

Building of Mathematical Model and Bang Bang Based PID Control for Fighter Aircraft with Real Flight Data: Modelling and Simulation

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ABSTRACT

The main purpose of this study is to understand the basic problems encountered in the modeling of an aircraft with an agility concept and to guide the creation of a mathematical model. The most challenging part of the construction of the aircraft's mathematical model is the determination of the forces and moments of continuously changing air flow according to the flight regime and environmental conditions, as well as the thrust force and the moment generated by the engine in the event of a deviation from the motor axis. For aircraft having a concept of agility, big differences are observed in the relevant parameters depending on flight regime, angle of attack and altitude. Bang-Bang controller applied as inner loop control of highly maneuverable fighter aircraft with high nonlinear characteristic. Obtained results is compared with multivariable control methods such as LQR and H^∞ controllers.

INTRODUCTION

Companies are required to carry out various operational and functional tests without imposing the products they have developed and during the development-certification phase. In particular, the hourly flight cost of the unit aircraft in the aviation sector can be up to 70,000 dollars. This has led to the development of Modeling Simulation as a natural consequence of this situation. In this study, the methods of high accuracy modeling in air vehicles and potential difficulties encountered will be discussed. Furthermore, flight control systems are undoubtedly one of the most critical units for airplanes with Fly-by-wire concept, which is not a direct link between the pilot and the aircraft. Flight control computer, actuator and hydraulic subsystem is a subsystem of flight control system. In this study, flight control computers and actuators which are modeled sections will be mentioned. In the modeled aircraft, Bang-Bang controller, an old time optimal control method, will be implemented.

MODELLING

Aircraft is a complex of highly complex systems. For this reason, systems need to be examined separately. There are many systems on the aircraft and their subsystems. In this study, the sub-systems to be examined within the scope of the modeling simulation are the autopilot function to be provided by the flight control system, propulsion system and the flight

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management computer (mission computer) under the avionics system. Top-level model of a fighter is shown in Figure 1.

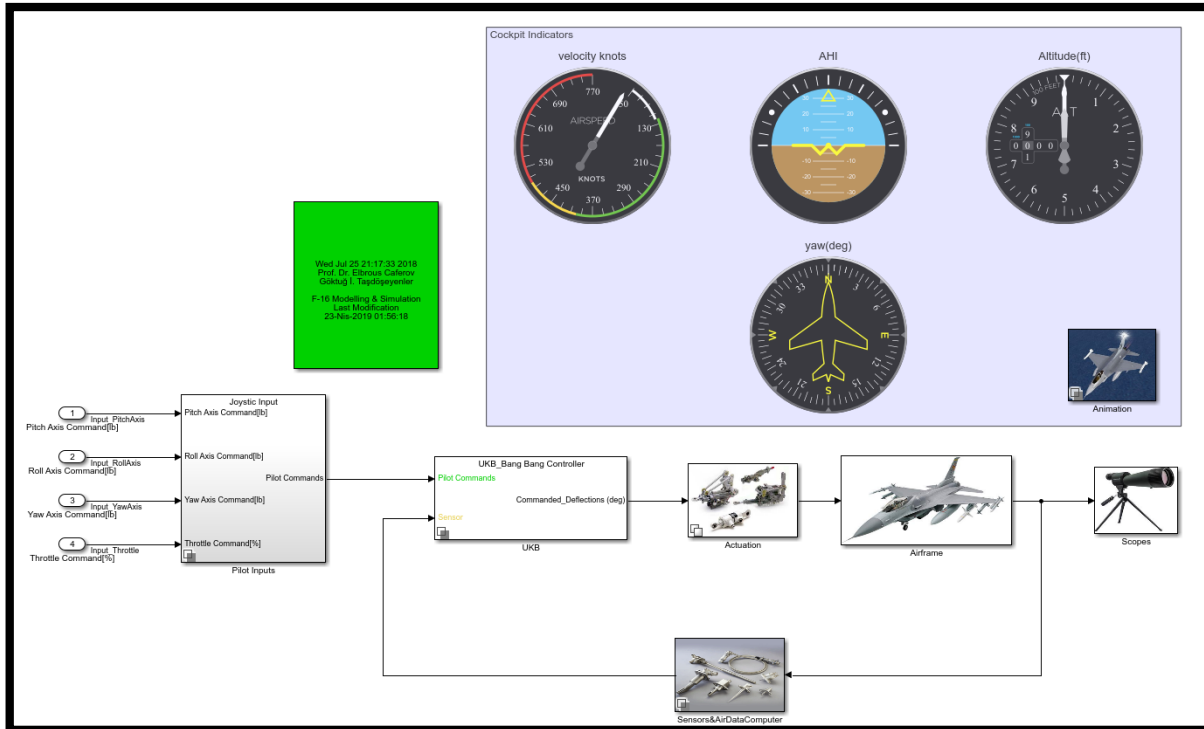


Figure 1: Top-level model of a fighter

Flight Control System

There are various types of interfaces in air vehicles such as mechanical, fly-by-wire, fly-by-light and fly-by-wireless. The control system varies according to the interface type. The performance of fly-by-wire, fly-by-light and fly-by-wireless airplanes are directly connected to the flight control system and there are many constraints such as frequency response performance in design optimizations.

