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3-D STATIC ANALYSIS AND DESIGN OF SPACECRAFT WITH CENTRAL CYLINDERS

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ABSTRACT

Central cylinder (CC) is backbone of the spacecraft. It is typical main structural elements for the spacecraft with main mission of orbiting around the earth such as geosynchronous communication and earth observation satellites. Structurally, it is the optimal alternative since it is one-piece main structure and has high structural stability, minimal weight and houses propulsion tanks properly. It can be approximated as straight cantilever beam/bar element since it is attached as a fixed structural element at the top of the launcher rocket. There are many structurally optimized types of CC such as lattice, iso-grid and CFRP sandwich etc. This study is an attempt to formulate the preliminary design of central cylinder and develop theoretical analysis tool for any type of long structures such as satellite CC, long stacks, pipes, etc.

INTRODUCTION

Central cylinders (CCs) are widely used as primary structure for spacecraft (SC) (Figure-1). It is backbone of the spacecraft and house propellant tanks. Structurally, it is the optimal structure with high structural stability, high stiffness, minimal weight and providing large volume for housing of large propulsion tanks. It can be ideally approximated as straight cantilever rod since it is nearly attached as a fixed-free structural element at the top of the launchers. The launcher static and dynamic loads are transmitted to the spacecraft. The spacecraft is attached to launcher interface adaptor via clamp band. This support can be considered almost as a fixed base since the displacements and rotations are prevented during the launching. Typically, the CCs has length of 2.0 to 6-meter height and there are mainly three standardized different diameters of 0.94, 1.2, 1.67 meters. Structurally it can be approximately considered as a fixed-free uncoupled bar and beam for axial and bending motion respectively since the spacecraft is designed to be symmetric with respect launcher vertical direction as a launcher requirement. The CC approximately represents whole spacecraft since it is main structural element with almost all stiffness of the spacecraft both in axial and lateral directions.

Maximum loadings of a spacecraft generally occur during the launching. The main loads are quasi-static axial and lateral loads, dynamic and acoustic loads caused by launcher during launching. The SC has to be designed according to these launcher load levels defined on the launcher manuals (Figure-2). Although loads and their levels are well defined, the dynamic

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and quasi-static loads on the spacecraft are inertial type and application points of the loads depends on the location of masses on which they are attached to the spacecraft. The loads depend on the SC mass distribution and differ from SC to SC. The launcher manuals specify quasi-static acceleration loads levels in axial and lateral directions. But the launcher lateral load directions are not specifically specified in x and y components because of randomness of direction of the lateral loads. In analysis, the same level of lateral loads is applied both x and y directions separately or divided into components in x-and y directions to find maximum critical loads.

Figure-1 The typical Central cylinder (CC)

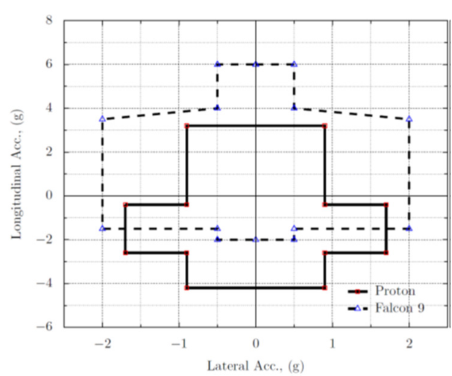


Figure-2 The quasi-static load envelopes for the Proton and Falcon 9 launchers

Initial design by simple engineering models may help to find the critical directions and determine precisely initial thickness and other structural parameters such as thickness variation along the height, material layout, material itself etc. Initial design of a CC can be done by considering it as statically determinate beam/bar carrying whole spacecraft itself with all subsystems are attached to it. The distributed loads can be assumed to uniformly distributed over a line/area and/or point loads acting on a point on the CC depending upon their size and location. The CC has to be highly stiff with high slenderness ratio to meet launcher requirements. These assumptions result in reasonable and fairly representative of loads for the preliminary design purposes. The CC structural characteristics and proper representation of loads make it possible to use engineering beam/bar model with various ideal loads and obtain main design parameters accurately.

This study is about 3-D modeling of the CC under point and distributed loads and may help the designers to find design parameters precisely. The study is mainly divided into three sections. The first part is description of 3-D static model and use of torsor concept for the model. The second part is model for section forces in x, y and z directions, section moments in bending and torsion. The last section is application for the CC.

