

ORBIT DETERMINATION, PROPAGATION AND DEFLECTION OF ASTEROID (99942) APOPHIS

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ABSTRACT

In this study, the initial orbit of asteroid (99942) Apophis is determined from both Gauss's and Laplace's methods. The differential correction method is applied with use of additional observations to increase accuracy. The improved orbit is propagated by numerically integrating the equation of motion with the solar radiation pressure and the third-bodies perturbation. To obtain the post-deflected orbit, kinetic impactor technique is used as orbit deflection method, and the Apophis-Earth distance at the time of closest approach (TCA) is calculated with different deflection conditions in order to find the most effective way.

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INTRODUCTION

The asteroid (99942) Apophis is a highly interested Near Earth Object (NEO) due to its close approach with Earth in 2029 at a distance near the altitude of geostationary satellites. There have been studies on Apophis related to orbit deflection strategies [Izzo et al., 2009; Yeomans et al., 2009].

The orbit deflection is, to change orbit of a potentially hazardous asteroid such a way that to increase asteroid-Earth distance at the time of closest approach (TCA). Many researches have been done on the asteroid deflection with several aspects such as mission design [Hernandez and Barbee, 2012; Sarli et al., 2017] and optimum deflection strategies [Vasile and Colombo, 2008]. In addition to the theoretical studies, ESA (European Space Agency) and NASA (National Aeronautics and Space Administration) have been working on a collaborative mission called as AIDA (Asteroid Impact and Deflection Assessment) which would be the first asteroid deflection mission.

There are several deflection methods such as kinetic impactor, gravity tractor, laser ablation and focused solar radiation. According to the current space technologies, most feasible one is the kinetic impactor technique. This method requires impact of a high velocity spacecraft to an asteroid and deflects its orbit based on the principle of momentum transfer.

For the asteroid deflection missions, orbit determination and propagation are important in terms of obtaining the post-deflected orbit and generating the ephemeris. The post-deflected orbit can be obtained by propagating the deflected state of the asteroid, or applying the orbit determination methods to post-deflected orbit with use of Earth-based observations. Besides, in order to design mission trajectory of the spacecraft, it is necessary to know position of the asteroid precisely at a given time. This can be acquired from the ephemeris data. An ephemeris is generated with the three main steps which are initial orbit determination, orbit estimation and orbit propagation, respectively. Initial orbit is determined from the optical observations with use of angles-only orbit determination methods. However, these methods provide coarse information about the orbit. In order to increase accuracy, estimation methods are applied by taking into account additional observations. Finally, the improved orbit is propagated until the requested time.

In this study, the orbit of Apophis is determined and its post-deflected orbit is investigated under the kinetic impactor technique with different impact directions and energies at different impact epochs. In this context, firstly the initial orbit is determined from both Gauss's and Laplace's methods. The differential correction method is applied with use of additional observations to increase accuracy. Then, the improved orbit is propagated by numerically integrating the equation of motion with the solar radiation pressure and the third-bodies perturbation. Finally to obtain the post-deflected orbit, kinetic impactor technique is used as orbit deflection method, and the Apophis-Earth distance at the TCA is calculated with different deflection conditions in order to find the most effective way.

METHODS

Methods used in this work are examined in the four title which are initial orbit determination, orbit estimation, orbit propagation and orbit deflection, respectively.

Initial Orbit Determination

In this study, Gauss's and Laplace's angles-only methods are used for the initial orbit determination. These methods require observation of two angles at three different epochs. Observed angles are right ascension α and declination δ . Aim of the both methods is to obtain orbital elements at one of the observation epochs from the angle and time data.

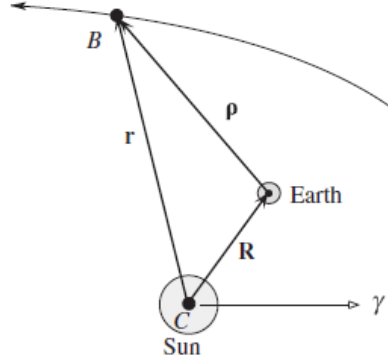


Figure 1: Geometry of the problem, where B symbolizes the asteroid. (Figure 5.13 in [Curtis, 2013])

Line of sight unit vector of the asteroid is written as

$$\hat{\rho}_i = \begin{bmatrix} \cos \delta_i \cos \alpha_i \\ \cos \delta_i \sin \alpha_i \\ \sin \delta_i \end{bmatrix}, \quad (1)$$

at three observation epochs by using the observed angles, where i represents the epoch number.

In Figure 1, $\vec{R}$ is the position vector of the Earth with respect to Sun and it is calculated from the solar ephemeris algorithm 12.2 in [Curtis, 2013]. Then, the position vector of the asteroid with respect to Sun is written as

$$\vec{r}_1 = \vec{R}_1 + \rho_1 \hat{\rho}_1, \quad (2a)$$

$$\vec{r}_2 = \vec{R}_2 + \rho_2 \hat{\rho}_2, \quad (2b)$$

$$\vec{r}_3 = \vec{R}_3 + \rho_3 \hat{\rho}_3, \quad (2c)$$

at three observation epochs. Here ρ is the range, the distance between Earth and asteroid. Vectors written in (2) are defined in the geocentric equatorial frame. Since the asteroid is orbiting around the Sun, orbital elements have to be calculated with respect to the heliocentric ecliptic frame. Ecliptic plane makes an angle with equatorial plane, close to 23° . It is called as obliquity and depends on time. Transformation between those frames is done with use of this angle. Section 2.6 in [Boulet, 1991] is followed for the implementation of the transformation. $\vec{R}$ and $\hat{\rho}$ are the known quantities in (2) and in order to find $\vec{r}$, therefore ρ , Gauss's and Laplace's methods are used.

