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HOVERING CONTROL OF A TILT-WING UAV

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ABSTRACT

In this study, the design and analysis of hovering controller of an UAV which is capable of doing vertical take-off and landing using the fixed six rotors placed on the tilt-wing and tilt-tail will be explained. The aircraft will have four rotors on the wing and two rotors on the tail. The main wing and horizontal tail will be capable of 90° tilting. Whole flight is separated into three flight modes, which are VTOL, Transition and Forward Flight, to have a robust control on aircraft. Only hover control of the VTOL mode will be explained in this study. Both aerodynamic and thrust forces will be used to control the aircraft during VTOL mode. MATLAB/Simulink simulations will be employed to show that the aircraft is capable of VTOL. By means of this study, it is aimed to bring an aircraft, which is capable of taking off from all platforms and reconnaissance and surveillance thanks to its hovering capability, in use.

Keywords: Tilt Wing Mechanism, VTOL, Unmanned Air Vehicles.

INTRODUCTION

Design of an air vehicle which is efficient, affordable, reliable and also capable of serving the needs throughout the lifecycle of it is one of the main objectives of the Aerospace Engineering. In the direction of this objective, several aircraft types have been designed and tried in the history of aviation. Within the boost of this performance criteria, the question of how the wings will move has been stood out. Aircrafts can be classified in three groups in terms of their wing motion. These are fixed wing, rotary wing, and flapping wing. There are advantages and disadvantages of all three groups with respect to each other. Therefore, one type can become dominant in compliance with the application area with respect to others. While fixed wing airplanes has a disadvantage about take-off and landing distances, rotary wing and flapping wing airplanes have disadvantages in terms of payload capacity and flying to long distances with respect to fixed wing airplanes. Main purpose of this study is to combine the advantages of all three types and reveal a new and more efficient type of aircraft.

There are a lot of studies on different types of UAV platforms, designed for combining the desired features of Fixed and Rotary Wing aircrafts. Platforms in this category like tail-sitter,

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tilt-rotor and tilt-wing are capable of level flight and VTOL. In general, these platforms can be classified as VTOL airplanes.

Combining all advantages of three groups is bringing some challenges in flight control. A flight control system is expected to stabilize the aircraft, follow guidance commands, reject disturbances, reduce sensitivity to parameter variations, provide robustness to uncertainties and be implementable to the real world applications.

NASA GL-10 [McSwain, R.G., Glaab, L. 2017] uses an L1 Adaptive Robust control technique to control the aircraft. The flight controller provides closed-loop feedback control utilizing body-axis angular rates and attitudes to help control the vehicle. It uses separate PID gains assigned during forward flight and hover. During hover the pilot input is mixed to assigned motor and servo outputs which correspond to a "Y-copter" multi-rotor configuration.

SUAVI [Cetinsoy, E., Dikyar, S., Hancer, C., Oner, K.T., Sirimoglu, E., Unel, M., Aksit, M.F. 2012] proposes a hierarchical control system architecture, which has position and attitude subsystems and uses a dynamic inversion method, with anti-aliasing filters. 100 hertz real time control loop gives sufficient closed loop stability to the aircraft. They implemented a Dryden Wind-Gust Model to 6-DoF model to increase the robustness of the PID controller. They also used LQR controller for the vertical flight. Aircraft, which has not an elevator, deviated from altitude by only 30 cm. Pitch control is made with thrust difference at VTOL.

[Onen A.S., Cevher L., Senipek M., Mutlu T., Gungor O., Uzunlar I.O., Kurtuluş D.F., Tekinalp O., 2015] developed a tilt rotor VTOL UAV in tri-copter configuration. [Guclu A., Arıkan K.B., Kurtulus D.F., 2016] introduced a Hybrid Air Vehicle which has the ability to vertically takeoff and landing in addition to fly horizontally as a fixed wing aircraft. [Kaya D., Kutay A.T., Kurtulus D.F., Tekinalp O., Simsek I., Soysal S., Hosgit G., 2016] presented the development activities of a Quadrotor, which has a weight of less than 4 kg, for search and rescue operations. [Çakır H., Kurtuluş D.F., 2016] presented the design of a VTOL tilt-wing aircraft which will have a weight about 10 kg.

Hover capability is required for VTOL maneuvers. With regard to hovering control; a conceptual aircraft study [D.A. Ta and I. Fantoni 2011], named as a convertible tail-sitter UAV, is designed and hover is performed using PID controllers. Another study [Önen, A.S. 2015] demonstrated hover for a tri-copter-fixed wing UAV by optimal control techniques. Hovering a tail-sitter has been also studied by [Matsumoto T., Kita K., Suzuki R., Oosedo A., Hoshino Y., Konno A. and Uchiyama M. 2010] who used quaternions with PID controllers and [J. Escareno and S. Salazar, 2006] who separated lateral, longitudinal and axial dynamics. [Garcia O. and Sanchez A., 2008] employed Lyapunov functions in controlling hover maneuver.

The main contributions of this study are to develop a solution approach to the problem of combining the benefits of Fixed Wing and Rotary Wing aircrafts in one platform. The aircraft will be designed with combined VTOL and Fixed Wing control elements that increase aerodynamic efficiency. Possessing VTOL and Forward Flight modes together, the aircraft is required to be operated in an enlarged flight envelope from hover to high speeds of level flight. High speed flight characteristics of VTOL mode are observed to provide extra benefits through utilization of aerodynamic surfaces. Available control methods will be applied to Tilt-Wing UAV in controlling the aircraft in level flight and hover separately.

