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Investigation of Mean Stress Effect for Equivalent Stress Methods and Critical Plane Approaches for High Cycle Fatigue Life Estimations

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ABSTRACT

Various critical plane methods were proposed and developed since Findley introduced the idea of using critical plane concept for estimation of high cycle fatigue life. However, still basic equivalent stress methods are used for industrial applications. Aim of this paper is to compare several equivalent stress and critical plane criteria regarding their performance in predicting fatigue life under in-phase and out-of-phase bending (or axial)-torsion loadings that are obtained from the literature. In the present study special attention is given to mean stress effect. To that end, we transform the multiaxial stress state into uniaxial fully reversed stress history by inclusion of well-known mean stress corrections such as Soderberg, Goodman etc. On the other hand, for critical plane methods, stress state is transformed into a damage parameter which is calculated for all material planes. Statistical analysis of estimations is made for comparison of methods.

INTRODUCTION

Multiaxial fatigue is an important failure mode that is experienced by many engineering parts such as crankshafts, turbine blades, pressure vessels and fasteners. Such engineering parts, usually designed in accordance with safe life methodology for high cycle fatigue, operate under multiaxial cyclic loading which is the main cause of this fatigue failure. For several decades, different methods are proposed and developed in order to estimate fatigue life of components. However, none of those methods are universally accepted and applicable to all material types and loading scenarios. Therefore, a good knowledge of methods is required for accurate life estimations.

Multiaxial fatigue problem was first investigated by Lanza when he performed combined rotating bending/torsion experiments [Lanza, 1886]. Later, several researchers such as Mason, Haigh, Nishiara and Kawamoto and Gough et al. correlated multiaxial test results with empirical relations [Mason, 1917; Haigh, 1923; Nishiara and Kawamoto, 1941 and Gough, 1951]. However, since testing a material under all combination of complex loading is

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not feasible, methods that describe multiaxial fatigue behavior with simple uniaxial test results are required. Therefore, investigators attempted to adapt static yield criteria such as maximum normal stress and maximum octahedral stress theory to multiaxial fatigue problem. These methods are called equivalent stress methods and the basic idea of equivalent stress methods is to transform the multiaxial stress state into a uniaxial stress history. Then, by comparing the alternating equivalent stress with uniaxial test endurance limits, life of the component is estimated. Although such criteria are successful in estimation of fatigue life for in-phase loading without mean stresses, their accuracy drops with increase in phase difference of load channels as shown by Engin and Coker and Papuga et al. [Engin and Coker. 2017; Papuga et al., 2012]. Critical plane methods, first introduced by Findley, are developed in order to solve those problems stated above [Findley, 1957]. Multiaxial stress state is transformed into a damage parameter and this calculation is made for all material planes in order to find the critical plane that experiences the maximum damage. Usually shear stress amplitude, which is assumed to be the main cause of crack initiation, and normal stress, which is considered as the main reason of crack opening, is included into damage parameters.

Most of the engineering parts experience not only cyclic stresses but also mean stresses as a result of deadweight of the structure, pre-tension on fasteners and residual stresses induced by several machining processes. It is well known that mean stresses decrease fatigue life. As a result multiaxial fatigue models should be capable of predicting the adverse effects of mean stresses. Mean stress effect is included in equivalent stress and critical plane methods differently. For equivalent stress methods, if equivalent stress history has mean component, a mean stress correction (Soderberg, Goodman, etc.) is performed for obtaining the fully reversed history [Soderberg, 1939 and Goodman, 1899]. For critical plane methods however, in order to take into account the mean stress effects, several parameters such as maximum normal stress ($\sigma_{n,max}$), maximum hydrostatic stress ($\sigma_{Hn,max}$), mean normal stress ($\sigma_{n,m}$) and mean shear stress ($\tau_{n,m}$) are added to damage parameters. Several authors (Smith and Sines) stated that mean torsion has little effect on fatigue life, thus most of the critical plane models do not include mean shear stress component [Smith, 1942 and Sines, 1959]. However it was shown by Krgo et al., Kallmeyer et al., Kluger and Lagoda and Niesłony et al. that mean torsion (shear) stress adversely effects fatigue life [Krgo et al., 2000; Kallmeyer et al. 2001, Kluger and Lagoda, 2014 and Niesłony et al. 2014]. As a result new critical plane methods include mean shear stress in their damage parameters.