Gauss's Method: Gauss's method assumes that orbit of the asteroid is defined under the two-body assumption, in other words all the three vectors in (2) are in the same plane. Based on that, one of the three vectors in (2) can be written as a linear combination of other two:

$$\vec{r}_2 = c_1 \vec{r}_1 + c_3 \vec{r}_3. \quad (3)$$

Then, the lagrange coefficients are used in order to write $\vec{r}_1$ and $\vec{r}_3$ in terms of $\vec{r}_2$ and $\vec{v}_2$

$$\vec{r}_1 = f_1 \vec{r}_2 + g_1 \vec{v}_2, \quad (4a)$$

$$\vec{r}_3 = f_3 \vec{r}_2 + g_3 \vec{v}_2, \quad (4b)$$

where f_1 , f_3 , g_1 and g_3

$$f_1 \approx 1 - \frac{1}{2} \frac{\mu_{sun}}{r_2^3} \tau_1^2, \quad (5a)$$

$$f_3 \approx 1 - \frac{1}{2} \frac{\mu_{sun}}{r_2^3} \tau_3^2, \quad (5b)$$

$$g_1 \approx \tau_1 - \frac{1}{6} \frac{\mu_{sun}}{r_2^3} \tau_1^3, \quad (5c)$$

$$g_3 \approx \tau_3 - \frac{1}{6} \frac{\mu_{sun}}{r_2^3} \tau_3^3, \quad (5d)$$

are the lagrange coefficients. Here, $\tau_1 = t_1 - t_2$ and $\tau_3 = t_3 - t_2$ where t_1 , t_2 and t_3 are the observation times. Also μ_{sun} is the gravitational parameter for the Sun.

So far, the total number of scalar equation is 18. These are nine from (2), three from (3) and six from (4). On the other hand, the total number of scalar unknown is also 18. These are $\vec{r}$ and ρ in (2), each $\vec{r}$ corresponds to three scalar unknown and each ρ corresponds to one scalar unknown, totally 12, also c_1 and c_3 in (3), lastly $\vec{v}_2$ and the lagrange coefficients in (4), here $\vec{v}_2$ corresponds to three scalar unknown and the lagrange coefficients correspond to one scalar unknown which is r_2 . As a result, number of unknowns are equal to number of equations. This means that Gauss's method provides a solvable set.

Section 5.10 in [Curtis, 2013] is followed for the discussion of the Gauss's method.

Laplace's Method: Laplace's method uses a different approach to solve the problem. The method starts by taking the first and second time derivatives of one of the vectors in (2):

$$\dot{\vec{r}} = \dot{\rho}\hat{\rho} + \rho\dot{\hat{\rho}} + \dot{\vec{R}}, \quad (6a)$$

$$\ddot{\vec{r}} = \ddot{\rho}\hat{\rho} + 2\dot{\rho}\dot{\hat{\rho}} + \rho\ddot{\hat{\rho}} + \ddot{\vec{R}}. \quad (6b)$$

From the two-body equation of motion, $\ddot{\vec{r}} = -\frac{\mu_{sun}}{r^3}\vec{r}$ where $\vec{r} = \vec{R} + \rho\hat{\rho}$. Then, (6b) turns into form of

$$-\frac{\mu_{sun}}{r^3}(\vec{R} + \rho\hat{\rho}) = \ddot{\rho}\hat{\rho} + 2\dot{\rho}\dot{\hat{\rho}} + \rho\ddot{\hat{\rho}} + \ddot{\vec{R}}, \quad (7)$$

by rearranging (7), one can reach:

$$\ddot{\rho}\hat{\rho} + 2\dot{\rho}\dot{\hat{\rho}} + \rho(\ddot{\hat{\rho}} + \frac{\mu_{sun}}{r^3}\hat{\rho}) = -\ddot{\vec{R}} - \frac{\mu_{sun}}{r^3}\vec{R}. \quad (8)$$

In order to calculate $\dot{\hat{\rho}}$ and $\ddot{\hat{\rho}}$, the lagrange interpolating polynomial is used. Firstly, the function of $\hat{\rho}$

$$\hat{\rho}(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)}\hat{\rho}_1 + \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)}\hat{\rho}_2 + \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)}\hat{\rho}_3, \quad (9)$$

is constructed with time dependently by using the line of sight unit vectors $\hat{\rho}_1$, $\hat{\rho}_2$, $\hat{\rho}_3$. Then, first and second time derivatives of (9) are taken as

$$\dot{\hat{\rho}}(t) = \frac{2t-t_2-t_3}{(t_1-t_2)(t_1-t_3)}\hat{\rho}_1 + \frac{2t-t_1-t_3}{(t_2-t_1)(t_2-t_3)}\hat{\rho}_2 + \frac{2t-t_1-t_2}{(t_3-t_1)(t_3-t_2)}\hat{\rho}_3, \quad (10a)$$

$$\ddot{\hat{\rho}}(t) = \frac{2}{(t_1-t_2)(t_1-t_3)}\hat{\rho}_1 + \frac{2}{(t_2-t_1)(t_2-t_3)}\hat{\rho}_2 + \frac{2}{(t_3-t_1)(t_3-t_2)}\hat{\rho}_3, \quad (10b)$$

where t_1 , t_2 and t_3 are the observation times. In addition, $\ddot{\vec{R}}$ in (8) can also be calculated with the same manner, or directly from $\ddot{\vec{R}} = -\frac{\mu_{sun}}{R^3}\vec{R}$.

