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AERODYNAMIC THRUST OPTIMIZATION WITH NONLINEAR MODELING FOR THE LEADING EDGE VORTEX OF A FLAPPING WING

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ABSTRACT

A nonlinear model for the optimization of the aerodynamic thrust of flapping wing MAVs is implemented. The method is developed for harmonic motions in general but for the thrust optimization the amplitudes for a simple harmonic motion of a pitching-plunging airfoil are determined. The relation between the pitch-plunge amplitudes are solved in an iterative manner as a nonlinear eigenvalue problem. The maximum eigenvalue of the problem gives the maximum thrust and the associated eigenvector gives the amplitudes for the motion. The method is applied for a finite wing also, using the available Wagner function as indicial admittance together with the circulation provided with the leading edge vortex. As the optimum solution is obtained the Wagner function determines the phase lag between the motion and the aerodynamic response of the wing to the motion. The method, with a little modification, is also applied to the cases having no constraint on the pitch angle.

INTRODUCTION

In recent years, the interest in the flapping wing MAVs has become quite intense because of less noise production and more efficient performances as opposed to the MAVs with fixed wings. The unsteady aerodynamic predictions for the thrust, lift and moments take considerably long time using numerical and experimental techniques. On the other hand fast methods are preferred for the flight dynamics and control applications. The fast optimization of the aerodynamic thrust also plays an important role in MAVs design. In this study, quick results are obtained while computing the leading suction velocity and the lift, via the Wagner function [Bisplinghoff, et.al, 1996], which are necessary to calculate the aerodynamic thrust of a flapping wing.

The thrust optimization of a flapping airfoil is first studied in details by [Tuncer ve Kaya, 2005] via 2-D Navier-Stokes solutions. Afterwards, their study is applied for the non sinusoidal periodic motions [Kaya ve Tuncer, 2007]. For the general simple harmonic motions, including morphing, the thrust optimization based on potential theory for the thin profiles is studied by [Walker, 2012] and [Gülçat, 2017].

The aim of this study is to calculate the optimum thrust with a nonlinear modeling of the leading edge vortex of a flapping wing in a periodic motion. The thrust optimization is reduced down to a solution of a nonlinear eigenvalue problem involving the amplitudes of the pitch and plunge where both amplitudes are constraint. As an

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additional application no constraint on the pitch amplitude is also made. The establishment of the nonlinear eigenvalue problem, the solution technique and the results will be presented in following sections.

METHOD

The aerodynamic thrust force for a pitching plunging airfoil is expressed in terms of leading edge suction velocity S and the lift L as follows [Garrick, 1936]

$$S = -(\pi\rho P^2 - \alpha L) \quad (1)$$

The Wagner function $\varphi(s) = 1 - a_1 e^{-b_1 s} - a_2 e^{-b_2 s} = 1 - 0.165 e^{-0.455s} - 0.335 e^{-0.3s}$ provides us with the leading edge suction velocity as shown in [Gülçat, 2017]

$$P(s) = \sqrt{2} \left[w(b/2, s) \varphi(0) + \int_0^s w(b/2, \sigma) \varphi'(s - \sigma) d\sigma \right] - \dot{\alpha} b/2, \quad (2)$$

and the sectional lift coefficient as

$$L(s) = \pi\rho b \left[\vec{h} + U\dot{\alpha} \right] - 2\rho U b \pi \left[w(b/2, s) \varphi(0) + \int_0^s w(b/2, \sigma) \varphi'(s - \sigma) d\sigma \right] \quad (3)$$

The Equation for the pitching plunging airfoil is $z_a(x, t) = -h(t) - \alpha(t)(x - a)$, and the corresponding downwash becomes

$$w(b/2, t) = \frac{\partial z_a}{\partial t} + U \frac{\partial z_a}{\partial x} = -\dot{h}(t) - \dot{\alpha}(t)[b/2 - a] - U\alpha(t) \quad (4)$$