Equivalent stress methods are still preferred in industrial applications for high cycle fatigue life estimations instead of critical plane methods since these methods are simple and fast. Aim of this paper is to determine performance of several equivalent stress methods such as Absolute Maximum Principal and Signed von Mises Stress and several critical plane criteria namely Findley and Matake under bending/torsion loading with mean stresses obtained from literature. Furthermore, for equivalent stress criteria mean stress inclusion methods are studied to determine which correlates the experimental data better.

Loading Type

A good understanding of loading type is important for multiaxial fatigue. Therefore, in this section brief information is given about loading types that causes multiaxial fatigue. Multiaxial loading can be classified as proportional and non-proportional. The main difference between two types of loading is the principal stress directions. During proportional loading, principal stress directions remain constant with time while for non-proportional loading opposite is true. A loading with two channels both having normal stresses, is always proportional since principal stress directions remain fixed in time. However, loading with normal and shear stresses is proportional only if there is no phase or frequency difference and mean stresses. In this work, mean stress effect is investigated for in-phase and out-of-phase bending (or axial)-torsion loadings. Figure 1 shows several loading histories and stress paths investigated in this study.

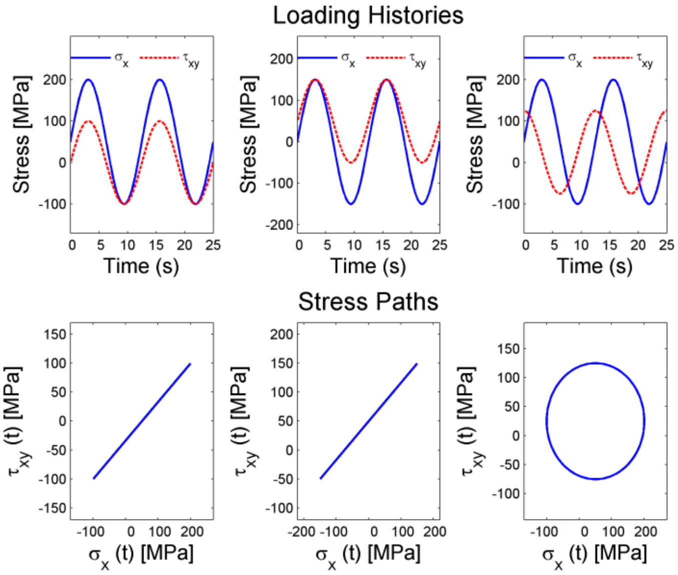

Figure 1 Multiaxial Loading Histories and related Stress Paths

METHOD

Equivalent Stress Methods

<u>Absolute Maximum Principal Stress Criterion</u>

Absolute Maximum Principal Stress method is an adapted version of the static failure criterion; maximum normal stress theory, to multiaxial fatigue problem. Multiaxial stress state is transformed into an equivalent uniaxial stress history from principal stresses and a signing procedure, in which the sign of equivalent stress at a time is the sign of absolute maximum principal stress, is applied.

$$\sigma_{eq} = sign * \sigma_{AMP} \quad (1)$$

where σ_{AMP} is the absolute maximum principal stress.

<u>Signed von Mises Stress Criterion</u>

The idea of Signed von Mises Stress method is similar to Absolute Maximum Principal Stress method. However, this time principal stresses are replaced with von Mises stress at a time. Like in Absolute Maximum Principal Stress method a signing procedure is needed in order to correctly simulate the real load spectrum. Several authors proposed different signings. Bishop claims sign should be the sign of absolute maximum principal stress and Papuga et al. suggests that signing should be applied according to the sign of the first invariant (I_1) [Bishop, 2000, Papuga et al. 2012]. In this study we implemented the suggestion of Bishop's. Signed von Mises stress at a time can be formulated in terms of principal stresses as follows:

$$\sigma_{eq} = sign * \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \tag{2}$$

