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A ZERO-EQUATION TRANSITION MODEL DEPENDING ON LOCAL FLOW VARIABLES

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ABSTRACT

A correlation-based zero-equation transition model that relies on local information is proposed. The model is qualified as an algebraic model by using only an intermittency function rather than an intermittency transport equation. The basic idea behind the model is that, instead of deriving new equations for intermittency convection and diffusion, already present convection and diffusion characteristics of the locally generated turbulent viscosity in Spalart-Allmaras turbulence model could be used. Multiplying the production term of the Spalart-Allmaras turbulence model with the newly introduced intermittency factor γ, the turbulence production is damped until it satisfies the turbulence onset requirements. Therefore, the present formulation being a local model yet bringing in the correlation data achieves a similar effect by using two less equations than similar transport equation based transition models like the γ - Re_θ model. Validation of the new model with well-known zero and variable pressure gradient flat plate test cases shows quite good agreement with the experimental data. The model is also tested against some low Reynolds number airfoil cases with very promising results. The results imply that the newly proposed model may become a viable alternative for some other higher order methods that is especially attractive in the design environment.

INTRODUCTION

Most of the industrial CFD analyses are practically carried out using full turbulence models which do not account for laminar-to-turbulent flow boundary layer transition. This is due to difficulty of incorporating different and very complex types of transition mechanisms into unified models. Yet, significant steps have been taken in the transition modeling in the last two decades by the use of correlation-based models coupled with the Reynolds Averaged Navier-Stokes equations (RANS). Before the correlation-based modeling approach, the well-known e^N method was the method of choice in many design applications [Drela and Giles, 1987]. Later, some two-equation low Reynolds number turbulence models were used; although these models lacked true physical ground and did not bring much help for a true predictive capability [Wilcox, 1994]. Correlation-based methods were later developed as viable alternatives for practical purposes. Very early example of this method is by Dhawan and Narasimha [Dhawan and Narasimha, 1958] that showed some success by using the

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concept of a generalized intermittency distribution function to blend the laminar and turbulent flow regions. Following this, an intermittency transport equation that mimics the experimental correlations was proposed by Steelant and Dick [Steelant and Dick, 1996]. Cho and Chung [Cho and Chung, 1992] developed k - ε - γ turbulence model and calculated spreading rates for free shear flows. Suzen and Huang [Suzen and Huang, 2000] greatly improved intermittency transport equation approach by combining the latter two methods, and proposed a model that is capable of obtaining a realistic variation of intermittency in both stream-wise and cross-stream directions. Although promising results were obtained, these models rely on non-local data, thus it is very difficult to implement them into the modern CFD codes since they require calculating the actual momentum thickness Reynolds number (an integral parameter) in order to compare it with a critical momentum thickness Reynolds number. For complex geometries, boundary layer edge is not well defined and therefore the integration process depends on a search algorithm and various assumptions.

Based on the promising outcomes of the correlation-based transition models that use intermittency transport equations, a number of quite successful models have been proposed for Reynolds averaged Navier-Stokes solvers. Foremost of these models is the “engineering transition modeling” by Menter et al. [Menter et al., 2004] that relies on local data to avoid all the complicated work present in the non-local models, and a slightly improved version [Langtry and Menter, 2005] of the original model. In the γ - Re_θ model, a triggering threshold is checked at every single computational cell. The cells in which the threshold value is exceeded generate intermittency, and the generated intermittency is convected and diffused to other cells to create the fully turbulent regions. Following the γ - Re_θ two-equation transition model of Menter et al., a number of successful two- or three-equation models appeared in the literature, such as physics-based k - k_L - ω model [Walters and Leylek, 2004], near/freestream intermittency model [Lodefier et al., 2003] and k - ω - γ model [Fu and Wang, 2008]. Menter et al. model [Menter et al., 2004] were further developed and extended into the realm of more physics such as cross-flow instability by Grabe and Krumbein [Grabe and Krumbein, 2014], the so-called secondary effects such as roughness by Dassler et al. [Dassler et al., 2012] and compressibility by Kaynak [Kaynak, 2012]. Most recently, Menter et al. [Menter et al., 2015] proposed a one-equation γ model which is a significantly simplified version of the γ - Re_θ model. In their work, it is stated that the new one-equation model achieves a Galilean invariant formulation while providing meaningful coefficients which can easily be fine-tuned to match specific application areas.

In this paper, a novel zero-equation transition model, herein called as the Bas-Cakmakcioglu (B-C) model is introduced which was inspired by the success of Menter et al. γ model by following a very pragmatic approach that provides a similar effect. An early version of this model has been formulated by Bas et al. [Bas et al., 2013] for low Reynolds airfoils. In the present paper, this novel idea is further developed and variety of flat plate and airfoil cases are documented with relevant data. In the original Menter et al. γ - Re_θ model, the need for non-local information is avoided by a link between the experimental correlation and the intermittency equation through the use of vorticity Reynolds number (Re_v). In other words, instead of using the momentum thickness Reynolds number (Re_θ) to trigger the onset of transition as is the case for the early correlation-based models, the authors used the vorticity Reynolds number. This was achieved through the basic experimental observation that the maximum value of vorticity Reynolds number is proportional to the momentum thickness Reynolds number in a Blasius boundary layer [Wilcox, 1993; Menter et al, 2002]. Besides, for moderate pressure gradients, the relative error between the momentum thickness Reynolds number and vorticity Reynolds number is less than 10%, as depicted in Figure 1, reproduced from [Menter et al, 2004]. Therefore, this approach can safely be used for the majority of the experimental data, since the relative error for these data fall within this range.

