9th ANKARA INTERNATIONAL AEROSPACE CONFERENCE **AIAC-2017-180**
20-22 September 2017 - METU, Ankara TURKEY

ANGLE ONLY ORBIT DETERMINATION USING A TELESCOPE SYSTEM

Prof. Dr. Ozan Tekinalp[1] and Abdulkadir Köker [2]
[2]TUBITAK UZAY
Ankara, Turkey
[1,2]Middle East Technical University
Ankara, Turkey

ABSTRACT

The aim of the study was to investigate the effect of the angle only measurements on orbit determination accuracy. Telescope angle measurements are simulated using System Tool Kit simulation software. Noise is added to the data as well. The effect of various factors on estimation accuracy such as, measurement frequency, observation duration, and number of observation sites (i.e., telescopes) are investigated. Estimation methods, extended and unscented Kalman filters are employed and compared.

INTRODUCTION

Number of methods, that uses different sensors, are proposed in the literature for orbit determination of non-collaborating objects. Radar, telescope and laser systems are the most famous of them. Although most accurate method is the laser system, its development cost is relatively high. Radar system is good at determining Low Earth Orbit (LEO) object orbits. On the other hand, its accuracy dramatically decreases when orbital estimation of Medium Earth Orbit (MEO), and Geostationary Orbit (GEO) objects are needed. Telescope systems may be used for all altitudes and their development costs are lower compared other systems. However, telescope can only give two angles, namely, azimuth and elevation, which is normally insufficient to determine the object position accurately. Range measurements are not available for telescope systems. In order to overcome this problem, there are some methods which include increasing measurement frequency and observation sites. Objects at different altitudes have different observation duration from Earth due to their orbital period. LEO objects are seen for a limited time of the day. However, GEO objects may be continuously observed. Therefore, orbit determination accuracy may be different for objects at different altitudes. Different estimation algorithms are also used for orbit estimation [Park, 2010].

In this manuscript, orbit estimation of LEO objects using telescope angle only measurement is addressed. Various factors, such as measurement noise, number of observation sites, and measurement duration in estimating the orbit accurately is investigated. Also investigated is the effect of estimation algorithm, namely Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) on the orbit prediction accuracy.

The observation method and mathematical models were given firstly in this study. According to presented estimation algorithms, simulation results were given and discussed.

[1] Professor, Aerospace Engineering Department, Email: tekinalp@metu.edu.tr
[2] M.S. Student, Aerospace Engineering Department, Email: kokabdul@hotmail.com

METHOD AND MATHEMATICAL MODELS

Angle measurements are simulated using System Tool Kit (STK) package program. As seen from Figure 1, the sensor is geographically placed in Ankara, gives the necessary angle and range measurements while the object is observable by the telescope. Then, the range information is removed, and only angle information is used to generate telescope measurements. This system also has constraint such that measurements are only available at nights. Thus, morning measurements are not taken into account since the object is only viewable during dark. The flow-chart of the data simulation and observation approach is given in Figure 2.

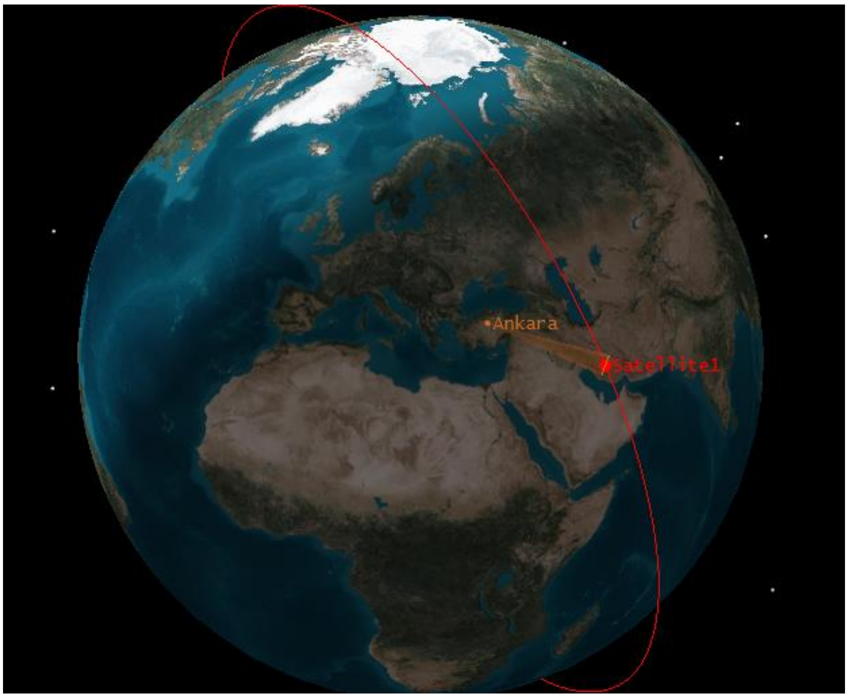

Figure 1: STK telescope simulation [STK User Manual, 2017]

Orbit determination process is shown below figure.