Inclusion of Mean Stress

Once the equivalent stress history is obtained, a transformation of the history to a fully reversed loading is required if mean stresses are non-zero. For this purpose several criteria were introduced. Soderberg and Goodman are often accepted since those give conservative results while there is a common belief that most of the experimental data lie between Goodman and Gerber curves. According to Dowling and Papuga, Goodman criterion is highly conservative and Smith, Watson and Topper (SWT) or Walker formulations should be implemented for mean stress correction [Dowling, 2004 and Papuga, 2005]. For comparing their performances, all criteria mentioned above are used in mean stress correction. Formulations of Soderberg, Goodman, Gerber, SWT and Walker are as follows:

$$\sigma_{a,Soderberg} = \frac{\sigma_a}{1-\sigma_m/S_y} \tag{3}$$

$$\sigma_{a,Goodman} = \frac{\sigma_a}{1-\sigma_m/S_u} \tag{4}$$

$$\sigma_{a,Gerber} = \frac{\sigma_a}{1-(\sigma_m/S_u)^2} \tag{5}$$

$$\sigma_{a,SWT} = \sqrt{\sigma_a*(\sigma_a+\sigma_m)} \tag{6}$$

$$\sigma_{Walker} = \sigma_a^{\gamma}*(\sigma_a+\sigma_m)^{1-\gamma} \tag{7}$$

where S_y and S_u are yield and ultimate strengths, σ_a and σ_m are alternating and mean stresses experienced by the component respectively while γ is a material constant that adjusts the sensitivity of the material to mean stress. As seen from formulations SWT does not include any material constant; however, Dowling states that the criterion gives fairly accurate results [Dowling, 2009]. Walker involves a material constant γ which makes SWT and Walker dissimilar and Walker criterion superior. An important thing to mention is that Walker reduces to SWT if γ=0.5. Dowling in his paper proposes an equation for estimations of material constant γ as:

$$\gamma = -0.0002000*S_u+0.8818 \tag{8}$$

where S_u is in MPa [Dowling, 2009]. Thus, this equation will be used for γ estimations in Walker formulation.

Critical Plane Methods

Findley Criterion

Findley proposed a shear-stress based critical plane damage and assumed both shear and normal stress contribute to fatigue failure. Therefore, Findley damage parameter is a combination of shear stress amplitude and maximum normal stress acting on the critical plane. Furthermore, effect of maximum normal stress is assumed to be different for each material type (ductile, brittle etc.); consequently maximum normal stress is multiplied with a material constant k that is to be calibrated with at least two uniaxial fatigue tests. Critical plane is defined as the material plane, which can be identified with two Euler angles θ and φ, that experiences the maximum damage. Findley damage parameter is as follows:

$$max_{\theta,\varphi}(\tau_a+k*\sigma_{n,max}) = f \tag{9}$$

In equation 9, k and f are material constants that can be found from fully reversed bending (or axial) and fully reversed torsion endurance limits as follows:

$$k = \frac{2-\sigma_{-1}/\tau_{-1}}{2\sqrt{(\sigma_{-1}/\tau_{-1})-1}}; f = \frac{1}{2}\sigma_{-1}\frac{1}{\sqrt{(\sigma_{-1}/\tau_{-1})-1}} \quad (10)$$

where σ_{-1} and τ_{-1} are fully reversed bending (or axial) and fully reversed torsion endurance limits.

<u>Matake Criterion</u>

Damage parameter proposed by Matake is similar to Findley criterion. Same formulation is used; however, critical plane is defined as the material plane that encounters the maximum shear stress amplitude. Formulation is as follows:

$$max_{\theta,\varphi}(\tau_a) + k * \sigma_{n,max} = f \quad (11)$$

Material parameters are again shown with k and f, but their values differ due to the difference in critical plane definition. For fully reversed bending (or axial) and torsion, material parameters can be obtained from:

$$k = \frac{2-\sigma_{-1}/\tau_{-1}}{\sigma_{-1}/\tau_{-1}}\ ; f = \tau_{-1} \quad (12)$$

There are several material parameter calibration and shear stress amplitude calculation methods. Different combination of those methods for Findley and Matake is investigated in Engin and Coker and it is concluded that for both methods using fully reversed bending (or axial) and torsion endurance limits for calibration of k and f and calculating shear stress amplitude by Maximum Rectangular Hull (MRH) method gives the best overall results.[Engin and Coker, 2017] Therefore, those are implemented in this study.

