VIBRATION ANALYSIS OF PASSIVELY ISOLATED JET AIRCRAFT AVIONICS

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ABSTRACT

This paper presents techniques for the vibration analysis of passively isolated airborne electronics to investigate them under adverse effects of vibration exposure. Establishment of an accurate mathematical model precedes the normal modes and random response analysis. In the mathematical model, elastic properties of isolators, their installation locations and orientations are allowed for arbitrary definition. Viscous damper is considered in the formulation as the damping mechanism to represent realistic conditions in the case of random excitation. Though the employed techniques herein are general, they are demonstrated through the application on jet aircraft avionics. Mathematical model and vibration analysis are verified through FE model results with very well agreement. Entire process is implemented in the open software Python environment and brought into use via a GUI.

INTRODUCTION

Avionics are defined as the electronic systems used in the air and space vehicles. In the modern aerospace industry, it is inevitable to utilize electronic systems which ensure the control of the air vehicles. Airborne electronics are subjected to adverse effects of vibration exposure. It is important to ensure the sustainability of such equipment through vibration isolation. Otherwise, it might result in a disaster in the vehicle. According to the failure history of electronic equipment hardware, which is observed and investigated by United States Air Force for about two decades, failures due to the operating environments are graded by percentage weight. Thus, high temperature and cycling through extreme temperatures get a percentage of 55, humidity gets a percentage of 20, and vibration and shock get a percentage of 20 [Steinberg, 2000].

Vibration isolation in order to reduce vibratory loads transmitted from the base structure, which acts as the vibration source, to the equipment through resilient mountings has been a much-discussed issue over years because of the fact that vibratory loads may eventually lead to fatigue failure on relevant parts of aircraft. Isolation relies on the separation of the equipment and excitation forces in phase by decoupling them through resilient supports. In

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this way, transmitted forces are reduced by resisting the motion or opposing the input excitation by inertia. Passive isolation mechanism distinguishes as the simplest practice for this purpose. A properly designed isolation system is critical to maintain the operational performance of the equipment [Agnes, 1993].

The system consists of six degrees of freedom rigid body which is mounted on resilient supports fixed at the rigid base. The simple Voigt model is employed with three orthogonal pairs of spring and damper connected in parallel. In the system with the internal damping mechanism, viscous damper is employed. In this way, realistic conditions can be represented more closely as random excitation is applied to the system. In the mathematical model, it is allowed for describing the system by resilient element stiffness, damping, position and orientation of installation.

Vibration analysis is implemented with the created mathematical model. It consists of two successive stages. They are free vibration and forced vibration analysis. A free vibration analysis ends up with modal properties or the eigenvalues and eigenvectors. Besides, response characteristics are obtained by implementing a forced vibration analysis. These stages of vibration analysis are achieved in three phases:

- Setting up the spatial properties i.e., the values of mass, stiffness and damping elements.
- Performing a free vibration analysis to obtain modal properties i.e., natural frequencies and mode shapes vectors as many as the number of degrees of freedom.
- Performing a forced response analysis to obtain response characteristics.

Mathematical model and vibration analysis techniques are verified through the FE model which is created in the MSC Patran/Nastran® developed by NASA and commonly employed in the aerospace industry. Begin with, literature is reviewed for studies which perform free vibration analysis of isolation systems. An isolation system from literature, whose natural frequencies are shared out explicitly, is modeled in an FE environment. Natural frequencies of the reference study and FE model are compared to be sure of that the finite element model is created properly. Once the FE model is made certain of, it is employed to verify the developed computer code. Natural frequencies and mode shapes are compared to ensure that the system model is defined correctly. Besides, random response analysis results for constant amplitude random excitation are compared to verify the employed techniques for the forced vibration analysis in the developed computer code. Results obtained from the FE model and developed computer code show well agreement.

Programming language environment used during this study is Python. It is open software and has a large usage in scientific computation. Entire analysis process is implemented by the developed computer code and brought into use via GUI which is presented in Appendix A. Expanded description of the analysis process can be found in [Eker, 2019].

METHOD

Mathematical Model

Coordinate frames

Three axis systems are primarily defined to construct the mathematical model with reference to them. The first system is the global axis system with the set of axes x, y, z. It is the basic frame in the space and fixed arbitrarily. The second coordinate system is the inertial axis system with the set of axes \overline{x} , \overline{y} , \overline{z} . Its origin is placed at the mass center of the isolated rigid boy at rest, and it is aligned with the basic frame. The last axis system with the set of axes X, Y, Z is fixed at the mass center of the isolated rigid body and it is coincident with the inertial frame when there is no motion. Primary coordinate frames are shown in **Figure 1**.

