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HIGH GAIN OBSERVER BASED ATTITUDE CONTROL OF A QUADROTOR

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ABSTRACT

In this study, we design a high gain observer to estimate the unmeasured system states based on the nominal system of a quadrotor aircraft. The desired closed-loop performance is described with a reference model. Using the observed unmeasured states, the attitude controller that gives the desired reference model performance is designed. The findings are illustrated with simulation results.

INTRODUCTION

Due to their broad field of applications, such as near field surveillance, exploration and survey, wind turbine and building inspection, quadrotor Unmanned Aerial Vehicles (UAVs) have been studied both in civil and military applications, indoor and outdoor environments. Quadrotor UAVs are preferred because they have ability to hover, simple mechanical structure, maneuverable and indoor flights, and cancellation of torque due to the reverse turning of neighboring rotors. In recent years, studies on design and control of quadrotors have been increasing [Achtelik (2010), Bouabdallah (2007), Cutler (2014), Hoffmann (2007), Kaya (2016). Kushleyev (2013)].

As in the many real-life applications, the full state measurement may not be available for the quadrotor platforms. In this case, the state observers can be designed to estimate the unmeasured system states as long as they are observable. These unmeasured states are estimated based on the nominal model of the actual system. However, inaccuracies in these nominal models may adversely affect the state estimations which may result in instabilities in the system. Therefore, in this study, we design a high-gain observer to estimate the unmeasured system states. By means of the high gain employment in the observer design, the effects of the model inaccuracies are rejected in the observations. However, the high gain observers suffer from the peaking phenomenon when the initial conditions of the observer and the actual system do not coincide. To eliminate this deficiency, the observed states are saturated at the boundaries of the sufficiently large compact set that we are interested in keeping the system states inside.

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METHOD

In this section, we give the basics of the quadrotor dynamics. Furthermore, we explain the specific input structure that we use in the controller design. This input structure allows us to assess the responses to the selected inputs, clearly. We also show that the unique input set is available that conforms to the specific input structure.

Having described the equations of motion of a quadrotor in the inertial frame, we give the methodology for the attitude controller design in the presence of unmeasured system states. First, we design a high-gain observer to access the unmeasured states. Then, the full-state feedback controller is employed based on the state estimations using a reference model that gives the desired closed-loop characteristics for each attitude channel.

Quadrotor Dynamics

Assumption 1. All the structures of the quadrotor are rigid. Furthermore, the center of gravity and the origin of the body-fixed frame are coincident.

Coordinate systems used in the derivation of equations of motion are body-fixed frame and earth-fixed frame. The use of body-fixed frame, Figure 1, is practical because inertia matrix is time-invariant and rotor and body aerodynamic forces can be expressed with respect to the body. Besides, it is easy to convert body-fixed frame to Earth frame, Figure 2.

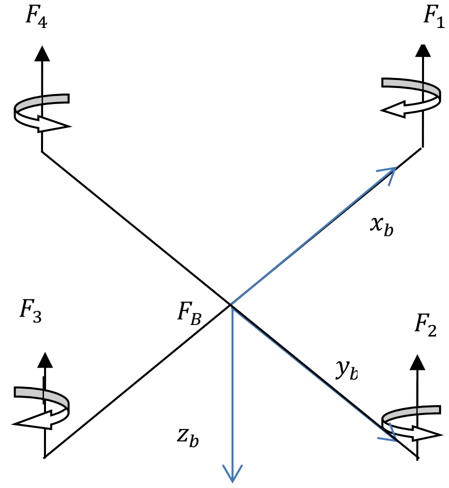

Figure 1. Body-fixed reference frame

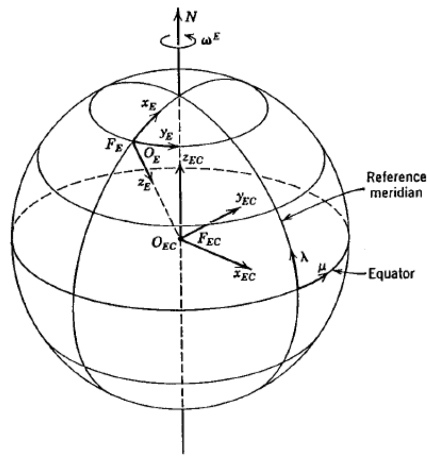

Figure 2. Earth-fixed reference frame

Equations of motion of a quadrotor in the body-fixed frame can be given as [Kaya (2015)]:

$$\dot{u} = \frac{\left(-\sum_{i=1}^{4} H_i\right)\cos\beta - \frac{1}{2}\rho\, S C_X \cos\beta\, V^2}{m} - g\sin\theta - qw + rv$$

$$\dot{v} = \frac{\left(-\sum_{i=1}^{4} H_i\right)\sin\beta - \frac{1}{2}\rho\, S C_X \sin\beta\, V^2}{m} + g\sin\phi\cos\theta - ru + pw \quad (1)$$

$$\dot{w} = \frac{-\sum_{i=1}^{4} T_i + \frac{1}{2}\rho\, S C_Z V^2}{m} + g\cos\phi\cos\theta - pv + qu$$

$$\dot{p} = \frac{(T_4 - T_2)l - h\left(\sum_{i=1}^{4} H_i\right)\sin\beta + \left(\sum_{i=1}^{4}(-1)^{i+1}R_i\right)\cos\beta + J_z q \Omega_r}{I_{xx}} + qr\frac{(I_{yy} - I_{zz})}{I_{xx}}$$

$$\dot{q} = \frac{(T_1 - T_3)l + h\left(\sum_{i=1}^{4} H_i\right)\cos\beta + \left(\sum_{i=1}^{4}(-1)^{i+1}R_i\right)\sin\beta - J_z p \Omega_r}{I_{yy}} + pr\frac{(I_{zz} - I_{xx})}{I_{yy}} \qquad (2)$$

$$\dot{r} = \frac{\sum_{i=1}^{4}\left((-1)^{i}Q_i\right) + l(H_2 - H_4)\cos\beta + l(H_3 - H_1)\sin\beta}{I_{zz}} + pq\frac{(I_{xx} - I_{yy})}{I_{zz}} + \frac{J_z\dot{\Omega}_r}{I_{zz}}$$

<u>Assumption 2.</u> Hub forces H_i and the rolling moments R_i are negligible.

Linearized equations of motion subject to assumption 2 for the hover flight regime are expressed in the inertial frame as:

$$\dot{\phi}, \dot{\theta}, \dot{\psi}, \phi, \theta, \psi = 0$$

$$U_2, U_3, U_4, \Omega_r = 0, \quad U_1 = mg$$

$$\ddot{\phi} = \frac{(T_4 - T_2)l}{I_{xx}}$$

$$\ddot{\theta} = \frac{(T_3 - T_1)l}{I_{yy}} \qquad (2)$$

$$\ddot{\psi} = \frac{\sum_{i=1}^{4}\left((-1)^{i+1}Q_i\right)}{I_{zz}}$$

$$\ddot{z} = g - \frac{\sum_{i=1}^{4} T_i}{m}$$

Thrust T_i and rotor torque Q_i are modeled as proportional to square of the angular velocity of rotor:

$$T_i = b\Omega_i^2, \qquad Q_i = d\Omega_i^2 \qquad (4)$$

The inputs for system are chosen as the total thrust and the three torques as follows:

$$U_1 \triangleq b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right)$$

$$U_2 \triangleq lb\left({\Omega_4}^2 - {\Omega_2}^2\right)$$

$$U_3 \triangleq lb\left({\Omega_3}^2 - {\Omega_1}^2\right) \qquad (5)$$

$$U_4 \triangleq d\left(-{\Omega_1}^2 + {\Omega_2}^2 - {\Omega_3}^2 + {\Omega_4}^2\right)$$

<u>Remark 1.</u> Having obtained the control inputs $\boldsymbol{U_i}$, we need to be able to compute the angular velocities of each rotor. Further, these computed velocities should be unique. Gather the equation (5) to form a system of linear equations as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}, \qquad G \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}, \ \bar{\Omega} \triangleq \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{6}$$

Realize that the range space of the matrix $\mathbf{G}$ is $\boldsymbol{\mathcal{R}}(\mathbf{G}) = \mathbf{R^4}$. Hence, there exists at least one solution for the vector $\bar{\boldsymbol{\Omega}}$. Furthermore, the null space of the matrix $\mathbf{G}$ is $\boldsymbol{\mathcal{N}}(\mathbf{G}) = \{\mathbf{0}\}$ which implies the uniqueness of the solution $\bar{\boldsymbol{\Omega}}$. Once the unique solution is obtained, taking its element-wise square root yields the inputs for all the rotors.