Flight Control Computer: Flight control computers require the highest level (A) of design assurance level. When modelling the flight control computer, command limiting, gain scheduling, and structural filters for the compensate frequency response characteristics according should not be neglected.

Actuators: Actuators are flight critical parts for aircraft, and the time required for the full lap with standard expressions in modeling is taken 1 second for the control surfaces and 3 seconds for the secondary control surfaces. Actuator representation is shown in Table 1.

Table 1: Actuator Specification

Actuator	Transfer Function	Saturation	Rate Limit
Dif. Horizontal Tail	$\frac{20}{s + 20}$	$\pm 25^\circ$	$\pm 70^\circ/\text{s}$
Flaperon	$\frac{20}{s + 20}$	$\pm 21.5^\circ$	$\pm 70^\circ/\text{s}$
Rudder	$\frac{20}{s + 20}$	$\pm 30^\circ$	$\pm 70^\circ/\text{s}$
Leading Edge Flap	$\frac{10}{s + 10}$	$\pm 25^\circ$	$\pm 30^\circ/\text{s}$

Airframe

Airframe model includes aerodynamics, thrust, gravity and equation of motions.

Propulsion: In the first phase, the physical modeling of the motor must be carried out. Then the thrust model should be arranged iteratively with the flight data, the model accuracy should be increased and iteratively transferred to the control system design in order to achieve high flight performance. General engine specifications shown in Table 2 [General Electric, 2019].

Table 2: Engine Specification

Parameter	Value
Type	A/B Turbofan
Length	4.6 m
Maximum Diameter	1.2 m
Weight	1778 kg
Compressor	9 High pressure compressor
Turbine	2 Low / 1 High pressure turbine
Maximum Thrust	129 kN (73 kN dry)
Airflow	122.4 kg/second
Bypass-ratio	0.76

Thrust data according to flight regime modelled. Throttle inputs and outputs can be modelled as function iteratively with Flight Data as shown in Figure 2.

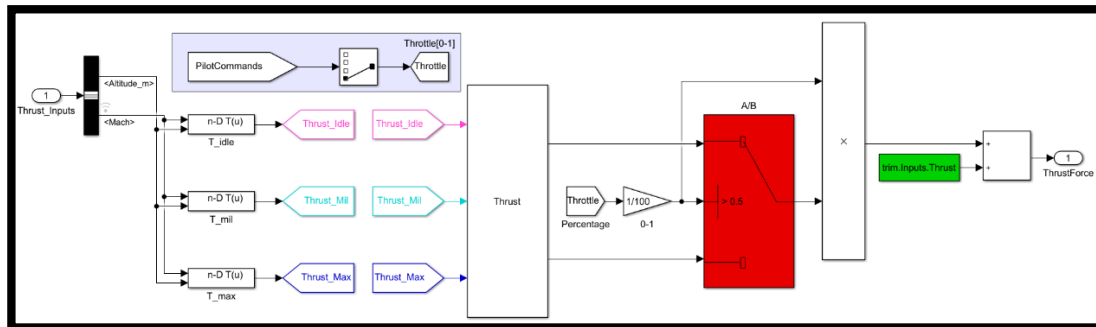


Figure 2: High precision propulsion model

Sensor: The sensor system is the system that transmits the signal with noise and delay in the absence of sensor malfunction.

Aerodynamic: In order to increase the accuracy of the model, modeling of aeroelastic effects should be low fidelity modeled by the structural and aerodynamic model in a manner that iteratively works [Gaétan Dussart, 2018]. Aerodynamic model can be setup as shown as Figure 3.