Wherein, a is the pitch point location. Now, we can write the expression for the quasi steady circulation as follows

$$\Gamma_{qs}(t) = bUC_{ls}(\alpha(t)) + \pi b(b/2 - a)\dot{\alpha}(t) + \pi b\dot{h}(t) \quad (5)$$

Here, $C_{ls} = A \sin 2\alpha$, [Taha, et.al. 2014] is the sectional lift coefficient at high angles of attack including the effect of the leading edge vortex and it is the replacement for the convective term in (4). Comparing Equations (4) and (5) gives us the relation between the downwash and the circulation as follows

$$w(b/2, t) = -\Gamma_{qs}(t) / \pi b \quad (6)$$

If we use Equation (6) in (2) and (3), the leading edge suction velocity and the lift read as

$$P(s) = -\sqrt{2} / (\pi b) \left[\Gamma_{qs}(s) \varphi(0) + \int_0^s \Gamma_{qs}(s) \varphi'(s - \sigma) d\sigma \right] - \dot{\alpha} b/2 \quad (7)$$

and

$$L(s) = \pi\rho b \left[\vec{h} + U\dot{\alpha} \right] + 2\rho U \left[\Gamma_{qs}(s) \varphi(0) + \int_0^s \Gamma_{qs}(s) \varphi'(s - \sigma) d\sigma \right] \quad (8)$$

Expanding (5) into the series in α we obtain the approximate expression for the steady lift coefficient as follows

$$C_{ls}(\alpha) \cong A(2\alpha - 4\alpha^3/3 + 4\alpha^5/15)$$

The relation between the steady lift coefficient and the quasi steady circulation is now used in (7) and (8) to obtain the aerodynamic thrust, Equation (1), as a nonlinear equation in terms of h and α which are function of time. Then we can take the time average of this equation for over a period to have

$$\bar{S} = -(\pi\rho \bar{P}^2 - \bar{\alpha} \bar{L}) \quad (9)$$

in terms of the amplitudes.

If we let $h = -\bar{h} \cos(\omega t)$ for plunging and $\alpha = -\bar{\alpha} \cos(\omega t + \phi)$ for pitching the average thrust in terms of the amplitude vector, $\bar{Q} = \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix}$, reads as

$$\bar{S} = \pi \rho U^2 b \{Q\}^T [H] \{Q\} \quad (10)$$

In order to find the maximum value of the thrust amplitude we have to set the gradient of (10) to zero to obtain an equation for the amplitude vector Q. However, this gives us a trivial solution because of H being a non-singular matrix. In order to remedy this, we need to put a constraint on the amplitude vector while setting the gradient of (10) into zero. Hence, we have now

$$\nabla \bar{S} = \vec{0} \quad \text{and} \quad \{Q\}^T \{Q\} \leq 1 \quad (11a,b)$$

If we let $f(Q) = \{Q\}^T \{Q\} - 1 = 0$, and apply the concept of Lagrange multiplier, λ , we have

$$T(q, \lambda) = \pi \rho U^2 b \{Q\}^T [H] \{Q\} - \lambda f(Q) \quad (12)$$

without changing the value of T, i.e. T=S. The gradient of T together with the constraint gives

$$\begin{aligned} \pi \rho U^2 b [H] \{Q\} - \lambda \{Q\} &= 0 \\ \{Q\}^T \{Q\} - 1 &= 0 \end{aligned}$$

which is a non-linear eigenvalue problem in the following matrix form

$$\begin{bmatrix} a_{11} - \lambda & a_{21} \\ a_{12} & a_{22} - \lambda \end{bmatrix} \begin{pmatrix} \bar{h} \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (13a,b)$$

$$\bar{h}^2 + \bar{\alpha}^2 - 1 = 0$$

In Equation (13), except a_{11} all entries depend on $\bar{\alpha}$. The contribution of plunging to the leading edge velocity is denoted with P_h , to pitching is P_α , contribution of the lift to the plunging is L_h , and finally contribution to pitching is shown with L_α . Hence, the entries of the matrix read as follows: $a_{11} = \pi P_h^2$, $a_{12} = a_{21} = (1 - 2\bar{\alpha}^2/3) 2\pi P_h P_\alpha - L_h/2$, $a_{22} = 16\pi(1 - 2\bar{\alpha}^2/3)^2 P_\alpha^2 - 2(1 - 2\bar{\alpha}^2/3)L_\alpha$. Now, with (13) we have a non-linear system with 3 equations for 3 unknowns. The solution of this non-linear eigenvalue system gives the optimum thrust for the maximum eigenvalue and the corresponding eigen vector provides the amplitudes for the motion.

