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NATURAL VIBRATION AND AEROELASTIC STABILITY OF REALISTIC LAUNCH VEHICLES

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ABSTRACT

Natural vibrations and aeroelastic stability of realistic launch vehicles are investigated in this paper. Firstly, a discrete mass system is developed for determining the equations of motions of free-free unsymmetrical beams. Natural frequencies, mode shapes and mode slopes, also bending moments are obtained for three-staged launch vehicle. Secondly, to figure out aeroelastic behavior of launch vehicles, dynamic divergence pressure and generalized static margin of launch vehicle are considered. Considering realistic launch vehicles that using NASA research at early space flight investigations, numerical results are represented for both cases.

INTRODUCTION

The dynamic and aeroelastic stabilities of launch vehicle have great importance in space flights. Stability analysis of a slender flexible aerospace vehicle is a crucial point in structural design, control systems, targeting, and carrying payload. Static and dynamic instability of aerospace vehicle is related natural vibration characteristics and forces acting to the structure. According to the altitude, a rocket or missile is subjected thrust and aerodynamic force that classified non-conservative forces. Vibration characteristics and dynamic-aeroelastic stabilities of realistic rockets have been carried out rigorously in many papers and NASA technical reports from early studies of space history that mentioned in following paragraphs.

Alley and Gerringer [Alley and Gerringer, 1962] reported a matrix method for the determination of the natural vibrations of free-free unsymmetrical beams with application to launch vehicles. They applied discrete-mass system to evaluate vibrational characteristics of the system, and also deflections, slopes and moments by considering influence coefficients. They illustrated numerical results in the application of three-staged typical launch vehicle, besides accounted effect of joints to obtain realistic results. In another study, Alley and

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Gerringer [Alley and Gerringer, 1966] developed a discrete-mass method to analyze the aeroelastic divergence behavior of unguided launch vehicles. They considered stability criteria according to both generalized static margin and divergence dynamic pressure for rigid and flexible vehicles. A generic rocket body is used to demonstrate numerical results on the effect of aeroelasticity by considering variations in fin-lift characteristics.

Young [Young, 1968] presented the analysis of the aeroelastic divergence of two experimental unguided launch vehicles. He examined the theoretical analysis of these vehicles that have failed because of aeroelastic divergence and compared flight results and analytical results. Another study that declared by Young [Young, 1967] showed us a numerical method to determine the aeroelastic divergence characteristics of unguided, slender-body, multistage launch vehicles. In this study, a matrix recurrence solution to the system equations based on a finite difference method yields the stability boundaries because of aeroelastic divergence. On the other hand, Nakano [Nakano, 1971] inspected the application of transfer matrix method to the structural dynamics of rocket vehicles. He showed that axial force or aerodynamic lift force could be incorporated to the rocket system and found the distribution of displacement, slope and aeroelastic divergence pressure of selected rocket vehicles.

Abbas et al. [Abbas et al., 2013] performed transfer matrix method to determine the natural vibration characteristics of realistic thrusting multistage launch vehicle. They calculated the frequencies and mode shapes of non-uniform free-free Timoshenko beam also described slope and rotation angle, mode moments, and mode shears. Joshi [Joshi, 1995] dealt with free vibration characteristics of variable mass rockets having large axial thrust/ acceleration. There are also numerous studies in the literature that on the dynamic and aeroelastic instabilities of general or realistic launch vehicles [Cihan and Kaya, 2015; Beal, 1965; Wu, 1975; Wu, 1976; Yoon and Kim, 2002; Trikha et al. 2010; Duan and Wang, 2014].

In this paper, we mainly aim to find the natural vibration and aeroelastic stability of realistic launch vehicles. Launch vehicles are modeled as free-free unsymmetrical beams and equations of motion are obtained by using discrete-mass systems for both analysis. For selected research launch vehicles from NASA technical reports, free vibration analysis and stability analysis of the system are carried out and numerical results are compered with the previous papers.

