QUANTIC BEHAVIOR OF TURBULENCE

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We seem to be left at present with the loose idea that

whenever oscillations in space are coupled with

oscillations in time through a dispersive relation,

we expect the typical effects of dispersive waves.

G.B. Whitham

(Linear and Non Linear Waves, p:369)

ABSTRACT

This paper is the third one as related to the application of a wave number definition in turbulence [Çıray C.,1980 and 2017]. This definition expresses the fluctuating component of the instantaneous velocity as the group velocity. Ultimately, it leads to relate the frequency and the wave number that has the capability to represent the totality of the wave number (or frequency) range of the spectrum consistent with various requirements of turbulent flow in question. The proposed definition, associated method of calculation and the ensuing numerical results are explained in more detail in previous papers. The test of the definition and associated calculations were performed for ten different cases that formed the basic material of the first two papers. The second paper contains some discussion on the perception of turbulence as a **discrete** *phenomenon" [Çiray C., 2017]. The present paper emphasizes the nature of turbulence as a dispersive wave train and the role of molecules in the process of formation of eddying behavior. This presentation begins with an "Introduction" reminding some historical works where the turbulence was treated as discrete phenomenon. Then, in "A Paradigm of Turbulence", possible physics for the formation of particles that emphasize the discrete nature of turbulence is described. The reasoning behind the proposed wave number definition completes this section. It is followed by "Calculation of the Spectrum in terms of Wave Number". The requirements to perform the mathematics are explained; it continues with the method of calculation. Only two out of ten Test Cases presented in previous papers are shown just for the sake of completeness and because of the connection of numerical results to the material of this paper. Some features of numerical results are briefly exposed with respect to some critical characteristics of turbulence.*

The paper continues with dispersion relation which is obtained from numerical results. The form of this relation requires two constants specific to the turbulent flow under consideration. One of these constants is reported in tabulated form for ten investigated test cases. In "Discussion" section the discrete nature of turbulence is once more highlighted in view of the overall evaluation of results. "Conclusion" terminates the paper.

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INTRODUCTION

It appears that more attention should be given to molecular behavior of fluids to have a better understand of the physics of turbulence. It may also be interesting and perhaps rewarding to learn as much as possible about the microscopic behavior of turbulence and any possible relation to molecular activity even if the macroscopic state does not seem to pose a problem.

Since the introduction of the notion of turbulence in the historical Reynolds' paper [Reynolds O, 1882], and the use of continuum formulations led to consider turbulence as a continuum phenomenon. Sometimes, statement like "turbulence is a continuum phenomenon" is pronounced [Lele,1994]. Formulations based on continuum approach are, no doubt, satisfactory for many needs of engineering. Turbulence modeling at various levels is applied with the help of powerful computers. These models rely on successful estimation of some so called "constants", mostly valid for the particular solution of the problem in hand. In fact, these constants are correcting coefficients to take care of the lack of knowledge about the physics of turbulence and perhaps to account for some processes or some boundary-initial conditions which are not known. It seems that different approaches, fundamentally different, may help to understand turbulent phenomenon in more depth. It may be appropriate to recall that simple averaging of any random process obscures the details of the physics of randomness. Perhaps a probabilistic approach reflecting more the details of the process may help to advance (one step more) in the study of the physics of turbulence.

The work and results presented here suggest that it is worthwhile to look at the physics of turbulence both from "a dispersive wave and discrete particle(s) aspects" that can be qualified as "quantic behavior".

The perception of turbulence as a discrete phenomenon is used at the onset indepen- dently and successfully by Taylor [Taylor G. I.1915] and by Prandtl [Prandtl L. 1925] in their **mixing length** theory of turbulence. They did not use the word "discrete" at least explicitly as it appears in this paper. Their common approach is that a lump of fluid conserves its identity for a while, where the conserved quantity is momentum in the case of Prandtl approach and vorticity in the case of Taylor (ref. cited).

In the case of Prandtl, there exist two lengths to be concerned. The first one is the size of the lump (he uses the word "ball") and the second is the distance along which the momentum is conserved. These two lengths are merged into a single one and it is named **mixture* length** "ℓ" which at the end of the development leads to the well known turbulent shear stress formula. A proper estimation of mixture length is:

$$\ell = \kappa y \quad \text{by Prandtl and} \quad \ell = K \frac{dU}{dy} \Big/ \frac{d^2U}{dy^2} \quad \text{by von Karman [Goldstein, 1957]}$$

These estimates show that the mixture length corresponds to the distance along which the identity of the ball of fluid is conserved, rather than the size of the ball. Otherwise, the size of eddies would be considered to increase with distance perpendicular to an impervious wall (say in boundary layers) which is in contradiction with experiments. Perhaps this argument may justify the term "mixing length" as it is commonly known, rather than "mixture* length". Indeed, results presented in the sequel show also that larger eddies are **closer** to walls

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NOTE: L. Prandtl uses the word "**mischung länge**" in his ESSENTIALS of FLUID DYNAMICS (1952, Blackie & Son1)**,** translated as **mixture length** (instead of more commonly used expression "**mixing length**") by Miss W. M. Deans.

Indeed, this expression appears in the authorized translation of the book**, (**for example**:** pp:118, line: -14)."Prof. H. Squire was our advisor on English technical expressions" quotes L. Prandtl in the Author Preface on page VI of his book referred above, (dated 1952, Göttingen).

A PARADIGM of TURBULENCE

General: Turbulence, in simplest consideration, is an assembly of randomly moving fluid particles. Simple as the sentence is, it represents the nature of turbulence provided that terms like "randomly moving" and fluid "particles" are clarified.

This paper uses the word turbulence to refer shortly to fluctuating part of turbulent flow, since this part of the turbulent flow is the main subject under consideration. Yet, it is clear that turbulence as such cannot exist and persist without the existence of mean flow. Mean flow or main flow velocity plays a central role, either at turbulence scale (in the sense used here) or at RANS or FANS equations level.