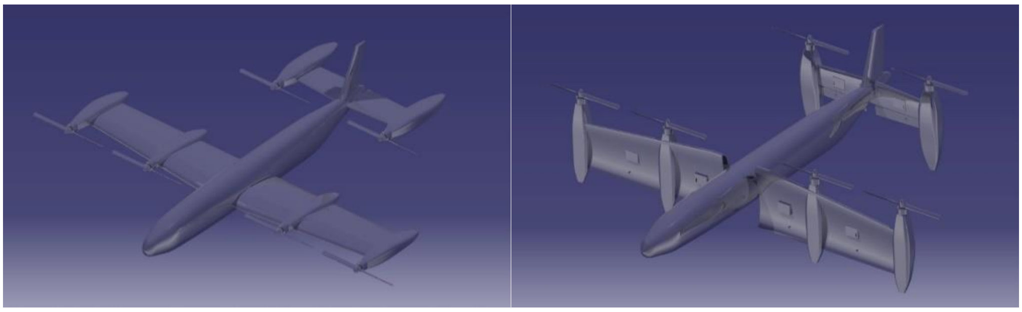

Figure 1*: VTOL Tilt-Wing Aircraft in Different Flight Conditions*

DYNAMIC MODEL

The vehicle is equipped with six rotors, four of which are mounted on the wing and two of which are mounted on the tail, and can be rotated from vertical to horizontal position and vice versa. The vehicle's airframe transforms into a quad-rotor like structure if the wings are at vertical position. During the vertical flight the wing will be positioned only in vertical positon and the motor differential thrust inputs will be used for attitude and altitude control. In addition to motor thrust, aerodynamic forces will be used as an input for yaw and position control.

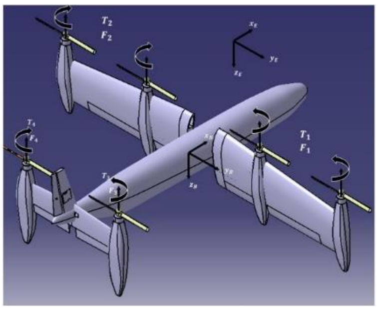

Figure 2*:* Forces and Moments in Hover

For modeling the vertical flight motion of the rigid-body air vehicles, two frames must be introduced. They are:

Earth-centered-Earth-fixed (ECEF) frame $\mathcal{F}_E\{E; \hat{\imath}_E, \hat{\jmath}_E, \hat{k}_E\}$,

Vehicle body-fixed frame $\mathcal{F}_B\{B; \hat{\imath}_B, \hat{\jmath}_B, \hat{k}_B\}$,

It is usually preferred to express positional dynamics with respect to the ECEF and the rotational dynamics with respect to Vehicle body-fixed frame. Since the aerial vehicle is assumed as a rigid body, its dynamic can be written with respect to ECEF as;

$$\left.\begin{bmatrix} \boldsymbol{F_t} \\ \boldsymbol{M_t} \end{bmatrix}\right|_E = \begin{bmatrix} \boldsymbol{mI} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I_B} \end{bmatrix} \begin{bmatrix} \dot{\vec{V}}{}_V^E \\ \dot{\vec{\omega}}{}_B^E \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \vec{\omega}_B^E \times \boldsymbol{I_B} \vec{\omega}_B^E \end{bmatrix} \tag{1}$$

The subscripts and superscripts E and B used in the above equation express the quantities in ECEF and Body Frames, respectively. Eq. (1) can be separated in two equations. One should be the force the other one should be the moment equation,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = m \begin{bmatrix} \dot{x}_V \\ \dot{y}_V \\ \dot{z}_V \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = I_B \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \left[\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \left[I_B \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right] \right] \tag{3}$$

LHS of Eq. (2) and Eq. (3) is the total force and moment acting on the vehicle and can be represented as follows;

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_{EB} \begin{bmatrix} \bar{q}S\left(\bar{C}_X + C_{X_q}q + C_{X_{d_e}}d_e + C_{X_{d_a}}d_a\right) \\ \bar{q}S\left(\bar{C}_Y + C_{Y_r}r + C_{Y_p}p + C_{Y_{d_e}}d_e + C_{Y_{d_a}}d_a\right) \\ \bar{q}S\left(\bar{C}_Z + C_{Z_q}q + C_{Z_{\dot{\alpha}}}\dot{\alpha} + C_{Z_{d_e}}d_e + C_{Z_{d_a}}d_a\right) \end{bmatrix} + R_{EB} \begin{bmatrix} 0 \\ 0 \\ -(T_1 + T_2 + T_3 + T_4 + T_5 + T_6) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = R_{EB} \begin{bmatrix} \bar{q}Sb\left(\bar{C}_l + C_{l_p}p + C_{l_r}r + C_{l_{d_e}}d_e + C_{l_{d_a}}d_a\right) \\ \bar{q}S\bar{\bar{c}}\left(\bar{C}_m + C_{m_q}q + C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{d_e}}d_e + C_{m_{d_a}}d_a\right) \\ \bar{q}Sb\left(\bar{C}_n + C_{n_r}r + C_{n_p}p + C_{n_{d_e}}d_e + C_{n_{d_a}}d_a\right) \end{bmatrix} + R_{EB} \begin{bmatrix} 0.975(-T_1 + T_4) + 0.4(-T_2 + T_3) + 0.325(-T_5 + T_6) \\ 0.08(T_1 + T_2 + T_3 + T_4) - 0.838(T_5 + T_6) \\ Moment\ coming\ from\ Angular\ Velocity\ of\ propeller \end{bmatrix} \tag{5}$$

