DESIGN OPTIMIZATION OF VARIABLE STIFFNESS COMPOSITE LAMINATES USING SURROGATE MODELS FOR COMPLIANCE AND BUCKLING LOAD

Hasan İnci[1]
TÜBİTAK-SAGE
Ankara, Turkey

Altan Kayran[2]
Middle East Technical University
Ankara, Turkey

ABSTRACT

In the design optimization process of composites, the analyses were mostly done for straight fiber composites. With the improved manufacturing capabilities, designers gained the ability to design more efficient composite structures. In the past two decades, variable stiffness (VS) composite laminates are introduced to the literature. Researchers working on the VS concept are progressively increasing, due to the advantages of variable stiffness laminated composites over the straight fiber laminated composites. The main disadvantage of VS composite laminates is the higher design optimization and manufacturing cost. Optimization of VS composites is too costly if function evaluations are performed by finite element analysis during the optimization process. In this study, to reduce the computational cost of the design optimization of VS composite laminates, surrogate models developed are used in the optimization process for function evaluations. The effectiveness of the surrogate models on the compliance and buckling load optimization of variable stiffness open hole composite laminates under in-plane bending and compression loads is demonstrated. It is shown that for the case of buckling load response 22% increase in the buckling load is obtained for the variable stiffness open-hole composite laminate with respect to straight fiber caseusing the surrogate model in function evaluations during the optimization process.

INTRODUCTION

In advanced composite manufacturing, Automated Fiber Placement (AFP), Automated Tape Laying (ATL) and Continuous Tow Shearing (CTS) methods allowed the design of VS composites with curvilinear fiber paths resulting in the modification of load paths and more favorable stress distribution. In the design of VS composites, iterative optimization methods are used together with FEA to determine the optimum values of the design variables defining the reference fiber path. Many studies have been made on the design optimization of variable stiffness composites with different structural performance requirements such as buckling capacity [Hyer and Lee 1991, Setoodeh et al. 2008], elastic behavior [Gürdal and Olmedo 1993], stiffness [Setoodeh et al. 2006], compressive buckling and first ply failure [Lopes et al. 2008], maximum fundamental frequency [Blom et al. 2008], post buckling

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[1] Senior Researcher, Email: hasan.inci@tubitak.gov.tr
[2] Prof. in Aerospace Engineering, Email: akayran@metu.edu.tr

progressive damage [Lopes et al. 2007]. Variable stiffness composite design also ensures flexibility for trade-off between different structural properties [Gürdal et al. 2008]. Use of FEA analysis for function evaluations in the optimization process increases the time required in design optimization studies substantially. Therefore, surrogate models are often used to reduce the optimization time [Nik, et.al. 2014].

In the present study, surrogate models are developed to be used in function evaluations, for the optimization of variable stiffness composite laminates. The surrogate models which are developed in the present study are based on Radial Basis Functions (RBF) approximation [Broomhead and Lowe 1998]. Optimizations with the surrogate models are performed by the Particle Swarm Optimization (PSO) [Shi and Eberhart 1998] compliance minimization and buckling load maximization of variable stiffness open hole composite laminates under in-plane bending and compression loads. It is shown that for the buckling case study, for the variable stiffness open-hole composite laminate buckling load is 22% higher compared to the straight fiber composite laminate.

METHOD

In the current study, design optimization of VS laminates is performed using surrogate models to reduce the computational cost of optimization. Two problem cases are studied in this study. First one is the compliance minimization of a rectangular plate with a circular hole under in-plane bending load; second one is the buckling load maximization of the same open-hole composite laminates under a compressive force.

Reference fiber path definition is based on the linear variation of the fiber angle along the composite laminate using with two variables. In the manufacturing of the VS laminate, shifted reference fiber path technique is used, and maximum curvature manufacturing constraint is also taken into account in the optimization process. Particle Swarm Optimization (PSO) is used as the optimizer. Radial Basis Function (RBF) approximation is used to generate the surrogate model.

Reference Fiber Path Definition

In the present study, reference fiber path definition that is used is first introduced by Gürdal and Olmedo. Orientation angle of the reference fiber path is changes linearly along the length of the composite laminate, as shown in Eqn. (1). There are three variables defining the reference fiber path which are $T_0, T_1, \emptyset$ [Gürdal and Olmedo, 1993].

$$\theta(x') = \begin{cases} \emptyset + \frac{2}{a}(T_1 - T_0)x' + T_0 - 2(T_0 - T_1), & for \ -a \leq x' \leq -\frac{a}{2} \\ \emptyset + \frac{2}{a}(T_0 - T_1)x' + T_0, & for \ -\frac{a}{2} \leq x' \leq 0 \\ \emptyset + \frac{2}{a}(T_1 - T_0)x' + T_0, & for \ \ 0 \leq x' \leq \frac{a}{2} \\ \emptyset + \frac{2}{a}(T_0 - T_1)x' + T_0 - 2(T_0 - T_1), & for \ \ \frac{a}{2} \leq x' \leq a \end{cases} \tag{1}$$

where;

$$x' = x \cos \emptyset + y \sin \emptyset \ ,$$

In Equation (1), As shown in Figure 1, T_0 defines the orientation angle of the midpoint on the reference fiber path, "a" is the width of the plate, T_1 defines the orientation angle of the point which is exactly at the end of the laminates at a/2, and $\emptyset$ is the rotation angle of the reference coordinate axis shown in Figure 1. All the aforementioned angles can take values in an interval of [0º,90º]. In this study only T_0, T_1 are taken as the design variables for the reference fiber path definition.