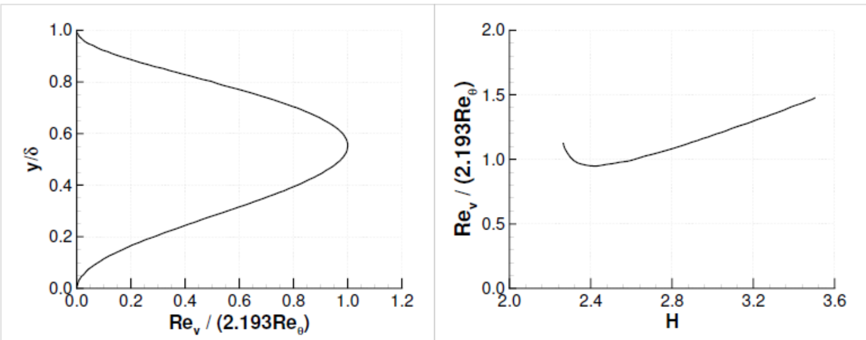

Figure 1: Scaled vorticity Reynolds number across Blasius boundary layer (left), and relative error between momentum thickness and vorticity Reynolds numbers against shape factor-H (right) (reproduced from [Menter et al, 2004])

The basic idea behind the B-C model is quite pragmatic: Instead of deriving new equations with suitable terms for intermittency convection and diffusion, already present convection and diffusion characteristics of locally generated turbulent viscosity could be used. The production term of the turbulence model could be damped until a considerable amount of turbulent viscosity is generated, and after that point on, the damping effect of the transition model would be disabled. From this idea, Spalart-Allmaras (S-A) one-equation turbulence model [Spalart and Allmaras, 1992] is chosen as the baseline turbulence model, and an intermittency function is proposed and implemented into S-A equation in order to control its production term. The resulting model is also local, and easy to implement for both two- and three-dimensional formulations. Also, it should be noted that, in comparison to the Menter et al. γ - Re_θ model, the number of equations solved for a three-dimensional problem reduces from 9 equations (1 continuity + 3 momentum + 1 energy + 2 turbulence + 2 transition) to just 6 equations (1 continuity + 3 momentum + 1 energy + 1 turbulence transport). Free stream turbulence intensity appears only in the critical momentum thickness Reynolds number equation, and model recalibration for any new experimental data set for different physical problems is quite easy. The details of the model are presented in the following sections.

METHOD

Flow Solver

Stanford University Unstructured (SU2), an open-source CFD solver by Aerospace Design Laboratory at Stanford University, is used as the flow solver [Palacios et al., 2013]. SU2 code is chosen; since the structure of the code is well-established and the code is easy-to-understand as it is written in plain language with clear constant/variable naming. The user/developer community of the SU2 is also growing each day, thus there is a potential that the model can be applied to many different needs of the users. SU2 can solve 2-D/3-D incompressible/compressible Euler/RANS equations with linear system solver methods like LU-SGS, BiCGSTAB and GMRES. The convective terms are discretized using central or upwind methods, and several state-of-art numerical schemes including JST, Roe, Lax-Friedrich, Roe-Turkel and AUSM are implemented. Currently, the code includes S-A and k - ω SST turbulence models. These features make SU2 an excellent platform for implementing and testing the B-C model. As of January 2017, SU2 version 5 is officially released, and the B-C model is available to the community.

Model Formulation

The B-C transition model is coupled with the S-A turbulence model. The original transport equation of the S-A turbulence model is given in Equation 1:

$$\frac{\partial \nu_t}{\partial t} + \frac{\partial}{\partial x_j}\left(\nu_t u_j\right) = C_{b1}(1 - f_{t2})S\nu_t - \left[C_{w1}f_w - \frac{C_{b1}}{\kappa^2}f_{t2}\right]\left(\frac{\nu_t}{d}\right)^2 + \frac{1}{\sigma}\left\{\frac{\partial}{\partial x_j}\left[(\nu_L + \nu_t)\frac{\partial \nu}{\partial x_j}\right] + C_{b2}\frac{\partial \nu_t}{\partial x_j}\frac{\partial \nu_t}{\partial x_j}\right\} \tag{1}$$

The proposed transition model achieves its purpose by modifying the production term in the S-A. As given in Equation 2, the production term is multiplied by the intermittency distribution function γ_{BC}:

$$\frac{\partial \nu_t}{\partial t}+\frac{\partial}{\partial x_j}(\nu_t u_j)=\boldsymbol{\gamma_{BC}}C_{b1}(1-f_{t2})S\nu_t-\left[C_{w1}f_w-\frac{C_{b1}}{\kappa^2}f_{t2}\right]\left(\frac{\nu_t}{d}\right)^2+\frac{1}{\sigma}\left\{\frac{\partial}{\partial x_j}\left[(\nu_L+\nu_t)\frac{\partial \nu}{\partial x_j}\right]+C_{b2}\frac{\partial \nu_t}{\partial x_j}\frac{\partial \nu_t}{\partial x_j}\right\} \qquad (2)$$

where the proposed γ_{BC} is given by:

$$\gamma = 1-e^{-(Term_1+Term_2)} \qquad (3)$$

$Term_1$ and $Term_2$ in the γ_{BC} equation are defined as:

$$Term_1=\sqrt{\frac{\max\left(\frac{Re_v}{2.193}-Re_{\theta c},0.0\right)}{\chi_1\,Re_{\theta c}}} \quad and \quad Term_2=\sqrt{\frac{\max(\nu_{BC}-\chi_2,0.0)}{\chi_2}} \qquad (4)$$

where,

$$Re_v=\frac{\rho\,{d_w}^2}{\mu}\,\Omega \quad and \quad \nu_{BC}=\frac{\nu_t}{U\,d_w} \qquad (5)$$

In the Equations 4 and 5 above, $Re_{\theta c}$ is the critical momentum thickness Reynolds number, Re_v is the vorticity Reynolds number, ρ is density, μ is molecular viscosity, Ω is the vorticity, d_w is the distance from the nearest wall and the constant appearing in $Term_1$, 2.193, is the proportionality constant that relates the momentum thickness Reynolds number to the vorticity Reynolds number as seen in Menter et al. [Menter et al, 2004]. ν_{BC} is a proposed turbulent eddy viscosity-like non-dimensional term. χ_1 and χ_2 appearing in the equations are calibration constants, which are currently assigned to be 0.002 and 5.0, respectively.