Finally, (8) is written in the following matrix form:

$$\begin{bmatrix} \hat{\rho} & 2\dot{\hat{\rho}} & \ddot{\hat{\rho}} + \frac{\mu_{sun}}{r^3}\hat{\rho} \end{bmatrix} \begin{bmatrix} \ddot{\rho} \\ \dot{\rho} \\ \rho \end{bmatrix} = - \left[\ddot{\vec{R}} + \frac{\mu_{sun}}{r^3}\vec{R} \right]. \quad (11)$$

Here, the unknowns are $\ddot{\rho}$, $\dot{\rho}$, ρ and r . But also, $r = \sqrt{\rho^2 + 2\rho\hat{\rho} \cdot \vec{R} + R^2}$ from (2). Then, the system of equation is reduced to 3 unknowns with 3 equations. In order to find unknowns, matrix equation (11) can be solved with use of Cramer's rule.

Section 7.3.1 in [Vallado, 2013] is followed for the discussion of the Laplace's method.

Orbit Estimation

In this study, an iterative method called as differential correction is used for the orbit estimation. Aim is to increase accuracy of the initial orbital elements found from Gauss's and Laplace's methods with use of additional observations. The idea of the correction is based on to minimize square of the residuals. The residual means the difference in computed and observed data. This difference occurs due to observational errors and mathematical model.

In order to calculate residuals, initial state is propagated to the each additional observation epoch. Propagation is done by solving the two-body equation of motion analytically and it is actually the mathematical model of the system. Orbital perturbations are not considered in the estimation part because duration of the propagation, in other words time intervals between the observations are not long enough to consider effect of perturbations, also another reason is due to the time efficiency of the computation. Those propagated states are called as computed data. On the other hand, observed data are known from the actual observations. Then, the residual matrix b can be written as

$$b = \begin{bmatrix} \alpha_{obs} \\ \delta_{obs} \end{bmatrix} - \begin{bmatrix} \alpha_{com} \\ \delta_{com} \end{bmatrix}, \quad (12)$$

where α_{obs} and δ_{obs} are the observed, α_{com} and δ_{com} are the computed right ascension and declination angles. Then, the computed angles can be related with the initial state X_0 by using the partial derivative matrix A :

$$A = \frac{\partial observations}{\partial X_0} = \begin{bmatrix} \frac{\partial \alpha_{com}}{\partial r_{I_0}} & \frac{\partial \alpha_{com}}{\partial r_{J_0}} & \frac{\partial \alpha_{com}}{\partial r_{K_0}} & \frac{\partial \alpha_{com}}{\partial v_{I_0}} & \frac{\partial \alpha_{com}}{\partial v_{J_0}} & \frac{\partial \alpha_{com}}{\partial v_{K_0}} \\ \frac{\partial \delta_{com}}{\partial r_{I_0}} & \frac{\partial \delta_{com}}{\partial r_{J_0}} & \frac{\partial \delta_{com}}{\partial r_{K_0}} & \frac{\partial \delta_{com}}{\partial v_{I_0}} & \frac{\partial \delta_{com}}{\partial v_{J_0}} & \frac{\partial \delta_{com}}{\partial v_{K_0}} \end{bmatrix}. \quad (13)$$

In order to find those partial derivatives, the finite differencing method is used. Initial state is modified slightly to make a small difference with nominal initial state. The mathematical expression of the finite differencing is given as

$$\frac{\partial observations}{\partial X_0} \cong \frac{f(X_0 + \Delta x) - f(X_0)}{\Delta x}, \quad (14)$$

where Δx is the difference between modified and nominal initial state.

All the observations include noises due to instruments used in the measurement. These noises are taken into account by constructing the weighting matrix W as

$$W = \begin{bmatrix} \frac{1}{\sigma_\alpha^2} & 0 \\ 0 & \frac{1}{\sigma_\delta^2} \end{bmatrix}, \quad (15)$$

where σ_α and σ_δ are the noises in the measurement of right ascension and declination. According to (15), less noise generates large weighting and it would be understood better by introducing the

covariance matrix.

The covariance matrix gives information about the uncertainty of the result. It is expressed as:

$$P = (A^T W A)^{-1}. \quad (16)$$

It is found small for the large weighting and small covariance matrix means that results are reliable. Actually, this is a consequence of (15) and (16). In order to have a large weighting, noises of the instrument should be small according to (15) and it is obvious that instrument with small noise would be more reliable.

Finally, the differential correction is calculated from the partial derivative matrix A , weighting matrix W and residual matrix b :

$$\Delta X = (A^T W A)^{-1} (A^T W b). \quad (17)$$

The initial state found from Gauss's and Laplace's methods is improved based on (17) at each iteration.

