

## FATIGUE LIFE ANALYSIS ON A MISSILE BODY EXPOSED TO FLIGHT LOADS

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### ABSTRACT

*In this study, fatigue analysis of a missile body structure during captive carriage at the underwing of a fixed wing aircraft platform is performed. The F-16 jet aircraft was determined as a fixed wing aircraft platform. As the external store, a data measurement store (DMS) was designed to measure loads during captive carriage for five different sorties. Acceleration and strain data were collected during the test. Accelerometers were used to generate power spectral densities (PSD) for the vibration tests and fatigue analyses. Strain gages were placed to the most critical locations for fatigue calculations. In data processing and analyses, Matlab<sup>®</sup>, Ncode<sup>®</sup>, MSC/Patran<sup>®</sup> and MSC/Nastran<sup>®</sup> were used.*

### INTRODUCTION

Fixed wing aircraft platforms have great importance and are very commonly used vehicles in militaries. However, without mounting external stores like missiles, bombs etc. on these aircrafts as shown in Fig. 1, usage of these aircrafts will be pointless. Therefore, external stores are indispensable necessities for these aircrafts. Consequently, design and production of external stores is very crucial for military applications. During the design of an external store, one must know the loads which the external store will encounter during its lifespan. During this lifespan, external stores encounter several types of loading like inertial loads, shock loads etc. Among these loads, random vibration loads are one of the most important ones. By definition, random vibration is the excitation of all frequencies while value of this random vibration is not predictable at any time. Moreover, some external stores can be exposed to these random vibration loads for a long time. For instance, some missiles must be attached to the aircrafts continuously for emergency situations; thus, random vibration exposure duration of these missiles can be very long such as several hundred or thousand hours. As a result, these loads cause structural failure of the external stores especially due to fatigue. By definition, fatigue is one of the reasons of structural failure which can occur in a metallic material due to the repeated application of stresses and strains which never reach a level sufficient to cause failure in a single application. Therefore, it is important to know the loads on the external stores to avoid fatigue.

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Figure 1: A fixed wing aircraft with external stores

Dreher worked on the development of random vibration test criteria for aircraft equipment [Dreher, 1982]. He collected acceleration data from 4 different aircrafts, two of them were fighter bomber types and two of them were cargo types. He showed variance of the vibration level by speed of the aircraft, aerodynamic pressure and surface geometry for different parts of aircraft. Findings of this study were generalized to pertinent, adaptable random vibration test criteria for aircraft equipment and this criterion is included in MIL-STD-810.

Eldred et al. presented the various sources of vibrations in space vehicles and equipment with formulation and experimental data [Eldred, Roberts and White, 1961]. They also investigated the response of the parts with both empirical and analytical approaches. Moreover, they discussed combining of various dynamic responses during the service life of the vehicle for vibration tests.

Lyon also presented dynamic environment of a flight vehicle, prediction of vehicle response and simulation of expected environment in the test [Lyon, 1967]. Bendat et al. did research on checking data for randomness, stationary and normality [Bendat, Enochson and Piersol, 1965]. Some basic inspection methods and tests were recommended in their study.

Halfpenny compared fatigue life results of different frequency domain fatigue analyses (Narrow Band and Dirlik methods) and time domain analysis [Halfpenny, 1999]. In his research, he found Dirlik method gives the best comparable results with the traditional time domain approaches. Dirlik method gives average discrepancy of only 4% from the fatigue life calculated in the time domain.

Topham evaluated and compared the fatigue damage from two different vibration events and an envelope of both events [Topham, 1991]. He used the Advanced Medium Range Air-to-Air Missile (AMRAAM) as the test item and the McDonnell Douglas F-15 Eagle as its platform for his study. Then, he used the same vibration test profile for the equipment test and the section test and compared fatigue damage from these both tests.

Behrooz et al. compared Findley's multi-axial stress based methods [Findley, 1959] with the test results [Behrooz, Ghajar and Moztarzadeh, 2007]. They calculated fatigue life for a steel shaft of a high speed micro generator rotor which was exposed to the torsional and bending stresses at the same time. They did multi-axial fatigue life analysis with Findley methods and compared the results with fatigue tests. Using Findley methods they found almost the same order of magnitude of life prediction with the test results.