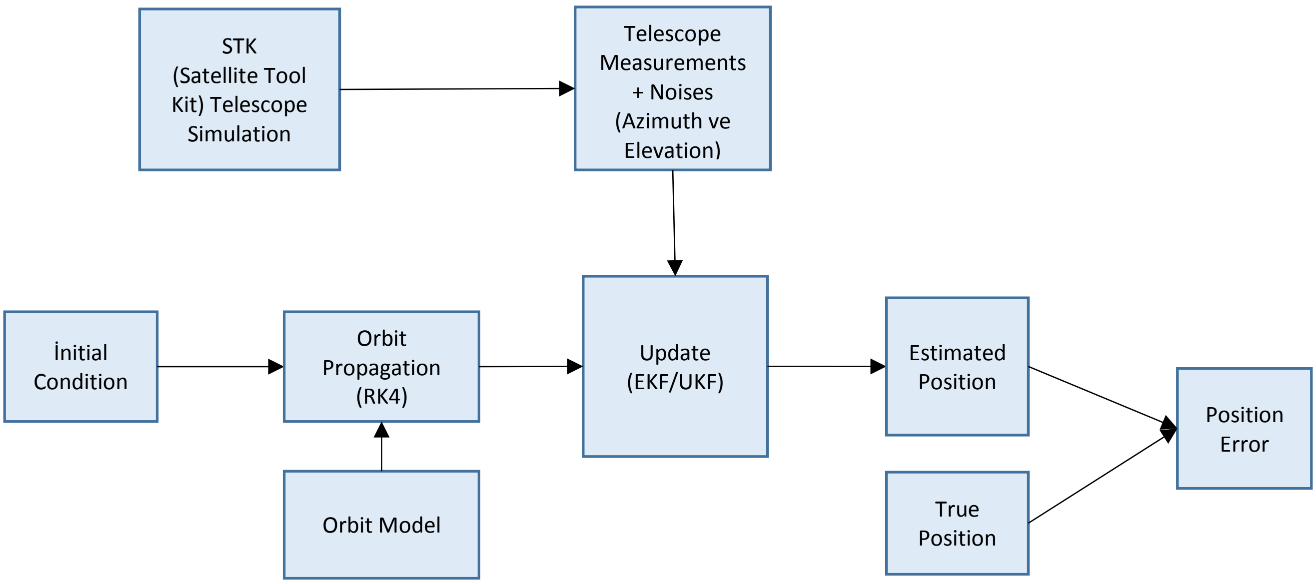

Figure 2: Orbit determination flow diagram

Orbit Determination

The orbit determination problem is to estimate accurately the ephemeris of an orbiting satellite at a chosen epoch. To achieve this goal, estimations of the states and the model parameters of the satellite are made based on a sequence of observations. The dynamic models of the equations of motion are usually integrated from a chosen epoch to each observation times to produce predicted observations. The differences between the predicted observations and true observations are defined as the observation residuals. Thus, solving the orbit determination problem requires [Park, 2010]:

- Equations of motion describing the forces acting on the satellite,
- The relationship between the observed parameters and the satellite's state vector
- An estimation algorithm.

Force Models

The equations of motion of a satellite are usually described in an inertial reference frame as being composed of a sum of gravitational, non-gravitational and empirical or un-modeled forces. In the current research, the equations of motion for an Earth orbiting satellite are given by

$$\dot{r} = v \tag{1}$$

$$\dot{v} = \frac{-\mu}{r^3} r + a_{geo} + a_{third-body} + a_{drag} + a_{SR} \tag{2}$$

where r and v are the position and velocity vectors in the inertial frame. The forces ($\dot{v}$) acting on the satellite consist of the two-body effect and the addictive perturbing accelerations. $\boldsymbol{a_{geo}}$ is the geo-potential force due to the gravitational force of the Earth and can be expressed as a spherical harmonic expansion of the gradient of the Earth's solid body distribution.

$\boldsymbol{a}_{third-body}$ is the lunar/ solar gravitational perturbation, which are usually modeled as point masses within the Newtonian framework. $\boldsymbol{a}_{drag}$ is the atmospheric drag force. $\boldsymbol{a}_{SR}$ is the force due to solar pressure on satellite. For simplicity, only two-body effect and J2 geo-potential force is chosen for analysis. All the equations of motion are numerically integrated by the Runge–Kutta fourth-order fixed step numerical integrator.

Measurement Models

We consider a ground tracking station that measures a range, azimuth and elevation of a satellite in orbit. Actually, telescope systems are also similar to ground tracking station without range information. The geometry associated with this observation is shown in Figure 3.

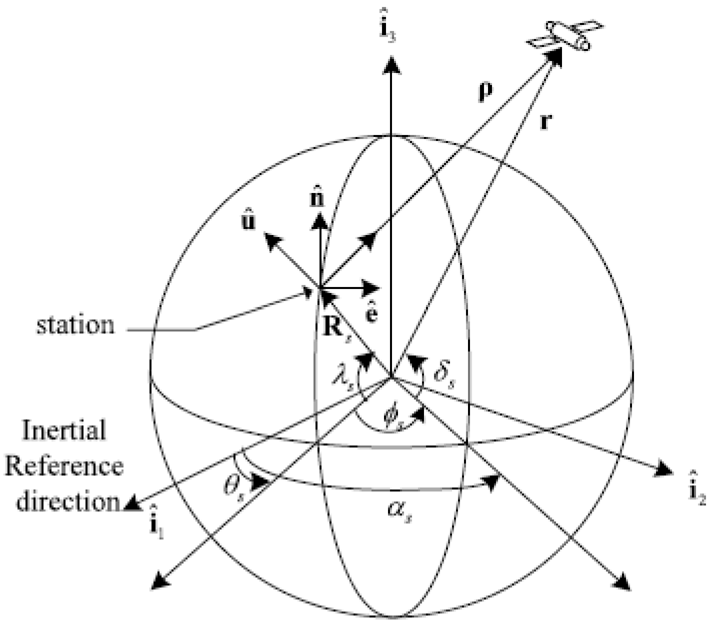

Figure 3 *Geometry of Earth observation of satellite motion* [Park , 2010]

ρ is the slant range vector, r is the radius vector locating the satellite, R_s is the radius vector locating the ground tracking station, α_s and δ_s are the right ascension and declination of the satellite, respectively, θ_s is the sidereal time of the ground station, λ_s is the latitude of the ground tracking station, and φ_s is the east longitude from the ground tracking station to the satellite. The fundamental observation is given by [Vallado, 2001]

$$\rho = r - R_s \tag{3}$$

In non-rotating equatorial components the vector ρ is given by

$$\rho = \begin{bmatrix} x - |R_s| \cos\lambda_s \cos\theta_s \\ y - |R_s| \cos\lambda_s \sin\theta_s \\ z - |R_s| \sin\lambda_s \end{bmatrix} \tag{4}$$

where x, y, and z are the components of the vector r. The ground tracking station coordinate system (up, east and north) is described in Figure 3. The conversion from the inertial to ground tracking station coordinate is given by

$$\begin{bmatrix} \rho_u \\ \rho_e \\ \rho_n \end{bmatrix} = \begin{bmatrix} \cos\lambda_s & 0 & \sin\lambda_s \\ 0 & 1 & 0 \\ -\sin\lambda_s & 0 & \cos\lambda_s \end{bmatrix} \begin{bmatrix} \cos\theta_s & \sin\theta_s & 0 \\ -\sin\theta_s & \cos\theta_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \rho \tag{5}$$