In this study, weighting matrix could not be formed due to lack of noise information belonging to observations, therefore (16) and (17) turn into form of $P = (A^T A)^{-1}$ and $\Delta X = (A^T A)^{-1} (A^T b)$, respectively. The unweighted covariance matrix would not provide meaningful information about the actual uncertainty of the result because it is not scaled according to noises of the observations. Therefore, covariance matrix is not presented in this work.

Section 10 in [Vallado, 2013] is followed for the discussion of the differential correction method.

Orbit Propagation

The initial state obtained from the orbit determination should be propagated for the calculation of future position of the asteroid. In the orbit estimation part, two-body analytical orbit propagator is used due to small propagation durations and time efficiency of the computation. But for the longer propagation durations, like order of years, orbital perturbations have to be considered. The essential perturbations that would effect motion of the asteroid are the solar radiation pressure and the third-bodies perturbation. The trajectory of the asteroid is calculated by numerically integrating the equation of motion

$$\ddot{\vec{r}} = -\frac{\mu_{sun}}{r^3} \vec{r} + \vec{p}_{srp} + \vec{p}_{third-bodies}, \quad (18)$$

with the solar radiation pressure $\vec{p}_{srp}$ and the third-bodies perturbation $\vec{p}_{third-bodies}$. (18) is a second order ordinary differential equation. In order to solve this equation, eighth order Runge-Kutta method [Cappelari et al., 1976] is used with fixed step size.

Third-Bodies Perturbation: In the keplerian orbit, only Sun-asteroid system is examined. However, the gravitational field of the planets causes deviation from the keplerian orbit. The acceleration due to those third-bodies is given as

$$\vec{p}_{third-bodies} = \sum_{i=1}^n \mu_i \left(\frac{(\vec{R}_i - \vec{R}_{ast})}{\|\vec{R}_i - \vec{R}_{ast}\|^3} - \frac{(\vec{R}_i)}{\|\vec{R}_i\|^3} \right), \quad (19)$$

where n is the number of third bodies, μ_i is the gravitational parameter for the corresponding planet, also $\vec{R}_i$ and $\vec{R}_{ast}$ are the position vectors of the planet and asteroid relative to Sun, respectively. The positions of the planets are calculated from the planetary ephemeris algorithm 8.1 in [Curtis, 2013]. Since Apophis has an orbit in the inner belt of the solar system, Mercury, Venus, Earth, Mars, Jupiter and Moon are considered as third-bodies.

Solar Radiation Pressure: The solar radiation applies a pressure to the surface of the asteroid due to momentum of the photons. The acceleration caused by this pressure is given as

$$\vec{p}_{srp} = \frac{S}{c} \frac{C_R A_s}{m} \hat{u}, \quad (20)$$

Here, S is the solar flux which is a function of distance

$$S = S_0 \left(\frac{R_0}{R} \right)^2, \quad (21)$$

where S_0 is the radiated power intensity of the Sun at a distance of R_0 , namely radius of the Sun, and also R is the distance between Sun and asteroid. In (20), $\hat{u}$ is the unit vector pointing from the Sun toward the asteroid. Actually, it is in the direction of $\vec{r}$ in Figure 1. Also in (20), A_s is the absorbing area and it is calculated based on the cannonball model which assumes that the asteroid has a spherical shape with radius r_{ast} , therefore the absorbing area is πr_{ast}^2 . Other parameter in (20) is the reflectivity C_R which ranges between 1 to 2. It is reasonable to choose an intermediate value because there is not any surface that absorbs ($C_R = 1$) or reflects ($C_R = 2$) all the incoming radiation. Therefore, $C_R = 1.5$ is chosen. The last two parameters are the speed of light c and mass of the asteroid m . It is assumed that asteroid always sees the Sun, so the shadow conditions due to planets are not considered.

Section 12 in [Curtis, 2013] is followed for the discussion of the orbital perturbations.

Orbit Deflection

In this work, kinetic impactor technique is used as orbit deflection method. It requires impact of a high velocity spacecraft to an asteroid and deflects its orbit based on the principle of momentum transfer. According to that, the instantaneous velocity change of the asteroid due to impact is given as [Sarli et al., 2017]

$$\Delta \vec{v} = \vec{v}_\infty \frac{m_{S/C}}{m_{S/C} + m_{asteroid}}, \quad (22)$$

where $\vec{v}_\infty$ is the arrival velocity of the spacecraft relative to asteroid which is $\vec{v}_{s/c} - \vec{v}_{ast}$, also $m_{S/C}$ and $m_{asteroid}$ are the mass of the spacecraft and asteroid, respectively.