Controller Design

In this study, we consider the linearized dynamics of the quadrotor UAV which is described in previous section. The goal is to design an attitude controller such that the quadrotor can maintain the desired attitude commands satisfactorily.

The system states are chosen as:

$$\begin{aligned} x &= [\phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi} \quad z \quad \dot{z}]^T \\ y &= [\phi \quad \theta \quad \psi \quad z]^T \end{aligned} \tag{7}$$

Then, the linear model of the quadrotor can be written in state-space form as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8}$$

where the control input is $\mathbf{u(t)} = [\mathbf{U_1} \quad \mathbf{U_2} \quad \mathbf{U_3} \quad \mathbf{U_4}]^\mathbf{T}$. The system matrix $\mathbf{A}$, and the input matrix $\mathbf{B}$ are given as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/I_{yy} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/I_{zz} \\ 0 & 0 & 0 & 0 \\ -1/m & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

The output matrix $\mathbf{C} \in \mathbf{R^{4\times8}}$ can be obtained accordingly from the Eqn. (8).

The control objective is to achieve the desired tracking response at each channel (roll, pitch, yaw, altitude) with the natural frequency of $\boldsymbol{\omega_n}$, and the damping ratio of $\boldsymbol{\xi}$. Hence, the reference model that characterizes the desired response is given by:

$$
A_r = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n
\end{bmatrix}, \tag{10}
$$

Hence, the controller gain for the feedback controller $\mathbf{u(t)} = -\mathbf{K_x x(t)}$ is obtained as follows:

$$
K_x = B^{\dagger}(A - A_r) \tag{11}
$$

where $\mathbf{B}^{\dagger}$ denotes the left pseudo-inverse of the input matrix $\mathbf{B}$.

High-Gain Observer Design

Since the full-state information is not available, we design a state observer so that the full-state feedback control is applied as $\mathbf{u(t)} = -\mathbf{K_x \hat{x}(t)}$ where $\mathbf{\hat{x}(t)}$ is the estimated state. The linear system may deviate from the actual nonlinear system due to assumptions made in modeling. It is known that the high-gain observers (HGO) can suppress the modeling errors effectively. Hence, in this study, we design a HGO to obtain the state estimations. The model for HGO is:

$$
\dot{\hat{x}}(t) = A_0\hat{x}(t) + B\phi_0(\hat{x}, u) + H(y - C\hat{x}), \quad \phi_0(\hat{x}, u) = u(t) \tag{12}
$$

where the nominal model $\boldsymbol{\phi_0}(\mathbf{\hat{x}}, \mathbf{u})$ is locally Lipschitz in its arguments. If the nominal model information $\boldsymbol{\phi_0}(\mathbf{\hat{x}}, \mathbf{u})$ is not available, then it can be assumed to be zero; i.e. $\boldsymbol{\phi_0}(\mathbf{\hat{x}}, \mathbf{u}) = \mathbf{0}$. In such a case, the observer becomes a linear HGO. Here, the matrix $\mathbf{A_0}$ differs from the system matrix $\mathbf{A}$ as it only contains the derivative chain information of the system states. In our linearized model, these two matrices coincide $\mathbf{A} = \mathbf{A_0}$. For more information, see the Ref. [Atassi (1999)].