It has to be made clear at the beginning that one dimensional treatment of turbulence kinematics of incompressible fluids forms the material of this paper.

Fluctuating velocity components have critical characteristics. Firstly, they are **integral part** of the instantaneous velocity vector. Secondly, they are **random** in space and time. Thirdly, their magnitudes are **smaller than that of mean velocity** by a factor less than 0.3.

The term "particle" used in this paper is a generic name to represent any coherent fluid agglomeration the size of which may range from sub-scales to dimensions commensurate with those of the domain in which turbulence is generated [Landau-Lifshitz, 1959. pp: 118*, first three lines*]. The logic behind this argument is that boundaries of the main flow limit also fluctuations associated with the coherent structure: either when the motion ceases because of a solid boundary or when a uniform flow (or no flow) becomes the boundary. Therefore, the size of a coherent structure cannot protrude from these boundaries. This paper accepts "coherence" strictly in the sense of meaningful correlation of velocities.

The life span of a particle is limited. This fact is connected with wave like behavior of the motion of fluid particles.

Particles are collection of sub-particles composed of fluid substance which choose to form a particle for a while. Sub-particles are the elements of the particle they form. Sub-particles may also be an agglomeration of sub-sub-particles and this may go down to reach molecules forming the first infant sub-sub-sub-……particles. Hinze considers the smallest sizes of sub particles (or group of molecules) to be smaller than the conventional "micro scale" of turbulence [Hinze J. O. 1959, page: 276]. The author of this paper has not noticed any explanation for stopping smallest size of particles at micro scale of turbulence.

Particles have velocities of order of magnitudes encountered in turbulent flows. While they move with a mean velocity of the region of flow they happen to occupy, they also have their own particular motion known as fluctuations. Fluctuations of a particle are an association of velocities of sub particles that form the particle.

All particles have **limited life span** and their particular motion is **random**. These two items are further studied in the sequel.

How particles are formed? One can begin from random motion of molecules. Molecules of fluids move randomly in three dimensions at velocities of about a few hundred m/s for gases*. Their motion will be influenced in case they are subjected to move altogether in a given direction with a certain velocity. The directional constrain forces some molecules to move closer to each other while they are also forced to reduce their individual random motion (and velocities) and form perhaps the first infant sub-sub-particles. In turn, they form higher order sub-particles in a cascading fashion till the formation of a particle. Now, we have a particle which is a collection of sub-particles having an almost common direction. This is coherence and it entails a discrete characteristic. Yet sub particles do not persist to maintain the coherence indefinitely and they disperse to end the life of the particle.

*Simulations of Argon at 300 K gave an average velocity of 290m/s. [Eneren P Ş., 2016]

Thus formed particles move randomly but at a much reduced level of frequency and perhaps speed (when compared to molecular activity). Velocities and frequencies associated with sub-particles (forming the particle) can be viewed as wave trains each with its own characteristics of wave-like behavior. So randomness exists both in space and time.

The kinetics of the particle is **reflected (or perceived)** as a dispersive wave formed by systems of pilot waves. The togetherness of sub-particles, i.e. the perception of the group as a discrete particle because of coherence, cannot persist as a consequence of dispersive character of such waves, resulting in a limited life span of the particle.

Group Velocity: Since a particle is composed of sub-particles, sub-sub-particles ..and so on, the kinetic energy of a particle can be depicted as the energy of a wave packet during its life span. Indeed, velocities of sub-particles (and of sub-sub particles…..) forming the coherent structure behave like "pilot waves" with close wave numbers and give rise to a wave package appearance. Therefore, the velocity of translation of the kinetic energy of this group is the "group velocity" which is at the same time the translational velocity of the particle [Lamb. 1962], [de Broglie, with reference to Incropera. 1974], [Whitham, 1974]. At the end of its life span, the matter of the particle integrates with the medium of the flow, till when it may be a part of another particle.

But during the life span of the group, the particle is of **discrete** nature. Also during this period, the identity of the particle, perhaps in terms of mass and/or momentum and/or in terms of circulation, is conserved at an accep table level. Such a particle is named an "eddy".

On the other end, the perception of the activity of the eddy through its velocity is reflected as a dispersive wave.

CALCULATION OF SPECTRUM in WAVE NUMBER

The definition of wave number in turbulence studies is generally expressed as:

$$\text{(1)} \qquad k = \frac{\omega}{\overline{U}}$$

where **"k"** and "ω" (or "f") are wave number and circular frequency (or natural frequency) of fluctuating part of instantaneous velocity "U". "$\overline{U}$" is the mean velocity. In view of the weak description of (1) accentuated in low frequency range of turbulence kinetic energy spectra, an alternative definition is proposed [Çıray1980 and 2017] with an appropriate calculation method.

As reported in these two papers, ten test cases were studied which gave a consistent description of turbulence structure for all of them.

Detailed results of two samples from the group of ten Test Cases are presented in the sequel. But first, a summary of the method which relates "f", "u" and "k" will be explained.

Wave number as the consequence of Group velocity: Turbulence paradigm described in previous section concludes with the statement:

The fluctuating velocity is the group velocity of the wavy pattern signaled by the

kinetics of the eddy, i.e:

$$\text{(2)} \qquad u = \frac{d\omega}{dk} \ .$$

"The fluctuating part of the instantaneous velocity

is at the same time the energy transport velocity of the particle".

4

In order to complete the method of calculation adopted in this paper, the following information is needed:

A: Turbulence kinetic energy spectrum should be available at the point of interest. In this study, spectra are obtained from experiments and are presented in Tables of Test Cases in the form of $\Delta G(f; f + \Delta)$ where "f" is the natural frequency instead of circular frequency "ω". $u'^2 \Delta G(f; f + \Delta f)$ is twice the kinetic energy measured in the band of "f" to "$f + \Delta f$". $\Delta G(f; f + \Delta)$ is a non-dimensional quantity.