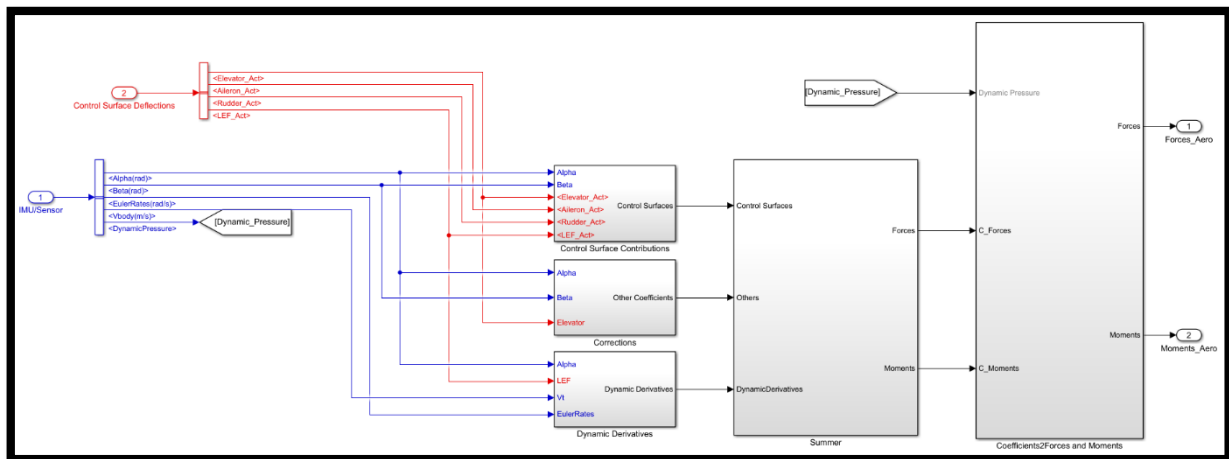


Figure 3: High precision aerodynamic model

Aerodynamic contributions modelled according to flight data. Aerodynamic parameters which effects included to model is shown in Table 3 [Stevens Brian L, 1992].

Table 3: Included aerodynamic contributions

Parameter	Description
$C_{L\alpha}$	Lift-curve slope (Determines response to turbulence)
$C_{m\alpha}$	Pitch stiffness (Negative for longitudinal static stability)
C_{mq}	Pitch damping (Negative for short period damping)

$C_{m\dot{\alpha}}$	Alpha-dot derivative
C_{Lq}	Pitch-rate-dependent lift
$C_{l\beta}$	Dihedral derivative (Negative for lateral static stability)
$C_{n\beta}$	Yaw stiffness (Positive for directional static stability)
C_{lp}	Roll damping (Negative for roll damping)
C_{nr}	Yaw damping (Negative for yaw damping)
C_{np}	Yawing moment according to roll rate
C_{lr}	Rolling moment according to yaw rate
C_{Yr}	Sideforce according to yaw rate
C_{Yp}	Sideforce according to roll rate

Sample maneuver has been prepared in order to show aerodynamic contribution of control surfaces and dynamic stability derivatives throughout flight regime. Aerodynamic parameters has been shown in Figure 4 with corresponding maneuver.