METHOD

Consider a straight rod/beam element subjected to distributed and point loads including moments in 3-D. It is assumed that the deformations are small and the material is linear elastic. In engineering mechanics, the axial, shear and bending moment (NSM) diagrams are used to show the internal force values caused by the external loading (Teribat 2016]. These diagrams are obtained using equilibrium equations of an isolated element of a beam (Inan 2015 and Svetlitsky 2000]. These equilibrium equations can be shown in vector form as

$$\sum_{n=1}^{N} \mathbf{F_n} = \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \sum_{n=1}^{N} \begin{Bmatrix} F_{xn} \\ F_{yn} \\ F_{zn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

$$\sum_{n=1}^{N} \mathbf{M_n} = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \sum_{n=1}^{N} \begin{Bmatrix} M_{xn} \\ M_{yn} \\ M_{zn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

Internal stress distribution can be calculated using beam model. The governing equations are

$$T_x = \int_A \sigma_{zx} dA, \qquad T_y = \int_A \sigma_{zy} dA, \qquad T_z = \int_A \sigma_{zz} dA \tag{3}$$

For the total lateral shear force on the section

$$M_{xx} = \int_A y\, \sigma_{zz} dA\,, \qquad M_{yy} = \int_A x\, \sigma_{zz} dA\,, \qquad M_{xx} = \int_A (-y\, \sigma_{zx} + x\sigma_{zy}) dA \tag{4}$$

where T_x, T_y and T_z are total traction forces acting on the section. The stresses on these equations can be simply determined using beam torsion and bar structural models. Some loads are transferred to the main body and can be represented as torsors (Berthelot 2012 and Teribat 2016]. The torsor is simply two vector pairs representing force and moments applied at the same point. It is shown as $\{T\}_p = \{\mathbf{F}, \mathbf{M_p}\}_p$ where $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$ is force vector acting on the point *P* and $\mathbf{M_p}$ is resulting moment vector.

Torsors

Torsor concept is not well-known and used in the in the English mechanics textbooks but it is well-recognized and used in French and Spanish textbooks. It represents two vectors as one pair, the first one is applied static load **F** relative to a given point P and the second one is equivalent moment $\{\mathbf{M_p}\}$ caused by the **F** at that point(Berthelot 2012]. Thus, any mechanical point load acting on body can be represented as torsor. The torsor concept is useful to define equivalent eccentric forces causing moment at the connecting portion of main structure. It is simply replacement of eccentric force by two vectors one is acting force vector and the other one is moment vector caused by the eccentricity of the applied load (Figure-3).

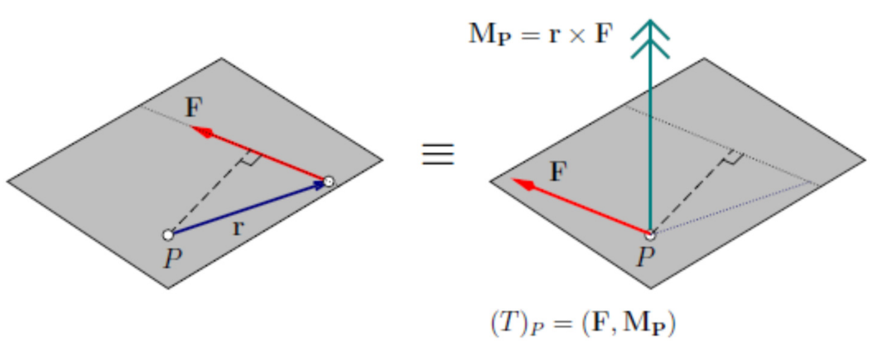


Figure-3 Torsor concept

Torsors concepts can be extended for distributed load $\boldsymbol{p}(z) = p_x\boldsymbol{i} + p_y\boldsymbol{j} + p_z\boldsymbol{k}$ and distributed moment $\boldsymbol{m}(z) = m_x\boldsymbol{i} + m_y\boldsymbol{j} + m_z\boldsymbol{k}$. In this study, torsors with point loads and point moments are only considered.