In the model, there are two exponents, which are effective in stream-wise and cross-stream directions. $Term_1$ determines the onset location of the transition by comparing the locally calculated vorticity Reynolds number to the critical momentum thickness Reynolds number. However, the locally calculated vorticity Reynolds number is a low value inside the boundary layer (close to wall), thus the only way to achieve the diffusion of the generated turbulence into the boundary layer is the addition of $Term_2$. $Term_2$ checks for the viscosity levels, and if it is larger than a critical value, it surpasses the damping effect of the intermittency function and activates the production term. It should be noted that the proposed model currently does not include any correlation to account for the transition length.

Any experimental critical momentum thickness Reynolds number ($Re_{\theta c}$) correlation can be used in the model. Currently, the correlation used in the B-C model formulation is the zero-pressure gradient version of the Menter et al. correlation [Menter et al, 2004]. Originally, the turbulence intensity appearing in the correlation is calculated locally by using turbulent kinetic energy; however, S-A turbulence model does not solve for it, thus upstream turbulence intensity is used for the calculations as Suluksna and Juntasaro [Suluksna and Juntasaro, 2008] suggested. The correlation is given in Equation 6:

$$Re_{\theta c}=803.73\,(Tu_\infty+0.6067)^{-1.027} \qquad (6)$$

This correlation is created by incorporating the desirable aspects of the other prominent correlations [Menter et al., 2004]. For turbulence intensities less than 1%, Drela's [Drela, 1995] e^N type model is curve fitted, for turbulence intensities between 1% and 3%, the correlation is comparable to that of Abu-Ghannam and Shaw [Abu-Ghannam and Shaw, 1980], and finally at high turbulence intensities (>3%), the correlation is close to the Mayle's [Mayle, 1991]. These correlations are depicted in Figure 2.

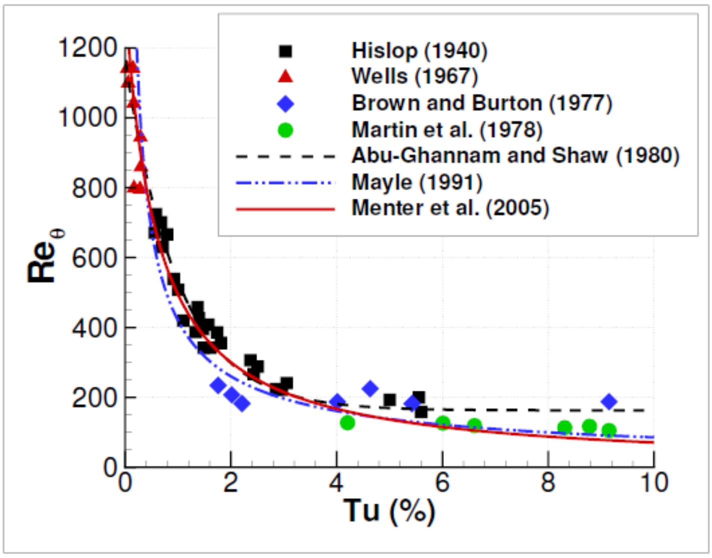

Figure 2: Comparison of transition onset correlations with experimental database

Model Calibration

The B-C model is calibrated against the zero-pressure gradient flat plate test case of Schubauer and Klebanoff [Schubauer and Klebanoff, 1955]. The tunnel used for the experiment generates a freestream turbulence intensity of only about 0.18%, thus the experiment represents a natural transition process. The model calibration is done by setting the χ_1 and χ_2 seen in Equation 4 such that the transition onset matches the experimental data. Different computational domains are generated with various number of points on the flat plate (200, 300 and 400 points) in order to ensure grid independence. In all these domains, y+ values are calculated as below 1.

Figure 3 compares the results with the experimental data. The figure also includes the numerical results of two-equation γ - Re_θ transition model of Menter et al. and the most recent one-equation γ model of Menter et al. [Menter et al., 2004; Menter et al., 2015]. As seen in Figure 3, 300 points on the flat plate seems sufficient enough to resolve the flow features. Although there is an abrupt rise observed in the calculations, the same sort of abrupt rise is observed in the calculations of Menter et al. As said previously, B-C model is an abrupt transition model because of not having incorporated a transition length correlation into the model.

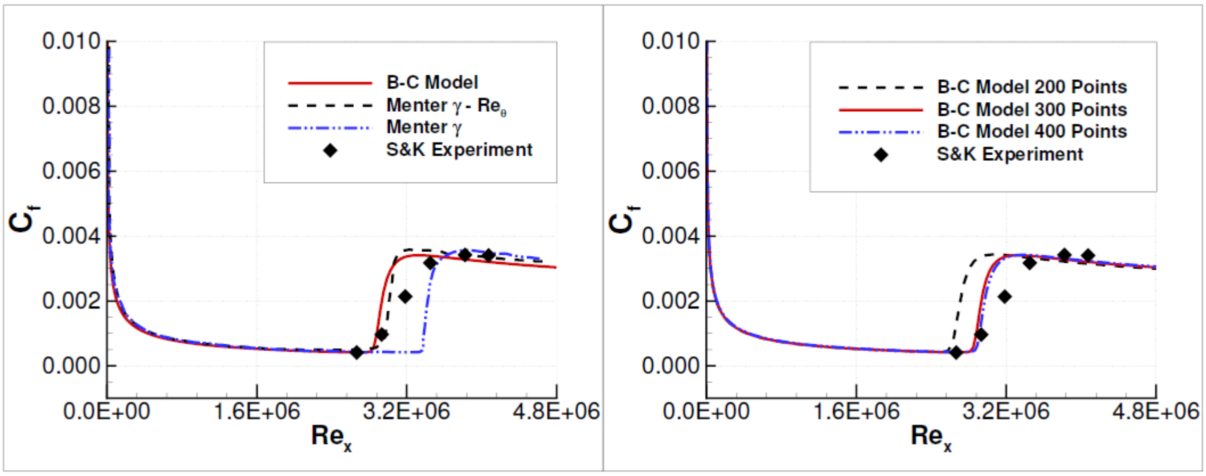

Figure 3: Comparison of skin friction coefficients for the Schubauer and Klebanoff case