FORMULATION

Structural Model

A rocket, which modeled as a discrete mass system, is demonstrated in Figure 1. The beam is considered as fixed at $x = 0$, free at $x = L$. Here, p is the number of discrete masses in the system; r is the station of discrete mass. y_0 and θ_0 are deflection and slope at the 0th station, respectively. The deflection of the rth station is defined as,

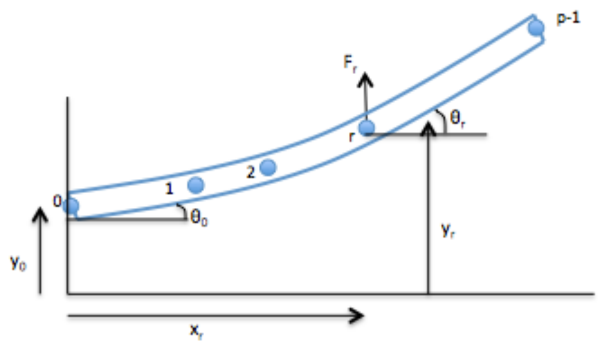

Figure 1: Discrete mass system

$$y_r = \sum_{n=1}^{p-1} F_n \sigma_{r,n} + \theta_0 x_r + y_0$$

$\sigma_{r,n}$ is total deflection influence coefficient accounting bending and joint effects at $x = x_r$ because of unit load applying at $x = x_n$. According to the laws of conservation of linear and angular momentum, we can drive the following equations:

$$\sum_{n=0}^{p-1} F_n = 0 \quad , \quad \sum_{n=0}^{p-1} F_n\, x_n = 0 \quad , \quad F_n = m_n \omega^2 y_n$$

Here, ω is the frequency of the beam. After necessary regulation, we can set the deflection and slope at the 0th station in the case of starting at $n = 1$, as;

$$\theta_0 = \frac{\omega^2 m}{\bar{x}} \frac{\bar{x}^2}{\bar{r}^2} \left\lfloor 1 - \frac{x_r}{\bar{x}} \right\rfloor \left\lfloor \frac{m_r}{m} \right\rfloor \left[\sigma_{r,n}\right] \left\lfloor \frac{m_n}{m} \right\rfloor \{y_n\}$$

$$y_0 = -\omega^2 m \left\lfloor 1 + \frac{\bar{x}^2}{\bar{r}^2} - \frac{\bar{x}^2}{\bar{r}^2} \frac{x_r}{\bar{x}} \right\rfloor \left\lfloor \frac{m_r}{m} \right\rfloor \left[\sigma_{r,n}\right] \left\lfloor \frac{m_n}{m} \right\rfloor \{y_n\}$$

Here, $\bar{x}$ is the distance of center of gravity to the origin, and $\bar{r}$ is the radius of gyration of total mass about the center of gravity of the beam. By substituting θ_0 and y_0 into the equation of total deflection y_r, we can obtain equations of motion in matrix form as;

$$\frac{1}{\omega^2 m}\{y_r\} = \left[\lfloor 1 \rfloor + \left[\frac{\bar{x}^2}{\bar{r}^2} \left\{ \frac{x_r}{\bar{x}} \right\} \left\lfloor 1 - \frac{x_r}{\bar{x}} \right\rfloor - \{1\} \left\lfloor 1 + \frac{\bar{x}^2}{\bar{r}^2} - \frac{\bar{x}^2}{\bar{r}^2} \frac{x_r}{\bar{x}} \right\rfloor \right] \left\lfloor \frac{m_r}{m} \right\rfloor \right] \left[\sigma_{r,n}\right] \left\lfloor \frac{m_r}{m} \right\rfloor \{y_r\}$$

$$(n = 1, 2,3 \ldots p-1)$$

$$(r = 1, 2, 3 \ldots p-1)$$

The solution of the above equation is classical eigenvalue problem. We can obtain natural frequencies and mode shapes of the system by generating the following equation.