Each resilient element in the isolation system has its own coordinate frame. An isolator coordinate frame is defined such that its x-axis is coincident with the axial direction of the isolator. Therefore, the axes y and z refer to the transverse directions of the isolator.

Attachment point matches with the elastic center of the isolator, which also corresponds to the origin of the related set of axes. These coordinate systems will be called the secondary frames. Each set of axes is named according to the corresponding resilient element e.g. First resilient element will have the set of axes X1, Y1, Z1, second resilient element will have the set of axes X2, Y2, Z2 and so on. Secondary coordinate frames are exemplified in **Figure 2**.



Figure 2: Secondary Coordinate Frames

The isolated body is considered as a six degrees-of-freedom rigid body. Degrees of freedom comprise three translational and three rotational displacements. It rests on rigidly connected resilient elements which are fixed on the rigid base structure. Thereby, a six degrees-of freedom mathematical model is created. For this purpose, system equations of motion are defined for the mass **m** and mass moment of inertia **I** of the isolated body, stiffness **k** and viscous damping **c** of resilient element and distance of the elastic center of the resilient element to the mass center **a**.

Resilient support is represented as having linear spring and viscous damping elements. The elastic center of the support is indicated as the intersection point of the principal elastic axes. This parameter is important because of that it determines the magnitude of the stiffness by

being used in the equations of motion. The torsional elements are neglected since the torsional stiffness is much smaller than the translational stiffness in general case. Therefore, there are no torsional springs and dampers to apply coupling to the isolated body.

Resilient element is represented by three mutually perpendicular members of Voigt model as shown in **Figure 3**. These members are oriented in axial and radial directions of principal axes of the resilient support.



Figure 3: Resilient Element of Three Mutually Perpendicular Members

The displacements of the center of gravity of the rigid body are indicated by x_g , y_g and z_g in \overline{x} , \overline{y} , and \overline{z} directions of the inertial frame, respectively. Besides, the rotations of the rigid body about these axes are denoted by α , β and γ , respectively. In the same manner, the displacements of the center of gravity of the foundation are indicated by u, v and w in \overline{x} , \overline{y} , and \overline{z} directions of the inertial frame, respectively. Besides, the rotations of the rigid body about these axes are denoted by α , β and γ , respectively. System diagram is shown in **Figure 4**.



Figure 4: Basic System Diagram

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System equation of motion for damped system in compact form is given by

$$[M]{\ddot{X}} + [C]{\dot{X}} + [K]{X} = {L}$$
⁽¹⁾

Equations of motion for six degree-of freedom system are given explicitly in [Piersol, 2010]. Damping forces applied on the rigid body by resilient members are not involved in these equations for convenience. They are to be included in each equation by appropriately adding damping terms analogous to corresponding stiffness terms.

$$m\ddot{x}_{g} + \sum k_{xx}(x_{g} - u) + \sum k_{xy}(y_{g} - v) + \sum k_{xz}(z_{g} - w) + \sum (k_{xz}a_{y} - k_{xy}a_{z})(\alpha - \underline{\alpha}) + \sum (k_{xx}a_{z} - k_{xz}a_{x})(\beta - \underline{\beta}) + \sum (k_{xy}a_{x} - k_{xx}a_{y})(\gamma - \underline{\gamma}) = F_{x}$$

$$I_{xx}\ddot{\alpha} - I_{xy}\ddot{\beta} - I_{xz}\ddot{\gamma} + \sum (k_{xz}a_{y} - k_{xy}a_{z})(x_{g} - u) + \sum (k_{yz}a_{y} - k_{yy}a_{z})(y_{g} - v) + \sum (k_{zz}a_{y} - k_{yz}a_{z})(z_{g} - w) + \sum (k_{xy}a_{z}^{2} + k_{zz}a_{y}^{2} - 2k_{yz}a_{y}a_{z})(\alpha - \underline{\alpha}) + \sum (k_{xz}a_{y}a_{z} + k_{yz}a_{x}a_{z} - k_{zz}a_{x}a_{y} - k_{xy}a_{z}^{2})(\beta - \underline{\beta}) + \sum (k_{xy}a_{y}a_{z} + k_{yz}a_{x}a_{y} - k_{yy}a_{x}a_{z} - k_{xz}a_{y}^{2})(\gamma - \underline{\gamma}) = M_{x}$$