MODEL VALIDATION

Flat Plate Test Cases

In the literature, ERCOFTAC (European Research Community on Flow, Turbulence and Combustion) flat plate experiments with zero pressure gradient and variable pressure gradients assembled by Savill [Savill, 1993] are widely used for transition code validation. In these experiments, a flat plate with a length of 1.5 meters is used. The upper wall profile of the working section of the tunnel can be adjusted to produce various pressure gradients including zero pressure gradient. The operating velocity range of the tunnel is up to 25 m/s, and the turbulence generating grids located at the entrance of the working section are designed to generate freestream turbulence intensities ranging from 0.5% to 7.0%. Since the upstream turbulence intensities of the majority of the test cases are around 3.0%, the kind of the transition process is bypass transition. Zero pressure gradient test cases are T3A, T3B and T3A- that have turbulence intensities of 3.0%, 6.0% and 0.9% respectively. Variable pressure gradient test cases are T3C1, T3C2, T3C3, T3C4 and T3C5 that represent an aft-loaded turbine blade with different freestream turbulence intensities (6.6% for T3C1 and 3.0% for the rest). For the T3C cases, the experimental data does not explicitly provide the upper wall profiles, however the freestream velocities and densities at different measurement stations along the flat plate are given. Simply applying the conservation of mass law, the upper wall profiles that would create the pressure gradients that matched the experimental data are obtained.

Zero Pressure Gradient Test Cases: Three zero pressure gradient bypass transition test cases; T3A, T3B and T3A- are tested. Table 1 summarizes the inlet conditions of the experiments. Figure 4 show the skin friction coefficient comparison of numerical results and the experimental data of T3A, T3B and T3A-, respectively. The figures also include the numerical data of Menter et al. γ - Re_θ transition model as well as Menter et al. γ transition model.

In general, the agreement between the numerical and experimentally measured skin friction coefficients is good. For the T3A case, the B-C model and Menter et al. γ model predict somehow late transition onset, whereas Menter et al. γ - Re_θ model is in good agreement with the experimental data. Another observation is that the overshoot in the immediate turbulent region is not well captured. For the T3B case, B-C model captures the transition onset with a slight delay. On the other hand, both Menter's models predict early onset of transition, but could not capture the deepest point. For the T3A- case, the experimental Tu% value is used rather than the inlet Tu% value, and the results are satisfactory.

Table 1: Inlet conditions for zero pressure and variable pressure gradient flat plate test cases

Case	U_∞ (m/s)	Re_∞	Tu (%)
S&K	50.1	3.3E6	0.18
T3A	5.4	3.6E5	3.0
T3B	9.4	6.3E5	6.0
T3A-	19.8	1.3E6	0.5
T3C1	5.9	3.9E5	6.6
T3C2	5.0	3.3E5	3.0
T3C3	3.7	2.5E5	3.0
T3C4	1.2	8.0E4	3.0
T3C5	8.4	5.6E5	3.0

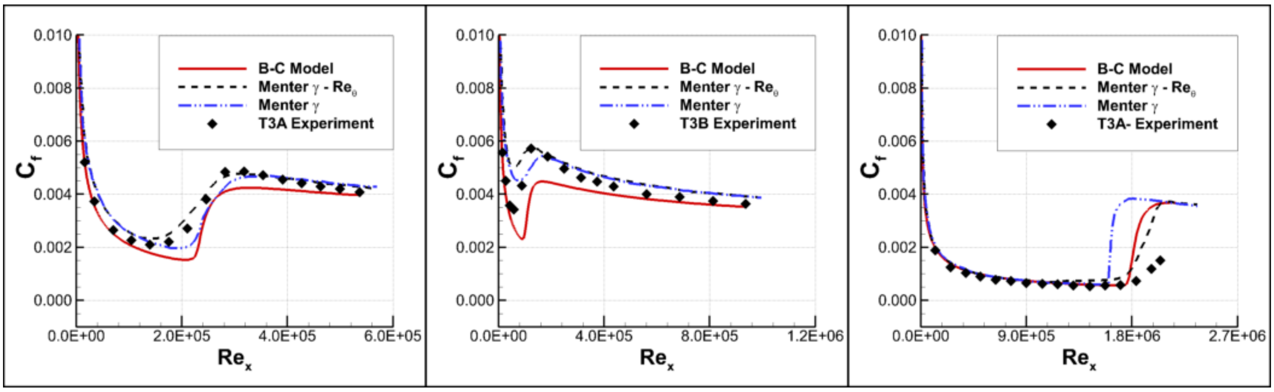

Figure 4: Skin friction coefficient for the Savill T3A, T3B and T3A- flat plate zero pressure gradient test cases

The new algebraic intermittency function γ_{BC} behaves quite in line with the rise in the turbulent viscosity and resultant rise in the skin friction coefficient. As shown in Figure 5 for the T3A case, the intermittency function γ_{BC} starts kicking in the production term in the Spalart-Allmaras turbulence model relatively early compared with the eddy viscosity and skin friction coefficient. This relatively earlier rise in γ_{BC} gives way to succeeding rise in both the turbulent eddy viscosity and skin-friction coefficient. Although the present model is an algebraic model, both the diffusion and convection effects are in fact simulated thanks to the corresponding Spalart-Allmaras model terms. In a way, the use of just a zero-equation modeling is vindicated by these results. Because, the crux of the matter is that the maximum value of vorticity Reynolds number is proportional to the momentum thickness Reynolds number in a Blasius boundary layer [Menter et al., 2004].