In the calculation of the post-deflected orbit, firstly the initial state vector obtained from the orbit determination is propagated to time that impact occurs. This propagated state vector could be shown as:

$$\vec{r}_{impact} = [r_{x,impact} \quad r_{y,impact} \quad r_{z,impact}]^T, \quad (23a)$$

$$\vec{v}_{impact} = [v_{x,impact} \quad v_{y,impact} \quad v_{z,impact}]^T. \quad (23b)$$

In order to define direction of the deflection, NTW frame is used. The unit vectors of the three axes $\hat{N}$, $\hat{T}$, $\hat{W}$ are written as [Vallado, 2013]

$$\hat{T} = \frac{\vec{v}}{\|\vec{v}\|}, \quad \hat{W} = \frac{\vec{r} \times \vec{v}}{\|\vec{r} \times \vec{v}\|}, \quad \hat{N} = \hat{T} \times \hat{W}, \quad (24)$$

where $\vec{r}$ and $\vec{v}$ are the position and velocity vector of the asteroid in ecliptic frame. It is seen that $\hat{T}$ is in the velocity direction and $\hat{W}$ is in the direction perpendicular to the orbital plane. The $\hat{N}$ is not exactly align with the radial direction except for the circular orbits. Since the orbit of Apophis is not highly elliptical ($e = 0.19$), $\hat{N}$ can be considered as radial direction. Then, the directions of the deflection are written in NTW frame as

$$\Delta \vec{v}_{NTW,N} = [\Delta v \quad 0 \quad 0]^T, \quad (25a)$$

$$\Delta \vec{v}_{NTW,T} = [0 \quad \Delta v \quad 0]^T, \quad (25b)$$

$$\Delta \vec{v}_{NTW,W} = [0 \quad 0 \quad \Delta v]^T, \quad (25c)$$

where Δv is the instantaneous velocity change of the asteroid. In this work, Δv is not calculated with details from (22). It is assumed a value on the order of m/s to observe trend of the post-deflected orbit clearly. The state vector in (23) is defined in the ecliptic frame but the directions of the deflection in (25) are defined in the NTW frame. Therefore, the transformation is required from NTW frame to ecliptic frame:

$$M_{NTW \rightarrow Ecliptic} = \begin{bmatrix} N_x & N_y & N_z \\ T_x & T_y & T_z \\ W_x & W_y & W_z \end{bmatrix}^T. \quad (26)$$

Then, the directions of the deflection are written in ecliptic frame as:

$$\Delta \vec{v}_{Ecliptic} = M_{NTW \rightarrow Ecliptic} \Delta \vec{v}_{NTW}. \quad (27)$$

Since it is assumed that impact occurs instantaneously, the state vector of the asteroid just after the deflection becomes like the following:

$$\vec{r}_{impact,new} = [r_{x,impact} \quad r_{y,impact} \quad r_{z,impact}]^T, \quad (28a)$$

$$\vec{v}_{impact,new} = [v_{x,impact} \quad v_{y,impact} \quad v_{z,impact}]^T + \Delta \vec{v}_{Ecliptic}. \quad (28b)$$

Finally, the new state vector in (28) is propagated to TCA and asteroid-Earth distance is calculated. This process is repeated for the different deflection times in order to find the optimum deflection.

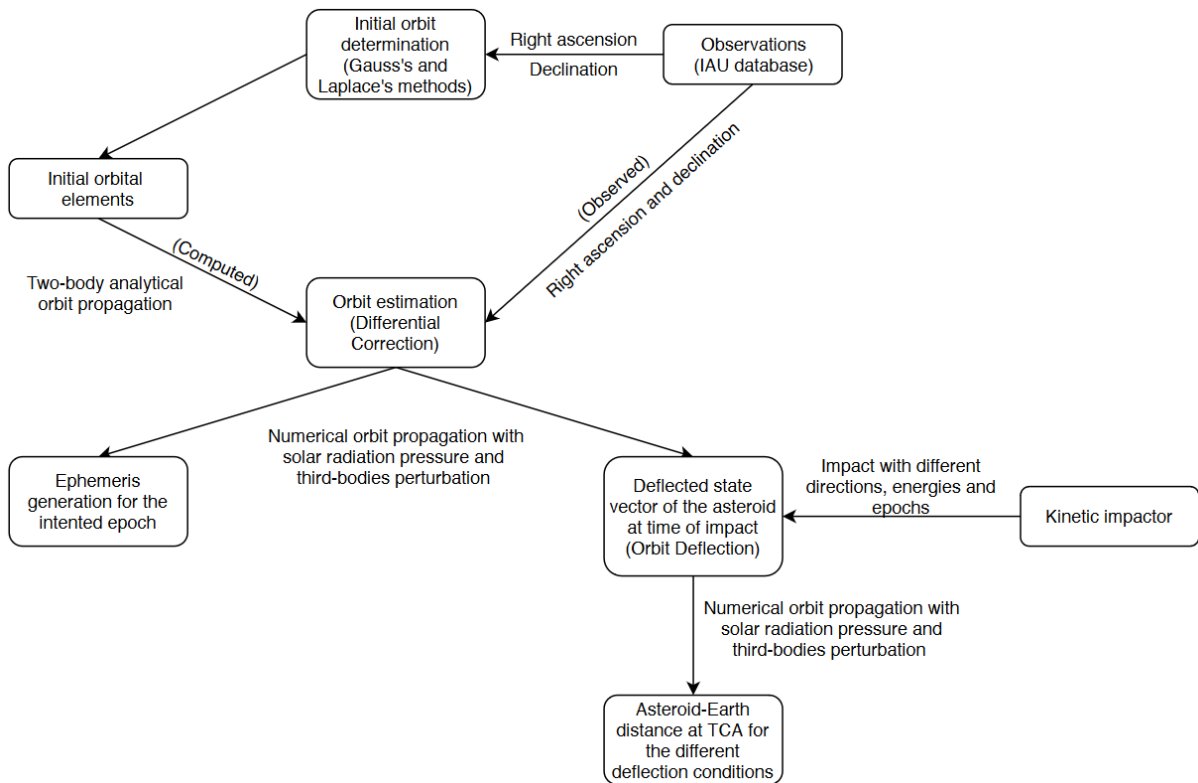


Figure 2: Overview of the methods used in this work.