$$(2)$$

$$m\ddot{y}_{g} + \sum k_{xy}(x_{g} - u) + \sum k_{yy}(y_{g} - v) + \sum k_{yz}(z_{g} - w) + \sum (k_{yz}a_{y} - k_{yy}a_{z})(\alpha - \underline{\alpha}) + \sum (k_{xy}a_{z} - k_{yz}a_{x})(\beta - \underline{\beta}) + \sum (k_{yy}a_{x} - k_{xy}a_{y})(\gamma - \underline{\gamma}) = F_{y}$$

$$I_{yy}\ddot{\beta} - I_{xy}\ddot{\alpha} - I_{yz}\ddot{\gamma} + \sum (k_{xx}a_z - k_{xz}a_x)(x_g - u) + \sum (k_{xy}a_z - k_{yz}a_x)(y_g - v) + \sum (k_{xz}a_z - k_{zz}a_x)(z_g - w) + \sum (k_{xz}a_ya_z + k_{yz}a_xa_z - k_{zz}a_xa_y - k_{xy}a_z^2)(\alpha - \underline{\alpha}) + \sum (k_{xx}a_z^2 + k_{zz}a_x^2 - 2k_{xz}a_xa_z)(\beta - \underline{\beta}) + \sum (k_{xy}a_xa_z + k_{xz}a_xa_y - k_{xx}a_ya_z - k_{yz}a_x^2)(\gamma - \underline{\gamma}) = M_y$$

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$$m\ddot{z}_{g} + \sum k_{xz}(x_{g} - u) + \sum k_{yz}(y_{g} - v)$$

$$+ \sum k_{zz}(z_{g} - w) + \sum (k_{zz}a_{y} - k_{yz}a_{z})(\alpha - \underline{\alpha})$$

$$+ \sum (k_{xz}a_{z} - k_{zz}a_{x})(\beta - \underline{\beta})$$

$$+ \sum (k_{yz}a_{x} - k_{xz}a_{y})(\gamma - \underline{\gamma}) = F_{z}$$

$$I_{zz}\ddot{\gamma} - I_{xz}\ddot{\alpha} - I_{yz}\ddot{\beta} + \sum (k_{xy}a_{x} - k_{xx}a_{y})(x_{g} - u)$$

$$+ \sum (k_{yy}a_{x} - k_{xz}a_{y})(y_{g} - v)$$

$$+ \sum (k_{yz}a_{x} - k_{xz}a_{y})(z_{g} - w)$$

$$+ \sum (k_{xy}a_{x}a_{z} + k_{yz}a_{x}a_{y} - k_{yy}a_{x}a_{z} - k_{xz}a_{y}^{2})(\alpha - \underline{\alpha})$$

$$+ \sum (k_{xy}a_{x}a_{z} + k_{xz}a_{x}a_{y} - k_{xx}a_{y}a_{z} - k_{yz}a_{x}^{2})(\beta - \underline{\beta})$$

$$+ \sum (k_{xx}a_{y}^{2} + k_{yy}a_{x}^{2} - 2k_{xy}a_{x}a_{y})(\gamma - \underline{\gamma}) = M_{z}$$

The mathematical model is created by defining spatial properties of the system i.e., mass, stiffness, and damping elements. These elements are derived with matrix algebra, which provides a compact method depending on the generalized matrix method [Smollen, 1966].

System mass matrix contains principal masses and mass moments of inertia.

$$[M] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{xy} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$
(3)

Translational stiffness matrix of the resilient supporting element according to its principal elastic axes is

$$\begin{bmatrix} \bar{k}^{tt} \end{bmatrix} = \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_q & 0 \\ 0 & 0 & k_r \end{bmatrix}$$
(4)

Directional cosines matrix of the principal elastic axes of resilient supporting element and reference axes of rigid body motion is

$$[\lambda] = \begin{bmatrix} \lambda_{xp} & \lambda_{xq} & \lambda_{xr} \\ \lambda_{yp} & \lambda_{yq} & \lambda_{yr} \\ \lambda_{zp} & \lambda_{zq} & \lambda_{zr} \end{bmatrix}$$
(5)