Straight rod model for CC

Consider straight structural member subjected to torsors, forces and moments. Alternatively, all point loads without any eccentricity can be considered as torsor with zero moment component, Similarly, all moments without force component can be represented as torsor with zero force component. Shortly, all kinds of forces can be represented as torsor pairs. The reaction forces caused by the external forces at the any arbitrary section along z-axis are the traction force $\boldsymbol{R}(z) = T_x\boldsymbol{i} + T_y\boldsymbol{j} + N_z\boldsymbol{k}$ and reaction moment $\boldsymbol{M}(z) = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$. The force and moment equilibrium equations for a straight rod element are obtained from 3-D equilibrium equations of differential element along the rod axis as follows

$$\frac{d\boldsymbol{R}}{dz} + \boldsymbol{p} + \boldsymbol{F}_r\langle z - z_r\rangle^{-1} = \boldsymbol{0} \tag{5}$$

$$\frac{d\boldsymbol{M}}{dz} + \left(-T_y\boldsymbol{i} + T_x\boldsymbol{j}\right) + \boldsymbol{M}_r\langle z - z_r\rangle^{-1} + \boldsymbol{m} = \boldsymbol{0} \tag{6}$$

where $\langle z - z_r\rangle$ is singularity function and z_r is the z-coordinate of the load $\boldsymbol{F}_r$ [Svetlitsky 2000]. Note that the vectors are shown in boldface. The solution of the equations is obtained by direct integration. Firstly, the traction force equations are solved then the moment equations are obtained by substitution of the traction components into the moment equations and by taking the integration [Inan 2015]

Traction forces

Traction forces are obtained by integration of the equilibrium equations

$$\boldsymbol{R} + \int_0^z \boldsymbol{p}\,dz + \sum_{r=1}^{n} \boldsymbol{F}_r\langle z - z_r\rangle^0 = \boldsymbol{0} \tag{7}$$

$$\boldsymbol{R} = \begin{Bmatrix} T_x(z) \\ T_y(z) \\ T_z(z) \end{Bmatrix} = \begin{Bmatrix} T_x(0) - \int_0^z p_x dz - \sum_{r=1}^{n} F_{xr}\langle z - z_r\rangle^0 \\ T_y(0) - \int_0^z p_y dz - \sum_{r=1}^{n} F_{yr}\langle z - z_r\rangle^0 \\ T_z(0) - \int_0^z p_z dz - \sum_{r=1}^{n} F_{zr}\langle z - z_r\rangle^0 \end{Bmatrix} \tag{8}$$

After the substitution of the traction force equation into the moment equation and integration result in

$$\frac{d\boldsymbol{M}}{dz} + \left(-T_y\boldsymbol{i} + T_x\boldsymbol{j}\right) + \boldsymbol{m} + \sum_{s=1}^{n} \boldsymbol{M}_s\langle z - z_s\rangle^{-1} = \boldsymbol{0} \tag{9}$$

These equations can be written in in explicit form as

$$\begin{aligned} \frac{dM_x}{dz} - T_y + \sum_{s=1}^{n} M_{xs}\langle z - z_s\rangle^{-1} + m_x = 0 \\ \frac{dM_y}{dz} + T_x + \sum_{s=1}^{n} M_{ys}\langle z - z_s\rangle^{-1} + m_y = 0 \\ \frac{dM_z}{dz} + \sum_{s=1}^{n} M_{zs}\langle z - z_s\rangle^{-1} + m_z = 0 \end{aligned} \tag{10}$$

The moments after integration become

$$\boldsymbol{M}(z) = \boldsymbol{M}(0) - \left[\int_0^z\left(-T_y\boldsymbol{i} + T_x\boldsymbol{j}\right)dz + \sum_{s=1}^{n} \boldsymbol{M}_s\langle z - z_s\rangle^0 + \int_0^z \boldsymbol{m}\,dz\right] \tag{11}$$

In matrix form as

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} M_x(0) + \int_0^y T_y dz - \sum_{s=1}^{n} M_{xs}\langle z - z_s\rangle^0 - \int_0^z m_x\,dz \\ M_y(0) - \int_0^y T_x dz - \sum_{s=1}^{n} M_{ys}\langle z - z_s\rangle^0 - \int_0^z m_y\,dz \\ M_z(0) - \sum_{s=1}^{n} M_{zs}\langle z - z_s\rangle^0 - \int_0^z m_z\,dz \end{Bmatrix} \tag{12}$$