## EXPERIMENTAL STUDY

### Data Measurement Store (DMS) and Sensor Placement

In this study, a data measurement store (DMS) as shown in Figure 2a and 2b was used to measure the vibration loads for fatigue calculations. Accelerometers were placed at critical parts and locations considering vibration tests and strain gages were placed to high stress expected locations. Accelerometers were used to generate power spectral densities (PSDs) for the vibration tests and fatigue analyses. Strain gages were used for fatigue calculations; therefore, they were placed at the most critical locations on the store. In total, 7

accelerometers and 51 strain gages were placed on the store. All accelerometers were 3 axial and dynamic accelerometers which measure the data above 1 Hz. However, one of the 7 accelerometers broke down during the flights and no meaningful data were retrieved from this accelerometer. Locations of other 6 accelerometers are shown in Fig. 2a. From 51 strain gages, 3 of them were rectangular rosette pattern, 16 of them were shear pattern and 32 of them were linear pattern strain gages. Locations of other strain gage are shown in Fig. 2b.

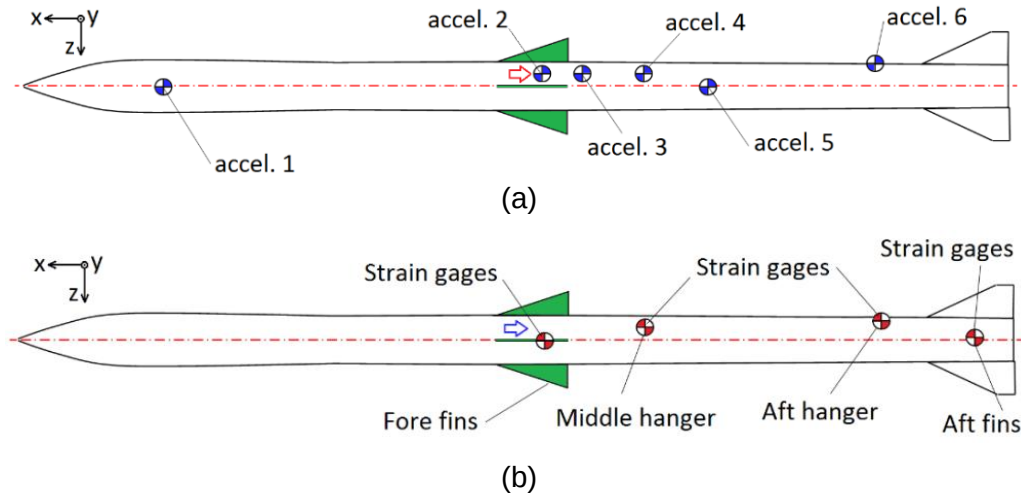


Figure 2: Locations of (a) the accelerometers and (b) the strain gages on the DMS

In Fig. 2b, two of the symbols which point at the wings of the store represent all 48 strain gages on the shafts of the wings. Other two symbols in the middle represent the rectangular rosette strain gages in front of the store hangers. Actually, there are three hangers (front, mid and aft hangers) in the store and there were three rosette strain gages in front of each of them. However, the rosette in front of the front hanger broke down and no meaningful data were retrieved from this rosette strain gage. Fig. 3 shows the locations of other two rosette strain gages. All gages in rosettes (3 gages for 1 rosette) were connected to the data acquisition system to form a quarter bridges.



Figure 3: Locations of rosette strain gages on the DMS

The DMS was mounted to the F-16C jet aircraft during sorties. In total, the DMS attached to the aircraft flew 5 sorties. In 2 of these sorties, the DMS was attached to 9th station of the aircraft and during other sorties, it was attached to the 8th station of the aircraft. In general, 3 types of data were obtained from the sorties. One of them is *Mux Bus* data obtained from the aircraft which consists of aircraft dynamic data such as altitude, speed, accelerations of the aircraft. Other two are acceleration and strain data obtained from the DMS.