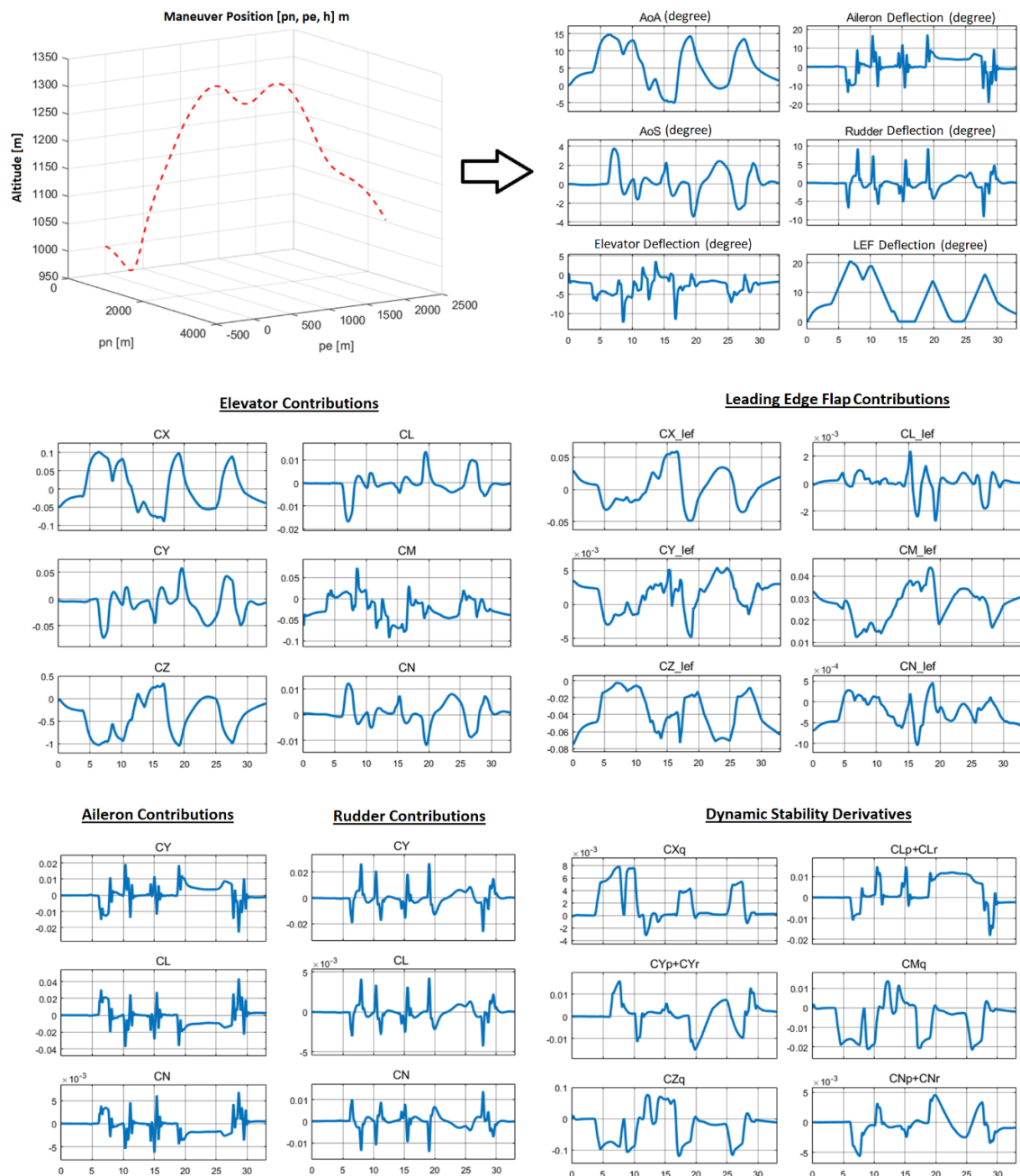


Figure 4: Aerodynamic contributions of control surfaces

Equation of Motion: This is the part where the acceleration, velocity, position, rotation information of the aircraft is generated and the six degrees of freedom equation of motion is built with Newton's second law using the input of the forces and moments acting on the aircraft. Quaternion method [Zipfel, 2007] was used to avoid singularity due to its high attack angle characteristic of fighter aircraft. Aircraft weight and inertia information was used in this section. Overall forces and moment equations can be shown as below with the assumption of aerodynamic center on the center of gravity and the engine produce thrust on the longitudinal axis only.

$$\begin{aligned}\Sigma X &= qS \left[0.5C_x(\alpha, \beta, \delta_{le}) + 0.5C_x(\alpha, \beta, \delta_{re}) + C_{x_{lef}}(\alpha, \beta) + \frac{C_{xq}(\alpha)q\bar{c}}{2V} \right] - mg\sin\theta + Thrust \\ \Sigma Y &= qS \left[C_y(\alpha, \beta) + C_{y_{lef}}(\alpha, \beta) + C_{y_{\delta a}}(\alpha, \beta)(\delta_{la} - \delta_{ra})/2 + C_{y_{\delta R}}(\alpha, \beta)\delta_R + [C_{y_p}(\alpha)p + C_{y_r}(\alpha)r]b/2V \right] + mg\cos\theta\sin\phi \\ \Sigma Z &= qS \left[0.5C_z(\alpha, \beta, \delta_{le}) + 0.5C_z(\alpha, \beta, \delta_{re}) + C_{z_{lef}}(\alpha, \beta) + \frac{C_{zq}(\alpha)q\bar{c}}{2V} \right] + mg\cos\theta\cos\phi \\ \Sigma L &= qSb \left[C_l(\alpha, \beta, \delta_e) + C_{l_{lef}}(\alpha, \beta) + C_{l_{\delta a}}(\alpha, \beta)(\delta_{la} - \delta_{ra})/2 + C_{l_{\delta R}}(\alpha, \beta)\delta_R + [C_{l_p}(\alpha)p + C_{l_r}(\alpha)r]b/2V + C_{l_\beta}(\alpha)\beta \right] \\ \Sigma M &= qS\bar{c} \left[C_m(\alpha, \beta, \delta_e) + C_{m_{lef}}(\alpha, \beta) + C_{m_q}(\alpha)q\bar{c}/2V + \Sigma C_z(\Delta x_{cg}) \right] \\ \Sigma N &= qSb \left[C_n(\alpha, \beta, \delta_e) + C_{n_{lef}}(\alpha, \beta) + C_{n_{\delta a}}(\alpha, \beta)(\delta_{la} - \delta_{ra})/2 + C_{n_{\delta R}}(\alpha, \beta)\delta_R + [C_{n_p}(\alpha)p + C_{n_r}(\alpha)r]\frac{b}{2V} + C_{n_\beta}(\alpha)\beta \right]\end{aligned}$$

BANG-BANG CONTROL

The optimal control theory, which emerged in the 17th century, caused a variety of control methods. One of them is the time optimal Bang-Bang controller [Zavoli, 2013]. The Bang-Bang controller works with the switching logic, and it emerged with two levels (on-off).

Two Level Bang-Bang Control

It works with the open-close logic and the switching is controlled by the hysteresis. If hysteresis is reduced by industries, comfort decreases, and in case of increasing hysteresis, it causes bad effects such as fatigue on control system. For this reason, it has been replaced by PIDs, which is the classical control method. The classical 2 level bang-bang controller structure is shown in the Figure 5.