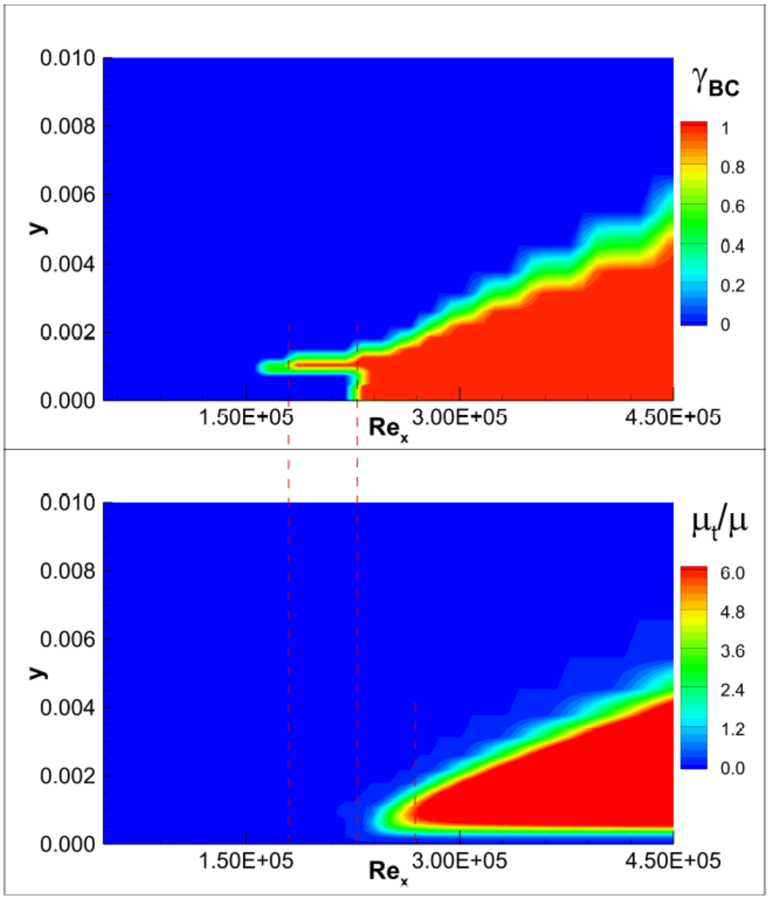

Figure 5: Intermittency function and viscosity ratio contours for the T3A test case

Variable Pressure Gradient Test Cases: Five variable pressure bypass transition test cases that represent actual turbine characteristics; T3C1, T3C2, T3C3, T3C4 and T3C5 are tested [Savill, 1993]. Table 1 summarizes the freestream conditions used in the experiments and the analysis; Figure 6 shows the comparison of the numerically obtained B-C model skin friction coefficient data against the experimental data as well as the comparison of the experimental and numerical upper wall velocity profiles along with upper wall contour.

Starting with the T3C1 case, the B-C model prediction for the skin friction seems to be quite good. The onset of transition is quite accurately predicted along with the subsequent variable pressure gradient sector. The T3C2 case represents a lower free stream turbulence case, and the B-C model is fairly good in the laminar region with good prediction of transition onset point but with an abrupt rise in turbulent stress. The T3C3 case is quite similar to the prior one although the onset of transition is slightly earlier. T3C4 case represents the lowest Reynolds number case in which although the laminar region is rather inaccurate, but the transition onset prediction is quite good. Finally, the favorable pressure gradient T3C5 case is in general in quite good agreement with the experiment in the laminar region, the onset of transition is fairly good with some delay and again quite good agreement in the subsequent variable pressure gradient region.