$$\lambda_s\{y_r(s)\} = [A]\{y_r(s)\}$$

In the discrete mass system, mode slopes and mode moments can be expressed respectively as following,

$$\{\theta_r\} = \omega^2 m \left[\frac{\bar{x}}{\bar{r}^2}\{1\} \left\lfloor 1 - \frac{x_r}{\bar{x}} \right\rfloor \left\lfloor \frac{m_r}{m} \right\rfloor \left[\sigma_{r,n}\right] + \left[\rho_{r,n}\right] \right] \left\lfloor \frac{m_r}{m} \right\rfloor \{y_r\}$$

$$\{M_s\} = \omega^2 m \left[[x_s - x_r] - \{x_s\} \lfloor 1 \rfloor \right] \left\lfloor \frac{m_r}{m} \right\rfloor \{y_r\}$$

Influence Coefficients

Total deflections and slopes of the stations will be stated as influence coefficients. Total influence coefficients consist of two parts; one of them is due to elementary beam flexure and the other one is due to elastic rotation of joints.

<u>Total deflection influence coefficients</u>

$\sigma_{r,n}$ is total deflection influence coefficient accounting bending and joint effects at $x = x_r$ because of unit load applying at $x = x_n$. Total deflection influence coefficient is consist of $\alpha_{r,n}$ and $\delta_{r,n}$. The number of joints denotes v.

$$[\sigma_{r,n}] = [\alpha_{r,n}] + [\delta_{r,n}{}^{(1)}] + [\delta_{r,n}{}^{(2)}] + \cdots + [\delta_{r,n}{}^{(v)}]$$

$$[\sigma_{r,n}] = [\sigma_{n,r}]$$

$\alpha_{r,n}$ is the deflection influence coefficient due to elementary beam flexure, deflection at $x = x_r$ because of unit load applying at $x = x_n$.

$$\eta_r = \int_0^{x_r} \frac{1}{EI}\, dx, \qquad \mu_r = \int_0^{x_r} \frac{x}{EI}\, dx, \quad \beta_r = \int_0^{x_r} \frac{x^2}{EI}\, dx$$

$$\alpha_{r,n} = \beta_r - (x_n + x_r)\mu_r + x_n x_r \eta_r \qquad (when\ n \geq r)$$

$$[\alpha_{r,n}] = [\{\{\beta_r\} - \lfloor x_r \rceil\{\mu_r\}\}\lfloor 1 \rfloor + \{\lfloor x_r \rceil\{\eta_r\} - \{\mu_r\}\}\lfloor x_n \rfloor]$$

$$\alpha_{r,n} = \alpha_{n,r}$$

$\delta_{r,n}$ is the deflection influence coefficient due to elastic rotation of joints, deflection at $x = x_r$ because of unit load applying at $x = x_n$. This coefficient is taken into account for each joint of beam and added to total deflection influence coefficient. The joint location on the beam is demonstrated in Figure 2. Here, κ_u is joint rotation constant for joint u and c_u is coordinate of joint u.

$$[\delta_{r,n}{}^{(u)}] = \kappa_u \left[\{\{x_r\} - c_u\{1\}\}\{\{x_n\} - c_u\{1\}\}^T\right]$$

$$\delta_{r,n}{}^{(u)} \neq 0 \qquad (when\ x_r, x_n > c_u)$$

$$\delta_{r,n}{}^{(u)} = 0 \qquad (when\ x_r, x_n \leq c_u)$$

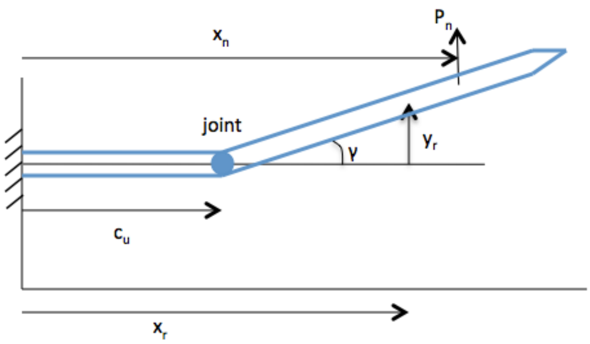

Figure 2: Joint location on the beam

<u>Total slope influence coefficients</u>

$\rho_{r,n}$ is total slope influence coefficient accounting bending and joint effects at $x = x_r$ because of unit load applying at $x = x_n$. Total slope influence coefficient is consist of $\nu_{r,n}$ and $\xi_{r,n}$.