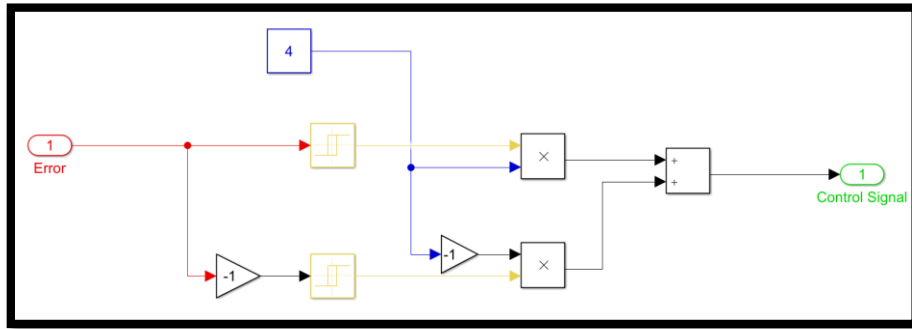


Figure 5: Two level bang-bang controller

Since the non-linear aircraft is a delayed system, the 2-level Bang-bang controller system is insufficient to control all regimes. Figure 6 shows the time response to the desired g commands for the g_command tracker controller designed in longitudinal axis.

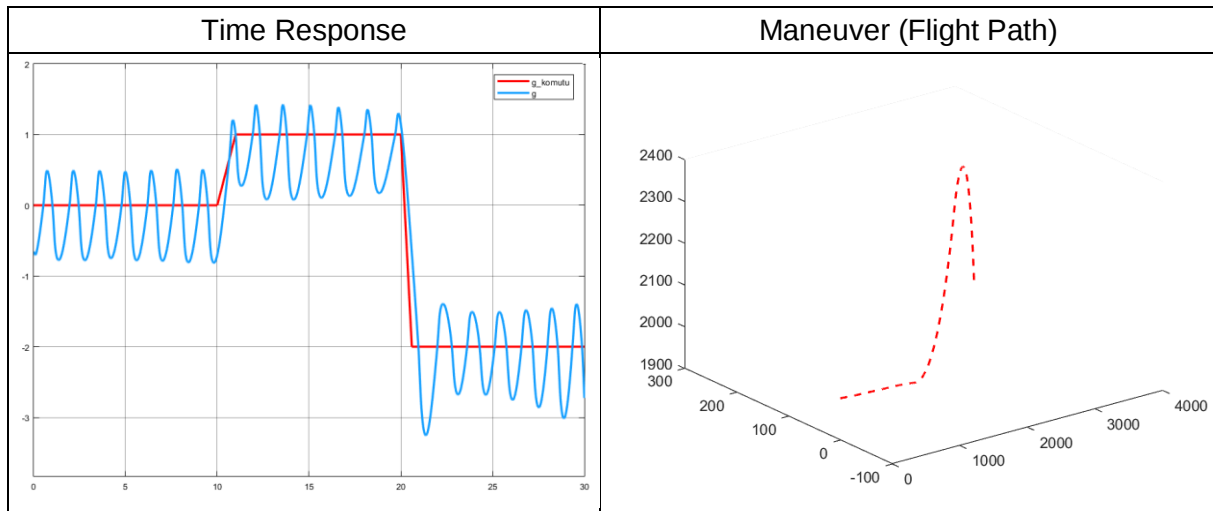


Figure 6: Time response of two level bang-bang controller and flight path

Multi-Level Bang-Bang Controller

The response of the multi-level bang bang controller is to adjust the control signal. In this way, switching takes place with different control signals. Multi-level bang bang controller is recommended on nonlinear systems. The example multi-level bang-bang controller structure is given in Figure 7.

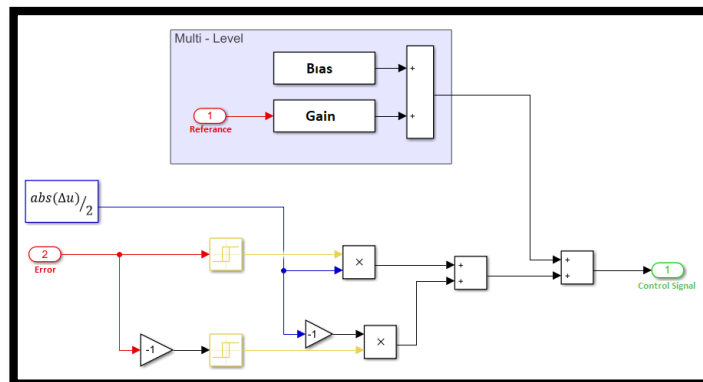


Figure 7: Multi level modified bang-bang controller

Bang-Bang Based PID Controller: Examples of multi-level Bang-Bang controllers are the Bang-Bang controller-based PID controllers. The size of the control signal generated by the Bang-Bang controller is provided by PID. In this way the switching takes place around different sized values. Figure 8 shows the structure of Bang-Bang based PID controller.