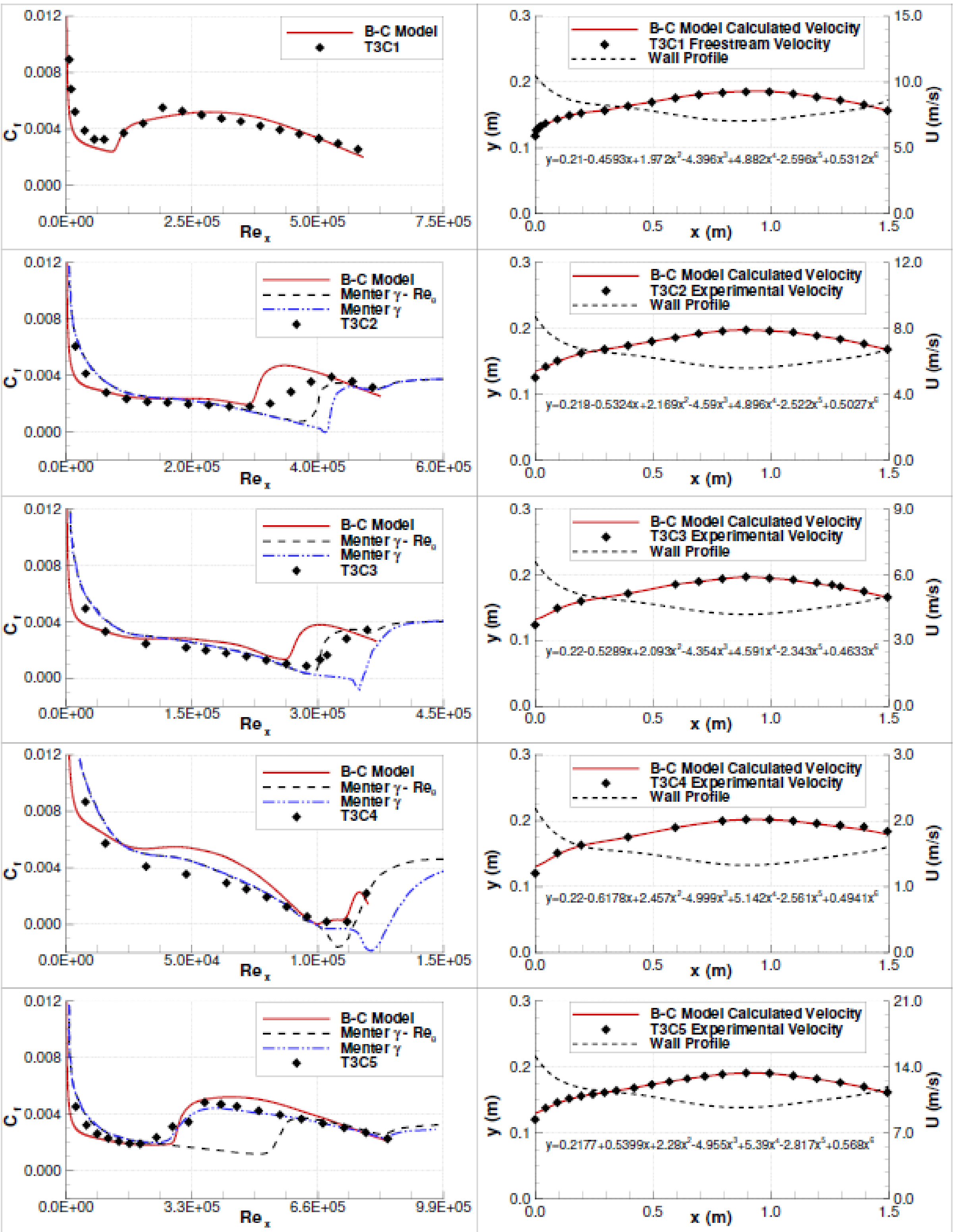

Figure 6: Skin friction coefficient for the Savill T3C1, T3C2, T3C3, T3C4 and T3C5 flat plate streamwise pressure gradient test cases (left), wall coordinates and velocities (right).

From the results, it appears that the present zero-equation model is a viable alternative for the Menter et al. original γ - Re_θ model, as well as the newly proposed γ model. Dropping the Re_θ equation in the Menter et al.'s newly proposed γ model seems to be a good decision as one consumes great efforts to tune the constants in the model against one set of data that does not work equally well for another set of data. Inspired by the Menter et al.'s γ model, further dropping of the γ equation as well to reduce the one-equation model into zero-equation may rather appear to be retrospective. However, having retained the crux of the matter, that is the observation that the maximum value of vorticity Reynolds number is proportional to the momentum thickness Reynolds number in a Blasius boundary layer, opens a quite pragmatic avenue for the present approach.

Eppler E387 Airfoil: The experimental data for E387 airfoil was obtained by Selig et al. [Selig et al., 1995] at UIUC for a range of low Reynolds numbers. In this study, numerical results are obtained with B-C transition model and S-A turbulence model for comparison at a low Reynolds number of 200,000. The computational grid having dimensions of 699x179 generated for the analysis is an O-type grid with first layer height from the airfoil surface is set to be 10^{-5} units for achieving y+<1 with an expansion ratio of 1.075 for better resolution of the boundary layer. The airfoil profile and the computational grid around it is given in Figure 7.

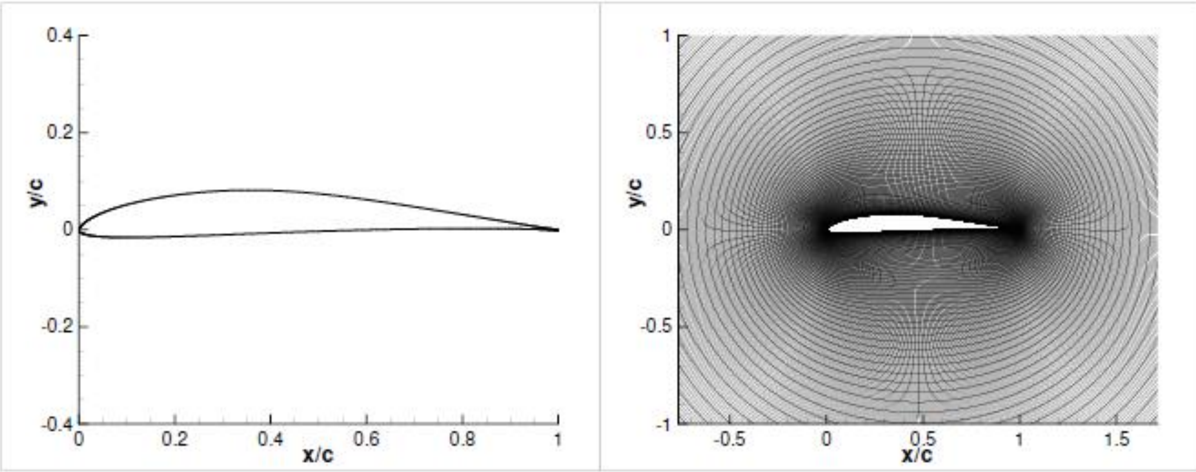

Figure 7: Eppler E387 airfoil profile (left), computational grid around E387 airfoil (right)

Figure 8 compares the experimental lift and drag coefficients with numerical results. As seen in the Figure 8, drag prediction of B-C transition model is in quite good agreement with the experimental data, whereas the fully turbulent solution is rather off-the-data in terms of drag coefficient.

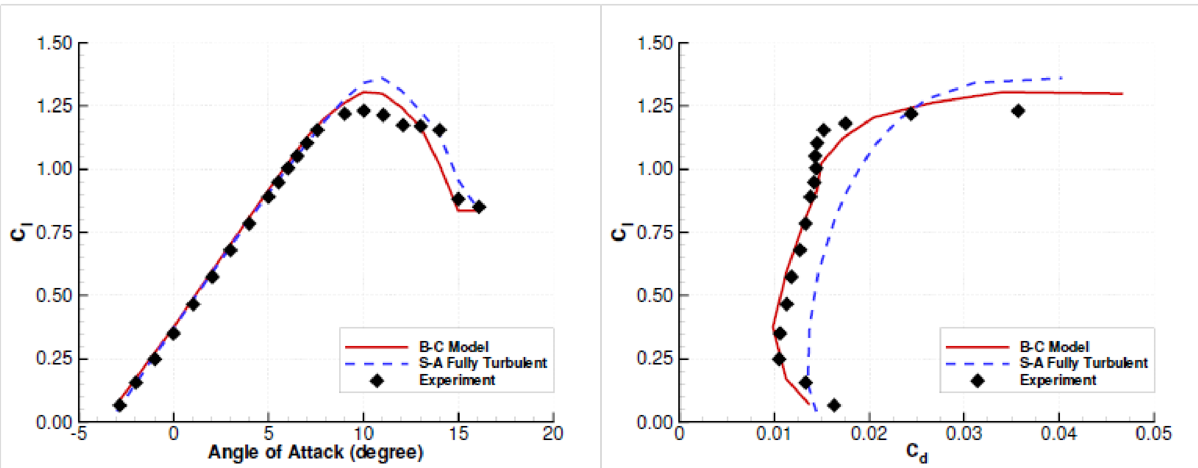

Figure 8: Comparison of B-C model calculations with fully turbulent numerical data and experimental data, lift coefficients (left), drag polar (right)