RESULTS AND DISCUSSION

In this section, firstly the initial orbital elements of Apophis calculated from the Gauss's and Laplace's methods are presented. Afterwards, the improved orbital elements obtained from the differential correction method are shown. Then, the graph of the Apophis-Earth distance during the orbit propagation

is provided. Finally, the results of the orbit deflection are given in the plots with different deflection conditions. In this work, all the results are obtained in the MATLAB environment.

In the calculation of the initial orbital elements, section 5.10 in [Curtis, 2013] and section 7.3.1 in [Vallado, 2013] are followed for the Gauss's and Laplace's methods, respectively.

	Epoch (year month day.dayfraction)	Right Ascension (hour min sec.secfrac)	Declination (degree arcmin arcsec.arcsecfrac)
1. Observation	2012 02 16.86087	02 42 24.70	+10 27 34.1
2. Observation	2012 02 26.063996	03 20 23.25	+13 17 22.2
3. Observation	2012 03 14.81842	04 29 52.80	+17 19 52.9

Table 1: Observations used in the initial orbit determination. They are taken from the International Astronomical Union (IAU) Minor Planet Center [Holman et al., 2019].

Initial Orbital Elements at Second Observation Epoch (2012 02 26.063996)			
	Gauss's Method (MATLAB code)	Laplace's Method (MATLAB code)	NASA JPL HORIZONS System
Semimajor axis	0.9286 <i>AU</i>	0.8645 <i>AU</i>	0.9223 <i>AU</i>
Eccentricity	0.1956	0.2015	0.1911
Inclination	3.3636°	3.0203°	3.3319°
Argument of perigee	128.66°	114.39°	126.43°
Right ascension of ascending node	204.03°	208.16°	204.43°
True anomaly	143.04°	158.11°	145.35°

Table 2: Orbital elements obtained from the initial orbit determination methods.

Results of the initial orbit determination are given in Table 2 together with the results of NASA JPL HORIZONS system [Chamberlin and Park, 2019a]. It is clearly seen that Gauss's method gives better result compared to Laplace's method if the NASA JPL Horizons system is taken as reference. The most likely reason of it is the lagrange interpolation formula that is used in the Laplace's method. This is a coarse approximation to find time derivatives of the line of sight unit vectors. The Gauss's method also uses approximation which is series expansion for the lagrange coefficients and it is valid under the small time interval. The time intervals between the observations given in Table 1 are relatively small compared to the orbital period ($T_{Apophis} \approx 325$ day). As consequence, it is observed that the lagrange coefficients give more accurate results compared to the interpolation formula. But still, the

orbital elements obtained from the Gauss's and Laplace's methods are not as accurate as reference values, so the differential correction method is applied.

In the implementation of the differential correction method, section 10.4 in [Vallado, 2013] is followed.

	Epoch (year month day.dayfrac)	Right Ascension (hour min sec.secfrac)	Declination (degree arcmin arcsec.arcsecfrac)
1. Observation	2012 02 16.86087	02 42 24.70	+10 27 34.1
2. Observation	2012 02 26.063996	03 20 23.25	+13 17 22.2
3. Observation	2012 03 14.81842	04 29 52.80	+17 19 52.9
4. Observation	2012 03 14.82797	04 29 55.00	+17 19 58.5
5. Observation	2012 03 14.83771	04 29 57.20	+17 20 05.0
6. Observation	2012 03 23.77567	05 03 25.35	+18 41 11.9
7. Observation	2012 03 23.78294	05 03 26.91	+18 41 14.9
8. Observation	2012 03 23.79015	05 03 28.50	+18 41 18.2
9. Observation	2012 03 24.78155	05 07 08.23	+18 48 40.5
10. Observation	2012 03 24.79028	05 07 10.16	+18 48 44.7
11. Observation	2012 03 24.79829	05 07 11.89	+18 48 47.7
12. Observation	2012 04 20.14870	06 41 12.81	+20 14 34.7
13. Observation	2012 04 20.14999	06 41 13.09	+20 14 34.4
14. Observation	2012 04 20.15125	06 41 13.34	+20 14 34.2
15. Observation	2012 05 08.25800	07 42 43.055	+19 21 57.56
16. Observation	2012 05 08.26263	07 42 43.972	+19 21 55.84
17. Observation	2012 05 08.26730	07 42 44.881	+19 21 54.37
18. Observation	2012 05 08.27193	07 42 45.811	+19 21 52.86

Table 3: Observations used in the differential correction method. They are taken from the International Astronomical Union (IAU) Minor Planet Center [Holman et al., 2019].

In total, 18 observations are used in the orbit determination with separated to 3 months. The first three are used in the initial orbit determination and the rests are used in the differential correction.