## VIBRATION FATIGUE ANALYSIS

Vibration fatigue analysis is a frequency based fatigue analysis because PSD loads are used instead of time loads during analysis. Hereby, the structural frequency responses are taken into account during fatigue calculations. Cycles are counted from stress PSDs instead of load time signals. In this paper, the Lalanne/Rice method [Lalanne, 2009] which is one of the most accurate one when compared with the time domain data was used for cycle counting from the power spectral densities. After finding cycles, rest of the analysis is similar to time domain fatigue analysis. However, in this study, firstly, stress PSDs for unit acceleration load were obtained by frequency response function (FRF) analysis for vibration fatigue analysis. Then, acceleration PSDs obtain from flights are used to obtain stress PSDs during flights. In Figure 4, vibration fatigue analysis steps are summarized.

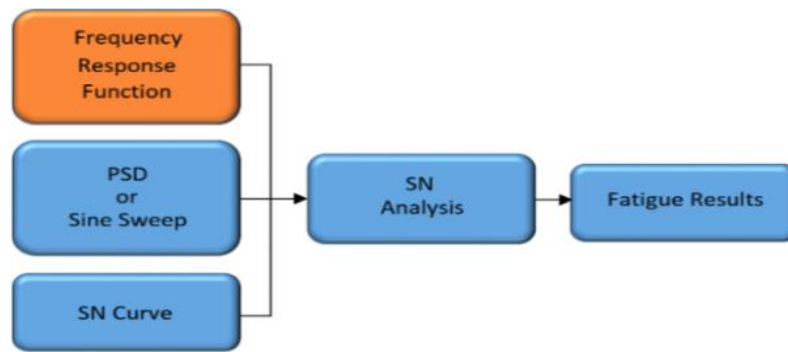


Figure 4: Vibration fatigue steps

As an example, fatigue analysis was done using PSD obtained from accelerometer 2 in this study. Accelerometer 2 location is right beside the front hanger of the DMS. For fatigue analysis, front part of the DMS was used and PSD obtained from accelerometer 2 was given as a base vibration. Model was also fixed from the base. The finite element model used for fatigue analysis is shown in Fig. 5. The model was meshed with solid TET10 meshes and RBE2 and RBE3 MPC elements were used instead of bolts to attach solid bodies and point masses, respectively.

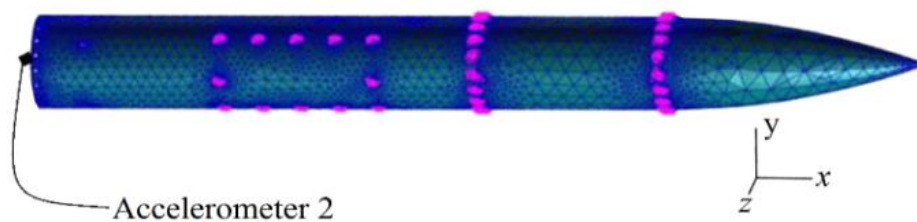


Figure 5: General view of finite element model for fatigue analysis

To make fatigue analysis, firstly stress PSDs of the model for every direction were obtained by FRF analysis choosing the unit load input location as accelerometer 2 location. FRF gives the relation between the input and output as follows:

$$H(f) = \frac{\sigma(f)}{\ddot{x}(f)} \quad (1)$$

where  $H(f)$  is the FRF,  $\sigma(f)$  is the stress (output) and  $\ddot{x}(f)$  is the base acceleration (input). After finding FRF, stress PSD can be found as follows:

$$PSD_{\sigma} = H(f) \cdot H^*(f) \cdot PSD_{\ddot{x}} \quad (2)$$

Where  $H^*(f)$  is the conjugate of the  $H(f)$ .  $PSD\ddot{x}$  corresponds to the PSDs in this case. After obtaining the stress PSDs for each axis, they are cycle counted by Lalanne method which is explained in Eq. 3. In this situation, stress probability density function was found instead of peak probability of the displacement as follows:

$$N(s) = \frac{\sqrt{1-r^2}}{\sigma_{rms}\sqrt{2\pi}} \cdot e^{-\frac{\Delta S^2}{2(1-r^2)}\sigma_{rms}^2} + \frac{r\Delta S}{2\sigma_{rms}} \left[ 1 + \operatorname{erf}\left(\frac{r\Delta S}{\sigma_{rms}\sqrt{2(1-r^2)}}\right) \right] \quad (3)$$

Where ( $S$ ) is the number of stress cycles,  $\Delta S$  is the stress range,  $\sigma_{rms}$  is the RMS value of the stress. In the above expression,  $r$  is irregularity factor and  $\operatorname{erf}$  is the error function respectively.