$$[\rho_{r,n}] = [\nu_{r,n}] + [\xi_{r,n}{}^{(1)}] + [\xi_{r,n}{}^{(2)}] + \cdots + [\xi_{r,n}{}^{(v)}]$$

$\nu_{r,n}$ is the slope influence coefficient due to elementary beam flexure, slope at $x = x_r$ because of unit load applying at $x = x_n$.

$$\nu_{r,n} = x_n \eta_r - \mu_r \qquad (r \leq n)$$

$$\nu_{r,n} = \nu_{n,n} = x_n \eta_n - \mu_n \qquad (r > n)$$

$\xi_{r,n}$ is the slope influence coefficient due to elastic rotation of joints, slope at $x = x_r$ because of unit load applying at $x = x_n$. This coefficient is taken into account for each joint of beam and added to total slope influence coefficient.

$$[\xi_{r,n}{}^{(u)}] = \kappa_u\{1\}\{\{x_n\} - c_u\{1\}\}^T \qquad (when\ \ x_r, x_n > c_u)$$

$$\xi_{r,n}{}^{(u)} = 0 \qquad (when\ \ x_r, x_n \leq c_u)$$

Stability Criteria

In this section, aeroelastic stability of an unguided launch vehicle that subjected to aerodynamic loads will be discussed. In order to determine the stability of flexible system under aerodynamic loads, obtaining the aeroelastic divergence of system is necessary but not adequate. In this context, the static margin of the system should be figured out. To define the stability of the system, both aeroelastic divergence and static margin should be considered in the case of following statements that mentioned before Reference [Alley and Gerringer, 1966] as,

$$\frac{q}{q_{div}} \leq \frac{1}{2}$$

$$\frac{L}{15} \leq x_{sm,rig} \geq D_{max}$$

Divergence Dynamic Pressure

The discrete mass system is used to figure out aeroelastic divergence of beam. A discrete element *r* subjected to aerodynamic and inertia forces and its coordinate system are demonstrated in Figure 3.

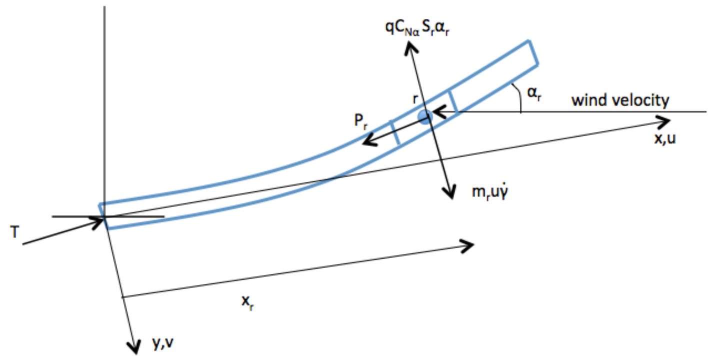

Figure 3: A discrete element of deflected beam that subjected to forces

The total transverse force F_r, the local angle of attack α_r at the r^th element and some regulations can be described as,

$$F_r = m_r u\dot{\gamma} - qC_{N_\alpha} S_r \alpha_r$$

$$\alpha_r \approx \frac{v}{u} - \left(\frac{dy}{dx}\right)_r$$

$$\left(\frac{dy}{dx}\right)_r = \sum_{n=1}^{p-1} \rho_{r,n} F_r$$

$$F_r = m_r u\dot{\gamma} - qC_{N_\alpha} S_r \left(\frac{v}{u} - \sum_{n=1}^{p-1} \rho_{r,n} F_n\right)$$