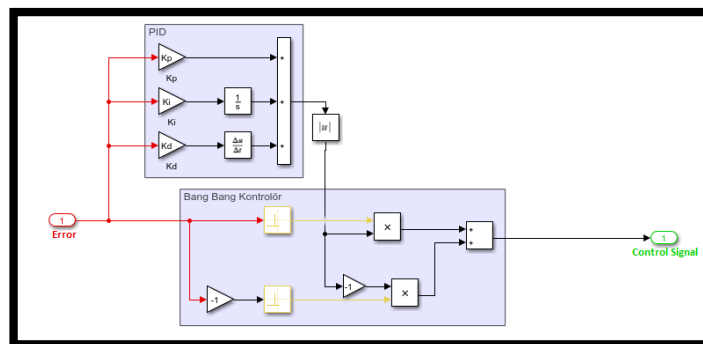


Figure 8: Bang-bang based PID controller

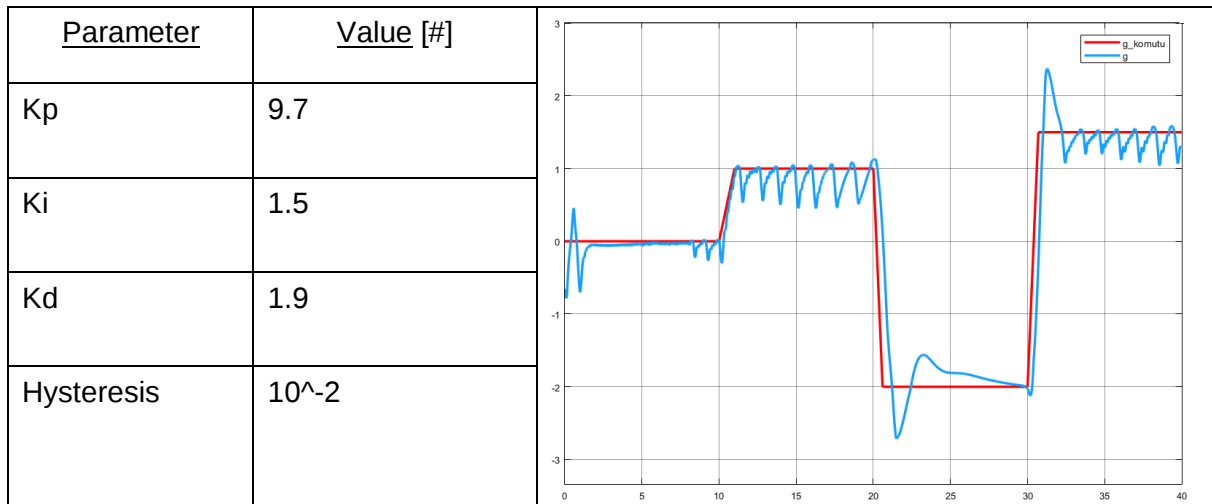


Figure 9: Time response of Bang-Bang based PID controller

The response characteristic is dependent on the control parameters, and it is possible to increase the damping ratio by the derivation coefficient. Also it is possible to decrease steady state error with the integral coefficient. Control parameters optimized according to ITSE performance index. Figure 10 shows the corresponding control signal and time response of different PID controller in Bang-Bang structure.

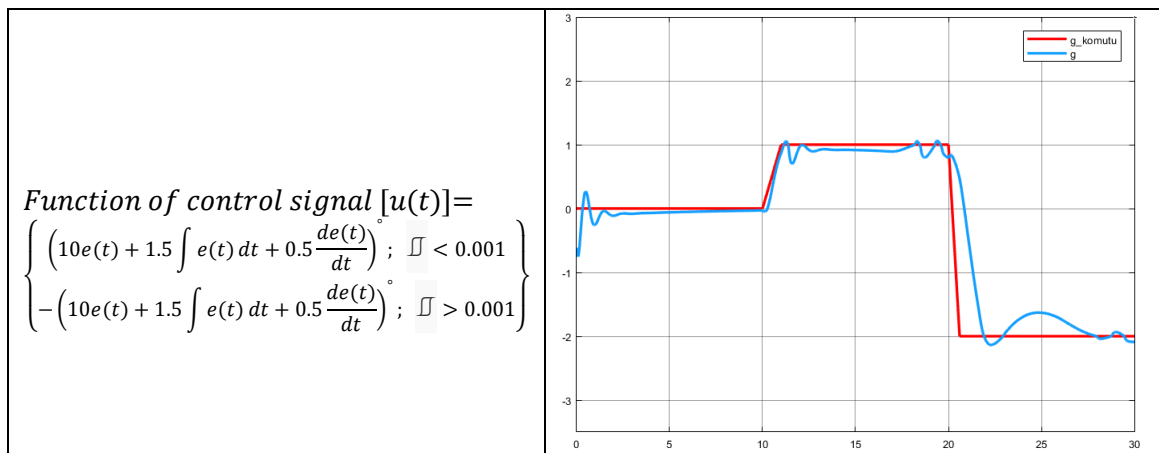


Figure 10: Control signal and time response of Bang-Bang based PID controller

Multi-Level Variable Hysteresis Controller: If it is intended to control a comprehensive regime, the definition of variable hysteresis is advantageous for the robustness of the control system. The variable hysteresis multi-level bang-bang controller is given in Figure 11.