The observation matrix $\mathbf{H}$ is given by:

$$
H = \begin{bmatrix}
\alpha_1/\varepsilon & 0 & \cdots & 0 \\
\alpha_2/\varepsilon^2 & 0 & & \\
0 & \alpha_1/\varepsilon & \ddots & \\
 & \alpha_2/\varepsilon^2 & 0 & \vdots \\
\vdots & 0 & \alpha_1/\varepsilon & \\
 & \ddots & \alpha_2/\varepsilon^2 & 0 \\
 & & 0 & \alpha_1/\varepsilon \\
0 & \cdots & 0 & \alpha_2/\varepsilon^2
\end{bmatrix} \tag{13}
$$

where $\boldsymbol{\varepsilon}$ is sufficiently small positive constant, and the positive constants $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ are chosen such that the roots of the following polynomial lie in the left half plane:

$$
s^2 + \alpha_1 s + \alpha_2 = 0 \tag{14}
$$

SIMULATION RESULTS

In this section, we give the preliminary results of the designed attitude controller. The identified system parameters for the quadrotor are as follows:

$$\mathrm{I_{xx}} = 0.060535602\ \mathrm{kg \cdot m^2}, \quad \mathrm{I_{yy}} = 0.060547676\ \mathrm{kg \cdot m^2}, \quad \mathrm{I_{zz}} = 0.109745533\ \mathrm{kg \cdot m^2}$$
$$\mathrm{J_z} = 3.1 \times 10^{-5}\ \mathrm{kg \cdot m^2}, \quad \mathrm{m} = 4\ \mathrm{kg}, \quad \mathrm{l} = 0.4\ \mathrm{m}, \quad \mathrm{b} = 5.38 \times 10^{-7}, \quad \mathrm{d} = 1.05 \times 10^{-8}$$

The desired response is characterized by the damping ratio of $\boldsymbol{\xi} = \mathbf{1.4}$ and the natural frequency of $\boldsymbol{\omega_n} = \mathbf{2.4}$.

For the observer design, we choose the design parameters as: $\boldsymbol{\varepsilon} = \mathbf{0.05}$, $\boldsymbol{\alpha_1} = \mathbf{8}$, $\boldsymbol{\alpha_2} = \mathbf{15}$ so that the roots of the polynomial in Eqn. (14) become $\mathbf{s_1} = -\mathbf{5}$ and $\mathbf{s_2} = -\mathbf{3}$.

The state observer estimations and the tracking performance for the roll attitude command are illustrated in Figure 3.

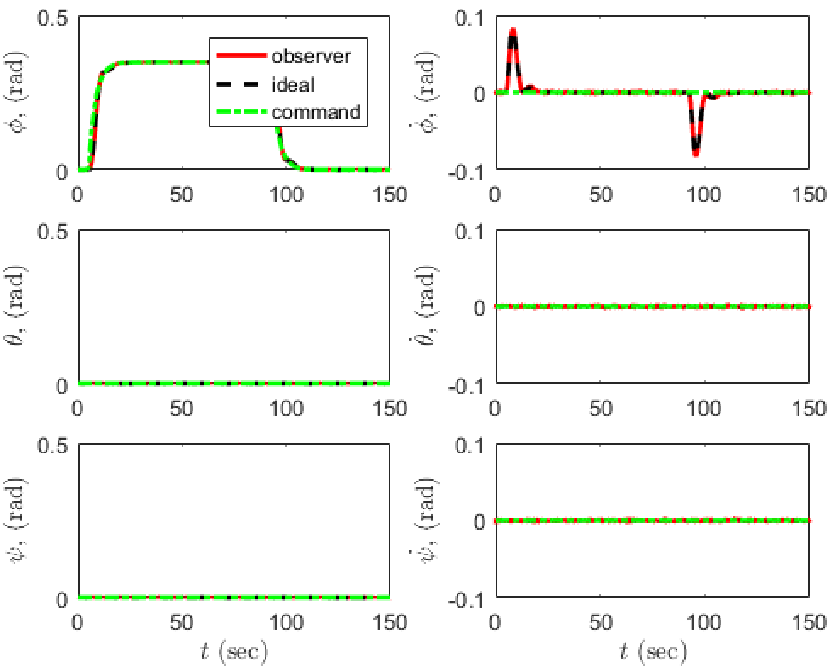

Figure 3. The State Observer Performance and the Roll Command Tracking Performance

Similarly, the pitch and yaw command tracking performances with the observed states are given in Figure 4 and Figure 5, respectively.