B: The probability-density function P(y) of instantaneous velocity (or of fluctuating velocity, P(x) is also known.

C: the relative turbulence intensity "I" and the mean velocity "$\overline{U}$" are known.

For convenience, following relations are used in non-dimensionalization:

$$U = \overline{U} + u = \overline{U}\left(1 + \frac{u}{\overline{U}}\right) = \overline{U}\left(1 + \frac{u'}{\overline{U}}\frac{u}{u'}\right) = \overline{U}(1 + Ix) = y\overline{U}$$

where: $x = \dfrac{u}{u'}$; $U = \overline{U} + u$; $I = \dfrac{u'}{\overline{U}}$ and $y = \dfrac{U}{\overline{U}} = 1 + Ix$

The equation used in calculations (Equation 4) is based on the following equality:

(3) $$\int_{-\infty}^{\infty} P(x)u^2 dx = u'^2 \int_{0}^{\infty} G(f)df$$

$G(f)$ is twice the non dimensional kinetic energy per unit mass and unit "f"; therefore, its integral shown in (3), is unity.

It is proposed to apply the relation (3) in intervals "-u" to "-u + Δu" and "u" to "u + Δu".

(4) $$\int_{-x_i}^{-x_{i+1}} P_L(\xi)\xi^2 d\xi + \int_{x_i}^{x_{i+1}} P_R(\xi)\xi^2 d\xi = \int_{f_i}^{f_{i+1}} G(\eta)d\eta$$

This equation states that bands of energy from "-u" to "-u + Δu" and from "u" to "u + Δu" occur at the same frequency band but they **may** occur at different probabilities. Indeed, P_L and P_R represent the left and right branches of the PDF if it is chosen to be skew. The PDF used in the application is an extended form of Maxwell distribution, namely:

(5) $$P(x) = P(1)y^n \exp\left\{A^n\left(1 - y^n\right)\right\}$$

with $y = 1 + Ix$ and:

(6) $$A = \frac{\Gamma\left(\dfrac{n+2}{n}\right)}{\Gamma\left(\dfrac{n+1}{n}\right)} \quad \text{and} \quad P(1) = I\frac{nA^{n+1}}{\Gamma\left(\dfrac{n+1}{n}\right)}\exp\left\{-A^n\right\}$$

Γ(z) is Gamma function of the argument "z" expressed as required in terms of "n" which appears in the PDF. The choice of this PDF is reminded by the probabilistic distribution of magnitudes of simple molecules without spin. It is plausible to think that rather larger velocities associated with molecules are still influential in the randomness of turbulent state (or regime).

5

Other, symmetric or skew PDF's may be tested and surely is worthwhile to do. The PDF expressed in (5) used in this work, appears to yield reasonable results. The constants A and P(1) are obtained from conditions that the PDF has to satisfy. These are: the "0" moment must be unity and the first moment must be the expectation or the mean velocity $\overline{U}$. The second moment "μ_2" is calculated starting from (5). It is at the same time equal to "$1+I^2$" i. e:

(6A)
$$\mu^2 = \frac{\Gamma\left(\dfrac{n+1}{n}\right).\Gamma\left(\dfrac{n+3}{n}\right)}{\left[\Gamma\left(\dfrac{n+2}{n}\right)\right]^2} = 1+I^2$$ .Since "I" ıs known, "n" can

be found. In practice, a Table with columns of μ_2, $I_{calculated}$, A and P(1) is prepared with "n" ranging 5(1)85. It helps to find the appropriate value of "n" once the turbulence relative intensity "I" of the investigated case is known. This known value of "I_{actual}" is matched with "$I_{calculated}$" of the Table; this exercise leads to find the appropriate value "n" of the problem. Consequently, the PDF proper to the problem is completely determined. The solution of algebraic form of (4) for successive intervals, yields a numerical relation between "u" and "f". Then, use of the proposed definition (2) helps to determine the relation between "ω" and the wave number "k", i.e: the dispersion relation. (See the following **Note**).

Note:

1: The author apologizes to squeeze the procedure of calculations in such a short space. Details and calculations are given to a certain length in [Çıray, 2017] and in in [Çıray, 1980].

2: The general expression of μ^κ is not reported for brevity, but (6A).

SOME TYPICAL RESULTS

Results of the prpose definition of wave number and of the procedure outlined in previous sections are presented both in tabulated and graphical form in the following pages. Two of the ten investigated Test Cases are shown as examples just to give an idea and an impression of the results. In reality, the method is applied to ten different Test Cases and reported in [Çıray, 1980 and 2017].

The two typical test cases (Test Case §2 and Test Case §8) appear in the following pages. Some observations can be made:

A: At low frequency the wavelength "$L=\dfrac{1}{k}$" is always commensurate with the intrinsic

dimensions of the domain where the turbulent flow takes place. Indeed, the max wave- length has never exceeded the size of the domain in ten TEST Cases investigated. This results of Test Case §8 is interesting. The measurement point is behind a grid made of 5mm bars with a mesh size of 2.5cm. The exact location of measurement is not known. Very likely it is in the region where turbulence can be considered to be homogeneous and isotropic. It is worth noting that the largest wavelength is 6mm and is almost the same as the diameter of the bars, 5mm, that the grid is made. If the point of measurement is really in the universal equilibrium zone of the spectrum, the turbulence structure at this point must not bear any sign of the boundary condition, hence of the geometry of the grid. Therefore, the equality of the size of largest wavelength with the diameter of grid bars may be accidental.