For both Findley and Matake criteria, only the effect of normal mean stress is considered as the term σ_{nmax} is the summation of mean and alternating values of normal stress on critical plane, and effect of mean torsion stresses are not included in their damage parameters.

RESULTS

In order to evaluate the results, fatigue index error (FIE) is introduced that shows the deviation of estimations from experimental results. FIE for equivalent stress methods is defined as:

$$FIE\ (\%) = \frac{\sigma_{eq,a}-\sigma_{-1}}{\sigma_{-1}} * 100 \quad (13)$$

where $\sigma_{eq,a}$ is the alternating equivalent stress obtained after mean stress correction and σ_{-1} is the fully reversed bending (or axial) endurance limit. For critical plane criteria, FIE is as follows:

$$FIE\ (\%) = \frac{DP-f}{f} * 100 \quad (14)$$

where DP is the damage parameter obtained by Findley or Matake criterion and f is the material allowable. Since experiments show the loading combinations for which specimens fail, negative value of FIE means that method predicted no failure, although it actually occurred in reality. As a result, positive values of FIE are desired for a multiaxial criteria to be conservative.

Test data is obtained from several resources [Zenner et al., 1985; Froustey and Lasserre, 1989; Froustey et al., 1989 and Gough, 1950]. Experimental data contains 72 tests which are resulted from constant amplitude in-phase/out-of-phase loading with mean stresses conducted on un-notched smooth specimens. Details of experiments investigated are shown in Table 1.

Table 1 Details of experiments investigated

Referance	Material Tested	Test Type	Number of tests
Zenner et al., 1985	42CrMo4	Plain bending-torsion	5
Zenner et al., 1985	34Cr4	Plain bending-torsion	8
Froustey and Lasserre, 1989	30NCD16	Plain bending-torsion	8
Froustey et al., 1989	30NCD16	Axial, Plane bending, Plane bending-torsion	22
Gough, 1950	S65A	Plane bending-torsion	29

Equivalent Stress Methods – Soderberg vs. Goodman vs. Gerber vs. SWT vs. Walker

For equivalent stress criteria, performance of mean stress correction methods is investigated. For that end, experimental data is regrouped according to phase difference as in-phase and out-of-phase.

In-phase (IP) Loading with Normal and Shear Mean Stresses

Figure 2 and Figure 3 shows mean, range and standard deviation of FIE (%) for Absolute Maximum Principal Stress (AMP) and Signed von Mises (SVM) criteria for in-phase loading with mean stresses (normal, shear or combination of both) respectively. For both methods, results obtained by Soderberg and Goodman seem to be highly conservative with high mean and range values. However, mean values for AMP is more acceptable. Results obtained by Gerber are conservative for SVM while for AMP mean value becomes negative with a value of -7.8 % which is actually tolerable as the error is less than 10 %. Again range and standard deviations are high for both methods. Estimations made by SWT are in 10 % for both methods. However, SVM gave much better predictions with a mean value of 4.2 % while AMP gave non-conservative results with a mean of -5.5 %. Both methods resulted in similar range (~100) and standard deviation (~25 %) values for SWT calculation. Predictions made by Walker method are similar for SVM but for AMP results are more non-conservative.