After some regulations for angle of attack, equation of motion in the matrix form can be defined as,

$$\{F_r\} = u\dot{\gamma}\{m_r\} - q\frac{v}{u}\lfloor C_{N_\alpha}S_r \rceil\{1\} + q\lfloor C_{N_\alpha}S_r \rceil\lfloor \rho_{r,n} \rceil\{F_n\}$$

$$\mathrm{r,n} = 1,2,3,\dots,\mathrm{p}-1$$

According to law of conservation of linear and angular momentum, following two relationships can be generated,

$$\lfloor 1 \rfloor\{F_r\} + F_0 = 0$$

$$\lfloor x_r \rfloor\{F_r\} = 0$$

Where,

$M = \lfloor 1 \rfloor\{m_r\} + m_0$

$Mx_{cg} = \lfloor x_r \rfloor\{m_r\}$

$\overline{C_{N_\alpha}S} = \lfloor 1 \rfloor\lfloor C_{N_\alpha}S_r \rceil\{1\} + C_{N_\alpha}S_0$

$x_{cp}\overline{C_{N_\alpha}S} = \lfloor x_r \rfloor\lfloor C_{N_\alpha}S_r \rceil\{1\}$

By using above equations and after necessary regulation, the equation of the motion of the system can be stated as,

$$\{F_n\} = q\overline{C_{N_\alpha}S}\left[\lfloor 1 \rfloor + \left\{\frac{m_r}{M}\right\}\left\lfloor\frac{x_{cp}-x_r}{x_{cg}-x_{cp}}\right\rceil + \left\lceil\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right\rfloor\{1\}\left\lfloor\frac{x_r-x_{cg}}{x_{cg}-x_{cp}}\right\rceil\right]\left\lfloor\frac{C_{N_\alpha}S_r}{\overline{C_{N_\alpha}S}}\right\rceil\lfloor \rho_{r,n} \rceil\{F_n\}$$

The solution of this equation is a classical eigenvalue problem and also we can find the natural frequencies of the system and divergence dynamic pressure that given as,

$$\frac{1}{q\overline{C_{N_\alpha}S}}\{F_n\} = [A]\{F_n\} \qquad (*)$$

$$\mathrm{q_{div}} = \frac{1}{\overline{C_{N_\alpha}S}\,\lambda}$$

Static Margin

The static margin is used to determine the stability of rigid structures, which is meant by the distance from the center of gravity to the center of aerodynamic center. For rigid vehicles, static margin is independent from the dynamic pressure. For the vehicles that elastically deformed, the static margin is expressed as, 1st mode

$$x_{sm} = x_{cg} - \frac{\lfloor x_r \rfloor\lfloor C_{N_\alpha}S_r \rceil\{\alpha_r\}}{\lfloor 1 \rfloor\lfloor C_{N_\alpha}S_r \rceil\{\alpha_r\} + C_{N_\alpha}S_0\alpha_0}$$

RESULTS

For the first part of numerical results, a three-staged realistic rocket that is given in NASA's technical report [Alley and Gerringer, 1962] is used and demonstrated in Figure 4. Discrete masses, locations, and stations are showed in this figure elaborately. In this configuration the rocket has 22 discrete masses and 9 joint rotations that are indicated black and blue dots in the figure, respectively. Flexural stiffness coefficients EI and joint rotation constants $\kappa_u$are given in [Alley and Gerringer, 1962]. According to above equations, natural frequencies and mode shapes of the system are figured out by using discrete mass system. Moreover, for the first three mode of the system, deflection y_r, slope θ_r, and the bending moment M_s of the rocket are calculated.

Natural frequencies and mode shapes, mode slopes and bending moments for first three natural modes of three-staged launch vehicle is demonstrated in Figure 5-7, respectively. Here, the results are perfectly matched up with the reference paper.