Translational stiffness matrix of the resilient supporting element transformed to the reference axes is

$$[k^{tt}] = [\lambda] [\overline{k}^{tt}] [\lambda]^T \tag{6}$$

Global stiffness matrix of the system for an isolator is calculated according to that isolator's stiffness terms and distance to the center of the rigid body motion. Global stiffness matrix in block matrices is

$$\begin{bmatrix} \mathbf{k} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \mathbf{k}^{tt} \end{bmatrix} & \begin{bmatrix} \mathbf{k}^{tr} \end{bmatrix} \\ \begin{bmatrix} \mathbf{k}^{rt} \end{bmatrix} & \begin{bmatrix} \mathbf{k}^{rr} \end{bmatrix}$$
(7)

Where, in compact form for unloaded configuration,

$$[\boldsymbol{k}^{tr}] = [\boldsymbol{k}^{tt}][\boldsymbol{A}_0]^T \tag{8}$$

$$[\boldsymbol{k}^{rt}] = [\boldsymbol{k}^{tr}]^T \tag{9}$$

$$[k^{rr}] = [A_0][k^{tt}][A_0]^T$$
(10)

Where

$$[A_0] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & a_x \\ -a_y & a_x & 0 \end{bmatrix}$$
(11)

Damping matrix can be written by adding the appropriate damping terms with analogous to the corresponding stiffness terms.

System stiffness matrix is

$$[K] = \sum_{i=1}^{n} [k_i]$$
(12)

System damping matrix is

$$[C] = \sum_{i=1}^{n} [c_i] \tag{13}$$

Where

$$[C] = A[M] + B[K] \tag{14}$$

Displacement vector consists of the unknowns of the equations of motion as given by

$$\{X\} = \left\{ x_g - u \quad y_g - v \quad z_g - w \quad \alpha - \underline{\alpha} \quad \beta - \underline{\beta} \quad \gamma - \underline{\gamma} \right\}^T$$
(15)

In this study, foundation on which resilient elements are attached is assumed to have no rigid body motion. Thus, translational and rotational displacement terms of base motion in the system equations of motion will vanish and reduce to

$$\{X\} = \{x_g \quad y_g \quad z_g \quad \alpha \quad \beta \quad \gamma\}^T \tag{16}$$

In the same manner, acceleration vector is given in the reduced form by

$$\{\ddot{\mathbf{X}}\} = \{\ddot{\mathbf{x}}_g \quad \ddot{\mathbf{y}}_g \quad \ddot{\mathbf{z}}_g \quad \ddot{\boldsymbol{\alpha}} \quad \ddot{\boldsymbol{\beta}} \quad \ddot{\boldsymbol{\gamma}}\}^T$$
(17)

Load vector is given by

$$\{L\} = \{F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z\}^T$$
(18)

Vibration Analysis

Vibration analysis will be summarized by indicating the identifiable aspects of the subject. It consists of two successive stages. They are free vibration and forced vibration analyses.

Free vibration analysis

The equation of motion for viscously damped system is written as

$$[M]{\ddot{x}} + [C]{\dot{x}} + [K]{x} = \{0\}$$
(19)

A solution to the system equation of motion is offered in the form of

$$\{x(t)\} = \{X\}e^{j\lambda t} \tag{20}$$

A complex eigenvalue problem is solved such that

$$([K] - \lambda^2[M]){X} = {0}$$
 (21)

The solution to the eigenvalue problem yields the modal properties of the system. Eigenvalue and eigenvector matrices are denoted by $[\lambda^2]$ and $[\varphi]$, respectively. Solution of eigenvalue problem leads to damped natural frequency λ_r . The eigenvalue λ_r^2 is related to the natural frequency or undamped natural frequency ω_r and modal damping ratio ζ_r of the system as follows:

$$\lambda_r^2 = \omega_r^2 (1 - \zeta_r^2) \tag{22}$$

Mode shape matrix is

$$[\boldsymbol{\varphi}] = [\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \cdots \quad \boldsymbol{\varphi}_n] \tag{23}$$

Modal mass, stiffness, and damping matrices are obtained by applying the orthogonality properties such that

$$[\boldsymbol{\varphi}]^T[\boldsymbol{M}][\boldsymbol{\varphi}] = [\boldsymbol{m}_r] \tag{24}$$

$$[\boldsymbol{\varphi}]^T[\boldsymbol{K}][\boldsymbol{\varphi}] = [\boldsymbol{k}_r] \tag{25}$$

$$[\boldsymbol{\varphi}]^T[\boldsymbol{\mathcal{C}}][\boldsymbol{\varphi}] = [\boldsymbol{c}_r] \tag{26}$$

Mode shapes are normalized to unit modal mass in keeping with the conventional approach.

$$[\boldsymbol{\phi}]^T[\boldsymbol{M}][\boldsymbol{\phi}] = [\boldsymbol{I}] \tag{27}$$

Or

$$[\boldsymbol{\phi}] = [\boldsymbol{m}_r]^{\frac{1}{2}}[\boldsymbol{\varphi}] \tag{28}$$

In which, ϕ_r is the mass-normalized mode shape of the rth mode.