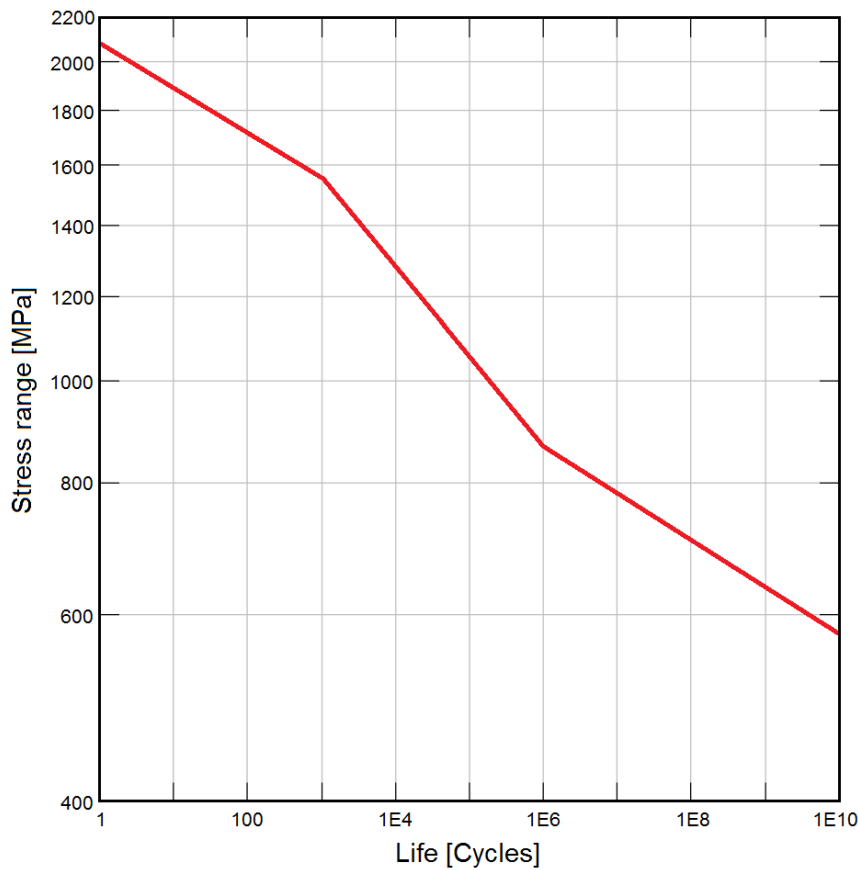


Figure 6: S-N curve of 4140 steel material

After cycle counting, S-N curve of the material was used to calculate life. The material of the model is 4140 steel with 36-42 HRC hardness and S-N curve of this material is shown in Figure 6.

## RESULTS AND DISCUSSION

Acceleration data steps to prepare data for fatigue analyses and tests are given briefly as follows. Also figures of these steps for accelerometer 2 shown in Figure 2 are shown as an example below the step explanation. The power spectral densities (PSD) of all the selected data were calculated separately as shown in Figure 7.