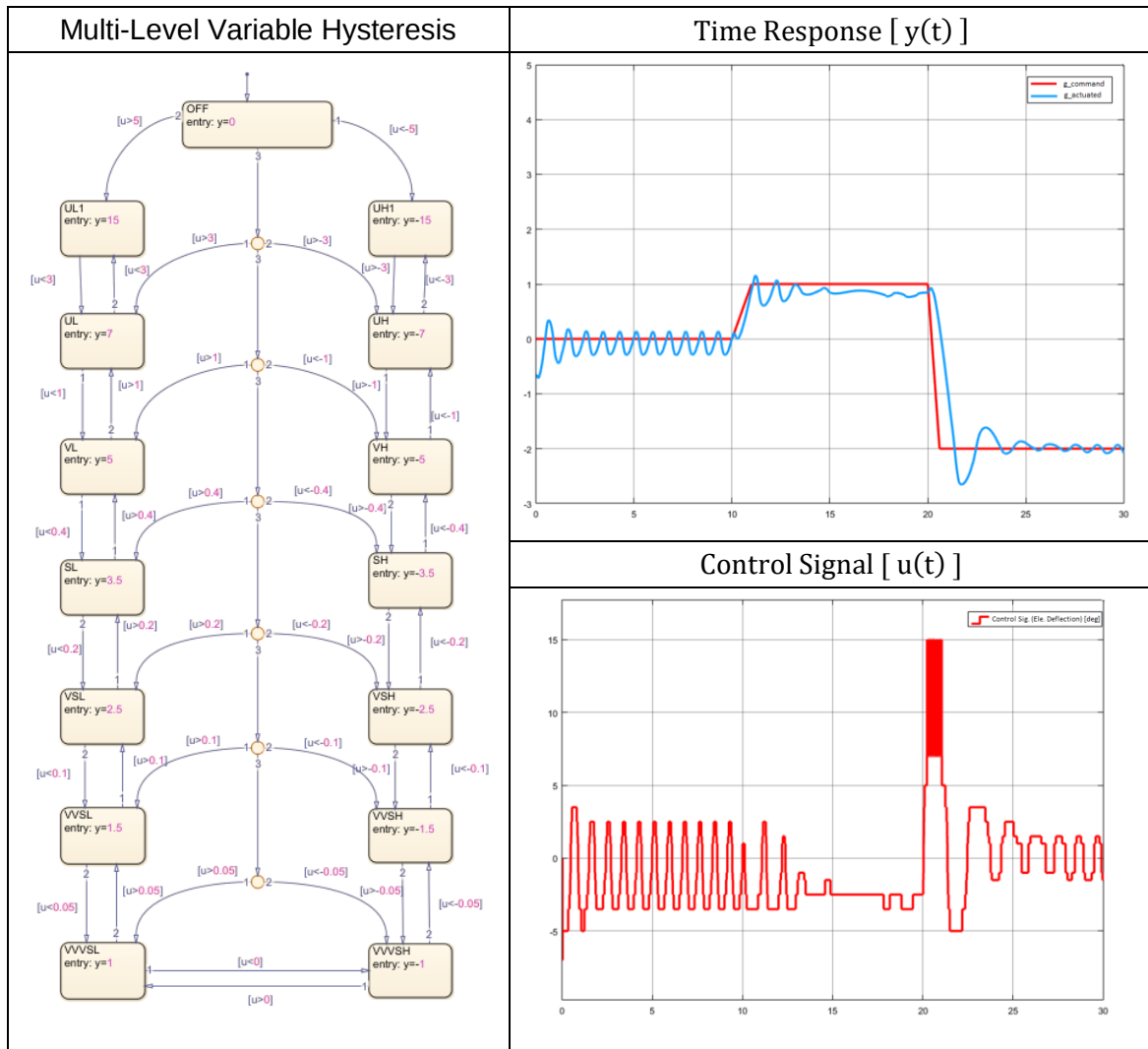


Figure 11: Control signal and time response of Bang-Bang based PID controller

DISCUSSION

All results has been obtained for highly nonlinear system. Therefore, bang-bang controller is not sufficient to control aircraft with high nonlinear characteristic in all regimes for inner loop of fighter aircraft. In order to compare result, multivariable feedback control design results has been shown below for the same model. The controller designs acquired in this section were obtained from the linearized system of the trim point and applied to the nonlinear system. LQR results for the same system is given in Figure 12.