A ground tracking station measures the azimuth (az), elevation (el), and range (ρ). The measurement equations are given by following equations:

$$\rho = \sqrt{{\rho_u}^2 + {\rho_e}^2 + {\rho_n}^2} \tag{6}$$

$$az = \tan^{-1}\left(\frac{\rho_e}{\rho_n}\right) \tag{7}$$

$$el = \tan^{-1}\left(\frac{\rho_u}{\sqrt{{\rho_n}^2 + {\rho_e}^2}}\right) \tag{8}$$

Since telescope system only gives angle measurements, range information is unavailable. Range is ignored in order to simulate telescope systems. Therefore, only azimuth and elevation angles are used as a measurement data in analysis.

Estimation Algorithms

Extended Kalman Filter

The extended Kalman filter provides the minimum variance estimate of the state based on statistical information about the dynamical and observation models. The continuous-time models can be converted into a discrete form through an approximate method. In this section the EKF algorithm is reviewed for discrete-time nonlinear equations of the form [Lee, 2007]

$$x_{k+1} = F(x_k, u_k, t_k) + w_k \tag{9}$$

$$y_k = H(x_k, t_k) + v_k \tag{10}$$

where x_k is the $L\,x\,1$ state vector, y is the $n\,x\,1$ observation vector, w_k is state noise vector, and v_k measurement noise vector. It is assumed that the noise vectors are zero-mean Gaussian processes satisfying

$$E\{w_k w_j^T\} = \begin{cases} Q_k, & k = j, \\ 0, & k \neq j, \end{cases} \tag{11}$$

$$E\{v_k v_j^T\} = \begin{cases} R_k, & k = j, \\ 0, & k \neq j, \end{cases} \tag{12}$$

$$E\{v_k w_j^T\} = 0, \qquad \forall\, k, j \tag{13}$$

where the measurement and process noise covariances Q_k , R_k are assumed to be positive definite. Given a system model with initial state and covariance values, the EKF propagates the state vector and the error covariance matrix recursively. Then, along with imperfect measurements, the EKF updates the state and covariance matrix. The update is accomplished through the Kalman gain matrix K, which is obtained by minimizing the weighted sum of the diagonal elements of the error covariance matrix. The EKF is based on the linearization by using the Taylor-series expansion of the nonlinear dynamical and measurement equations about the current estimate. For the nonlinear models in Eqs. (14) and (15) the predictions of the state estimates and covariance are accomplished by

$$\hat{x}_{k+1}^- = F(\hat{x}_k, t_k) \tag{14}$$

$$P_{k+1}^- = F_k P_k F_k^T + Q_k \tag{15}$$

where F_k is the Jacobian matrix of the nonlinear function. Measurement update equations are expressed by

$$\hat{x}_{k+1}^+ = \hat{x}_{k+1}^- + K_{k+1}(\tilde{y}_{k+1} - y_{k+1}^-) \tag{16}$$

$$\hat{P}_{k+1} = P_{k+1} - K_{k+1}P^{xy}{}_{k+1}K^T{}_{k+1} \tag{17}$$

$$K_{k+1} = P^{xy}{}_{k+1}(P^y{}_{k+1})^{-1} \tag{18}$$

Where $\tilde{y}_{k+1}$ the measurement is vector $(n \times 1)$,y_{k+1}^{-} is the predicted measurement vector $(n \times 1)$, K_{k+1} is gain matrix $(L \times n)$. Cross covariance is expressed by

$$P^{xy}{}_{k+1} = P_{k+1}^{-}H_{k+1}^{T} \tag{19}$$

$$P^y{}_{k+1} = H_{k+1}P_{k+1}^{-}H_{k+1}^{T} + R_{k+1} \tag{20}$$

Where $\bar{P}^y{}_k$ is measurement covariance matrix $(n \times n)$, $\bar{P}^{xy}{}_k$ is cross covariance matrix $(L \times n)$. In EKF algorithm the state distribution is approximated by a Gaussian random variable, which is then propagated through the first-order linearization of the nonlinear functions. These approximations, however, can introduce large errors in the true posterior mean and covariance. The UKF uses different approach to overcome this problem that is discussed in the next section.

Unscented Kalman Filter

Estimation algorithms are unscented Kalman filter (UKF) and nonlinear least square. UKF represents a derivative-free alternative to the extended Kalman filter(EKF), provides better performance for highly nonlinear systems. In orbit determination problem, orbit mechanics include very nonlinear force models. Therefore, UKF addresses nonlinearity problem by using unscented transformation (UT). UT is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation [Julier, 1997]. Consider propagating a random variable x (dimension L) through a nonlinear function, $y = f(x)$. Assume x has mean $\bar{x}$ and covariance P_x . To calculate the statistics of , we form a matrix X of $2L+1$ sigma vectors (with corresponding weights W_i), according to the following [Julier, 1997]:

$$\begin{aligned} &X_0 = \bar{x} \\ &X_i = \bar{x} + \left(\sqrt{(L+\lambda) * P_x}\right)_i \quad i = 1, \dots, L \\ &X_i = \bar{x} - \left(\sqrt{(L+\lambda) * P_x}\right)_{i-L} \quad i = L+1, \dots, 2L \\ &W_0{}^{(m)} = \lambda/(L+\lambda) \\ &W_0{}^{(c)} = \lambda/(L+\lambda) + (1 - \alpha^2 + \beta) \\ &W_i^{(m)} = W_i^{(c)} = 1/(2(L+\lambda)) \quad i = 1, \dots, 2\,L \end{aligned} \tag{21}$$

where $\lambda = \alpha^2(L+\kappa) - L$ is a scaling parameter. α determines the spread of the sigma points around $\bar{x}$. κ is a secondary scaling parameter which is usually set to 0, and β is used to incorporate prior knowledge of the distribution of (for Gaussian distributions $\beta = 2$, is optimal). $\left(\sqrt{(L+\lambda) * P_x}\right)_i$ is the ith row of the matrix square root. These sigma vectors are propagated through the nonlinear function,