Finite Wings

For the finite wings, the spanwise integration of the sectional value of the lift given with (7-8) provides the wing's lifting force. The time wise integration of the Duhamel integral is performed Matlab as shown in the Appendix. Then, Equations (11a,b) for the wings can be put into the following form

$$\begin{aligned} \pi \rho U^2 A/2 [H] \{Q\} - \lambda \{Q\} &= 0 \\ \{Q\}^T \{Q\} - 1 &= 0 \end{aligned} \quad (14a,b)$$

Here, A is the wing area. For rectangular wings the Wagner function effect remains the same at every section to keep the Equations (13a,b) the same. For the case of elliptical wings the Wagner function is altered to cause the changes in the coefficient matrix. There exist, however, expressions for the Wagner functions for the elliptical wings of the following aspect ratios [Bisplinghoff, et.al, 1996]:

$$\text{For } AR = 3 \quad \varphi(s) = 0.60 - 0.17e^{-0.54s}$$

$$\text{For } AR = 6 \quad \varphi(s) = 0.74 - 0.267e^{-0.381s}$$

No constraint on AoA

When we put a constraint on angle of attack with (13-b), we end up with low values for it. For a non-linear application we have to let $\bar{\alpha}$ be free so that the optimization is realistic. This can approximately be achieved by fixing the plunge amplitude while letting the pitch to be free in the first and second rows of (13a) with no λ being involved. Hence, we get

$$a_{11}\bar{h} + a_{12}\bar{\alpha} = \pi P_h^2 \bar{h} + \left[(1 - 2\bar{\alpha}^2 / 3) 2\pi P_h P_\alpha - L_h / 2 \right] \bar{\alpha} = 0 \quad (15)$$

If we fix the amplitude of plunge, \bar{h} , arbitrarily, we can find the optimum value for the AoA amplitude from (15). We have to note here that the second row of (13-b) may give slightly different result obtained by (15) since the matrix \mathbf{H} of (12) is non-singular. Hence, we can call this partial optimization!

APPLICATIONS

The method based on the non-linear modeling of the leading edge vortex is first implemented for a simple harmonically pitching plunging thin airfoil. The circulation created by the leading edge vortex is given as $C_l^{qs} = 1.833 \sin 2\alpha$ [Taha, et.al. 2014], and the sine term is expanded into the series in terms of angle of attack, Figure 1 shows the variation of quasi steady lift in terms of powers of angle of attack and shows with the expression given with sine. As seen in (Figure 1) the linear

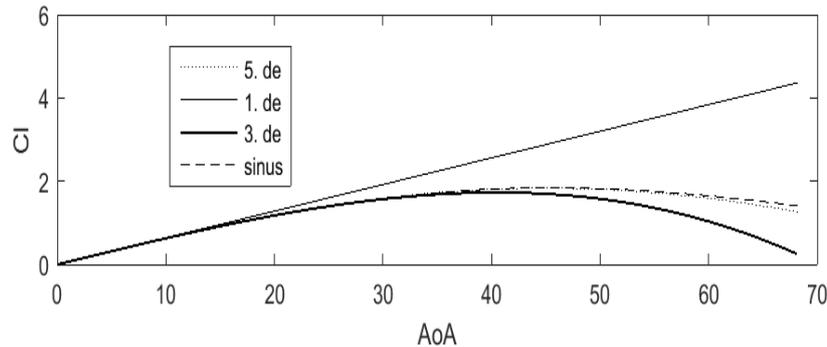


Figure 1: Linear, non-linear and the sine curves for the quasi-steady lift

approach represents the sine curve in 0^0 - 20^0 range, whereas the 3rd degree approach is good for 20^0 - 50^0 . The iterative solution of the Equations (13a-b) with the 3rd degree approach gives the value of maximum average thrust $S = 3.63$, in non dimensionalized form. The corresponding motion is determined from the corresponding eigenvector as