If we arrange Eq. (4) and Eq. (5) and combine with the Eq. (2) and Eq. (3),

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix}=\begin{bmatrix}(rv-qw)\\ (pw-ru)\\ (qu-pv)\end{bmatrix}+\begin{bmatrix}-gS_\theta\\ gS_\phi C_\theta\\ gC_\phi C_\theta\end{bmatrix}+0.1057\left(\begin{bmatrix}X_{Aero}\\ Y_{Aero}\\ Z_{Aero}+Z_{Thrust}\end{bmatrix}\right) \tag{6}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}-0.983qr\\ 0.971pr\\ 0.254pq\end{bmatrix}+\begin{bmatrix}0.4274 & 0 & 0\\ 0 & 0.7143 & 0\\ 0 & 0 & 0.2703\end{bmatrix}\begin{bmatrix}L_{Aero}+L_{Thrust}\\ M_{Aero}+M_{Thrust}\\ N_{Aero}+N_{\Omega}\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}\dot{\boldsymbol{\phi}}\\ \dot{\boldsymbol{\theta}}\\ \dot{\boldsymbol{\psi}}\end{bmatrix}=\begin{bmatrix}\mathbf{1} & \boldsymbol{S_\phi T_\theta} & \boldsymbol{C_\phi T_\theta}\\ \mathbf{0} & \boldsymbol{C_\phi} & \boldsymbol{-S_\phi}\\ \mathbf{0} & \boldsymbol{S_\phi Sec_\theta} & \boldsymbol{C_\phi Sec_\theta}\end{bmatrix}\begin{bmatrix}\boldsymbol{p}\\ \boldsymbol{q}\\ \boldsymbol{r}\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}\dot{P}_{x_V}\\ \dot{P}_{y_V}\\ \dot{P}_{z_V}\end{bmatrix}=\begin{bmatrix}C_\psi C_\theta & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi\\ S_\psi C_\theta & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi\\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix} \tag{9}$$

Now 12 equations and 12 states written in the state space format has been gathered. These non-linear equations will be used in the state space model.

Mass, Center of Gravity and Inertia Tensor

Mass, center of gravity and inertia tensor are taken from CATIA model. Tilt-wing mechanism tilts the wing from centerline of the wing spar. Spar center and center of gravity of the wing is placed in the same position. Since rotation line and center of gravity of the wing falls into same line, inertia of the whole body will not change during the transition phase. This feature will help us to eliminate the risk of having a time variant system. Reference Frame is placed at CoG.

$$\boldsymbol{m = 9.464\ kg} \tag{10}$$

$$\boldsymbol{G}=\begin{bmatrix}\mathbf{-0.00013}\\ \mathbf{-0.07}\\ \mathbf{0.00022}\end{bmatrix}\boldsymbol{mm} \tag{11}$$

$$\boldsymbol{I_B}=\begin{bmatrix}\mathbf{2.34} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{1.4} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{3.7}\end{bmatrix}\boldsymbol{kg.m^2} \tag{12}$$

Model Inputs, Outputs and States

Dynamic model inputs are four engine throttle levels (Front-Left, Front-Right, Rear-Left, Rear-Right), one aileron deflection and one elevator deflection for the whole flight regime. This means that there are six inputs for total aircraft in the hover phase.

The six DoF model has 12 states which are body angular rates, Euler angles, body velocities and position vector. For the hover case all states will be tracked, means that there will be 12 outputs which are exactly the states.

Table 1: State Variables in Equations of Motion

Dynamics						**Kinematics**					
Translational Velocities			Rotational Velocities			Euler Angles			Position Coordinates		
u	v	w	p	q	r	ϕ	θ	ψ	P_x	P_y	P_z

Table 2: Input Variables in Equations of Motion

Force Inputs					
δ_e	δ_a	δ_{T1}	δ_{T2}	δ_{T3}	δ_{T4}

CONTROLLER DESIGN

Trimming and Linearization

While linearizing the nonlinear model, MATLAB/Simulink Linear Analyses Toolbox is used. The inputs and outputs of the model are used for the analysis inputs and outputs and the trim point that is found. Numerical perturbation is used as a linearization algorithm and 'linoptions' is the command that is used to define in the MATLAB code. After defining the type of the algorithm, 'linearize' command is used to linearize the system around the specified operating point.