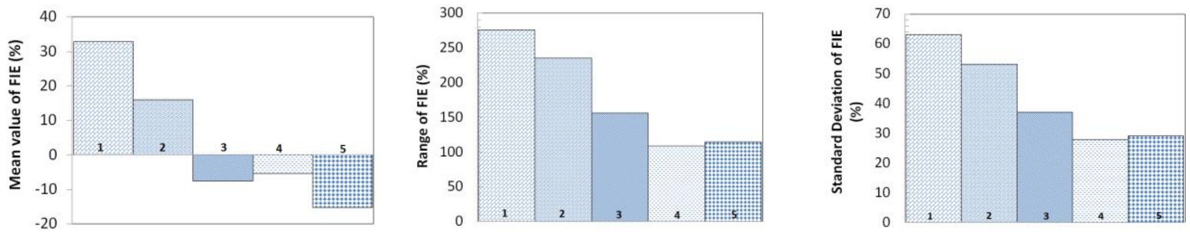

Figure 2 Comparison of Equivalent Stress Methods using AMP criterion under IP Loading with Normal and Shear Mean Stresses (1 Soderberg, 2 Goodman, 3 Gerber, 4 SWT, 5 Walker)

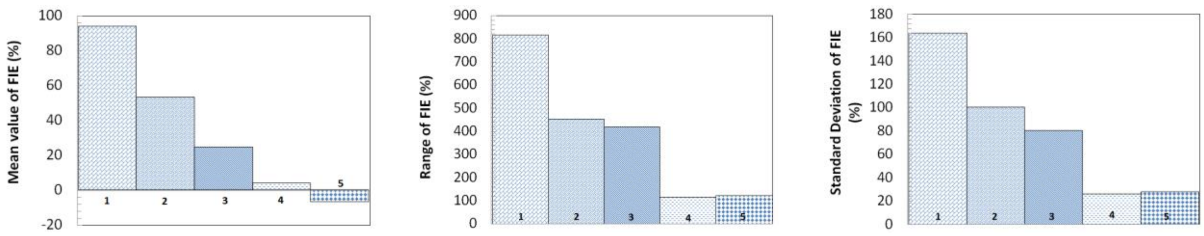

Figure 3 Comparison of Equivalent Stress Methods using SVM criterion under IP Loading with Normal and Shear Mean Stresses (1 Soderberg, 2 Goodman, 3 Gerber, 4 SWT, 5 Walker)

Out-of-phase (OP) Loading with Normal and Shear Mean Stresses

Figure 4 and Figure 5 shows mean, range and standard deviation of FIE (%) of AMP and SVM criteria for out-of-phase loading with mean stresses (normal, shear or combination of both) respectively. For both methods, results obtained by Soderberg seem to be conservative with mean values below 20 %. Results obtained by Goodman are promising with mean values below 10 % (-2 % for AMP, 8 % for SVM), much lower ranges (71 % for AMP and % 63 for SVM) and standard deviation (19 % for AMP and 15 % for SVM). Best results for SVM are obtained by SWT mean stress correction with mean value of 3 %, range of 62 % and standard deviation of 11 %. SWT calculation gives better range and standard deviation for AMP while mean value become more non-conservative (-7 %) but still lower than 10 % which is acceptable.

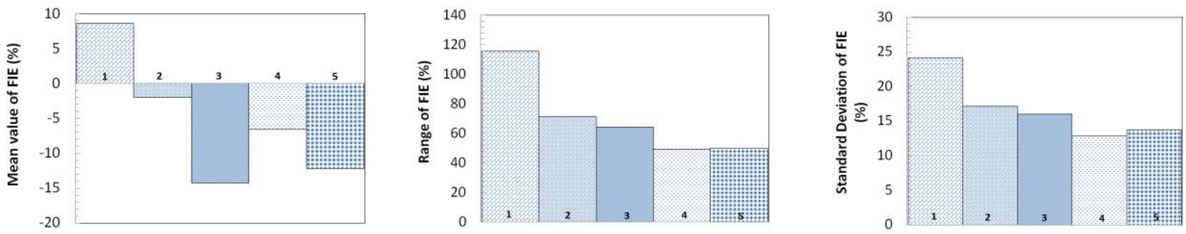

Figure 4 Comparison of Equivalent Stress Methods using AMP criterion under OP Loading with Normal and Shear Mean Stresses (1 Soderberg, 2 Goodman, 3 Gerber, 4 SWT, 5 Walker)