Data and Characteristics of TEST CASE §2

$\overline{U}$ = 32.22 m/s I_{actual} = 0.0354   n=35   A= 0. 98355565

u'=135 cm/s $I_{n=35}$ = 0.0352   P(1) = 0.4088

| f	ΔG	x	k	L	$P_R(x)$	$P_L(x)$
c/s	-	-	cm^{-1}	cm	-	-
0		-	-	-	0.4088	0.4088
	0.02041					
1		0.4369	0.1308	7.645	0.4562	0.3051
	0.100					
2		0.4968	0.2305	4.338	0.4540	0.2901
	0.294					
5		0.6219	0.4801	2.083	0.4407	0.2597
	0.0237					
10		0.6983	0.8327	1.20 1	0.4267	0 2418
	0.0452					
20		0.8136	1.448	0.6904	0.3971	0.2160
	0.0820					
50		0.9734	3.011	0.3321	0.3412	0.1833
	0.0.866					
100		1.116	5.239	0.1909	0.2810	0.1572
	0.1710					
200		1.345	9.022	0.1108	0.1778	0.1213
	0.2430					
500		1.642	18.37	0.0544	0.0699	0.0851
	0.0926					
1000		1.812	31.84	0.0314	0.0330	0.0691
	0.0878					
2000		2.006	56.23	0.0178	0.0108	0.0541
	0.1140					
5000		2.305	121.0	0.008	0.0010	0.0369
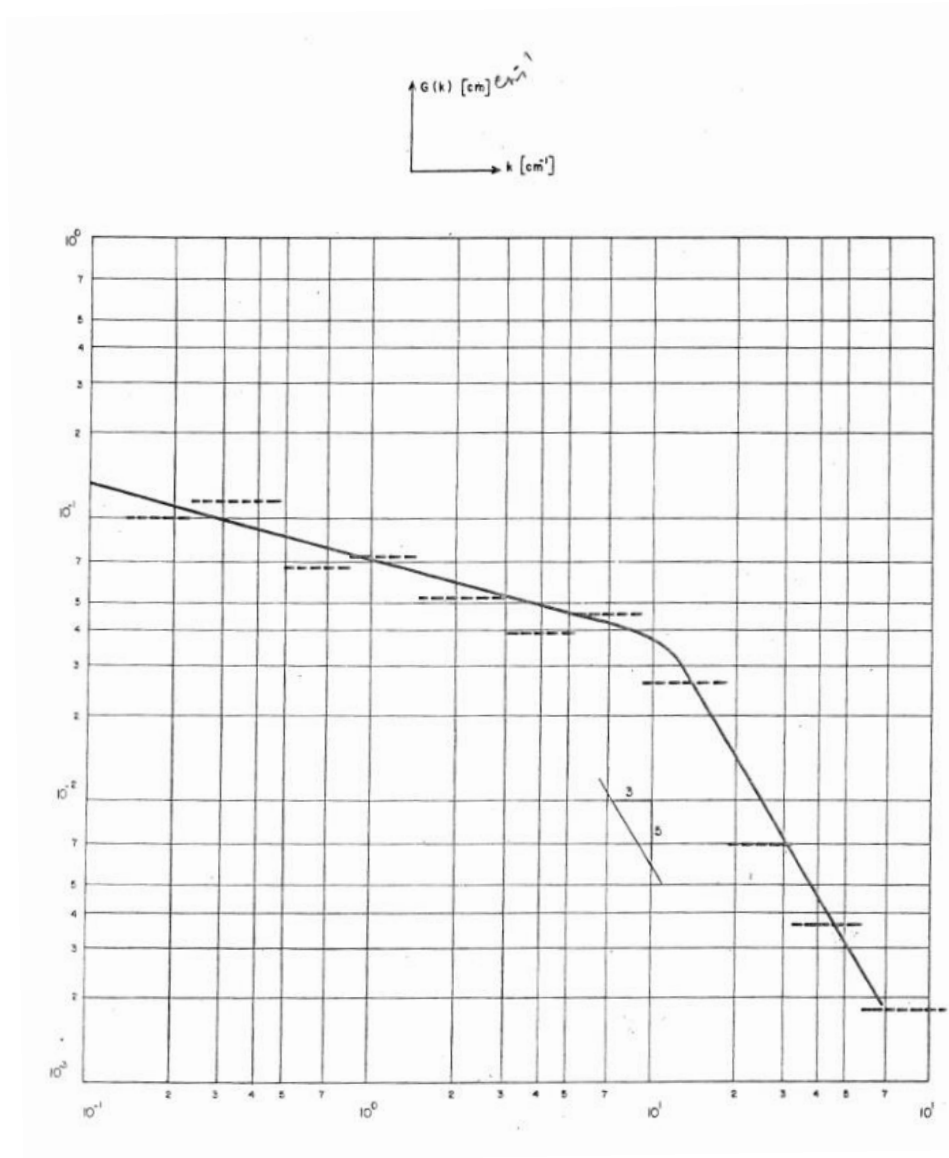
Figure 1: One dimensional spectrum for Test Case §2.

Test Case §2 (an example of series §1, §2, §3) belongs to a point above a 30 cm chord NACA 0012 airfoil placed in a wind tunnel with AOA=12°. The pressure gradient was maintained at zero within test section with the help of a false wall. In TEST CASE 2 the hot-wire was placed at 0.67cm from the upper surface and 4.0cm from leading edge of the airfoil*

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low pass filters of DISA 55D25 Auxiliary unit, 55D35 rms meter and HP 322 Dual Channel recorder, [Çıray,1980].