The system is linearized around that operating point with the classical Taylor series approach and after linearizing the 6 DoF equations of motion, a continuous time state-space model is obtained.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{13}$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \tag{14}$$

$$\boldsymbol{A} = \left.\frac{\partial f}{\partial x}\right|_{(x_0,u_0)} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{(x_0,u_0)} \\ \cdots & \cdots & \cdots \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{(x_0,u_0)} \end{bmatrix} \tag{15}$$

$$\boldsymbol{B} = \left.\frac{\partial f}{\partial u}\right|_{(x_0,u_0)} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial u_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial f_1}{\partial u_m}\right|_{(x_0,u_0)} \\ \cdots & \cdots & \cdots \\ \left.\frac{\partial f_m}{\partial u_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial f_m}{\partial u_m}\right|_{(x_0,u_0)} \end{bmatrix} \tag{16}$$

$$\boldsymbol{C} = \left.\frac{\partial g}{\partial x}\right|_{(x_0,u_0)} = \begin{bmatrix} \left.\frac{\partial g_1}{\partial x_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial g_1}{\partial x_n}\right|_{(x_0,u_0)} \\ \cdots & \cdots & \cdots \\ \left.\frac{\partial g_n}{\partial x_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial g_n}{\partial x_n}\right|_{(x_0,u_0)} \end{bmatrix} \tag{17}$$

$$\boldsymbol{D} = \left.\frac{\partial g}{\partial u}\right|_{(x_0,u_0)} = \begin{bmatrix} \left.\frac{\partial g_1}{\partial u_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial g_1}{\partial u_m}\right|_{(x_0,u_0)} \\ \cdots & \cdots & \cdots \\ \left.\frac{\partial g_m}{\partial u_1}\right|_{(x_0,u_0)} & \cdots & \left.\frac{\partial g_m}{\partial u_m}\right|_{(x_0,u_0)} \end{bmatrix} \tag{18}$$

Non-linear Equations of motion were found in the previous chapter. We can write Eqs. (6)-(9) in body coordinates with an explicit form;

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} (rv - qw) \\ (pw - ru) \\ (qu - pv) \end{bmatrix} + \begin{bmatrix} -gS_\theta \\ gS_\phi C_\theta \\ gC_\phi C_\theta \end{bmatrix} + 0.1057\left(\begin{bmatrix} \bar{q}S\left(\bar{C}_X + C_{X_q}q + C_{X_{d_e}}d_e + C_{X_{d_a}}d_a\right) \\ \bar{q}S\left(\bar{C}_Y + C_{Y_r}r + C_{Y_p}p + C_{Y_{d_e}}d_e + C_{Y_{d_a}}d_a\right) \\ \bar{q}S\left(\bar{C}_Z + C_{Z_q}q + C_{Z_{\dot{\alpha}}}\dot{\alpha} + C_{Z_{d_e}}d_e + C_{Z_{d_a}}d_a\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -(T_1 + T_2 + T_3 + T_4 + T_5 + T_6) \end{bmatrix}\right) \tag{19}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -0.983qr \\ 0.971pr \\ 0.254pq \end{bmatrix} + \begin{bmatrix} 0.4274\bar{q}Sb\left(\bar{C}_l + C_{l_p}p + C_{l_r}r + C_{l_{d_e}}d_e + C_{l_{d_a}}d_a\right) + 0.4274\left(0.975(-T_1 + T_4) + 0.4(-T_2 + T_3) + 0.325(-T_5 + T_6)\right) \\ 0.7143\bar{q}S\bar{\bar{c}}\left(\bar{C}_m + C_{m_q}q + C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{d_e}}d_e + C_{m_{d_a}}d_a\right) + 0.7143\left(0.08(T_1 + T_2 + T_3 + T_4) - 0.838(T_5 + T_6)\right) \\ 0.2703\bar{q}Sb\left(\bar{C}_n + C_{n_r}r + C_{n_p}p + C_{n_{d_e}}d_e + C_{n_{d_a}}d_a\right) + moment\ from\ angular\ velocity \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi Sec_\theta & C_\phi Sec_\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \dot{P}_{x_V} \\ \dot{P}_{y_V} \\ \dot{P}_{z_V} \end{bmatrix} = \begin{bmatrix} C_\psi C_\theta & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ S_\psi C_\theta & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{22}$$

Aerodynamic force and moment coefficients in Eq. (19) and Eq. (20) are found using ANSYS Fluent. These coefficients are also employed in our MATLAB Simscape model. For controller design, attitude dynamics can be linearized in hover conditions; which lead Euler Angles to be equal to zero and where angular accelerations in body and ECEF Frames are assumed to be equal. The states like $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}$ will also be equal to zero in hover. If we linearize our non-linear equations around trim point we can easily end up with linear equations of motion as given in Eqs. (23)-(26).

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} = \begin{bmatrix}-qw_0\\ pw_0\\ 0\end{bmatrix} + \begin{bmatrix}-gC_{\theta_0}\theta\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0.105\delta X\\ 0.105\delta Y\\ 0.105\delta Z\end{bmatrix} \tag{23}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = -\begin{bmatrix}0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0.42\delta L\\ 0.71\delta M\\ 0.27\delta N\end{bmatrix} \tag{24}$$

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{25}$$

$$\begin{bmatrix}\dot{P}_x\\ \dot{P}_y\\ \dot{P}_z\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix} \tag{26}$$