$$y_i = f(x_i) \;\; i = 0, \dots, 2L,$$

and the mean and covariance for y are approximated using a weighted sample mean and covariance of the posterior sigma points,

$$\bar{y} = \sum_{i=0}^{2L} W_i^{(m)} y_i \tag{22}$$

$$P_y = \sum_{i=0}^{2L} W_i^{(c)} (y_i - \bar{y})(y_i - \bar{y})^T \tag{23}$$

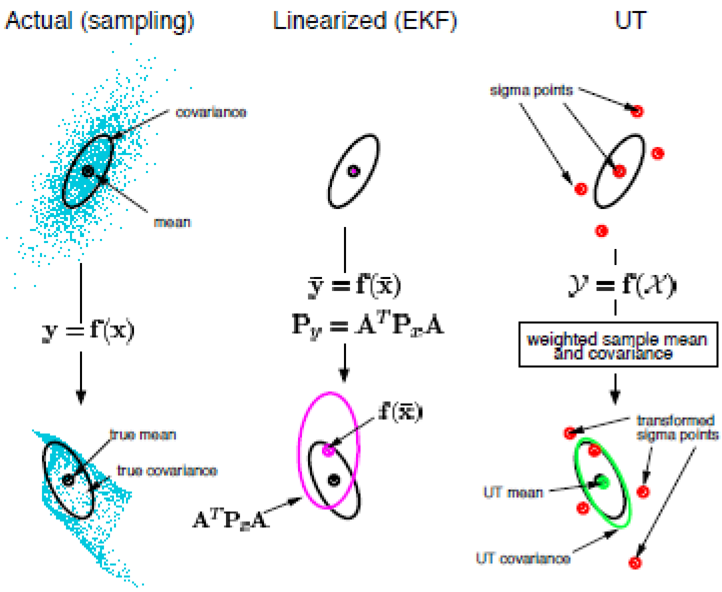

Figure 4: Example of the UT for mean and covariance propagation [Julier, 1997].

a) actual, b) first-order linearization (EKF), c) UT.

In state estimation process, UKF has two main part; first one is state prediction, which is orbit propagation, and second one is updating state with measurements, which are azimuth and elevation angles. Process can be implemented to angle-only orbit determination problem by using following equations:

Basic framework of nonlinear dynamic systems for kalman filter can be represented as

$$x_{k+1} = F(x_k, u_k, t_k) + w_k \tag{24}$$

$$y_k = H(x_k, t_k) + v_k \tag{25}$$

where x_k is state of nonlinear system, y_k is measurement, u_k is input, w_k and v_k are process and measurement noises respectively. In standard orbit determination process, states are positions and velocities,

$$x_k = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} \tag{26}$$

measurements are azimuth, elevation angles.

$$y_k = \begin{bmatrix} az \\ el \end{bmatrix} \tag{27}$$

F represents system functions related to equation of motion for satellite. H is measurement functions related to conversion between states (positions) to azimuth and elevation angles. Normally, H function calculates az, el and range from position but range is voluntarily ignored in order to simulate angle only measurements. UKF equations are given with assuming addictive noise in Figure 5 [Lee, 2007] :

Initialize with:

$$\hat{x}_0 = E[x_0]$$
$$P_0 = E[(x_0 - \hat{x}_0)]$$

for $k \in \{1, \dots, \infty\}$,

Calculate sigma points:

$$\hat{x}_{i|k} = [x_k \; x_k \pm \left(\sqrt{(L+\lambda) * P_k}\right)_i] \quad i = 1, \dots, 2L$$

where L is the state dimension

Time (state) Update:

$$\bar{x}_{i|k+1} = F(\hat{x}_{i|k}, t_k)$$
$$\bar{x}_{k+1} = \sum_{i=0}^{2L} W_i^{(m)} \bar{x}_{i|k+1} \; i = 1, \dots, L$$

$$\bar{P}_{k+1} = \sum_{i=0}^{2L} W_i^{(c)} (\bar{x}_{i|k+1} - \bar{x}_{k+1})(\bar{x}_{i|k+1} - \bar{x}_{k+1})^T + Q_{k+1}$$

Measurement Update:

$$\gamma_{i|k+1} = H(\bar{x}_{i|k+1}, t_{k+1})$$

$$\bar{y}_{k+1} = \sum_{i=0}^{2L} W_i^{(m)} \gamma_{i|k+1} \; i = 1, \dots, L$$

$$\bar{P}^y{}_{k+1} = \sum_{i=0}^{2L} W_i^{(c)} (\gamma_{i|k+1} - \bar{y}_{k+1})(\gamma_{i|k+1} - \bar{y}_{k+1})^T + R_{k+1}$$

Cross covariance:

$$\bar{P}^{xy}{}_{k+1} = \sum_{i=0}^{2L} W_i^{(c)} (\bar{x}_{i|k+1} - \bar{x}_{k+1})(\gamma_{i|k+1} - \bar{y}_{k+1})^T$$

$$K_{k+1} = \bar{P}^{xy}{}_{k+1} (\bar{P}^y{}_{k+1})^{-1}$$

$$\hat{x}_{k+1} = \bar{x}_{k+1} + K_{k+1}(\tilde{y}_{k+1} - \bar{y}_{k+1})$$

$$\hat{P}_{k+1} = \bar{P}_{k+1} - K_{k+1} \bar{P}^{xy}{}_{k+1} K^T{}_{k+1}$$

where $\tilde{y}_{k+1}$ is observed measurement vector and its dimension $(n \times 1)$ (n is measurement number , K_{k+1} is is gain matrix $(L \times n)$, $\bar{y}_k$ is predicted measurement $(n \times 1)$, $\bar{P}^y{}_k$ is measurement covariance matrix $(n \times n)$, $\bar{P}^{xy}{}_k$ is cross covariance matrix $(L \times n)$.