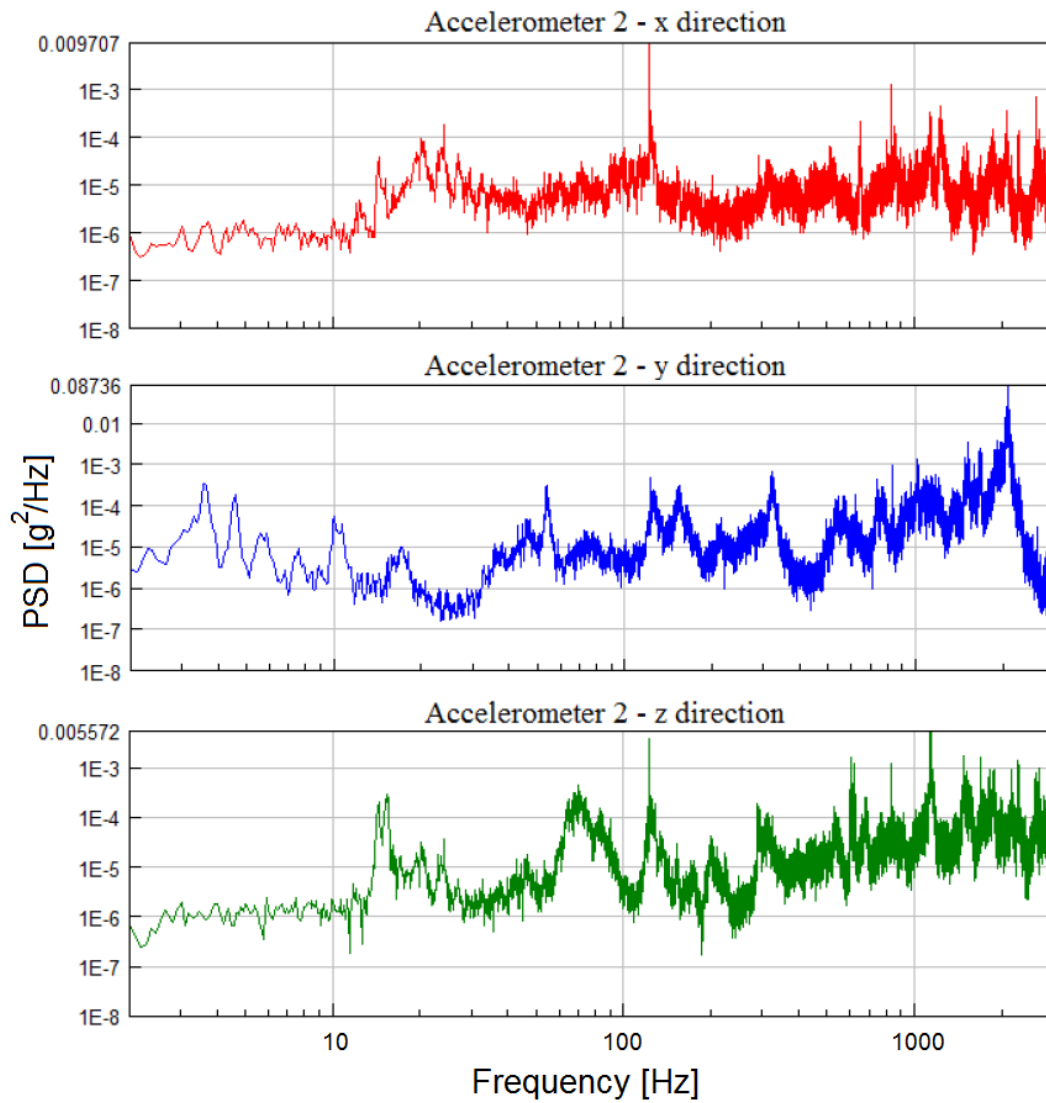


Figure 7: PSD of the acceleration data from accelerometer 2 (30 kft – 0.8 M)

PSD envelope is composed from the real environment PSDs to use in analyses and tests as shown in from Figure 8 to Figure 10.

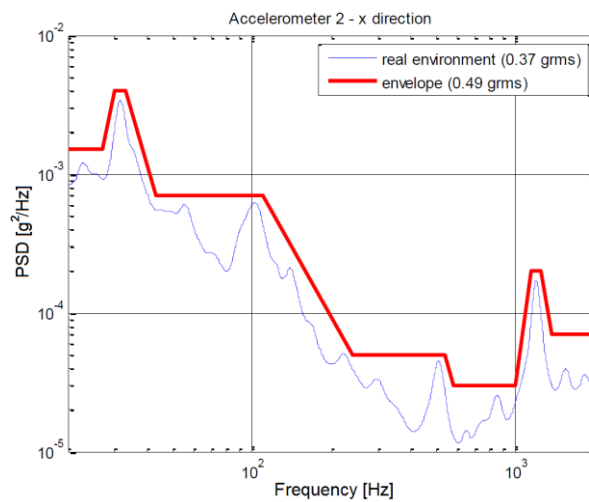


Figure 8: PSD envelope for the acceleration data from accelerometer 2 (x direction)