Data and Characteristics of TEST CASE §8

$\overline{U}$ =12.25 m/s I_{actual}= 0.0200   n=63   A= 0.991485

u' =24.5 cm/s $I_{n=63}$= 0.0199   P(1) = 0.4104

| f	ΔG	x	k	L	$P_R(x)$	$P_L(x)$
c/s	-	-	cm^{-1}	cm	-	-
0				-- -	0.4104	0.4104
	0.00298					
1		0.2270	1.666	0.6002	0.4502	0.3563
	0.00298					
2		0.2841	2.668	0.3748	0.4565	0.3418
	0.00893					
5		0.3854	4.963	0.2015	0.4630	0.3157
	0.0149					
10		0.4862	7.900	0.1266	0.4625	0.2899
	0.0298					
20		0.6157	12.55	0.797	0.4504	0.2579
	0.0893					
50		0.8469	23.05	0.0434	0.3945	0.2055
	0.130					
100		1.069	36.41	0.0275	0.3053	0.1623
	0.186					
200		1.317	57.87	0.0173	0.1879	0.1227
	0.290					
500		1.657	109.50	0.0091	0.0607	0.0821
	0.134					
1000		1.893	181.7	0.0055	0.0177	0.0614
	0.0 595					
2000		2.058	311.3	0.0032	0.0057	0.0500
	0.0513					
5000		2.231	669.4	0.0015	0.0012	0.0401
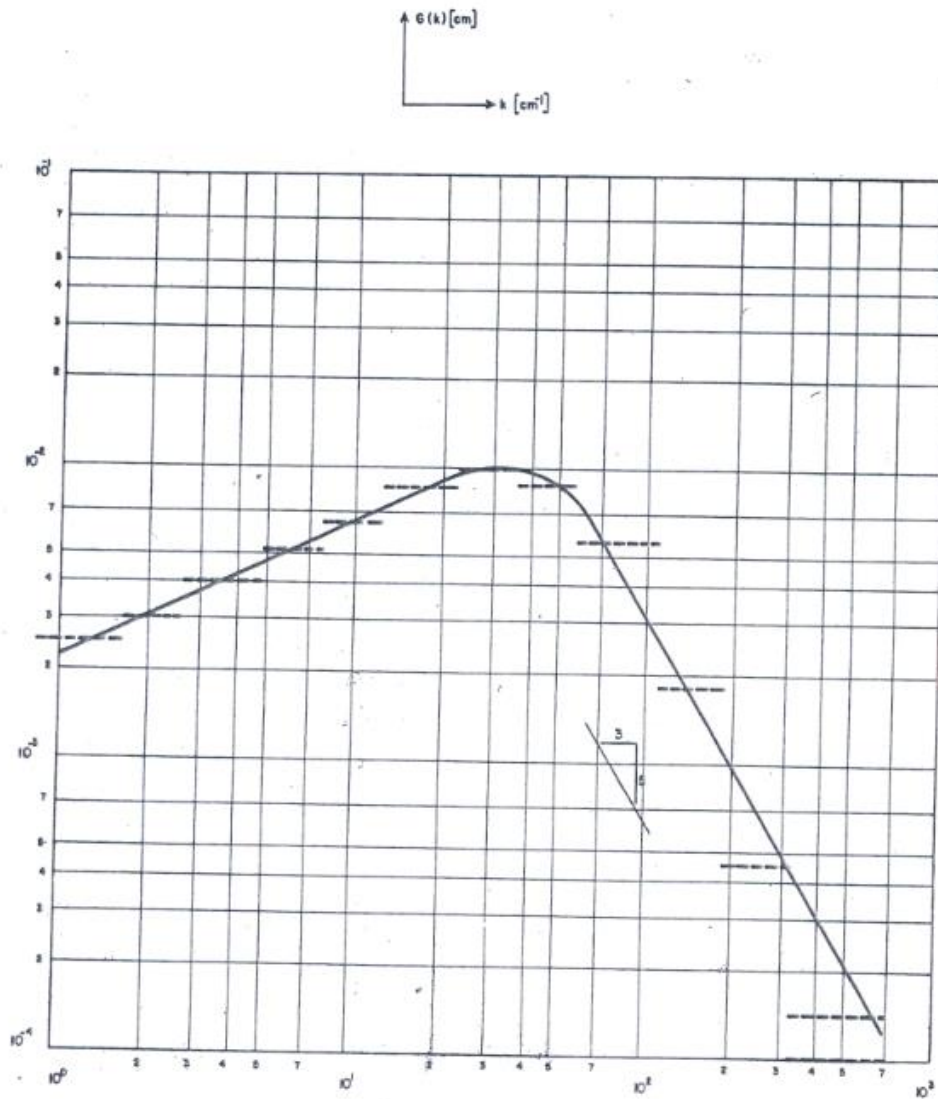
Figure 2. One dimensional spectrum for Test Case §8

Test Case §8 (and Test Cases §9 and §10 which are not reproduced here) is calculated with the help of data obtained from available literature

The information belongs to a turbulent flow behind a grid made of 5mm cylindrical bars with a mesh size of 2.5cm. The data belongs to a paper by Favre, [Hinze, pp: 61, Figure1.18].

B: At highest frequency range the dependence of spectral values to "$k^{-\frac{5}{3}}$" can be seen in the graphical presentations of sample Test Cases. This dependence repeats in all of the remaining eight Test Cases.

C: Spectra show the intrnsic peculiarities when they are presented with respect to wave number, contrary to representation w.r. to frequencies wher spectra remain similar for all cases.