$$\begin{aligned} h(t) &= -0.48 \cos \omega t \\ \alpha(t) &= -49^0 \cos(\omega t + \pi / 2) \end{aligned} \quad (16a-b)$$

In (16a-b) the phase difference between the plunge and pitch is taken as 90^0 together with $\omega = 1$. The iterations begin with the values taken from the linear solution, and continues with these new values substituted in their proper places in the coefficient matrix. The linear solution gives the maximum value of the thrust as $S = 4.26$. The iterations, on the other hand, converges to 4 digit accuracy [Matlab, 2015]. If we increase the accuracy with the 5th degree approximation we obtain the value of the maximum thrust to read as $S = 3.66$. This means increasing the number of non-linear terms does not improve the accuracy of the thrust value that much. In (Figure 2), the variation of the quasi steady circulation with respect to 3rd and 5th degree representations and also for the sine dependence. Accordingly, the 5th degree approximation and the sine representation gives almost the same circulation change by with time.

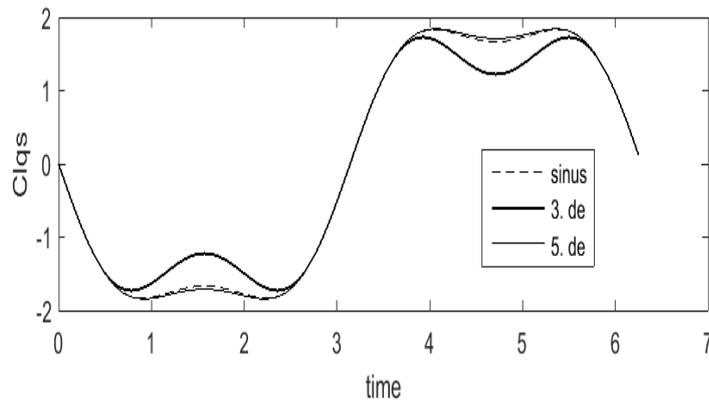


Figure 2: Change in quasi steady circulation , Γ_{qs} , by time (sine -----, 3rd and 5th___ degree approximations)

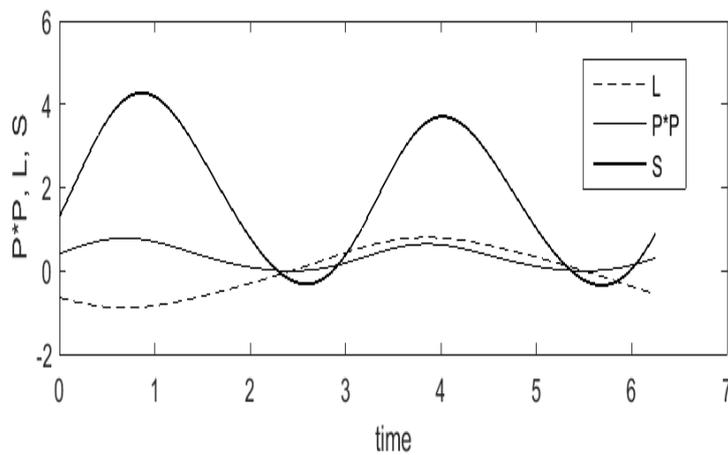


Figure 3: Time variation of maximum aerodynamic thrust S, lift L, and P*P

The time variation of maximum thrust S, associated lift L and the effect of the leading edge suction force P*P, in non-dimensional forms, are shown in Figure 3. As seen from Figure 3, the propulsive force S is positive most of the period covered except for a very short duration at around the half and full period locations.