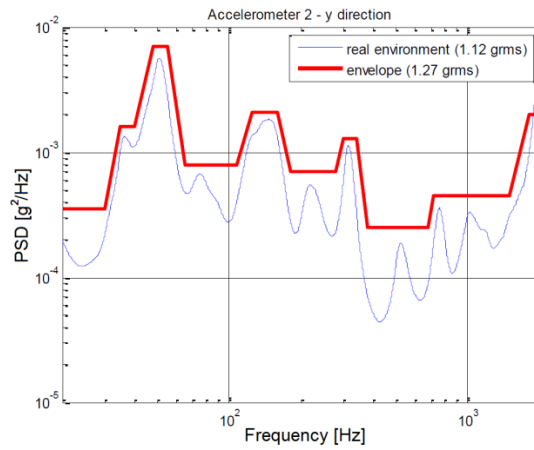


Figure 9: PSD envelope for the acceleration data from accelerometer 2 (y direction)

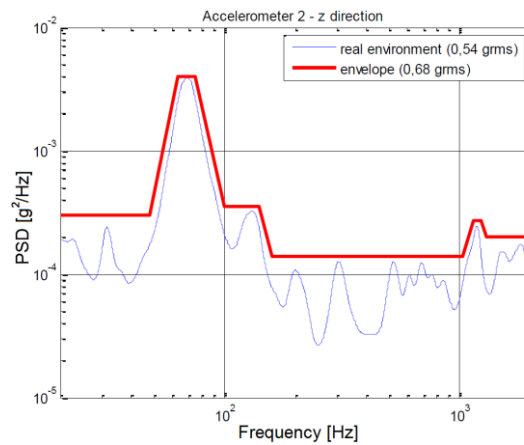


Figure 10: PSD envelope for the acceleration data from accelerometer 2 (z direction)

Von Mises RMS stress results of the model obtained from the  $PSD\sigma$ . These results were obtained for every axis separately because of software limitations. Therefore, multi-axial load effects were not taken into account. However, from figures, it can be seen that high stress locations are different for each axes vibration. Also, magnitudes of the von Mises RMS stresses are very different for each axis. Therefore, the results of uniaxial fatigue analysis would not be too different from the results of multi-axial stresses.