In the second part of the analysis, NASA research vehicle RAM III is used for the numerical considerations. Flexural stiffness coefficients EI, joint rotation constants κ_u, weight distribution, basic trajectory parameters and distribution of $C_{N_\alpha}S_r$ for RAM III vehicle according to selected stations are given in [Alley and Gerringer, 1966].

In this part of analysis the stability of the vehicle is carried out. Dynamic divergence pressure q_{div} and static margin x_{sm} of the rocket is needed to determine the stability criteria. In this regard, the rate of dynamic pressure of the airstream to dynamic divergence pressure q/q_{div} with respect to $C_{N_\alpha}S_0$ is showed in Figure 8. On the other hand, static margin of the system x_{sm} according to $C_{N_\alpha}S_0$ is demonstrated in Figure 9.

Primarily, to obtain divergence dynamic pressure we need to solve Equation (*) that containing mass and aerodynamic characteristic of rocket. These characteristics are captured for a specific flight time and Mach number. All data in Reference [Alley and Gerringer, 1966] are given at Mach 4 for RAM III because of the fact that the maximum dynamic pressure is comprised in this flight condition. $C_{N_\alpha}S_0$ is zeroth station aerodynamic coefficient, which is consisted both body and fin contributions but fin contribution has a superior effect relatively. For this reason, we consider the stability index $q/_{q_{div}}$ with changing $C_{N_\alpha}S_0$ values in Figure 8.

For flexible vehicles when the stability index is equal to one $\left(q/_{q_{div}} = 1\right)$, aeroelastic divergence occurs and the divergence normal force coefficient at zeroth station is depicts as $C_{N_\alpha}S_{div}$. As seen in the figure, the value of $C_{N_\alpha}S_{div}$ is 0.85*m*^{2}*/radian* where $q/_{q_{div}} = 1$. Further result from the figure we show, which is the nominal aerodynamic coefficient, is $C_{N_\alpha}S_0 = 1.59\, m^2/radian$ seen with red dot line on the curve. As mentioned before, $q/_{q_{div}}$ should be less and equal to 1/2 for nominal design to set stability criteria. Here, we can say that nominal design of RAM III vehicle is compatible by considering stability index that refer to aeroelastic divergence stability of rocket.

Second criteria for stability of a rocket is static margin that used for rigid structures. In Figure 9, it is apparently seen that when the $C_{N_\alpha}S_0$ increases the static margin also increases. On the other hand, the crucial point is $C_{N_\alpha}S_0 = 0.85\, m^2/radian$ where aeroelastic divergence occurs at $q/_{q_{div}} = 1$. In this point, the static margin value is found as $x_{sm} = 0.23m$.

For both analyses we can say that the result of the q/q_{div} and $\boldsymbol{x_{sm}}$ are compared with the reference paper [Alley and Gerringer, 1962] and our results are overlapped with given.

Stability criteria of launch vehicles is determined according to flexibility and rigidity of structures. For flexible structures, it is necessary criteria that obtaining the aeroelastic divergence, $q/_{q_{div}} \leq 1/_2$. In addition to this, for the rigid structures, static margin of the vehicle should be determined, as mentioned previous section.

For the RAM III vehicle, when nominal design value is considered as $C_{N_\alpha}S_0 = 1.59\, m^2/radian$, $q/_{q_{div}} = 0.4754$. This value is smaller than the limit value according to aeroelastic divergence criteria. Furthermore, the static margin of the RAM III x_{sm} , considering rigid body approximation is bigger than the maximum diameter of the vehicle, also bigger than the one fifteenth of the length of the vehicle $(L = 12.58m)$. In this context, we can say that the RAM III vehicle is stable for both stability criteria.