Perturbations of forces and moments in Eq. (23) and Eq. (24) can be expressed as follows;

$$\begin{bmatrix}\delta X\\ \delta Y\\ \delta Z\\ \delta L\\ \delta M\\ \delta N\end{bmatrix} = \begin{bmatrix}X_{\dot{u}} & 0 & X_{\dot{\alpha}} & 0 & X_{\dot{q}} & 0\\ 0 & Y_{\dot{\beta}} & 0 & Y_{\dot{p}} & 0 & Y_{\dot{r}}\\ Z_{\dot{u}} & 0 & Z_{\dot{\alpha}} & 0 & Z_{\dot{q}} & 0\\ 0 & L_{\dot{\beta}} & 0 & L_{\dot{p}} & 0 & L_{\dot{r}}\\ M_{\dot{u}} & 0 & M_{\dot{\alpha}} & 0 & M_{\dot{q}} & 0\\ 0 & N_{\dot{\beta}} & 0 & N_{\dot{p}} & 0 & N_{\dot{r}}\end{bmatrix}\begin{bmatrix}\dot{u}\\ \dot{\beta}\\ \dot{\alpha}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} + \begin{bmatrix}X_u & 0 & X_w & 0 & X_q & 0\\ 0 & Y_v & 0 & Y_p & 0 & Y_r\\ Z_u & 0 & Z_w & 0 & Z_q & 0\\ 0 & L_v & 0 & L_p & 0 & L_r\\ M_u & 0 & M_w & 0 & M_q & 0\\ 0 & N_v & 0 & N_p & 0 & N_r\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\end{bmatrix} + \begin{bmatrix}X_{\delta_e} & X_{\delta_a} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ Z_{\delta_e} & Z_{\delta_a} & Z_{\delta_{T1}} & Z_{\delta_{T2}} & Z_{\delta_{T3}} & Z_{\delta_{T4}}\\ 0 & 0 & L_{\delta_{T1}} & L_{\delta_{T2}} & L_{\delta_{T3}} & L_{\delta_{T4}}\\ M_{\delta_e} & M_{\delta_a} & M_{\delta_{T1}} & M_{\delta_{T2}} & M_{\delta_{T3}} & M_{\delta_{T4}}\\ 0 & 0 & N_{\delta_{T1}} & N_{\delta_{T2}} & N_{\delta_{T3}} & N_{\delta_{T4}}\end{bmatrix}\begin{bmatrix}\delta_e\\ \delta_a\\ \delta_{T1}\\ \delta_{T2}\\ \delta_{T3}\\ \delta_{T4}\end{bmatrix} \tag{27}$$

Since we are not in the airplane mode, we can simplify Eq. (27);

$$\begin{bmatrix}\delta X\\ \delta Y\\ \delta Z\\ \delta L\\ \delta M\\ \delta N\end{bmatrix} = \begin{bmatrix}X_u & 0 & X_w & 0 & X_q & 0\\ 0 & Y_v & 0 & Y_p & 0 & Y_r\\ Z_u & 0 & Z_w & 0 & Z_q & 0\\ 0 & L_v & 0 & L_p & 0 & L_r\\ M_u & 0 & M_w & 0 & M_q & 0\\ 0 & N_v & 0 & N_p & 0 & N_r\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\end{bmatrix} + \begin{bmatrix}X_{\delta_e} & X_{\delta_a} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ Z_{\delta_e} & Z_{\delta_a} & Z_{\delta_{T1}} & Z_{\delta_{T2}} & Z_{\delta_{T3}} & Z_{\delta_{T4}}\\ 0 & 0 & L_{\delta_{T1}} & L_{\delta_{T2}} & L_{\delta_{T3}} & L_{\delta_{T4}}\\ M_{\delta_e} & M_{\delta_a} & M_{\delta_{T1}} & M_{\delta_{T2}} & M_{\delta_{T3}} & M_{\delta_{T4}}\\ 0 & 0 & N_{\delta_{T1}} & N_{\delta_{T2}} & N_{\delta_{T3}} & N_{\delta_{T4}}\end{bmatrix}\begin{bmatrix}\delta_e\\ \delta_a\\ \delta_{T1}\\ \delta_{T2}\\ \delta_{T3}\\ \delta_{T4}\end{bmatrix} \tag{28}$$

Now we can combine Eq. (23), Eq. (24) and Eq. (27);

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-qw_0\\ pw_0\\ 0\\ 0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}-gC_{\theta_0}\theta\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0.105\\ 0.105\\ 0.105\\ 0.42\\ 0.71\\ 0.27\end{bmatrix}\left(\begin{bmatrix}X_u & 0 & X_w & 0 & X_q & 0\\ 0 & Y_v & 0 & Y_p & 0 & Y_r\\ Z_u & 0 & Z_w & 0 & Z_q & 0\\ 0 & L_v & 0 & L_p & 0 & L_r\\ M_u & 0 & M_w & 0 & M_q & 0\\ 0 & N_v & 0 & N_p & 0 & N_r\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\end{bmatrix} + \begin{bmatrix}X_{\delta_e} & X_{\delta_a} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ Z_{\delta_e} & Z_{\delta_a} & Z_{\delta_{T1}} & Z_{\delta_{T2}} & Z_{\delta_{T3}} & Z_{\delta_{T4}}\\ 0 & 0 & L_{\delta_{T1}} & L_{\delta_{T2}} & L_{\delta_{T3}} & L_{\delta_{T4}}\\ M_{\delta_e} & M_{\delta_a} & M_{\delta_{T1}} & M_{\delta_{T2}} & M_{\delta_{T3}} & M_{\delta_{T4}}\\ 0 & 0 & N_{\delta_{T1}} & N_{\delta_{T2}} & N_{\delta_{T3}} & N_{\delta_{T4}}\end{bmatrix}\begin{bmatrix}\delta_e\\ \delta_a\\ \delta_{T1}\\ \delta_{T2}\\ \delta_{T3}\\ \delta_{T4}\end{bmatrix}\right) \tag{29}$$

