AIAA Applied Aerodynamics Conference. doi:10.2514/6.2017-3397

[4] Fresconi, F., & Celmins, I. (2018). Experimental Flight Characterization of Spin-Stabilized Projectiles at High Angle of Attack. 2018 AIAA Atmospheric Flight Mechanics Conference. doi:10.2514/6.2018-0769

[5] Fresconi, F., & Rogers, J. (2014). Guidance and Control of a Man-Portable Precision Munition Concept. doi:10.21236/ada606639

[6] Fresconi, F., Cooper, G., Celmins, I., Despirito, J., & Costello, M. (2010). Flight Mechanics of a Novel Guided Spin-Stabilized Projectile Concept. AIAA Atmospheric Flight Mechanics Conference. doi:10.2514/6.2010-7638

[7] Kendall, M.G. (1979). The Advanced Theory of Statistics, 4th Ed., Macmillan.

[8] MATLAB and Aerospace Toolbox Release 2015b, The MathWorks, Inc., Natick, Massachusetts, United States.

[9] McCoy, Robert L. Modern Exterior Ballistics the Launch and Flight Dynamics of Symmetric Projectiles. Schiffer Publ., 2012.

[10] Montgomery, D. C.; Runger, G. C.; Huberle, N. F. Engineering Statistics, 3rd ed.; John Wiley and Sons: Hoboken, NJ, 2006.

[11] Seifert, J. (2012). A review of the Magnus effect in aeronautics. Progress in Aerospace Sciences, 55, 17-45. doi:10.1016/j.paerosci.2012.07.001

[12] Stevens, B. L. & Lewis F. L. (2003). Aircraft Control and Simulation, Second Edition, John Wiley & Sons.

[13] Strohm, L. S. (2013). An Introduction to the Sources of Delivery Error for Direct-Fire Ballistic Projectiles. doi:10.21236/ada588846

[14] Zipfel, P. H. (2007). Modeling and Simulation of Aerospace Vehicle Dynamics. Second Edition, AIAA Education Series,