Figure 5 Unscented Kalman Filter Algorithm

RESULTS AND DISCUSSION

In the following, LEO and GEO satellites are tracked with telescope is addressed. Observation parameters were shown at Table 1. In the first analysis, only one observation site (Ankara) was used to estimate orbit for LEO and GEO satellites. After that, the effect of additional site on orbit estimation accuracy was investigated and compared to single-site site case.

Table 1: Observation Parameters

Sensor Type	Rectangular°
Observation Sites	Ankara-Antalya
Angle measurement noise (1 sigma)	0.00138 degree
Elevation mask	10 degree

LEO observation case study with one observation site

Simulation parameters related to the LEO satellite are listed in Table 2 .

Table 2*:* Analysis Parameters

Noise characteristics	Gauss
Initial position error	3.464 km
Initial velocity error	3.464 m/s
Orbit propagation model	Two-body, J2 potential
Propagation Method	Runge Kutta 4

<u>LEO orbit estimation with one-day observation</u>

LEO satellite can be observed for 2 to 4 minute per day with a telescope system at Ankara. Orbit estimation process was conducted with the consideration of the observation duration, observation frequency. Observation duration cases include 200, 100, 50, 25, 20 observation data point when observation frequency was 1 second. Observation frequency cases include 1, 2, 5, 10, 20, 40 second measurement periods with fixed 200 second orbit duration. All of analysis was done for one pass during one-day observation. Number of observation stations were fixed as a one. Estimation results from UKF and EKF was compared with consideration of orbit accuracy.

As shown Figure 6, increase in observation frequency improves position and velocity accuracy significantly. Both UKF and EKF methods give similar results since observation duration is quite short. Therefore, nonlinear terms don't introduce large errors. However, in Figure 7 it is shown that for velocity estimation, low observation frequency cases couldn't reduce initial velocity error enough. For 40, 20, 10 and 5 observation points cases, estimated error was close to initial error. When observation frequency was higher than 5 second, initial errors could effectively be reduced.

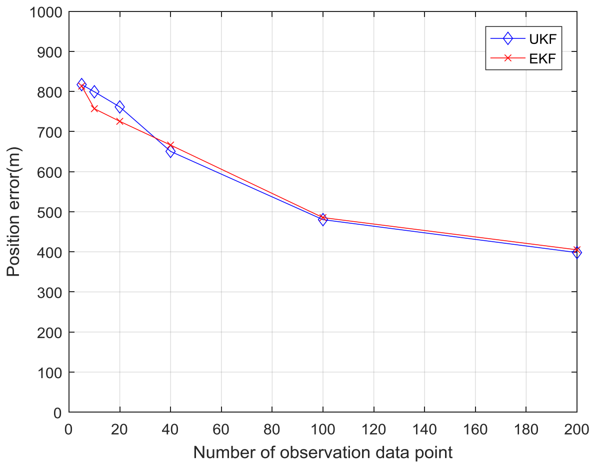

Figure 6. Position error for LEO frequency case using 1 day observation

Minimum position error reduction was about 400 meter and using 200 measurement data points with EKF and UKF. Further error reduction requires more observation or additional observation site.

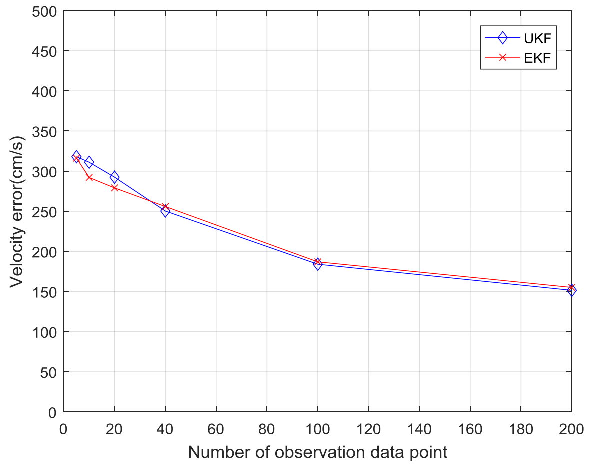

Figure 7. Velocity error for LEO frequency case using 1 day observation

Figure 6 shows that for frequency case, minimum velocity error achieved was 150 cm/s with 200 measurement points.

For duration case, Figure 8 and Figure 9 shows orbit determination results with different observation duration with fixed 1 second frequency. UKF and EKF give similar results as seen previous case. It was clearly seen that increase in observation points improve position estimation accuracy moderately. However, the effect of number observation points has more significant effect on velocity error reduction compared to position error reduction. When observation points were less than 150, both EKF and UKF couldn't reduce velocity errors sufficiently although position errors were relatively small.

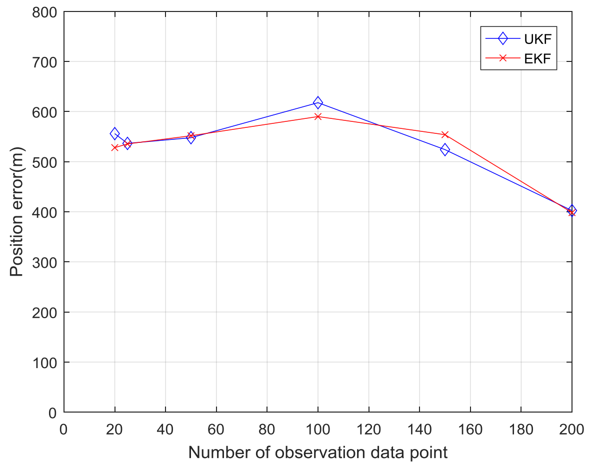

Figure 8. Position error for LEO duration case using 1 day observation

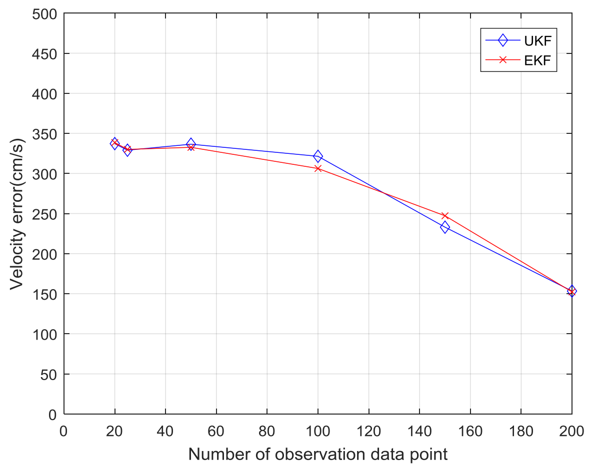

Figure 9. Velocity error for LEO duration case using 1 day observation

LEO observation case study with two observation site

In this analysis, orbit estimation was conducted with same initial conditions as used for one observation site case from Table 2. Observations were generated with using two observation site located at Antalya and Ankara. It was assumed that observations from two sites are synchronizes with each other.