If we write Eq. (25) and Eq. (29) in matrix form

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}0.105X_u & 0 & 0.105X_w & 0 & -w_0+0.105X_q & 0 & 0 & -g & 0\\ 0 & 0.105Y_v & 0 & 0.105Y_p & 0 & 0.105Y_r & 0 & 0 & 0\\ 0.105Z_u & 0 & 0.105Z_w & w_0 & 0.105Z_q & 0 & 0 & 0 & 0\\ 0 & 0.42L_v & 0 & 0.42L_p & 0 & 0.42L_r & 0 & 0 & 0\\ 0.71M_u & 0 & 0.71M_w & 0 & 0.71M_q & 0 & 0 & 0 & 0\\ 0 & 0.27N_v & 0 & 0.27N_p & 0 & 0.27N_r & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\\ \phi\\ \theta\\ \psi\end{bmatrix}+$$

$$\begin{bmatrix}0.105X_{\delta_e} & 0.105X_{\delta_a} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0.105Z_{\delta_e} & 0.105Z_{\delta_a} & 0.105Z_{\delta_{T1}} & 0.105Z_{\delta_{T2}} & 0.105Z_{\delta_{T3}} & 0.105Z_{\delta_{T4}}\\ 0 & 0 & 0.42L_{\delta_{T1}} & 0.42L_{\delta_{T2}} & 0.42L_{\delta_{T3}} & 0.42L_{\delta_{T4}}\\ 0.71M_{\delta_e} & 0.71M_{\delta_a} & 0.71M_{\delta_{T1}} & 0.71M_{\delta_{T2}} & 0.71M_{\delta_{T3}} & 0.71M_{\delta_{T4}}\\ 0 & 0 & 0.27N_{\delta_{T1}} & 0.27N_{\delta_{T2}} & 0.27N_{\delta_{T3}} & 0.27N_{\delta_{T4}}\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta_e\\ \delta_a\\ \delta_{T1}\\ \delta_{T2}\\ \delta_{T3}\\ \delta_{T4}\end{bmatrix} \tag{30}$$

We can easily substitute the aerodynamic coefficients, which should be given in body coordinates, to Eq. (30) and get the state space matrix form of linear equations of motion. Note that, in A matrix there is only one unknown which is w_0, this value will be zero in hover, however for the vertical flight, it will be around one meter per second.

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\\ \dot{p}\\ \dot{q}\\ \dot{r}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ \dot{P}_x\\ \dot{P}_y\\ \dot{P}_z\end{bmatrix}=\begin{bmatrix}-0.598 & 0 & 0 & 0 & -w_0 & 0 & 0 & -9.81 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ -0.0727 & 0 & 0 & w_0 & -0.028 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.086 & 0 & 0 & 0 & -0.0025 & 0 & 0 & 0 & 0 & 0 & 0\\ 0.174 & 0 & 0 & 0 & -2.14 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -0.051 & 0 & -0.0756 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u\\ v\\ w\\ p\\ q\\ r\\ \phi\\ \theta\\ \psi\\ P_x\\ P_y\\ P_z\end{bmatrix}+$$

$$\begin{bmatrix}-0.00026 & -0.00054 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -0.01 & -0.01 & -0.0015 & -0.0015\\ 0 & 0 & -0.0275 & 0.0275 & -0.0019 & 0.0019\\ -0.0001 & -0.00019 & 0.0054 & 0.0054 & -0.0076 & -0.0076\\ 0 & 0 & 0.0001 & -0.0001 & 0.00005 & -0.00005\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta_e\\ \delta_a\\ \delta_{T1}\\ \delta_{T2}\\ \delta_{T3}\\ \delta_{T4}\end{bmatrix} \tag{31}$$

Altitude and Attitude Controllers for Hovering

In the VTOL (Vertical Take-off and Landing) mode, desired altitude and desired position will be employed to control the aircraft. It is aimed that the vehicle should not lose the desired altitude and then stay in the vicinity of the reference position at the desired altitude. In order to develop altitude and attitude controllers, we first recall the state space representation of equations of motion;

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx+Du\end{aligned} \tag{32}$$

In this study, since there will not be any input which directly effects to the outputs, $D=0$. A and B matrices are found in Eq. (31). Linear Quadratic Regulator (LQR) command in MATLAB is used to determine the gain matrix of controller. LQR command need A, B, Q, R and

N matrices and yields the gain matrix. We already have found A and B matrices above, Q and R matrices will be determined by us considering the penalty level of state errors and input values.