DISPERSION RELATION

Dispersion relation, that is the relation between the frequency "f" and the wave number "k" can be obtained from numerical results displayed in tables of Test Cases. These relations are shown in graphical form for eightof ten Test Cases in Figure 3. The frequency range is: $1 < f < 10^4$ c/s.
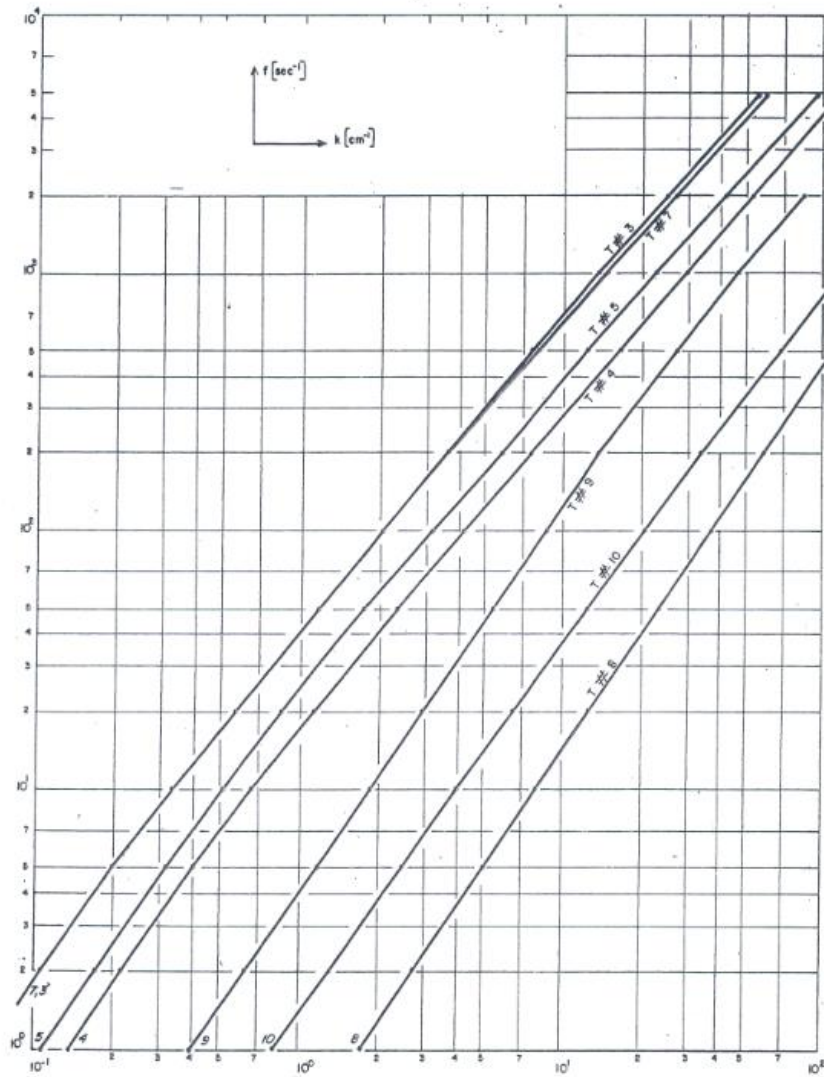

Figure 3: Dispersion relations for eight of studied ten Test Cases.

(Cases §1 and §6 are omitted for a clearer figure)

The straight line relationship between "f" and "k" in log-log coordinates seen in Figure 3, suggest the dispersion relation to be:

(7)
$$f = Ck^{\beta}$$

Two constants "C" and "β" can be determined either from Tables of Test Cases or from Figure 3. With the help of proposed definition, ie: 2, one obtains:

(8)
$$u = \beta Ck^{\beta-1} = \beta \frac{f}{k}$$

It seems necessary to point out the difference between "u" and "$\frac{\omega}{k}$". Whereas the former is the group velocity of group of waves centered around the wave number "k" and frequency "f", the latter is the phase velocity of the solitaire wave with wave number "k" and the same frequency "f", that is one of the pilot waves. The power "β" is reported below for two ranges of "k" or "corresponding "f", for which the log-log linearity appears as a good fit*.

DISPERSION COEFFICIENT (β)

TEST §	Lower Range	Higher Range
	1-100 (or 500) c/s	500-5000 c/s
1	1.8450	1.224
2	1.2438	1.221
3	1.4125	1.1625
4	1.4641	1.1682
5	1.4837	1.1520
6	1.8215	1.1325
7	1.5221	1.1325
8	1.4793	1.2723
9	1.5083	1.2107
10	1.4577	1.1628

* Values of "β" in this Table are inverses of corresponding parameter in [Çıray 1980]

N.B: It has to b noted that "f" in Figure 3 is natural frequency whereas "ω" in formula (7) is circular frequency.

DISCUSSION

A: It is observable that no wavelength for any integer frequency exceeds the geomet-rical dimension of the flow region. Yet, at high frequency range, $k^{-\frac{5}{3}}$ dependence of the spectrum is recovered.

Use of the relation (1) yields non-acceptable wave lengths in low frequencies. For example in TEST CASE 8, within the range 1 to 100c/s, the corresponding wave lengths should range from 12.5m to 12.5cm if (1) is adopted. Landau and Lifshitz [Ref. cited] on the other end remedy this by taking the largest intrinsic dimension of the domain as the starting point and find the corresponding frequency as the frequency of the largest eddy. If, in TEST CASE 8 the mesh size (= 2.5cm) is taken as the intrinsic largest dimension, then, the corresponding lowest frequency (again adopting (1)) becomes 500c/s. Yet, there exist sizeable energy in the range 1-500c/s as observed in the measured spectrum.