<u>LEO orbit estimation with one-day observation</u>

Same cases with one observation site was investigated using both EKF and UKF methods. Since observations from two site are synchronizes, in estimation algorithm, measurement updates for same epoch was made twice. This means that number of measurement data point was doubled while other observation frequency and duration parameters are unchanged. For frequency cases, orbit determination results are shown at Figure 10 and Figure 11. In this case, observation frequency is 1 second. Every case showed better performance than one observation site case. Both UKF and EKF gave generally similar results while UKF showed slightly better performance for low data point. Minimum achieved position and velocity errors are less than 50 m and 50 cm/s respectively. These values are superior when compared to one-site case errors which are 400 m and 150 cm/s.

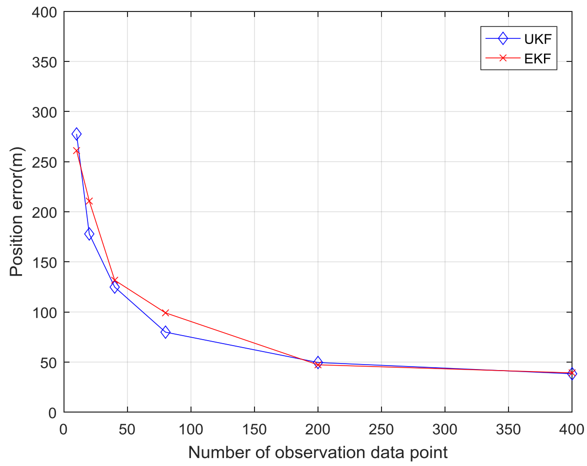

Figure 10 Position error for LEO frequency case using 1 day observation from two site

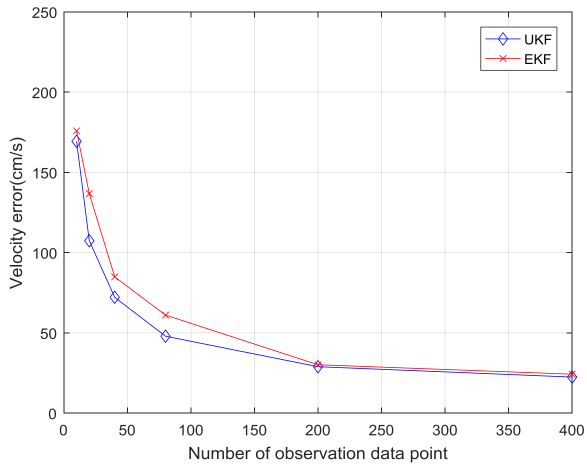

Figure 11 Velocity error for LEO frequency case using 1 day observation from two site

For duration cases, observation duration was reduced as one-site case with fixed 1 sec observation frequency. Results are shown at Figure 12 and Figure 13. In position estimation, every case gives similar result around 30-40 m. Therefore, increasing data point doesn't improve position accuracy much when observation form second site is available. However, it is clearly seen that velocity error can be reduced effectively with increasing measurements.

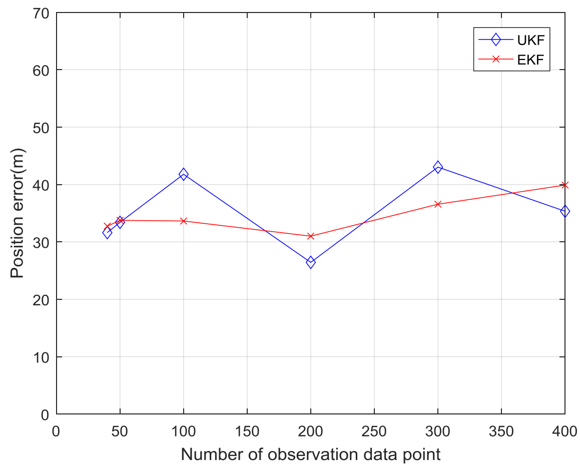

Figure 12 Position error for LEO duration case using 1 day observation from two site

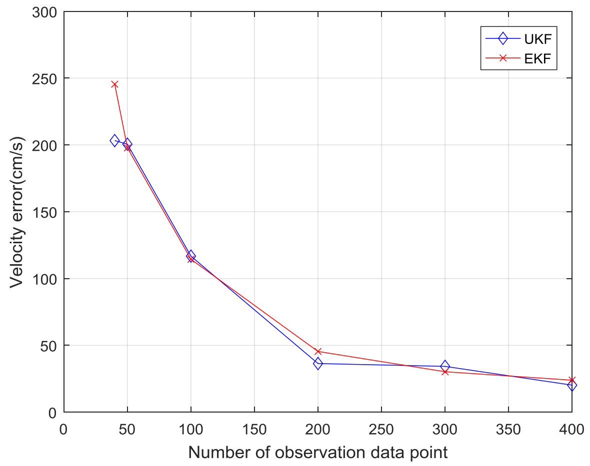

Figure 13 Velocity error for LEO duration case using 1 day observation from two site

GEO observation case study with one observation site

Analysis parameters related GEO satellite are shown at Table 3.

Table 3: Analysis Parameters (GEO)

Noise characteristics	Gauss
Measurement Noise	0.00138 degree
Initial average position error	17.32 km
Initial average velocity error	17.32 m/s
Orbit propagation model	Two-body, J2 potential
Propagation Method	Runge Kutta 4

GEO orbit estimation with one-day observation

GEO satellite can be observed continuously whole night with a telescope system at Ankara assuming weather conditions are available. Similar to LEO case, orbit estimation process was conducted with different observations, ranging from 600 to 10 samples. To test the effect of observation frequency same 600-minute observation period is sampled at different frequencies. These frequencies include 60, 72, 80, 120, 300, 600, 1800, 3600 second measurement periods. These periods represent 600, 500, 450, 300, 120, 60, 20, 10 observation data points.