After substitution of the traction vector $\boldsymbol{R}$ (Eq. (8)) into the moment vector $\boldsymbol{M}$ (Eq. 12), the moment vector can be written in the expanded form as

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} M_x(0) + zT_y(0) + \sum_{r=1}^{n} F_{yr}\langle z - z_r\rangle - \int_0^z\left[\int_0^z p_y\,dz\right]dz - \sum_{s=1}^{n} M_{xs}\langle z - z_s\rangle^0 - \int_0^z m_x\,dz \\ M_y(0) - zT_x(0) - \sum_{r=1}^{n} F_{xr}\langle z - z_r\rangle - \int_0^z\left[\int_0^z p_x\,dz\right]dz - \sum_{s=1}^{n} M_{ys}\langle z - z_s\rangle^0 - \int_0^z m_y\,dz \\ M_z(0) - \sum_{s=1}^{n} M_{zs}\langle z - z_s\rangle^0 - \int_0^z m_z\,dz \end{Bmatrix} \quad (13)$$

Calculation for numerical applications requires numerical math formulas for the singularity functions. Some numerical computer programs have the built-in the singularity or Macaulay functions definitions and some programs requires user defined programed functions to express these functions. For example, definition of the Macaulay functions for MS Excel has to be required [Wilson 2003]. A real CC and its idealized model for the 3-D analysis is shown on the Figure-4.

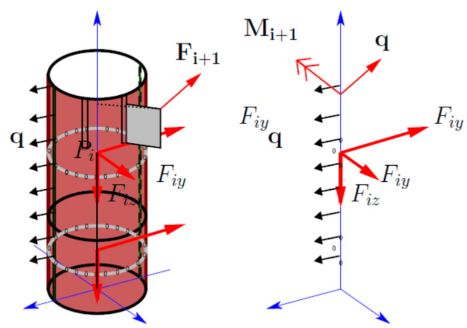


Figure-4 A real CC and its idealized model for the 3-D analysis

An example, shear diagram is shown at Figure-5.

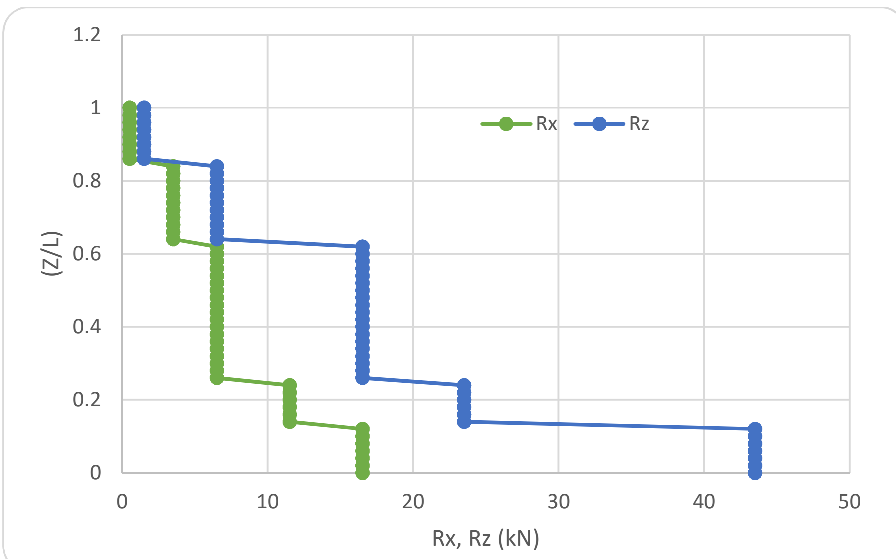
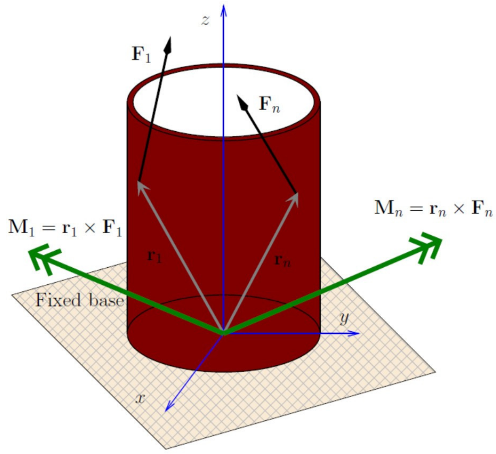


Figure-5 Example shear-diagram

Flux calculations

The force distribution at the base is critical because the spacecraft connection to launcher is provided with clamp band and forces caused by SC must not exceed the capacity of the clamp band. This requirement is given as a line load (force/length), sometimes named as flux, not to be exceeded in launcher manuals (Figure 3). The clamp bands are also used to fix the SC to the carrying dolly during static, dynamic, acoustic and thermal tests. Thus, the load carrying capacity of the adaptor limits the flux loads caused by spacecraft loads during the launching.