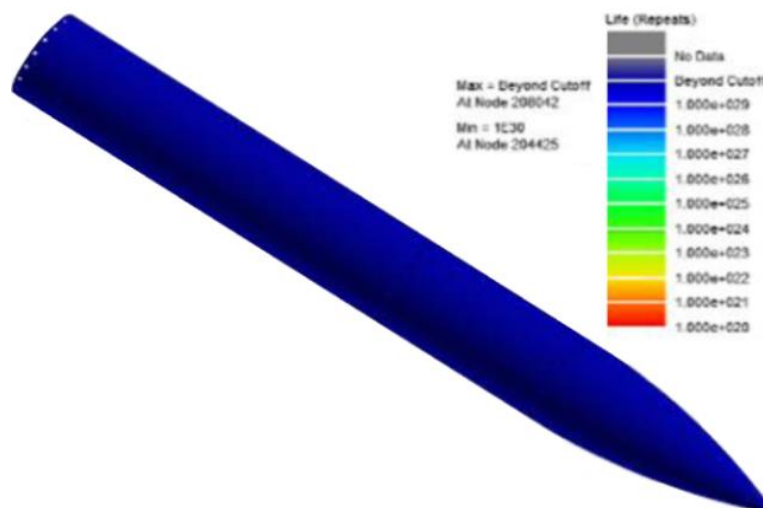


Figure 11: Life – x direction



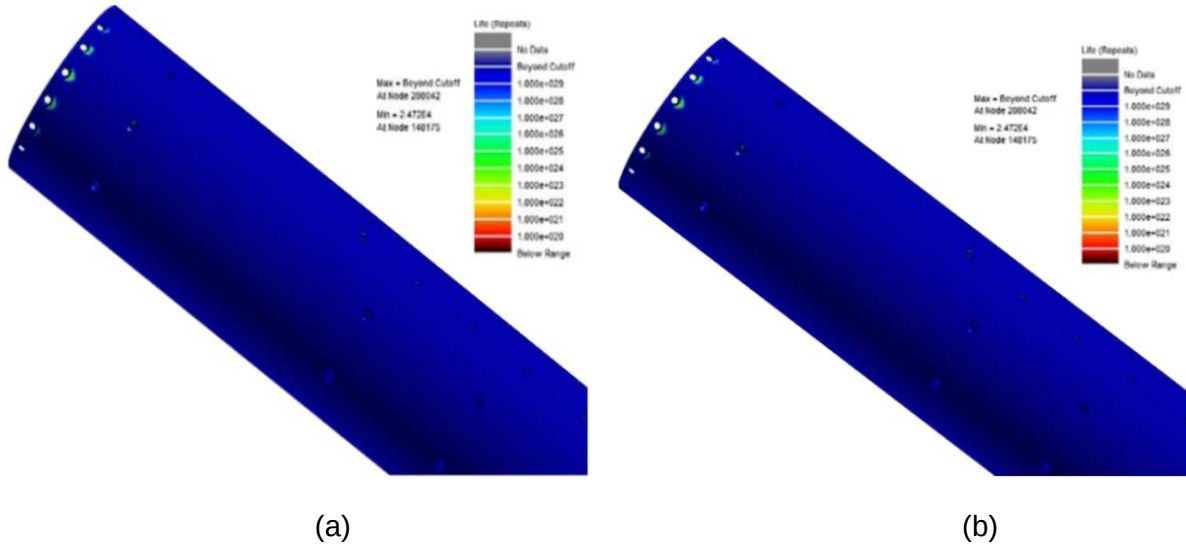


Figure 12: (a) Life – y direction and (b) Life – z direction

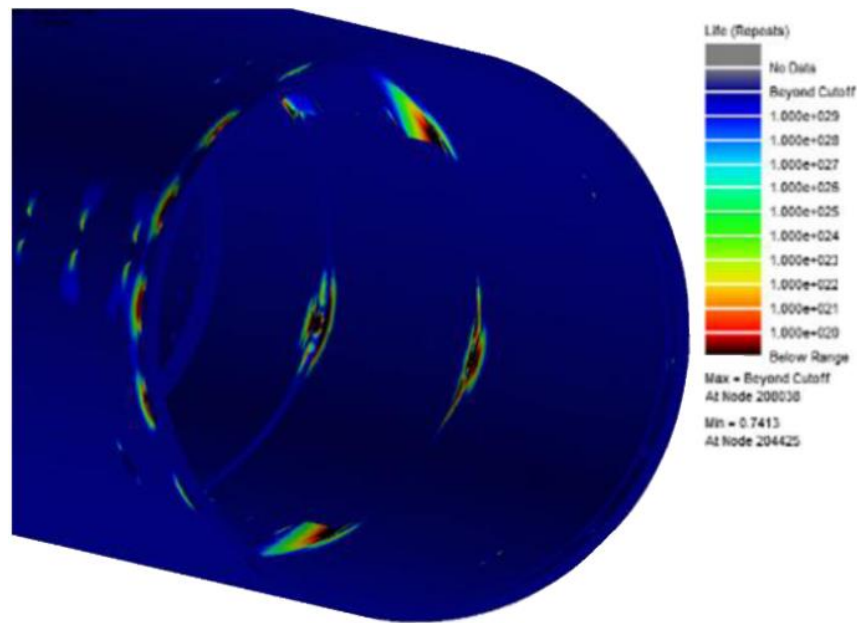


Figure 13: Life – z direction (inner detail)

Life results in Fig. 11 – Fig. 13 show that high stresses on the components because of vibrations are local only. Not too many locations have lower life than  $1e20$  seconds. The most critical vibration direction seems to be the z direction because of the gap in the middle of the cylinder shell and low thickness of the gap hatch and shell in this location. Results also show that multi-axial fatigue analysis would not change the results explicitly because z direction is much more dominant than the other two axes.

## CONCLUSION

As a result of this study, fatigue behavior of an external store under fixed wing aircraft platforms during random vibration is investigated. For this purpose, a data measurement store (DMS) was produced. Strain and acceleration data from the DMS was collected during 5 different sorties. Acceleration data were used for PSD calculation for an ideal average sortie for F-16C jet aircraft. The most critical vibration direction seems to be the z direction



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