For frequency case ,as seen from Figure 14, when observation frequency lower than 2 minute with 300 measurement points, EKF results are diverged from true values. This shows that nonlinear terms start to become important and ignoring these terms introduce high errors. However, UKF can reduce initial error significantly even with lowest observation point case with one measurement per hour.

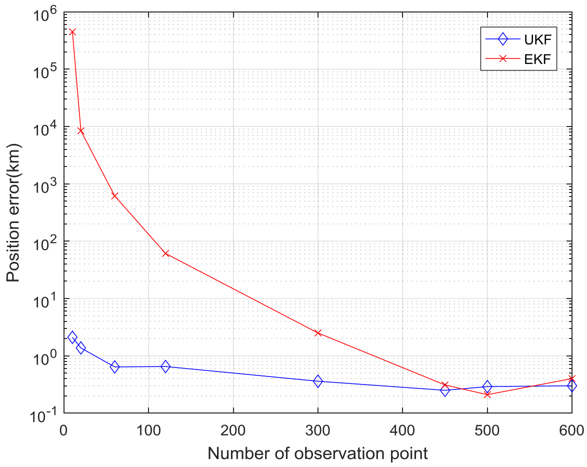

Figure 14. Position error for GEO frequency case using 1 day observation

Figure 14 shows that increase in observation frequency can have clearly important effect on orbit estimation accuracy for GEO satellites. EKF method is not suitable when time between measurements are relatively large. Velocity error results are shown in Figure 15. It is seen that UKF can reduce error sufficiently even with low observation point with large measurement sample duration.

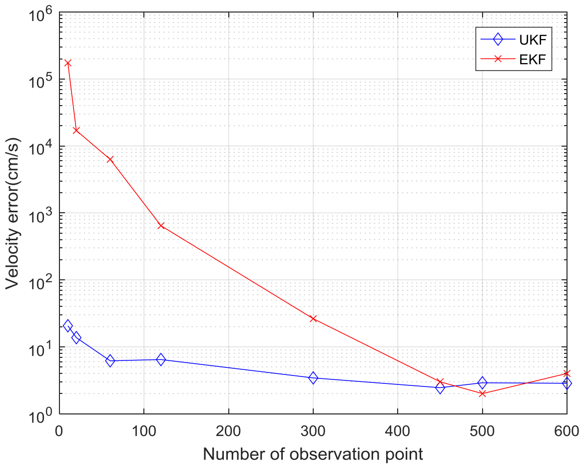

Figure 15. Velocity error for GEO frequency case using 1 day observation

For duration case, to test the effect of observation duration, data is sampled at 1 min intervals. Thus, when 60 samples are used in estimation, the observation period is 60 minutes. Results are shown in Figure 16 and Figure 17. When number of observation point are higher than 150 points, both EKF and UKF give relatively better estimation result. However, for 150 points and lower case, EKF can't reduce initial position error sufficiently, but, UKF can reduce error moderate level. For velocity error reduction, UKF is superior compared to EKF. Velocity error generally can be reduced significantly for GEO case analysis. İn GEO case analysis, it is clearly seen that UKF is generally better EKF when the number of observation points drop certain number. In this analysis, 150 point can be considered as a break point.

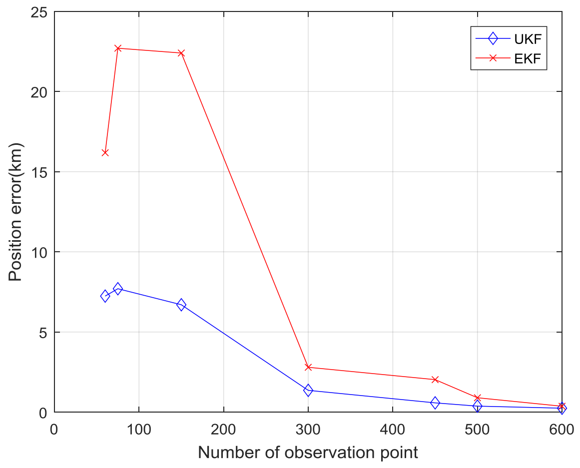

Figure 16. Position error for GEO duration case using 1 day observation

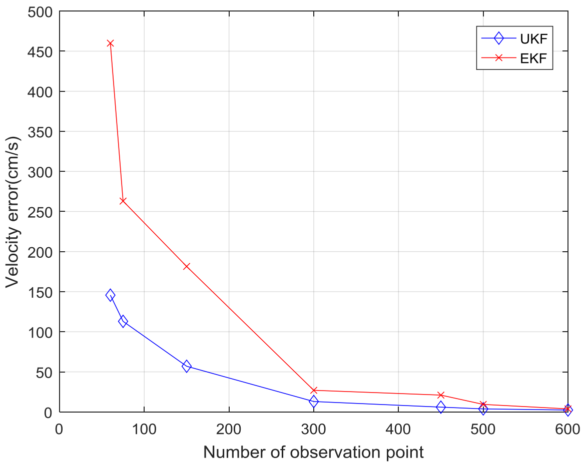

Figure 17. Velocity error for GEO duration case using 1 day observation

GEO observation case study with two observation site

Same analysis parameters are used as shown at Table 3. Since geo satellite can be observed continuously with long duration, two-site observation is expected to have better accuracy than one-site case.

<u>GEO orbit estimation with one-day observation</u>

Number of observation observations data points are doubled due to additional observation site. Other observation duration and frequency parameters are used same with one-site case. For frequency cases, analyses are conducted with 1200, 1000, 900, 600, 240, 120, 40, 20 observation data points. When 20 point is used, it means that the observations are performed once per hour from two-site. Figure 18 indicates that EKF and UKF can reduce position error significantly with addition of synchronize observation from second site. For one-site case, EKF results were diverged for less than 300 data points as shown previously in Figure 14. In two-site analysis, EKF converges for all cases. In Figure 19, all velocity error are below 25 cm/s. This shows that additional site can reduce necessity to perform dense observation. When observation frequency is 10 min, that is 120 data point from two-site, position and velocity errors are relatively low. Therefore, there is no need to perform denser observations.