It can be asserted that the group velocity understanding of the fluctuating component of turbulent instantaneous velocity, gives an acceptable wavelength representation for the **totality** of the spectrum.
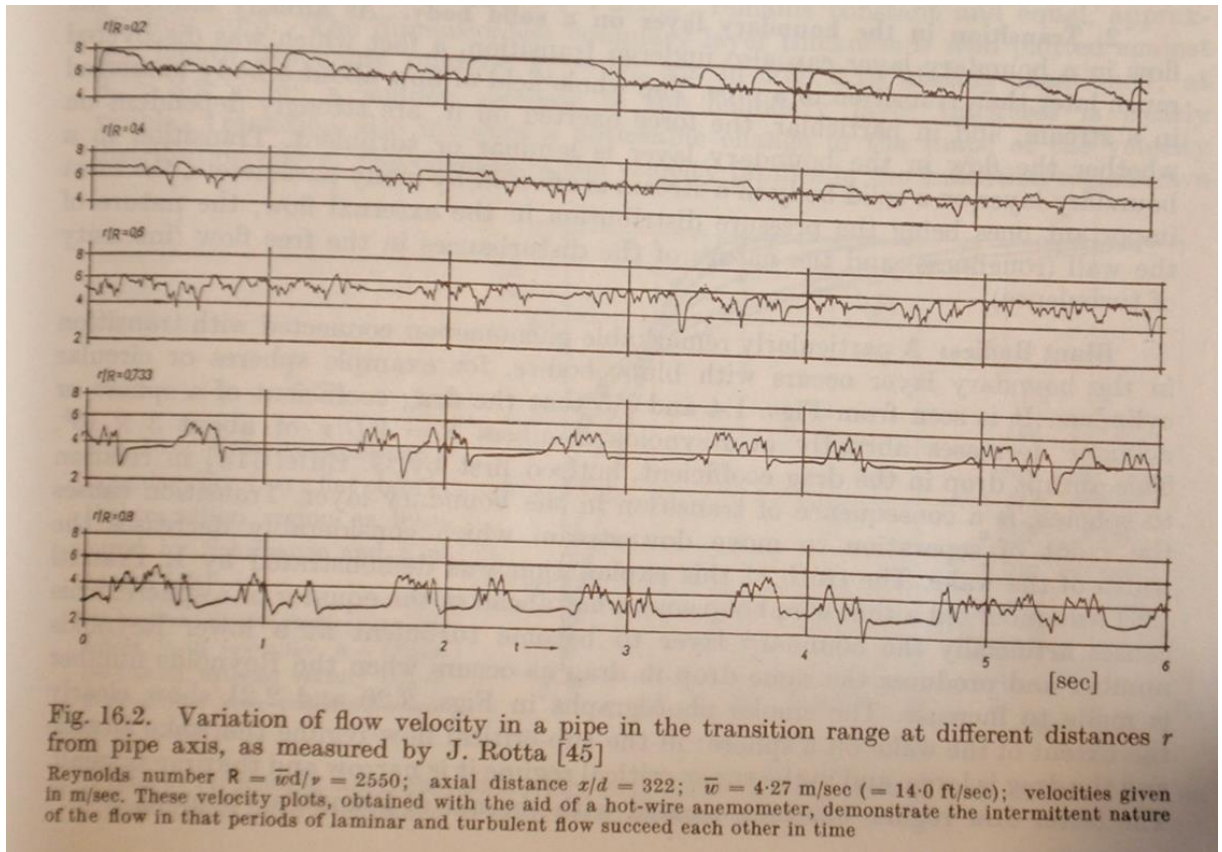

Fig. 16.2. Variation of flow velocity in a pipe in the transition range at different distances r from pipe axis, as measured by J. Rotta [45]

Reynolds number $R = \bar{w}d/\nu = 2550$; axial distance $x/d = 322$; $\bar{w} = 4\cdot27$ m/sec ($= 14\cdot0$ ft/sec); velocities given in m/sec. These velocity plots, obtained with the aid of a hot-wire anemometer, demonstrate the intermittent nature of the flow in that periods of laminar and turbulent flow succeed each other in time

Figure 4 [Schlichting, 1960]

(This is a very low Reynolds number air flow ($R_E = 2550$) in a pipe of approximately 0.6cm in diameter.)

B: Numerical results in TABLES of TEST CASES show that the magnitude of fluctu-   ating velocity increases by a factor of ten with increasing frequency (or wave number) whereas the corresponding probability decreases severely. Perhaps, this observation can best be seen in

13

Figure 4 where the development of intermittency is shown along the radius of a pipe flow at a highly low Reynolds number. Indeed, the sparseness of high frequencies shows that they occur at low probability but with large amplitudes.

C: The magnitude of turbulence fluctuations may be expressed in terms of wave num- ber through the dispersion relation. Indeed, in conjunction with the definition (2), one can obtain:

(8-repeat)
$$u = \beta C k^{\beta-1}$$

or shortly:
$$u = \propto k^{\beta-1}$$

We define "α" as: $\alpha = \dfrac{1}{\beta - 1}$. Then:

(10)
$$u^\alpha \propto \frac{1}{L}$$

where, "L" is the wave length. The value of "α" ranges as:

$$1. <\alpha < 4.0 \quad \text{for} \quad 1< f<100(\text{or } 500)c/s \quad \text{and}$$

$$3.7 <\alpha < 7.5 \quad \text{for} \quad 500< f<5000c/s$$

On the other end, the "Kolmogorov- Obukhov law" states:

(11)
$$u^3 \propto \epsilon L$$

This relation (shown with symbols of this paper), and its derivation can be found in [Landau-Lifshitz, 1959; pp:121].

The difference is noticed on the RHS of relations (10) and (11). Indeed, whereas (11) corresponds to dissipation and can be obtained starting from the definition of dissipation, the relation (10) is strictly kinematical.

D: The rate of decay of kinetic energy of turbulence is reported to be:

(12)
$$\frac{du^2}{dt} = -\frac{Au^3}{L}$$

on grounds of experimental evidences. ("A" is a number of the order of unity (which may vary slightly with the time of decay and the initial conditions of the turbulence and the choice of L) quotes Batchelor, [Batchelor, 1960, p: 103]. The number "A" is:

(13)
$$-\frac{\dfrac{du^2}{dt}}{\dfrac{u^3}{L}} = A$$

Left hand values side calculated from experimental results are plotted with respect to distance from grid. The figure shown below is obtained.