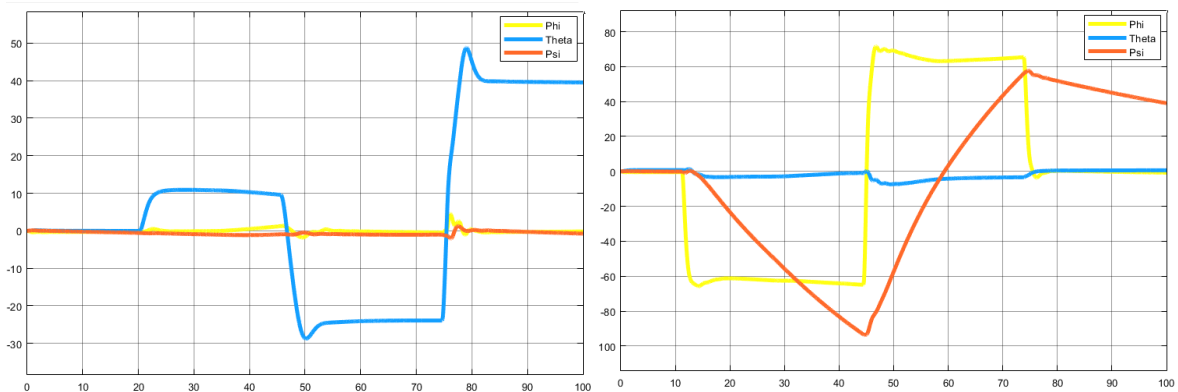


Figure 12: LQR maneuver results

H^∞ controller synthesis has done [Skogestad & Postlethwaite, 2001] for the pitch rate control in longitudinal axis. Results obtained is given in Figure 13.

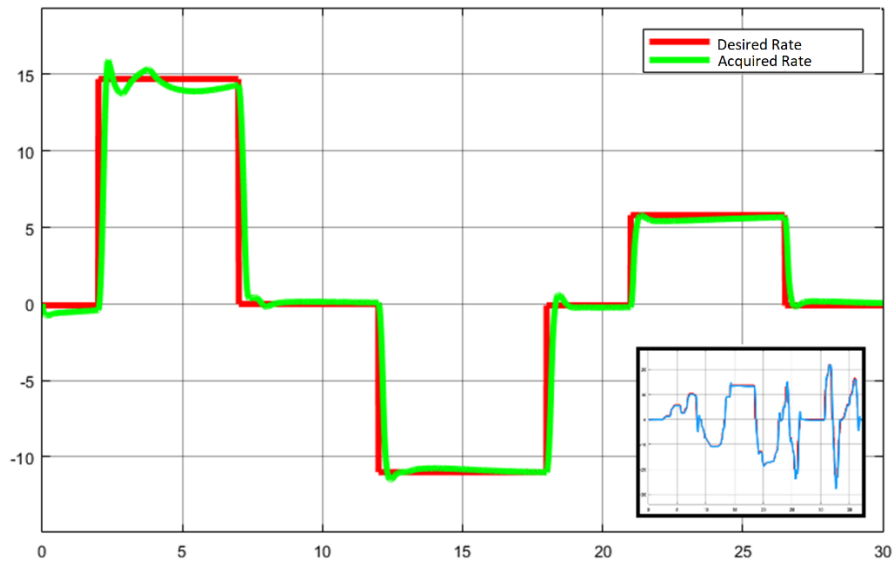


Figure 13: H^∞ controller maneuver results

Especially 2-level bang-bang controller is not appropriate for the inner loop control of high maneuverable fighter aircraft as compared multivariable control methods. As a less important control authority section like navigation control as outer loop control, Bang-Bang control can be applied.

References

- Gaétan Dussart, V. P. (2018). Flight Dynamic Modelling and Simulation of Large Flexible Aircraft. *Intech*, 49 - 72.
- General Electric. (2019). *F110-GE-129 Turbofan Engines Datasheet*. Ohio: GE.
- Skogestad, S., & Postlethwaite, I. (2001). *Multivariable Feedback Control Analysis and Design*. John Wiley & Sons.
- Stevens Brian L, a. F. (1992). *Aircraft Control and Simulation*. New York.
- Wei, Q. (2013). Bang-Bang Control Applied in Airfoil Roll Control with Plasma Actuators. *Journal of Aircraft*, 670 - 676.
- Zavoli, A. (2013). *Indirect Optimization of Bang-Bang Control Problems and Applications to Formation Flying Missions*. Rome.
- Zipfel, P. H. (2007). *Modelling and Simulation of Aerospace Vehicle Dynamics*. Gainesville, Florida: AIAA Education Series.