Figures-6 A CC with external forces represented by torsors.

Consider a CC with all forces are represented as tursors (Figure-6). The axial stresses at the base simple summation of the stresses caused by direct normal stress and stresses caused by the bending stresses around x- and y axes.

$$\sigma_{zz} = \frac{F_z}{A_z} + \frac{M_{xx}}{I_{xx}} y - \frac{M_{yy}}{I_{yy}} x \tag{14}$$

$$\sigma_z = \frac{F_z}{2\pi Rt} + \frac{M_{xx}}{\pi R^3 t} Rsin\theta - \frac{M_{yy}}{\pi R^3 t} Rcos\theta \tag{15}$$

where $\theta = \arctan(y/x)$. The resultant moment is $M = \sqrt{M_{xx}^2 + M_{yy}^2}$ and angle between the moment components is $\beta = \arctan\left(M_{yy}/M_{xx}\right)$. The flux in polar coordinates are as follows

$$q_z(\theta) = \frac{1}{\pi D}\left(F_z + 4\frac{\sqrt{M_{xx}^2 + M_{yy}^2}}{D} \mathbf{sin}\left(\theta - \mathbf{arctan}\left(\frac{M_{yy}}{M_{xx}}\right)\right)\right) \quad \mathbf{0} \le \theta \le 2\pi \tag{16}$$

The maximum flux occurs at $\theta_{\max} = \beta + \pi/2$ and it is calculated as

$$q \frac{1}{\pi D}\left(F_z + 4\frac{\sqrt{M_{xx}^2 + M_{yy}^2}}{D}\right)_{z,max} \tag{17}$$

A representation of axial flux distribution at the base of a CC is shown at Figure-6. The torsion flux can be ignored since it is generally considered to be not critical because of symmetrical mass distribution around z-axis and no external torsional loads are applied. However, the lateral shear fluxes due to F_x and F_y for thin walled cylinders are calculated as

$$q_{zx} = \tau_{zx} t = \frac{F_x Q}{I_{yy} t} t = \frac{2F_x}{\pi D} = \mathbf{0.634}\frac{F_x}{D} \tag{18}$$

$$q_{zy} = \tau_{zx} t y \frac{F_y Q}{I_{xx} t} t = \frac{2F_y}{\pi D} = \mathbf{0.634}\frac{F_y}{D} \tag{19}$$

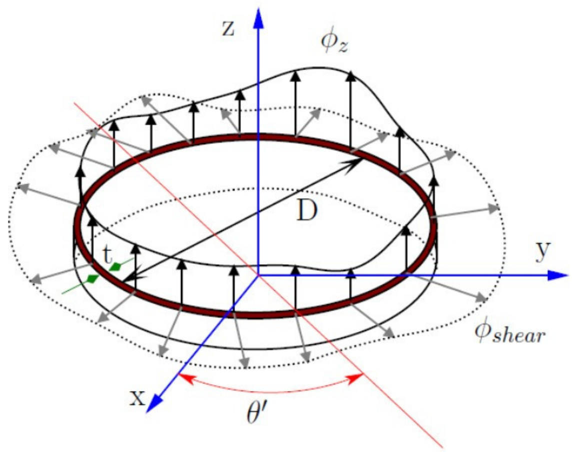


Figure-6 Representation of axial flux distribution at the base of a CC

Conclusion

This study is an attempt to formulate the preliminary design of central cylinder and develop theoretical analysis tool for any type of long structures such as satellite CC, long stacks, pipes, etc. 3-D analysis method for static test configurations, design and analysis of the CC is presented. The formulations for flux calculations are also given. The study is further going to be implemented for numerical calculations as an engineering tool using spreadsheets for preliminary design and design of static test loading configurations as well.

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