It is possible to express the energy transport with a decreasing trend as;

(14a, 14b)
$$\frac{du^2}{dt} = -u_G \frac{du^2}{dx} \quad \text{or} \quad \frac{du^2}{dt} \propto -u_G \frac{u^2}{L}$$

since the energy transport velocity on the right hand side must be group velocity.
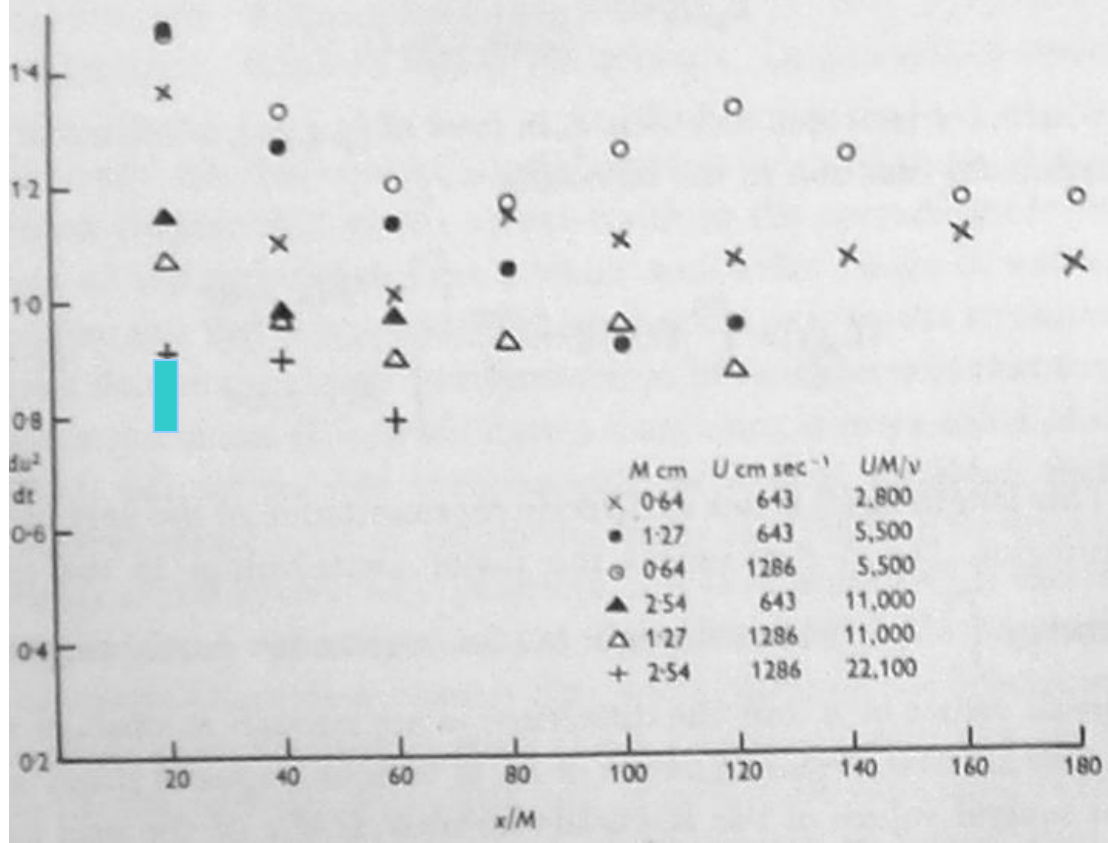

Figure 5. [Batchelor,1960, p:106]*

Symbolically, whereas "u_G is the group velocity, "u" indicates a measure of kinetic energy. The last relation is also:

(15)
$$\frac{1}{u^2}\frac{du^2}{dt} \propto -\frac{u_G}{L}$$

Using (8) : $u_G = \beta \dfrac{f}{k}$.   If $u \approx \dfrac{f}{k}$, then $u_G = \beta u$ and (15) becomes:

$$-\frac{\dfrac{1}{u^2}\dfrac{du^2}{dt}}{\dfrac{u}{L}} = \beta$$

in terms of symbols of the reference cited. It appears that "A" of (13) and "β" of (15) are quite alike, if not the same. The range of the number "A" in Figure (5) is:

_____

* It is taken from [Batchelor, 1960]. The data is the work of [Dryden, 1943]

$$0.8 \leq A \leq 1.50$$

The range of "β" is:

$$1.13 \leq \beta \leq 1.85$$

The data in Figure* (5) is obtained for decaying period of turbulent flow behind a gird.

Data related to "β" is obtained for almost the whole range of spectra and for a variety of flows including grid turbulence, namely Test Case §8.

Therefore, the manner of transmitting the kinetic energy of turbulence, as formulated In (13), is not restricted to decay period, but to all wave numbers and type of flows that are considered in this study.

E: Life span of an eddy: Since the existence of an eddy begins with the formation of the dispersive wave of a certain frequency and ends with the dispersion of the wave group, the life span of the eddy is as long as the the existence of the associated wave d group. Therefore, the life span of the eddy can be estimated by dividing the wave-length by group velocity.

The two test cases given above yield the following values.

	1c/s	5000 c/s
Test Case §2	0.13s	26 µs
Test Case §8	0.11s	27 µs

F: The perception of wave-like and particle-like behavior is not uncommon in nature. The dispersive wave-group consideration appears as a valuable tool.

G: In this paper, the method uses a PDF which is an extended form of Maxwell distribution. Further work on the same line will be highly usefu:.

1: Other PDF forms may be used within the method described here.

2: A search for a more fundamental approach may be devised to find the appropriate

   PDF of the cases investigated.

3: It seems worthwhile to study the tail of energy spectrum for higher frequency using the method adopted in this paper. It would be highly interesting to investigate frequencies in the band of Giga hertz.

CONCLUSION

A paradigm of turbulence is depicted where:

a: Discrete particles of varying sizes but of limited life span are formed by sub particles . The particles maintain their identity for a certain while They are "eddies" of turbulent flows.

b: Velocities associated with sub (sub-sub, …..) particles (forming the eddy) are wave trains of random character but highly correlated which helps to define the eddy .

c: The velocity perceived as related to the motion of eddy is the group velocity of the wave trains of sub, sub-sub, … particles. Therefore, it is at the same time the fluctuating component

of instantaneous velocity and the energy transport velocity since the eddy is at the same time an energy package.

d: The nature of turbulence bears discrete and dispersive wave characteristics. Yet, particles are neither of perpetuating kind nor of constant and same mass. They form and disappear in a random fashion but consistent with dispersive wave behavior. This nature of turbulence can be named "quantic behavior of turbulence".

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