CONCLUSION & FUTURE DIRECTIONS

In this paper, we summarized the works on the attitude controller for a quadrotor UAV. In the absence of full state measurements, a high gain observer (HGO) is designed based on the nominal model of the quadrotor. Although the nominal model has limited information about the quadrotor dynamics, satisfactorily state estimations are achieved with HGO. Using these state estimations, the full-state feedback controller is constructed, and the preliminary results are illustrated. In the future studies, this controller will be extended to full control of the quadrotor UAV including trajectory tracking. Furthermore, to attenuate the effects of mismatched points between the nominal model and the actual system, an adaptive input will be designed. Finally, all these works will be implemented on the quadrotor, and the flight tests will be performed.

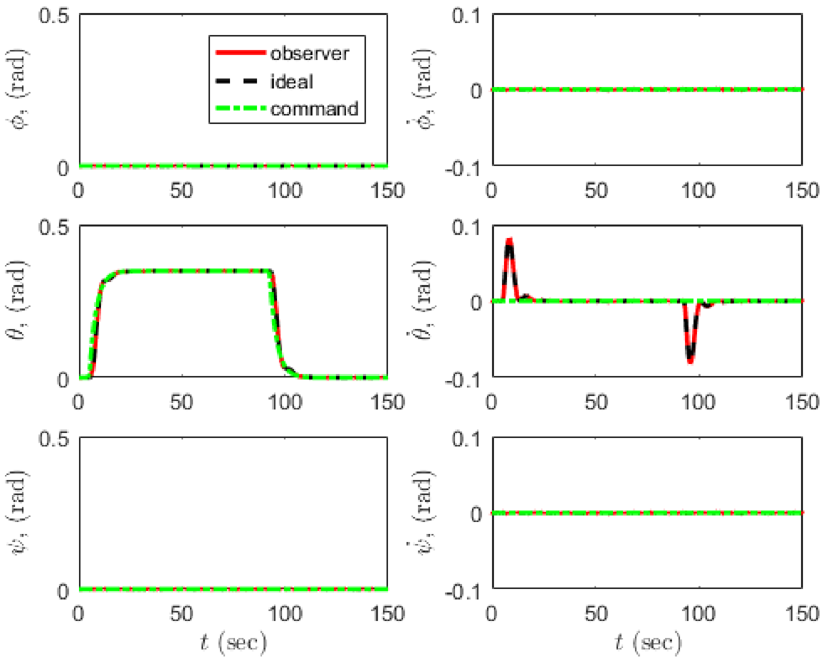

Figure 4. The State Observer Performance and the Pitch Command Tracking Performance

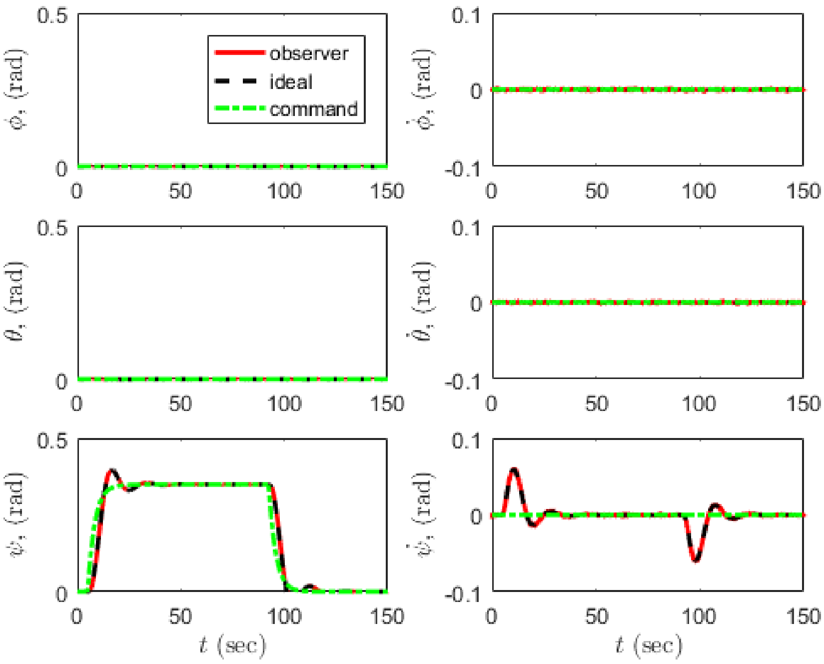

Figure 5. The State Observer Performance and the Yaw Command Tracking Performance

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