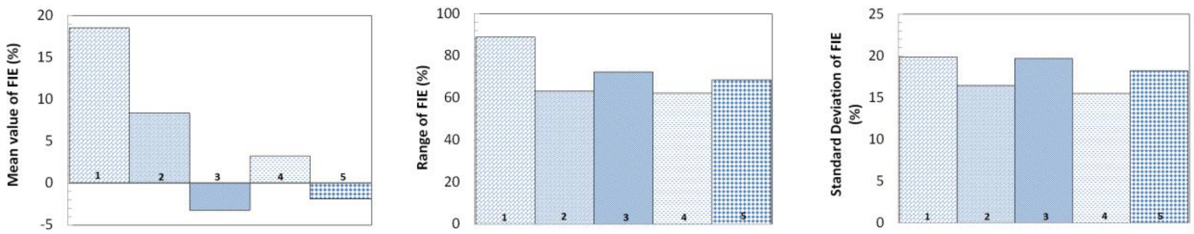

Figure 5 Comparison of Equivalent Stress Methods using SVM criterion under OP Loading with Normal and Shear Mean Stresses (1 Soderberg, 2 Goodman, 3 Gerber, 4 SWT, 5 Walker)

For in-phase and out-of-phase loading with mean stresses, results obtained by SWT mean stress correction seem to give the minimum positive mean value with least range and standard deviation. Therefore, for comparison of equivalent stress and critical plane methods, results with SWT for AMP and SVM methods would be preferred.

Equivalent Stress Methods (AMP-SWT, SVM-SWT) vs. Critical Plane Criteria (Findley, Matake)

For evaluation of equivalent stress and critical plane criteria experimental data is regrouped according to phase difference and mean stresses. Six different cases are investigated which are listed below:

1. IP,M : In-phase loading with means stresses (only normal, only shear or both)
2. IP,oNM : In-phase loading with only normal stresses
3. IP,oSM : In-phase loading with only shear stresses
4. OP,M : Out-of-phase loading with means stresses (only normal, only shear or both)
5. OP,oNM : Out-of-phase loading with only normal stresses
6. OP,oSM : Out-of-phase loading with only shear stresses

IP, M (50)

Figure 6 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for in-phase loading with mean stresses. All methods give mean values below 15 %; however, equivalent stress methods resulted in almost twice in values for range and standard deviation of FIE (%). Critical plane methods, Findley and Matake resulted in similar ranges (~40) and standard deviations (~10) which is quite well while mean value obtained by Matake is 1.9 % and it is the best among all other methods.

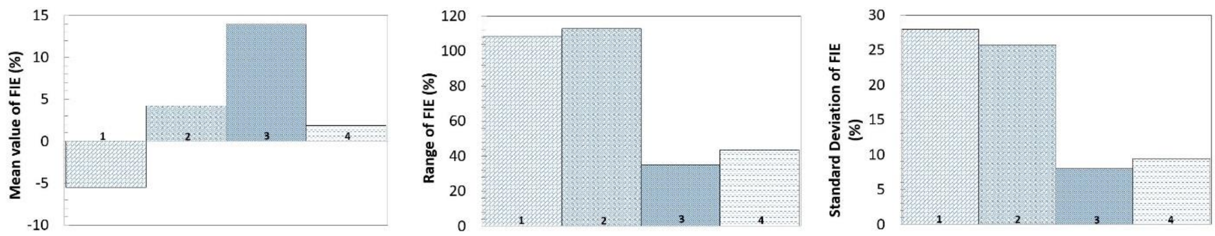

Figure 6 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under IP Loading with Normal and Shear Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

IP, oNM (29)

Figure 7 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for in-phase loading with only normal mean stresses. From figures it is clear that equivalent stress methods give non-conservative estimations; however, mean values of both equivalent stress and critical plane criteria are in the range of ±15%. Range (~85%) and standard deviation (~27%) of equivalent stress methods are again high while range of critical plane methods are close to 35% and their standard deviation is below 10%. For this loading case, Matake gave the least mean value of 2%.