Modal matrix is treated with mass matrix to give diagonal identity matrix:

$$[\boldsymbol{\phi}]^T[\boldsymbol{M}][\boldsymbol{\phi}] = [\boldsymbol{I}] \tag{29}$$

Modal matrix is treated with stiffness matrix to give diagonal eigenvalue matrix:

$$[\boldsymbol{\phi}]^T[\boldsymbol{K}][\boldsymbol{\phi}] = \left[\boldsymbol{\omega}_r^2\right] \tag{30}$$

8 Ankara International Aerospace Conference Modal matrix is treated with damping matrix to give:

$$[\boldsymbol{\phi}]^{T}[\boldsymbol{\mathcal{C}}][\boldsymbol{\phi}] = [2\zeta\omega_{r}] \tag{31}$$

Thus, modal properties can be obtained by free vibration analysis [He, 2001].

Forced vibration analysis

Vibration exposure for military aircraft is available in military standards. MIL-STD-810-G defines the vibration environment of the jet aircraft [DoD, 2008]. Acceleration spectral density level for turbulent air flow which is the main source of interior noise problems in jet aircraft, which leads the upper limit of acceleration spectral density level $W_0 = 0.20 g^2/Hz$ [Dreher, 1982] [Hall, 1980]. This vibration level is used in the vibration exposure offered by MIL-STD-810-G as presented in Figure 5.



Eventually, break points of the jet aircraft vibration exposure are given in Table 1.

Frequency [Hz]	$ASD \left[g^2 / Hz \right]$
15	0.04
89	0.04
300	0.2
1000	0.2
2000	0.05

Table 1: Jet Aircraft Vibration Exposure for All Other Material

System is diagonalized by utilizing the orthogonality properties of eigenmodes which are the constituents of the modal matrix. Thus, modal transformation is achieved by applying the separation approach as given by

$$\{X\} = [\varphi]\{q(\omega)\}$$
$$\{P\} = [\varphi]\{L(\omega)\}$$
$$(32)$$
$$[\varphi] = [\varphi_1 \quad \varphi_2 \quad \cdots \quad \varphi_n]$$

Herewith, the system of multiple degrees of freedom is decoupled to a system of single-mass oscillators as many as the number of degrees of freedom. Each single mass oscillator represents one of the eigenvectors of the system. Following the modal transformation, system is represented in general coordinates q as given by

$$m_{r}x_{r} + k_{r}x_{r} = l_{r}$$

$$m_{r}\varphi_{r}\ddot{x}_{r} + k_{r}\varphi_{r}x_{r} = \varphi_{r}l_{r}$$

$$m_{r}\ddot{q}_{r}(\omega) + k_{r}q_{r}(\omega) = p_{r}$$

$$\ddot{q}_{r}(\omega) + \frac{k_{r}}{m_{r}}q_{r}(\omega) = \frac{p_{r}(\omega)}{m_{r}}$$

$$\ddot{q}_{r}(\omega) + \omega_{r}^{2}q_{r}(\omega) = \frac{p_{r}(\omega)}{m_{r}}$$
(33)

As of now, damping may be included in the equations by employing the modal damping ratio ζ by starting with

$$\ddot{q}_{r}(\omega) + 2j\zeta\omega_{r}q_{r}(\omega) + \omega_{r}^{2}q_{r}(\omega) = \frac{p_{r}(\omega)}{m_{r}}$$
(34)

This differential equation can be solved easily and the solution for the displacement in general coordinates is obtained as given by

$$q_r(\omega) = \frac{\frac{p_r(\omega)}{m_r}}{\omega_r^2 - \omega^2 + 2j\zeta\omega_r\omega}$$
(35)

All the single mass oscillators are combined linearly to give the system solution by

$$X(\omega) = \sum_{r=1}^{N} \varphi_r q_r(\omega)$$
(36)