Improved Orbital Elements at Second Observation Epoch (2012 02 26.063996)			
	Gauss's Method (MATLAB code)	Laplace's Method (MATLAB code)	NASA JPL HORIZONS System
Semimajor axis	0.9223 AU	0.9223 AU	0.9223 AU
Eccentricity	0.1916	0.1916	0.1911
Inclination	3.3541°	3.3541°	3.3319°
Argument of perigee	126.54°	126.54°	126.43°
Right ascension of ascending node	204.22°	204.22°	204.43°
True anomaly	145.32°	145.32°	145.35°

Table 4: Orbital elements obtained after the differential correction method.

Results of the differential correction are given in Table 4. From the comparison of Table 2 and Table 4, it is observed that orbital elements are improved and almost four significant figure is matched with the NASA JPL HORIZONS system after the differential correction. In addition, Laplace's method gave the coarse initial orbital elements compared to Gauss's method but it also converged to the reference values as much as Gauss after the differential correction. It is concluded that the differential correction method is a powerful tool for the orbit improvement.

In the orbit propagation, epoch and orbital elements given in Table 4 are used as initial state. The orbit is propagated from this initial state until the closest approach at 2029. The duration of the propagation, in other words time since epoch, is approximately 17 years. This time interval is large enough to consider effect of orbital perturbations. In the numerical integration, different time steps are considered as shown in Table 5.

Time step of the numerical integration (dt)	Apophis-Earth distance (d) at TCA
$dt = 0.3 \text{ day}$	$d = 0.00403 \text{ AU}$
$dt = 0.6 \text{ day}$	$d = 0.00027 \text{ AU}$
$dt = 1.2 \text{ day}$	$d = 0.00869 \text{ AU}$

Table 5: Apophis-Earth distance at TCA for the different time steps.

References	Apophis-Earth distance (d) at TCA
NASA	$d = 0.00025 \text{ AU}$
ESA	$d = 0.00026 \text{ AU}$
IAU	$d = 0.00016 \text{ AU}$

Table 6: Apophis-Earth distance at TCA according to NASA [Chamberlin and Park, 2019b], ESA [ESA, 2019] and IAU [Holman et al., 2019].

It is observed that the Apophis-Earth distance at TCA for the $dt = 0.6$ day is the minimum and it is verified with the reference values of NASA and ESA given in Table 6. However, results are diverged from the reference values for the time steps smaller than $dt = 0.3$ day and larger than $dt = 1.2$ day. The most likely reason is the instability of the numerical integration for the small time steps and the decrease in accuracy of the numerical integration for the large time steps. In addition to those, the Apophis-Earth distance during the orbit propagation is plotted in Figure 3. It can be seen that there are two close approaches before the 2029.

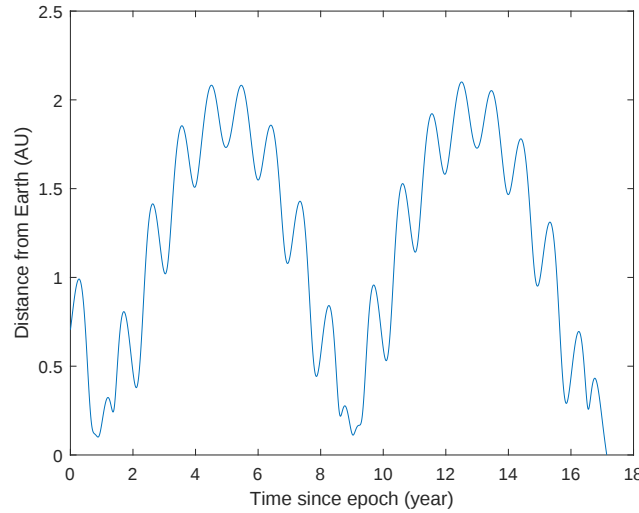


Figure 3: Graph of the Apophis-Earth distance during the orbit propagation.

The post-deflected orbit is investigated by calculating the Apophis-Earth distance at TCA for the different deflection conditions. The results are given in Figures (4a) and (4b) in terms of radius of Earth, for $\Delta v = 0.5$ m/s and for $\Delta v = 1$ m/s, with the directions of the deflection defined in NTW frame. In addition, the Apophis-Sun distance is plotted in Figure (4c) at different deflection times. The maximum and minimum points correspond to apogee and perigee, respectively. In all three figures, horizontal axis represents the deflection times, in other words times before the TCA. In Figures (4a) and (4b), it is seen that each direction has optimum deflection times which change periodically. If it is looked at those optimum deflection times for each direction, the deflection in the velocity direction (T) is the most effective way except for the case when there is not so much time before the TCA compared to orbital period ($T_{Apophis} \approx 325$ day). However, in that case the deflection in the direction perpendicular to the orbital plane (W) is reasonable. The deflection in the radial direction (N) has not any significant effect on the distance compared to other two direction. It is observed that the optimum deflection times in the velocity direction happen near the perigee points. This is because the deflection in the velocity direction is resulted with the change in semimajor axis and the maximum change in semimajor axis occurs at perigee where the orbital velocity is highest. For the deflection in the velocity direction, it is also observed that the Apophis-Earth distance is increase when the deflection times are getting far from the TCA. The reason is that the change in semimajor axis effects the mean motion and therefore mean anomaly. When the time interval is increase, the Apophis-Earth distance is also increase due to the growing change in mean anomaly. The deflections in the other two directions (N and W) are not resulted with the change in semimajor axis, so they have not such kind of trend. The detailed discussion about the effect of impact directions on the orbital elements can be found in [Vasile and Colombo, 2008]. Lastly, it is obtained that when the velocity change of the asteroid is doubled, results of the deflection in the velocity direction scale accordingly.