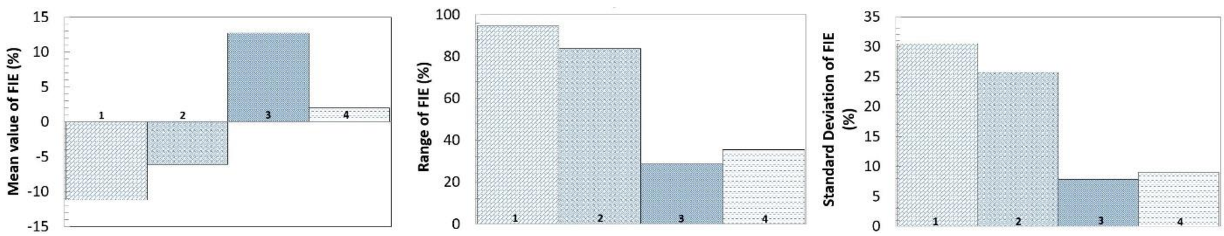

Figure 7 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under IP Loading with only Normal Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

IP, oSM (6)

Figure 8 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for in-phase loading with only shear mean stresses. Mean values except SVM is below 10% and for all methods ranges are below 35 % and standard deviations are below12%. Least range (17%) and standard deviation (6%) is obtained by Findley method; however, Matake resulted in the least mean value of 0.35.

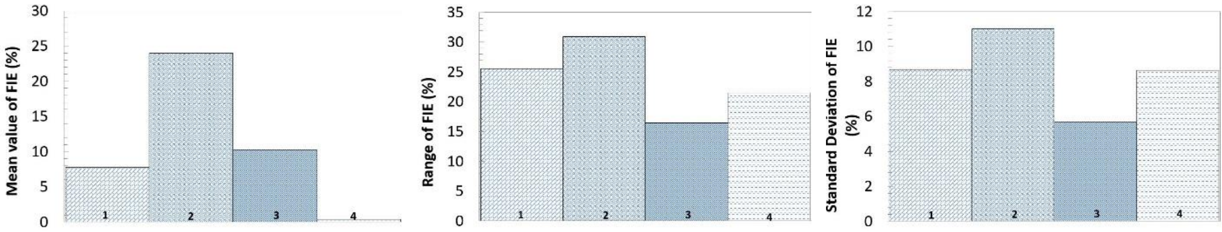

Figure 8 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under IP Loading with only Shear Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

OP, M (22)

Figure 9 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for out-of-phase loading with mean stresses. For all methods mean values are below 10%; however, AMP overestimates the fatigue life with a negative mean of -6.6%. Ranges are close to 55% for equivalent stress methods while for critical plane methods range value is approximately 35%. Standard deviations are close to 15% for equivalent stress methods while for critical plane methods its value is below 10%. Lowest mean value is obtained by SVM, but Matake resulted in a very close value (4.3%).

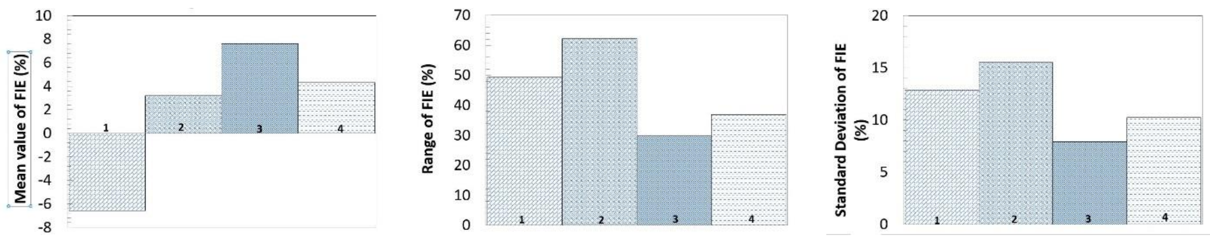

Figure 9 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under OP Loading with Normal and Shear Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

OP, oNM (17)

Figure 10 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for out-of-phase loading with only normal mean stresses. As seen from the figure mean values are below 10% for all methods, but equivalent stress methods give non-conservative results. Standard deviations are below 14% for equivalent stress methods, while for critical plane criteria it is below 10%.