Frequency response function is obtained by dividing the response of the system by the excitation force. Response might be displacement, velocity or acceleration. Frequency response function for displacement i.e. receptance is given by

$$H(\omega) = \frac{X(\omega)}{L(\omega)} = \sum_{r=1}^{N} \frac{\varphi_r \varphi_r^T}{\omega_r^2 - \omega^2 + 2j\zeta_r \omega_r \omega} \frac{1}{m_r}$$
(37)

10 Ankara International Aerospace Conference Frequency response function might also be written in terms of mass normalized modal matrix, and it comprises the contributions of all individual modes as given by

$$H(\omega) = \frac{X(\omega)}{L(\omega)} = \sum_{r=1}^{N} \frac{\phi_r \phi_r^{T}}{\omega_r^{2} - \omega^{2} + 2j\zeta_r \omega_r \omega}$$
(38)

Furthermore, the receptance for the ith node (degree of freedom) with a single excitation force at kth degree of freedom can be calculated by

$$H_{ik}(\omega) = \frac{X(\omega)}{L(\omega)} = \sum_{r=1}^{N} \frac{\phi_{ir}\phi_{kr}}{\omega_r^2 - \omega^2 + 2j\zeta_r\omega_r\omega}$$
(39)

Conversion from displacement to velocity and acceleration may be achieved through multiplication by ω and ω^2 , respectively. The response can be found using the receptance to calculate the response and mean square of the response [Yang, 1986].

Response at coordinate i of an MDOF system to a single stationary random excitation at coordinate k is calculated by

$$S_i(\omega) = |H_{ik}||H_{ik}|S_k(\omega) \tag{40}$$

The mean square of spectral response may be calculated by

$$\sigma_i^2 = \int \mathbf{S}_i(\boldsymbol{\omega}) \, d\boldsymbol{\omega} \tag{41}$$

Verification of the Mathematical Model and Analysis Process

Literature is reviewed for studies which perform free vibration analysis of isolation systems. An isolation system from literature, whose natural frequencies are shared out explicitly, is modeled in an FE environment. Natural frequencies of the reference study and FE model are compared to be sure of that the finite element model is created properly. Once the FE model is made certain of, random response analysis is conducted and response results for displacement, velocity and acceleration are obtained in the FE environment.

The reason to use an FE environment is the facility to simulate lots of different isolation systems. Furthermore, random response analysis data is not readily accessible in literature. This makes the complete verification of the developed tool difficult. However, it is relatively easier to reach modal properties of various isolation systems in literature, and this fact is utilized to verify whether the FE model is created correctly, which is required to continue with the random response analysis. Hereby, both the free vibration and forced vibration procedures of an entire vibration analysis are achieved through the FE model. Eventually, analysis results which are obtained from the FE model are used to verify the developed computer code.

There exist plenty of finite element software packages such as Feemap®, SESAM®, Abaqus®, Hypermesh®, Ansys®, Nastran®, etc. Nastran® is developed by NASA and commonly employed in the aerospace industry. The finite element environment used in this study is MSC Patran/Nastran®. It is a multidisciplinary structural analysis software which can perform static, dynamic, and thermal analysis in both linear and nonlinear domains.

Center-of-gravity installation arrangement of four coplanar identical resilient elements with three planes of vibrational symmetry is considered in this example [Vane, 1958]. Isolation subjects to a solid homogeneous rectangular body, and the results are found by analytical calculations.

Isolation system is shown in **Figure 6**. Isolation system properties are given in inertial axis system \overline{XYZ} , and center-of-gravity and mounting positions are given in global axis system xyz. Mounting orientations of resilient elements are also given with respect to the global axis system xyz.





Positions of center-of-gravity and mounting points beside the directional cosines of mounts and body dimensions are presented in **Table 2**.

				Directional
Parameter	x [mm]	y[mm]	z[mm]	Cosine
CG	635.0	406.4	254.0	N/A
Isolator I	1219.2	889.0	254.0	<0,0,1>
Isolator II	50.8	889.0	254.0	<0,0,1>
Isolator III	50.8	-76.2	254.0	<0,0,1>
Isolator IV	1219.2	-76.2	254.0	<0,0,1>
Body Dimension	1270.0	812.8	508.0	N/A

Inertial properties of the isolation system which comprise of mass and mass moments of inertia in principal directions are given in **Table 3**.

Table 3: Inertial Properties of the Isolation Systems

Component	m [Mg]	$I_{\overline{X}} [Mgmm^4]$	$I_{\overline{Y}}[Mgmm^4]$	$I_{\overline{Z}}[Mgmm^4]$
Body	1.8144	138906.2888	282884.9983	343754.0462

Elastic properties of resilient elements of the isolation system are presented in Table 4.