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix} \quad (33)$$

$$R = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix} \quad (34)$$

By using LQR command in MATLAB we have found the gain matrix which is given below;

$$K = \begin{bmatrix} -1.40 & 0 & 0.20 & 2.25 & 0.25 & -1.45 & 0.01 & 4.32 & -0.01 & -1.85 & 0 & 0.14 \\ -1.75 & 0 & 0.29 & -2.25 & -0.58 & 1.45 & -0.01 & 2.92 & 0.01 & -2.47 & 0 & 0.21 \\ -6.87 & 0 & -52.4 & -0.52 & 22.8 & -0.25 & 0 & 75.8 & 0 & -10.5 & 0 & -29.7 \\ -6.89 & 0 & -52.3 & 0.52 & 22.8 & 0.25 & 0 & 75.9 & 0 & -10.5 & 0 & -29.7 \\ 60 & 0 & -41.8 & -0.71 & -165 & -1.69 & 0 & -583 & -0.01 & 66.6 & 0 & -23.6 \\ 59.9 & 0 & -41.8 & 0.69 & -165 & 1.71 & 0 & -583 & 0.01 & 66.5 & 0 & -23.6 \end{bmatrix} \quad (35)$$

In simulation, it was expected from the aircraft to go to desired altitude, which was chosen as three meters, and stay around the desired position coordinates with zero angular rates. Figure 3 shows the deviation of the aircraft from the desired position and altitude with respect to time. Aircraft came to the desired altitude in 10 seconds and stayed in the vicinity of the desired position.