Elliptical wing

For the elliptical wing the aerodynamic thrust S for a wing with aspect ratio of 3 is obtained from Equations (14a,b) as shown in Figure 4 wherein the shapes of the curves are similar to that of given in Figure 3.

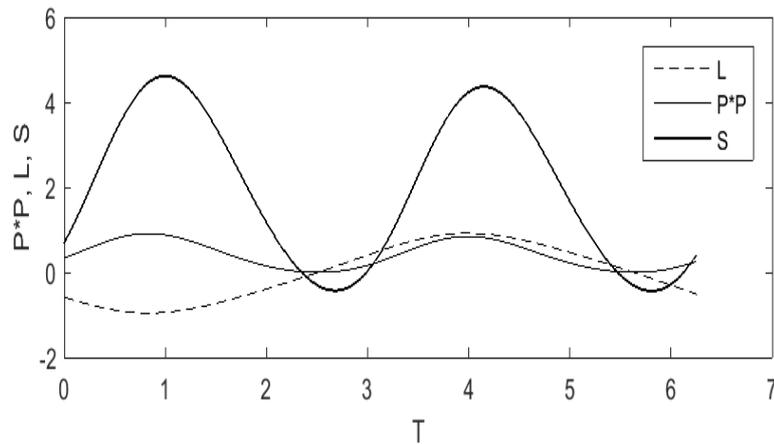


Figure 4: Time variation of maximum aerodynamic thrust S, lift L, and P*P for an elliptical wing with AR=3

Here, the results obtained for Figure 3-4 are non-dimensional quantities. In Figure 3 for non-dimensionalization the half chord is used for a characteristic length, whereas for Figure 4 half the wing area is taken as the characteristic area. In both figures the free stream speed is employed as the characteristic speed.

No constraint on AoA

We employ (15) for the case of no constraint on $\bar{\alpha}$. Hence in terms of the plunge amplitude (15) becomes

$$\bar{\alpha} = -\pi P_h^2 \bar{h} / [(1 - 2\bar{\alpha}^2 / 3) 2\pi P_h P_\alpha - L_h / 2] \quad (17)$$

Assigning values for \bar{h} in (17) makes the equation a cubic polynomial to be solved for $\bar{\alpha}$. The method here is tested with the plunge values close to the value taken from (16a). Results then found for the optimum AoA values are given in the following **Table1**:

Table1: Optimum AoA values for various \bar{h}

\bar{h}	$\bar{\alpha}$ (deg)
-0.40	42
-0.48	54
-0.50	58
-0.55	71

It seems from **Table1** that as the plunge amplitude increases with small amounts, the optimum pitch amplitudes increase considerably.

CONCLUSION

A non-linear model of the leading edge vortex is implemented to find the maximum aerodynamic thrust for a flapping wing. The implementation is based on a solution of a non-linear eigenvalue problem. The procedure is based on an iterative solution which converges to desired accuracy in 4-5 iterations.

It is worth noting that the quasi steady formula gives the maximum lift at 45° angle of attack, whereas the nonlinear optimized unsteady solution gives the maximum lift at 49°.

The aerodynamic thrust optimization employs the Wagner function to consider the time lag between the flapping motion and the aerodynamic response of thin profiles or wings with a constraint on flapping. The method is satisfactorily extended to handle the cases without having any constraint on pitch amplitude for more realistic optimization for large AoA. For larger AoA higher degree approximation looks necessary.

Appendix: Duhamel integral with the sine term calculated for the elliptical wing with AR=3.

$$\int_0^t \sin(\sigma) \dot{\phi}(t - \sigma) d\sigma$$

faa=int((sin(x))^1*(0.17*0.54*exp(0.54*(x-t))),x,0,t)

fs(t) =(201*exp(-(3*t)/10))/2180 + (5282933469138125*exp(-(91*t)/200))/849368334410448896 - (45559957544805335149*cos(t))/462905742253694648320 + (564570522729881746163*sin(t))/18516229690147785932800;

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