Parameter	K _{axial} [N/mm]	K_{radial} [N/mm]	ζr
Isolator I	4027.92	4027.92	0.05
Isolator II	4027.92	4027.92	0.05
Isolator III	4027.92	4027.92	0.05
Isolator IV	4027.92	4027.92	0.05

Table 4: Elastic Properties of the Isolation System

Insofar, all the parameters required for a complete definition of the isolation system are provided. Based on these parameters, FE model is created in the MSC Patran/Nastran® software.

System FE model

FE Model contains CBUSH elements to represent resilient mounts in the system. Besides, CONM2 element to represent the rigid body by introducing mass and inertia properties is rigidly connected to the CBUSH elements with RBE2 elements in all degrees of freedom. Model generation is performed in the MSC Patran® pre-processor environment.

As the solution sequence, SOL 111 included in the MSC Nastran® analysis solver is employed. Excitation is introduced to the system from the fixed ends of CBUSH elements, which represent the rigid connection points of resilient mounts to the base. Response is measured from the center-of-gravity of the isolated body which corresponds to the position of the CONM2 element.

FE model is shown in Figure 7.



Figure 7: Representation of the FE Model

According to the free vibration analysis results of both FE model and reference application natural frequencies are compared in **Table 5**. The results are demonstrated to be in good agreement.

	Natura	_	
DOF	FE Model	Reference [Vane, 1958]	Difference [%]
1	14.998~15.0	15.0	0.0
2	14.998~15.0	15.0	0.0
3	14.998~15.0	15.0	0.0
4	22.189~22.2	22.3	0.4
5	26.109~26.1	26.1	0.0
6	26.159~26.2	26.1	0.4

Table 5: Natural Frequency	Comparison	of Reference	[Vane,	1958]	& FE Model
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Following the confirmation of that the FE model is created properly by correct definition of mass and stiffness terms, modal property results of the FE model are compared with those of the mathematical model. According to the free vibration analysis results of both the FE model and mathematical model, modal properties are compared in **Table 6** and in **Table 7** in terms of natural frequencies and eigenmodes, respectively.

	Natura	_	
DOF	FE Model	Mathematical Model	Difference [%]
1	14.998	14.998	0.0
2	14.998	14.998	0.0
3	14.998	14.998	0.0
4	22.189	22.189	0.0
5	26.109	26.109	0.0
6	26.159	26.159	0.0

Table 6: Natural Frequency Comparison of Mathematical Model & FE Model

Table 7: Eigenmode Comparison of Mathematical Model & FE Model

Natural		Eigenmode Constituent					
Frequency [Hz]		1	2	3	4	5	6
	FEM	-0.7424	0.0	0.0	0.0	0.0	0.0
14.998	TOOL*	-0.7424	0.0	0.0	0.0	0.0	0.0
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.0
	FEM	0.0	0.7424	0.0	0.0	0.0	0.0
14.998	TOOL	0.0	0.7424	0.0	0.0	0.0	0.0
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.0
	FEM	0.0	0.0	0.7424	0.0	0.0	0.0
14.998	TOOL	0.0	0.0	0.7424	0.0	0.0	0.0
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.0
22.189	FEM	0.0	0.0	0.0	0.0	0.00188	0.0

	TOOL	0.0	0.0	0.0	0.0	0.00188	0.0
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.0
	FEM	0.0	0.0	0.0	0.0	0.0	0.00170
26.109	TOOL	0.0	0.0	0.0	0.0	0.0	0.00171
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.6
	FEM	0.0	0.0	0.0	-0.00268	0.0	0.0
26.159	TOOL	0.0	0.0	0.0	-0.00268	0.0	0.0
	DIFF [%]	0.0	0.0	0.0	0.0	0.0	0.0

Table 7 continued.

*TOOL refers to the developed computer code which implements vibration analysis through the mathematical model

Result of modal properties show that mathematical model is created properly by correct definition and formulation of mass and stiffness elements. Mode shapes obtained from the FE model and mathematical model are illustrated in **Figure 8** and **Figure 9**, respectively.



Figure 8: Mode Shapes by the FE Model

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Figure 9: Normalized Mode Shapes by the Mathematical Model

Modal assurance criterion is also applied to visualize the identity extent of the mode sets. Yellow and purple boxes in **Figure 10** correspond 1 and 0, respectively. This indicates that MAC values are 100% or that the mode sets of the FE model and mathematical model are identical.