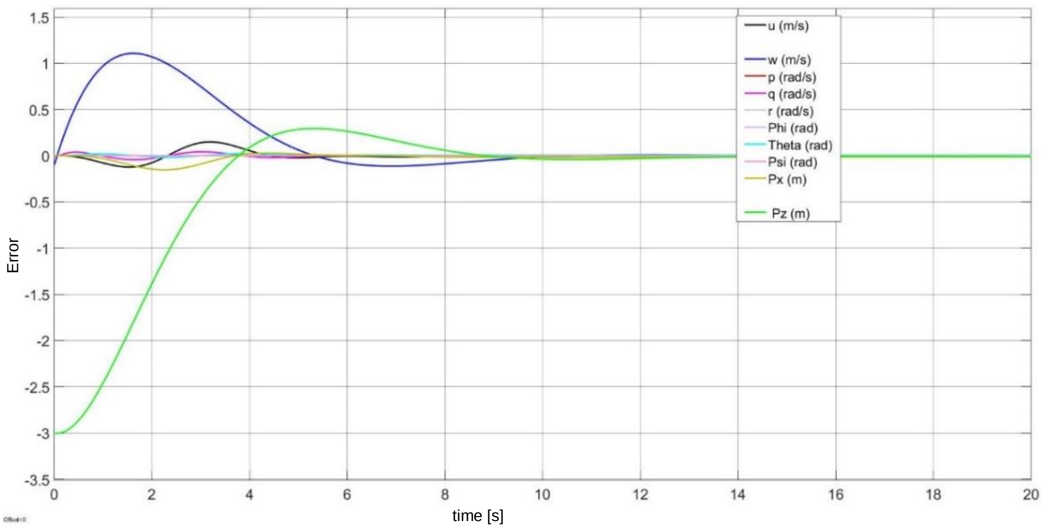

Figure 3: *Deviation from Desired States in Time*

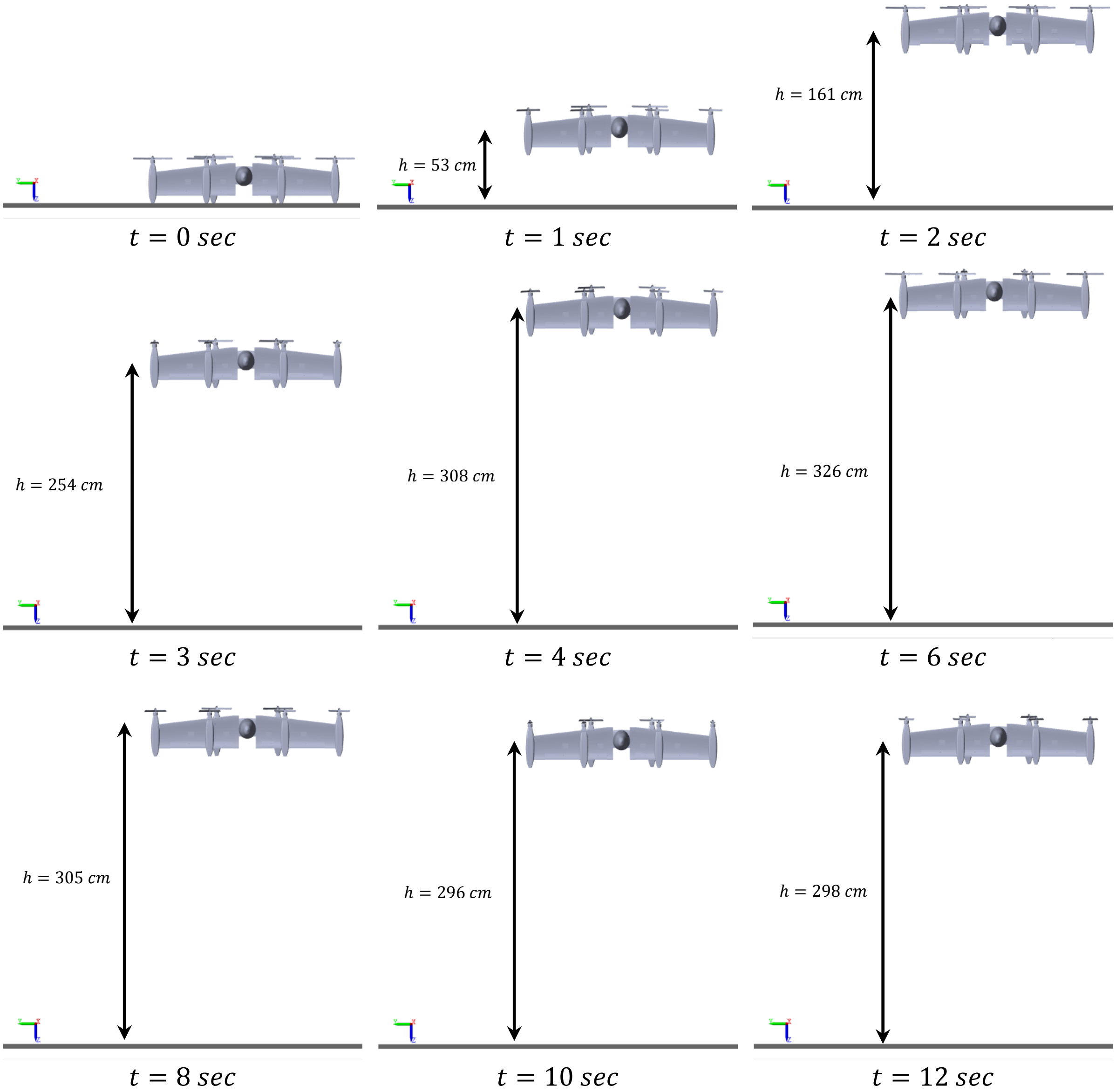

Figure 4*: Vertical Flight; from Zero Second to Twelve Seconds.*

Figure 4 gives the first 12 seconds of flight of VTOL Tilt-Wing UAV, subsequently. Tilt-Wing UAV climbs to three meters' altitude and hovers. Note that, Figure 3 and 4 show the same flight conditions. After 12 seconds, aircraft is settling down and hovering. Error in altitude and x direction was only 2 cm and 1 cm respectively.

CONCLUSION AND FUTURE WORK

This study has enabled a quad tilt-wing aerial vehicle to hover in the vicinity of a given point in the simulation. Simulation results was satisfactory. Hovering performance of the vehicle is planned to be improved by utilizing a disturbance observer. As a future work, it has been planned to focus on the controllers for the other phases, which are transition and the forward flight.

References

Cetinsoy, E., Dikyar, S., Hancer, C., Oner, K.T., Sirimoglu, E., Unel, M., Aksit, M.F. (2012), Design and construction of a novel quad tilt-wing UAV. Faculty of Engineering and Natural Sciences, Sabanci University, Orhanli-Tuzla, Istanbul, Turkey (2012)

Çakır H., Kurtuluş D.F. (2016), Dikey İniş/Kalkış Yapabilen Hareketli Kanatlı İnsansız Hava Aracının Tasarımı, VI. Ulusal Havacılık ve Uzay Konferansı (2016)

Escareno J. and Salazar S. (2006), Modelling and Control of a Convertible VTOL Aircraft, in Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, USA, (2006)

Garcia O. and Sanchez A., Tailsitter UAV having one tilting rotor Modeling, Control and Real Time Experiments (2008), in Proceedings of the 17th World Congress, The International Federation of Automatic Control, Seoul, Korea (2008)

Guclu A., Arıkan K.B., Kurtulus D.F. (2016), Attitude and Altitude Stabilization of a Fixed Wing VTOL Unmanned Air Vehicle AIAA Modeling and Simulation Technologies Conference Washington, D.C., US (2016)

Kaya D., Kutay A.T., Kurtulus D.F., Tekinalp O., Simsek I., Soysal S., Hosgit G. (2016), Propulsion System Selection and Modeling for a Quadrotor with Search and Rescue Mission, AIAA Modeling and Simulation Technologies Conference Washington, D.C., USA, AIAA 2016-3378 (2016)

Matsumoto T., Kita K., Suzuki R., Oosedo A., Hoshino Y., Konno A. and Uchiyama M. (2010), A Hovering Control Strategy for a Tailsitter VTOL UAV that Increases Stability Against Lönenarge Disturbance, in 2010 IEEE International Conference on Robotics and Automation, Anchorage, USA (2010)

McSwain, R.G., Glaab, L., Greased Lightning (GL-10) Performance Flight Research – Flight Data Report. Langley Research Center, Hampton, Virginia (2017)

Onen A.S., Cevher L., Senipek M., Mutlu T., Gungor O., Uzunlar I.O., Kurtuluş D.F., Tekinalp O. (2015), Modeling and controller design of a VTOL UAV, 2015 International Conference on Unmanned Aircraft Systems (ICUAS), 329-337 (2015)

Önen, A.S. (2015), Modeling and Controller Design of a VTOL Air Vehicle. M.S. Thesis, Middle East Technical University METU (Ankara-Turkey) (2015)

Ta D.A. and Fantoni I. (2011), Modeling and Control of a Convertible Mini UAV, in Proceedings of 18th IFAC World Congress, Milano, Italy (2011)