The pressure coefficient distribution over E387 airfoil for various angles of attack is also compared against the experimental data and given in Figure 9. B-C model seems to capture the separation bubble remarkably well for all cases, whereas the fully turbulent solution, as expected, could not do so. This is depicted in Figure 10, which shows the comparison of the streamline patterns displaying the laminar separation bubble obtained by the B-C transition model and the S-A turbulence model for 2 degrees angle of attack. Whereas the separation bubble is swept away in the fully turbulent S-A model, it is captured by the B-C model.

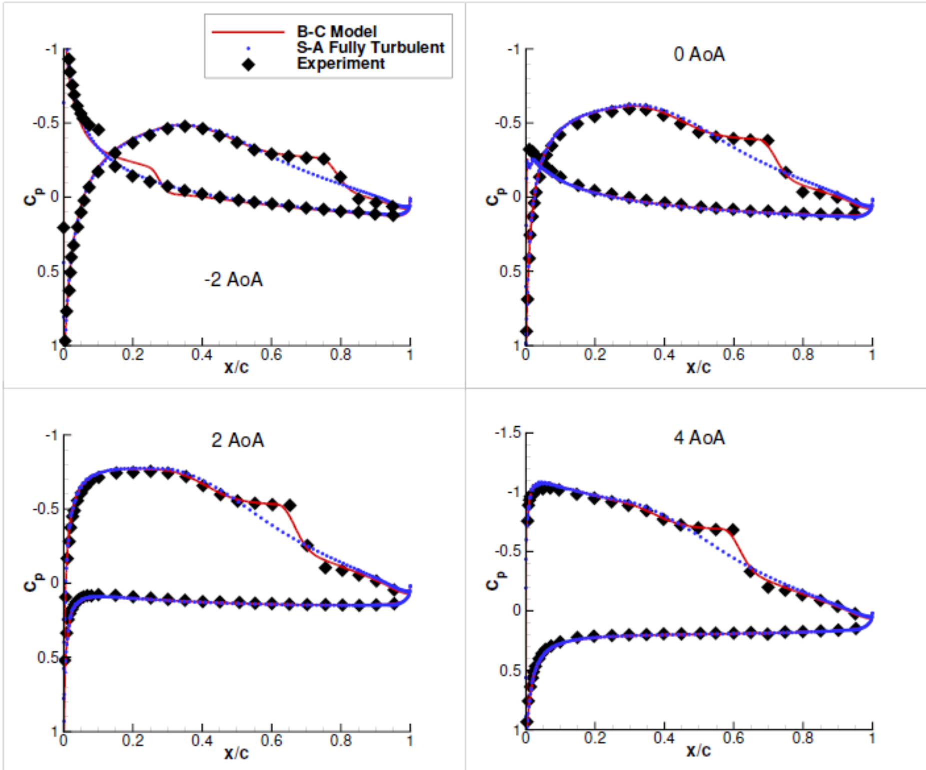

Figure 9: Comparison of pressure coefficient distribution for different angles of attack

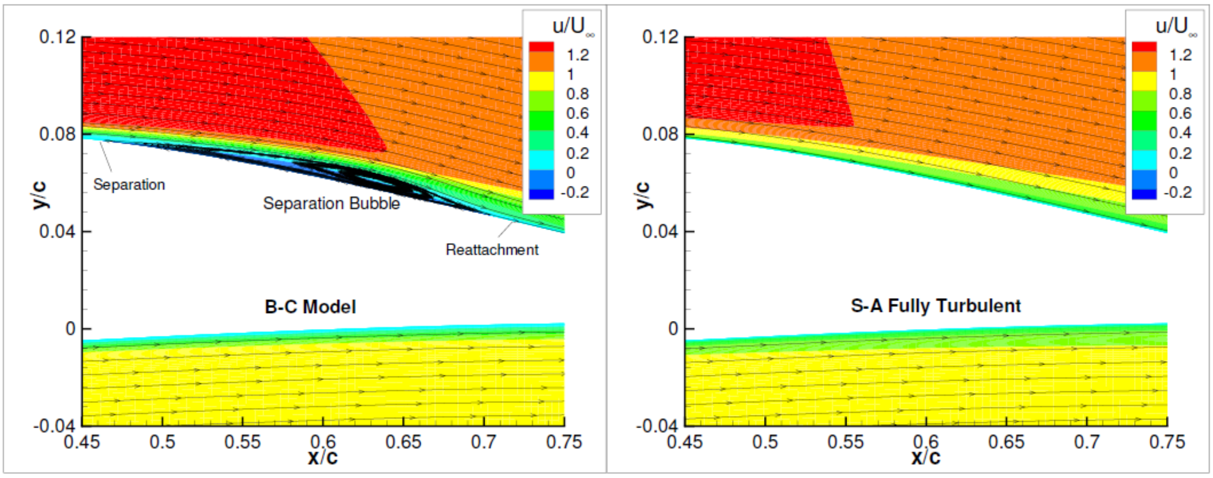

Figure 10: Comparison of velocity flow field contours for 2 degrees angle of attack; B-C transition model (left), S-A turbulence model (right)

<u>NACA 0021 Airfoil:</u> The experimental data for NACA 0021 21% thick symmetrical airfoil at low Reynolds number of 270,000 is obtained at Monash University wind tunnel [Swalwell, 2005]. The turbulence intensity at the inlet of the wind tunnel is reported to be around 0.6%. For the numerical solution, an O-type grid as seen in Figure 11 with dimensions of 699x179

is generated; the same grid generation parameters of Eppler E387 airfoil in the previous section is applied.