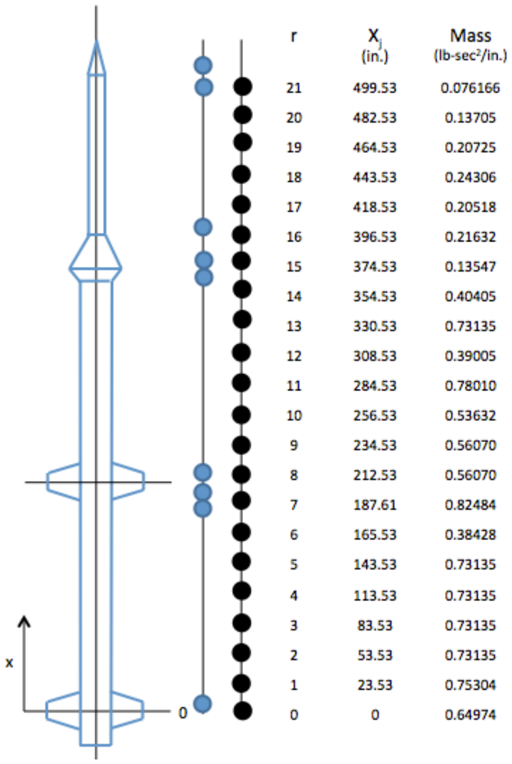

Figure 4: Discrete mass representation of a typical three-staged launch vehicle

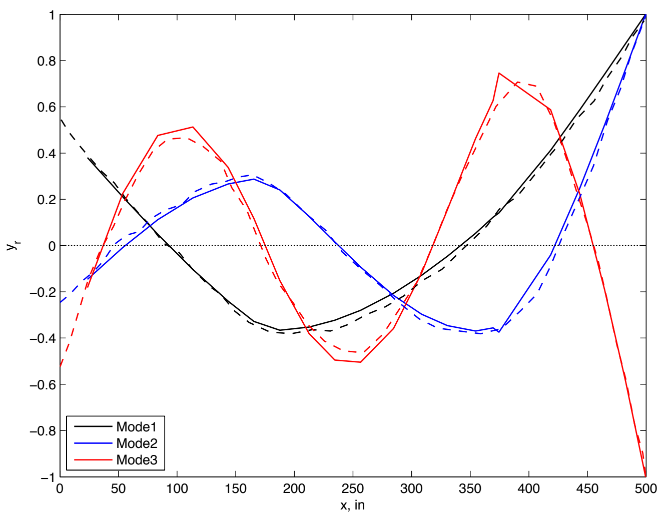

Figure 5: Natural frequencies and mode shapes for the first three modes of generic launch vehicle