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Random response analysis of the mathematical model is also to be verified. A random excitation of acceleration spectrum is considered to demonstrate the capability of the theoretical model to execute a random response analysis. Acceleration power spectral density function has units of acceleration g^2/Hz versus frequency Hz. In the meantime, acceleration may be expressed in metric units as $(mm/s^2)^2/Hz$. System is subjected to the constant acceleration of 0.01 g^2/Hz over the entire frequency range as shown in Figure 11.



Figure 11. ASD Input in Z-Direction

System response is obtained as acceleration response, velocity response and displacement response for comparison. Random response graphs are presented in **Figure 12** through. **Figure 14**. There is a remarkable agreement between the response curves.



Figure 12: Response Comparison for Displacement in Z-Direction







Figure 14: Response Comparison for Acceleration in Z-Direction

Root mean square (RMS) values of response spectrums are also compared in Table 8.

Table	8.	RMS	Comparison
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Response		Degree-of-Freedom
		Ζ
	FEM [mm]	1.774
Displacement RMS	TOOL* [mm]	1.774
	DIFF [%]	0.0
	FEM [mm/s]	159.592
Velocity RMS	TOOL [mm/s]	159.592
	DIFF [%]	0.0
	FEM [mm/s2]	14779.08
Acceleration RMS	TOOL [mm/s2]	14779.11
	DIFF [%]	0.0

*TOOL refers to the developed computer code which implements vibration analysis through the mathematical model

In conclusion, vibration analysis of a passively isolated equipment is completed by developing a computer code which is brought into use via a GUI. A mathematical model with realistic viscous damping is created with pre-determined spatial values. Mathematical modeling of the system is followed by normal modes and random response analyses. Eventually, the mathematical modeling and performed vibration analysis are verified by comparing results with those of created FE model. Modal property and random response results of the developed computer code and FE model show well agreement. Therefore, a convenient vibration analysis environment with the opportunity of a GUI is developed by utilizing open software Python environment for passively isolated jet aircraft avionics.

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Appendices

A. GUI of Developed Computer Code

GUI is composed of three main sections:

Section for the definition of spatial properties

Section for the preparation of the optimization problem

Section for the setup of the uncertainty simulation parameters

In the first section, job name and description are optionally required for convenience. Furthermore

i Mass

ii **Geometric Shape** may be either point or other basic shapes such as cube. This option is provided for automatic calculation of mass moment of inertia.

- iii Geometric Dimensions if (ii) exists.
- iv CG
 - a. At Centroid is available if (ii) exists

v Inertia Tensor

a. Automatic Calculation is available if (ii) exists

- vi Isolator Connection requires position and orientation of each single isolator.
 - a. Natural Coord option is available if (ii) exists
 - b. List button enables checking and modifying added isolators
- vii **Elastic Properties** can be entered separately for each single isolator previously added.

History window at the bottom enables following the taken actions on the window.

	Ceometri 🗆 🛙		
Definition of Isolation System & Analysis Parameters			
ystem definiton: Rigid body resting on resilient elements	O Point		
lob Name	Cube		
lob-1 Set	Rectangular Prism		
lob Description	Right Rectangular P	yramid	
Set Isolator Design Parameters Project: HJT, Eqp: FSBLWL Set	Right Circular Cone		
SPATIAL PROPERTIES	Right Circular Cyline	ler	
Mass	Hollow Right Circula	r Cylinder	
Set	Sphere		
Geometric Shape	Hollow Sphere		
tight Circular Cone Select	Hemisphere		
Geometric Dimensions	Set		
Yı SPI-R	L		
SP1: SP2:	Inertia Matrix		X-
	in inertia Matrix		Constant of the second se
The second secon	Ixx	Ixy	Ixz
P3: \$P4:			
1	Іух	Іуу	Iyz
Set			
26	Izx	Izy	Izz
At Centroid X: Y: Z: Set			
Inertia Tensor		Set	
Automatic Calculation Create			
isolator Connection			
Natural Coord 💿 Global Coord 👔 Select 🗸	Isolator List	- 0 -X	
	1		
slastic Properties			
seett			
aatai Kradai Çi Set			
OPTIMIZATION PROBLEM			
UNCERTAINTY SIMULATION			
Apply Close	Remove Selected R	temove All Set	
>>Equipment geometric shape created			_
Right Circular Cone			