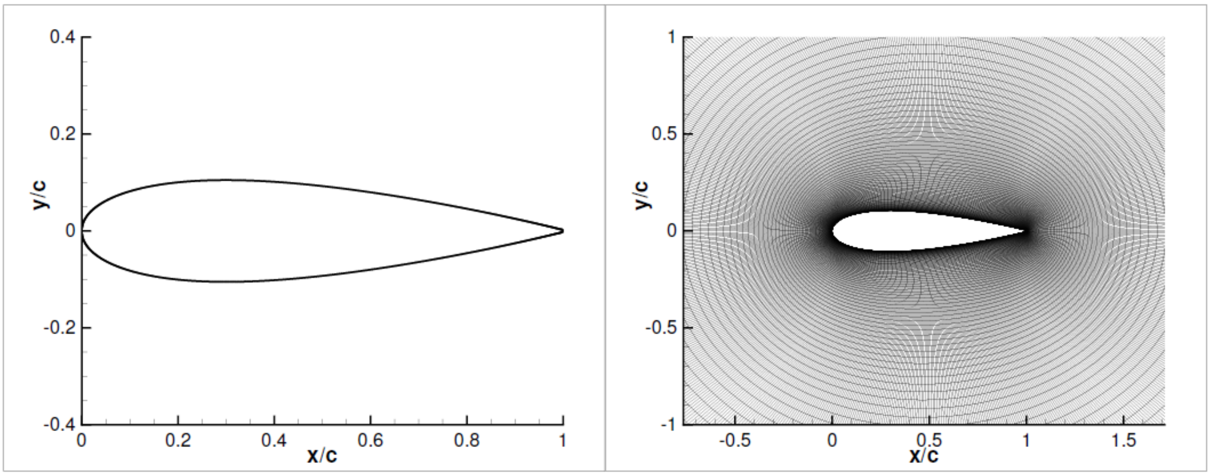

Figure 11: NACA 0021 airfoil profile (left), computational grid around NACA 0021 airfoil (right)

The experimental data is compared against the numerical data obtained with B-C model as well as the numerical data of Menter et al. γ - Re_θ model and γ model as presented in Figure 12. From Figure 12, it seems that the B-C model better predicted the maximum lift coefficient although the angle of attack at which it occurred is 4 degrees later than the experimental data. The deep stall, which is around 18 degrees of angle of attack in the experiment, is captured earlier by the B-C model whereas both γ - Re_θ model and γ model captured it with small delay. After the deep stall, quite remarkable match between the experimental data and the B-C model numerical data is observed. Considering the Eppler E387 airfoil case and the NACA 0021 airfoil case together, the B-C model seems to be a quite useful model for predicting the performance of airfoils operating at low Reynolds numbers.

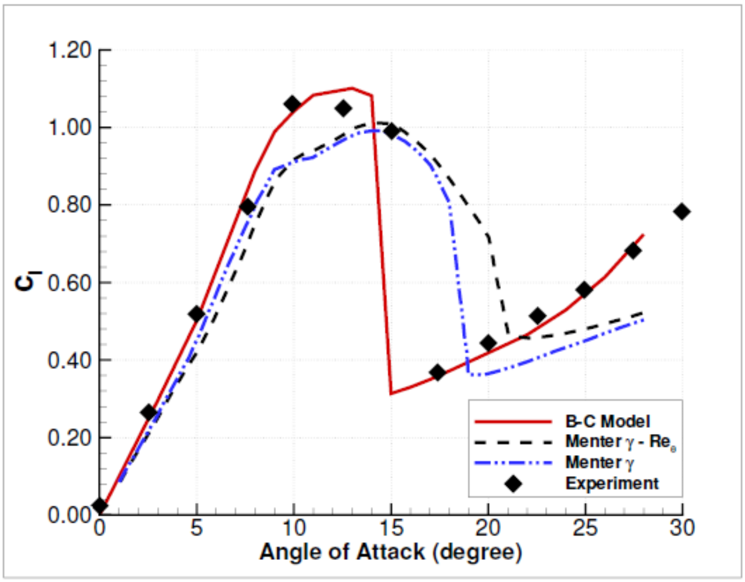

Figure 12: Comparison of experimental lift coefficients with numerical data for NACA 0021 airfoil at Re=270,000

CONCLUSIONS

A newly developed zero-equation transition model, namely the B-C model, has been validated. The B-C model does not solve for an extra intermittency equation with diffusion and convection terms, but instead already present turbulent convection and diffusion characteristics of the Spalart-Allmaras turbulence model is used. This way, the number of equations solved for a three-dimensional transition problem reduces from 9 equations to just 6 equations in comparison with the Menter et al. γ - Re_θ model. The B-C model is first calibrated against the zero-pressure gradient natural transition flat plate test case of Schubauer and Klebanoff. After that, the calibrated model is used to validate the T3A-, T3A and T3B zero pressure gradient bypass transition flat plate test cases as well as the T3C1, T3C2, T3C3, T3C4 and T3C5 variable pressure gradient flat plate test cases. In addition, the B-C model is employed to predict the aerodynamic performances of two airfoils, Eppler E387 and NACA 0021, operating at low Reynolds numbers. The results are generally in good agreement with the experimental data. Since the B-C model solves less equations than its counterparts, it is computationally cheaper. Moreover, calibration of the model for any application areas different than the presented ones is easier due to the fact that only two calibration constants appear in the model formulation. These two aspects are especially attractive in the design environment. Along with the very promising results, the B-C model may become a viable alternative for the state-of-art models like the γ - Re_θ model, and could be used in the industrial CFD applications as well.

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