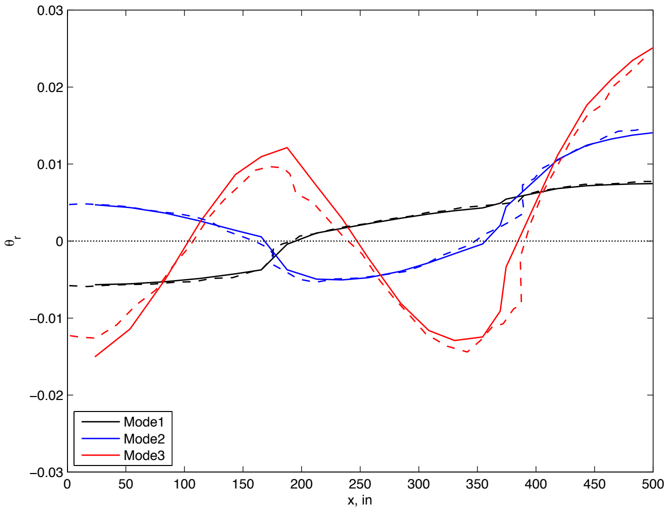

Figure 6: Mode slopes for the first three modes of generic launch vehicle

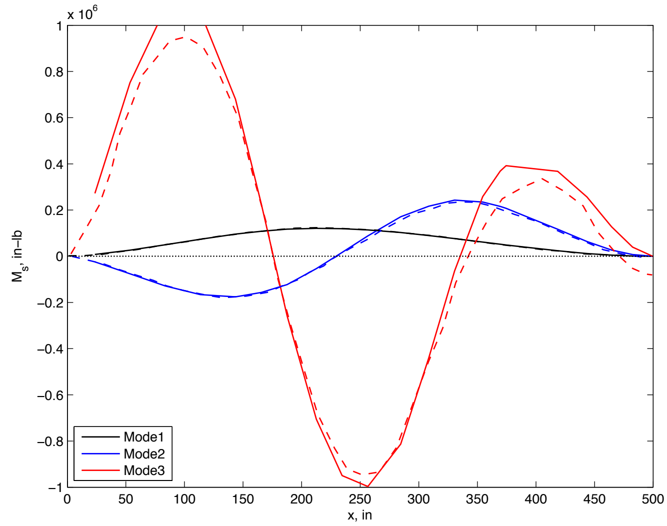

Figure 7: Moment curves for the first three modes of generic launch vehicle

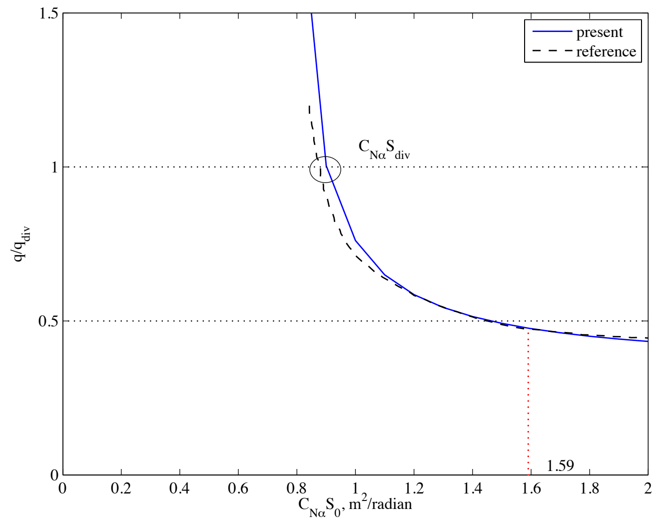

Figure 8:Variation of q/q_{div} with zeroth panel normal force coefficient for the RAM-III vehicle

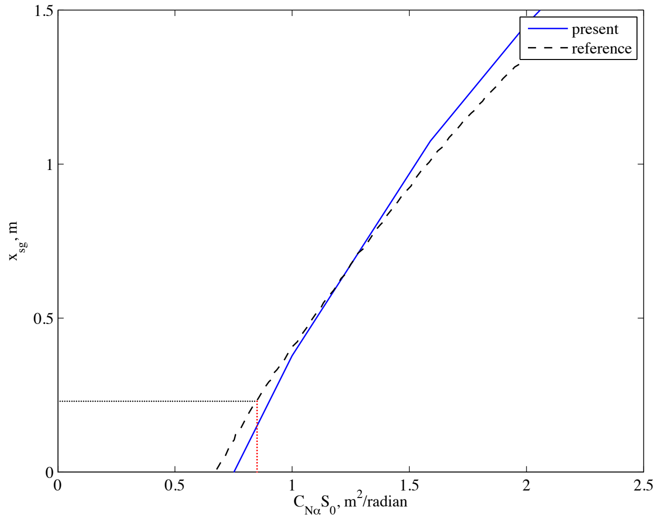

Figure 9: Static margin of the RAM-III vehicle

CONCLUSION

Natural vibrations and aeroelastic stability criteria are studied in this paper. The numerical solutions are obtained by implemented of discrete mass systems. Natural frequencies and mode shapes by considering deflections, slopes and bending moments of the launch vehicle, are figured out. Additionally, static margin and dynamic divergence pressure of a generic launch vehicle that subjected to aerodynamic forces are calculated. For both analysis two different generic launch vehicles, which experienced in NASA research, are used to verify our results. In conclusion, we achieved perfect match with literature for our results.

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