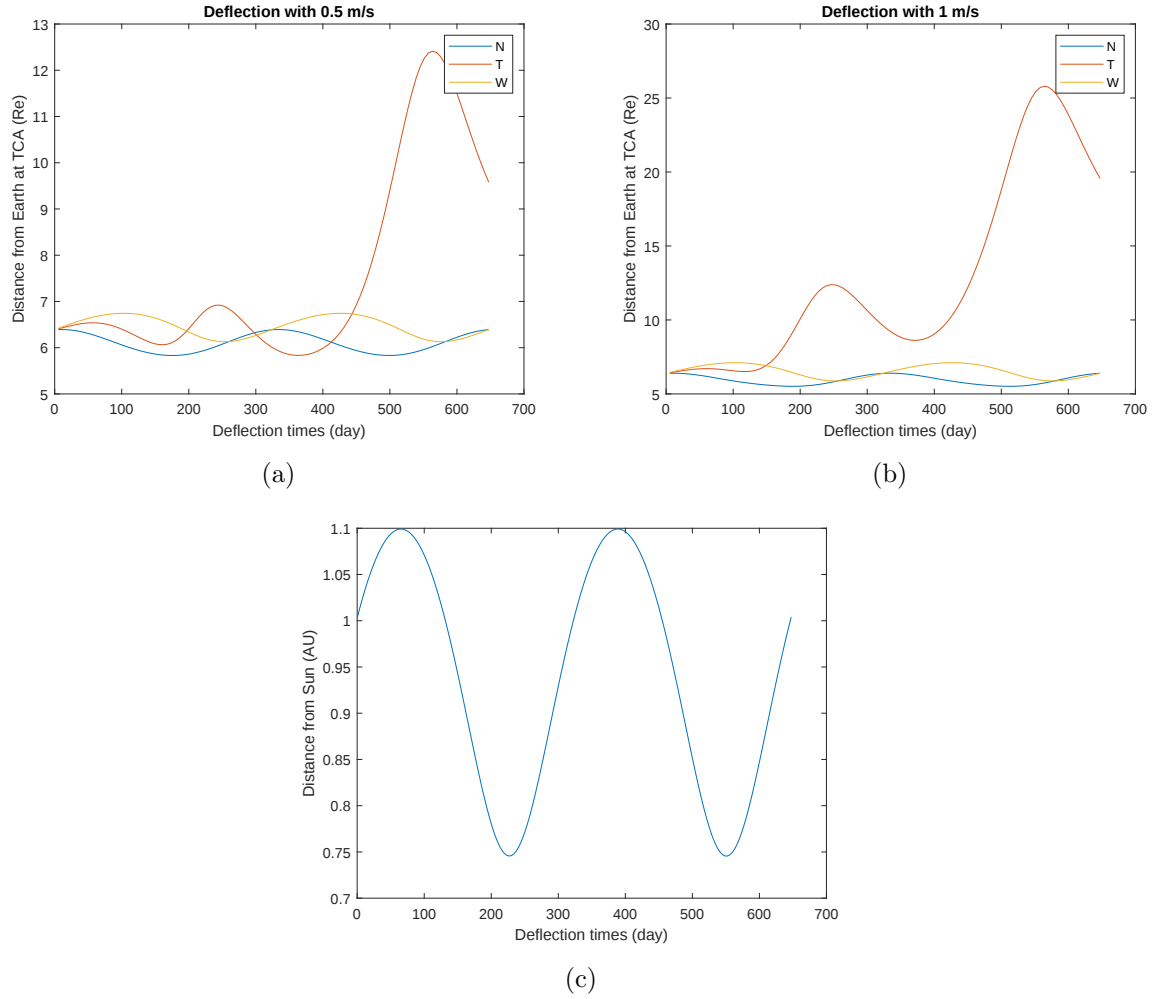


Figure 4: The graphes (4a) and (4b) are the results of the orbit deflection for $\Delta v = 0.5 \text{ m/s}$ and $\Delta v = 1 \text{ m/s}$, respectively. The graph (4c) is the radius of the orbit at different deflection times.

CONCLUSIONS

In this study, firstly the initial orbital elements of Apophis is calculated with use of Gauss's and Laplace's methods. Then, the differential correction method is applied in order to increase accuracy. Results are compared with NASA JPL HORIZONS system. It is observed that Gauss's method gives better results compared to Laplace's method in the initial orbit determination, but after the differential correction, both methods converged to the same values and almost four significant figure accuracy is obtained. Then, the improved orbit is propagated to TCA by considering the solar radiation pressure and the third-bodies perturbation. The Apophis-Earth distance at TCA is calculated, and verified with the reference values of NASA and ESA. Finally, the post-deflected orbit is investigated by calculating the Apophis-Earth distance at TCA with different deflection conditions. It is observed that each impact direction has optimum deflection times which change periodically. Also, it is concluded that if there is not so much time before the TCA compared to orbital period, deflection in the direction perpendicular to the orbital plane is reasonable, otherwise deflection in the velocity direction near the perigee points is the most effective way.

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