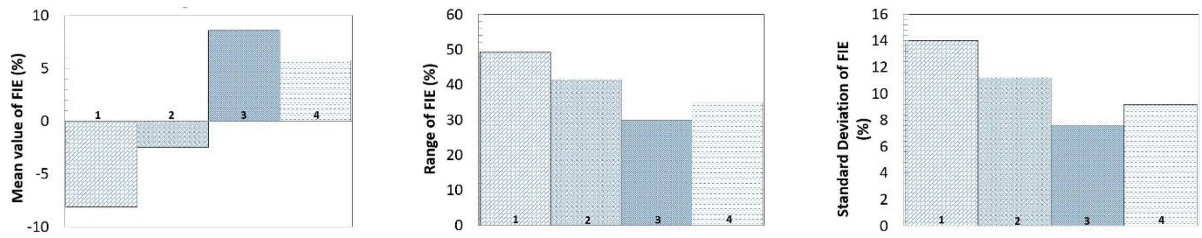

Figure 10 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under OP Loading with only Normal Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

OP, oSM (4)

Figure 11 shows mean, range and standard deviation of FIE (%) for equivalent stress and critical plane criteria for out-of-phase loading with only shear mean stresses. All methods except SVM resulted in mean values below 5%. Lowest mean value is again obtained by Matake (-0.5 %).

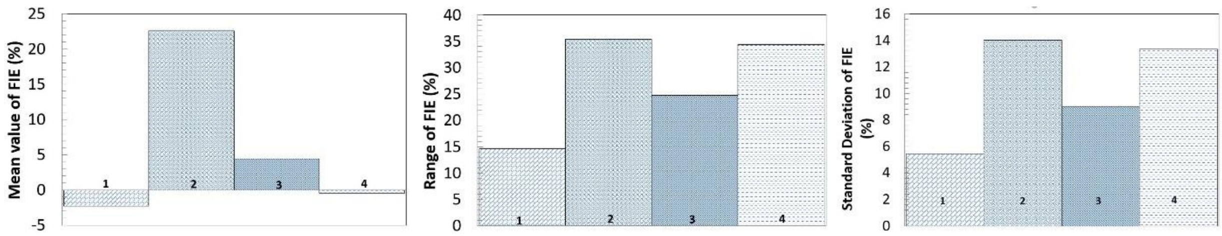

Figure 11 Comparison of Equivalent Stress Methods (AMP-SWT and SVM-SWT) with Critical Plane Methods (Findley and Matake) under OP Loading with only Shear Mean Stresses (1 AMP-SWT, 2 SVM-SWT, 3 Findley, 4 Matake)

DISCUSSION and CONCLUSION

Several equivalent stress and critical plane criteria are compared in this study. Special attention is given to mean stress effect, thus experimental data which include loadings with mean stresses are obtained from literature. For equivalent stress methods, Absolute Maximum Principal Stress and Signed von Mises criteria are chosen as they interest industrial community due to their simplicity and speed. Furthermore, different mean stress inclusion methods are investigated for obtaining the best performance of equivalent stress methods. Findley and Matake criteria are studied for critical plane methods as their formulations are same but they differ in critical plane definitions. Therefore, both effect of critical plane definition and mean stress could be examined.

- For equivalent stress methods, estimations obtained from Soderberg are conservative, but with high range and standard deviations while other methods may shift to non-conservative side.
- Mean stress correction with SWT gives the best overall results for equivalent stress methods.
- Signed von Mises criterion resulted in tolerable mean FIE (%), range and standard deviation compared to Absolute Maximum Principal criterion which tends to give high mean values and standard deviation with wide ranges.

- Critical plane methods, Findley and Matake resulted in estimations with low mean values, range and standard deviation of FIE (%) for all cases. However, Matake method gave the best predictions with mean values of FIE (%) less than 6% and standard deviation below 10%.

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