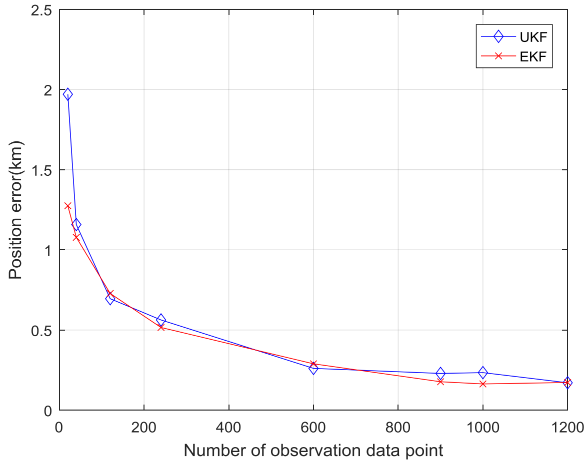

Figure 18 Position error for GEO frequency case using 1 day observation from two site

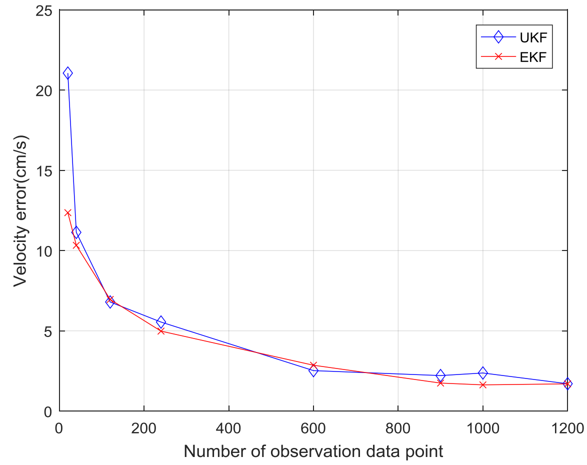

Figure 19 Velocity error for GEO frequency case using 1 day observation from two site

For duration case, data is sampled at 1 min intervals with different observation points ranging from 120 to 1200. Both UKF and EKF shows similar results except 120 data point case. It may be observed from Figure 20 and Figure 21, the accuracy increases with increasing data size, as expected.

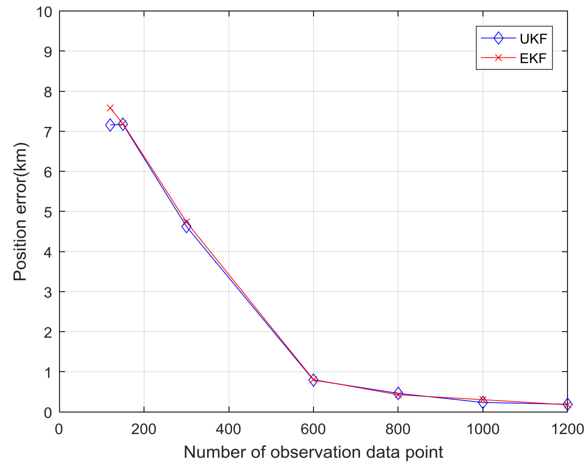

Figure 20 Position error for GEO frequency case using 2 day observation

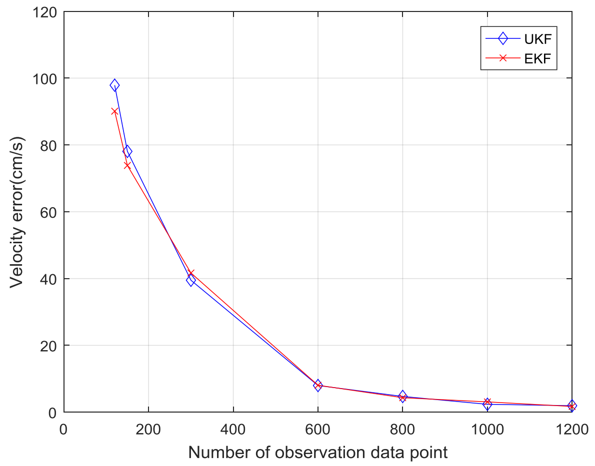

Figure 21 Velocity error for GEO frequency case using 2 day observation

CONCLUSION

In this study, the orbit estimation accuracy of angle-only measurements were analyzed and investigated for LEO and GEO objects. Both UKF and EKF methods were implemented and results were compared. Various factors were considered in analysis such as measurement frequency, observation duration and number of observation site. In LEO cases, when one-site measurements were used, UKF and EKF show similar results, but, velocity error may not be reduced sufficiently. Thus, high uncertainty in velocity may cause bigger error on orbit prediction analysis. However, when additional observation site is available, orbit estimation results were significantly improved, especially velocity error. To maintain velocity error reasonable range, measurements from two-site were necessary for LEO objects. In GEO cases, for one-site observation, UKF outperformed EKF for low observation data point case. Since EKF neglect nonlinear terms, it diverges when duration between measurements were increased. When additional site was added, EKF could converge for all cases and give close results with UKF.

References

Julier S. J. and Uhlmann J. K. A New Extension of the Kalman Filter to Nonlinear Systems. In Proc. of AeroSense: The 11th Int. Symp. on Aerospace/Defence Sensing, Simulation and Controls., 1997.

Lee D.-J. and Alfriend K. T., “Sigma Point Filtering for Sequential Orbit Estimation and Prediction,” J. Spacecr. Rockets, vol. 44, no. 2, pp. 388–398, 2007.

Park E.-S., Park S.-Y. , Roh K.-M., Choi K.-H., Satellite orbit determination using a batch filter based on the unscented transformation, Aerospace Science and Technology 14 (2010) 387–396.

STK User Manual, http://help.agi.com/stk/, 2017.

Vallado D.A., Fundamentals of Astrodynamics and Applications, 2nd ed., Kluwer Academic Pub., Netherlands, 2001.