During the optimization a manufacturing constraint is also implemented and minimum curvature constraint of the fiber placement machines is taken into account. In the present study, curvature calculation introduced by Tatting and Gürdal is used [Tatting and Gürdal

2003]. If the curvature constraint is not satisfied during the design process, a penalty is applied to the objective function for avoiding infeasible solutions.
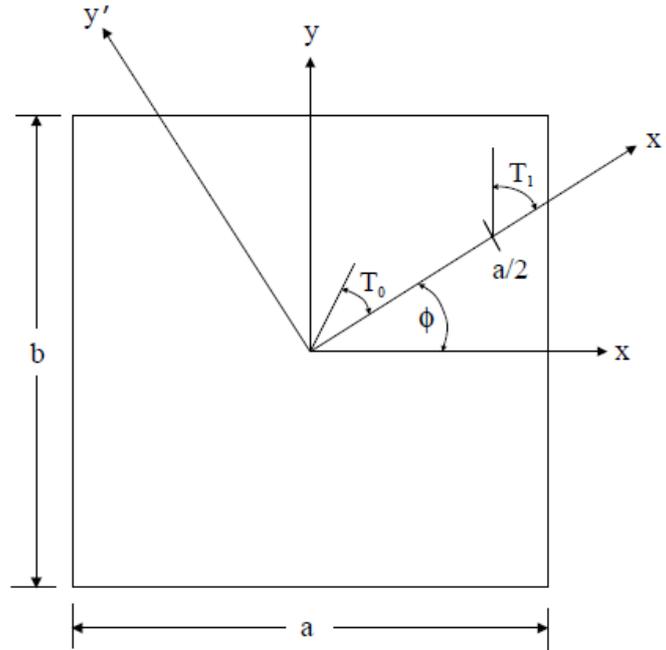

Figure 1 – Variables in the fiber path definition [Waldhart 1996]

Particle Swarm Optimization (PSO)

For the purposes of the study, an optimizer, based on Particle Swarm Optimization (PSO) algorithm, is developed. PSO simulates the behavior of a school of birds. Each individual is named as a "particle" which, in fact, represents a potential solution to a problem. Each particle is treated as a point in a D-dimensional space. Each particle adjusts its flying according to its own flying experience and its companions' flying experience. The *i*th particle is represented as $X_i = (x_{i1}, x_{i2}, ..., x_{iD})$. The best previous position (the position giving the best fitness value) of any particle is recorded and represented as $P_i = (p_{i1}, p_{i2}, ..., p_{iD})$. The index of the best particle among all the particles in the population is represented by the symbol *g*. The rate of the position change (velocity) for particle *i* is represented as $V_i = (v_{i1}, v_{i2}, ..., v_{iD})$. The particles are manipulated according to Equations (2) and (3),

$$v_{id} = w * v_{id} + c_1 * rand() * (p_{id} - x_{id}) + c_2 * rand() * (p_{gd} - x_{id}) \qquad (2)$$

$$x_{id} = x_{id} + v_{id} \qquad (3)$$

where c_1 and c_2 are two positive constants, rand( ) is a random function in the range [0,1], *w* is the inertia weight which reduces linearly as the iteration goes on. Inertia weight is between [0.9, 0.4] [Shi and Eberhart 1998]. The first part of the Eqn. (2) lets the particles to move in the search space globally with the inertia weight. The second part of the Eqn. (2) is the "cognition" part, which represents the private thinking of the particle itself. The third part is the "social" part, which represents the collaboration among the particles. Equation (2) is used to calculate the particle's new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group's best experience. Then, the particle flies toward a new position according to Eqn. (3). The performance of each particle is measured according to a predefined fitness function, which is related to the problem to be solved.

In the initial phase of the study, optimizer is coupled with the commercial FEA code NASTRAN and objective function evaluations are performed by FEA in each iteration. In the PSO algorithm, particles are defined as the reference fiber path variables T_0 and T_1. During the optimization process, after each FEA, manufacturing constraint is checked for constraint

violation and the objective function is penalized if constraint violation is encountered. Termination condition of the optimization process is based on a prescribed tolerance which is calculated utilizing the Euclidian distance of each particle.

Radial Basis Functions (RBF) Approximation

Because of the high computational cost of FEA in each iteration during the optimizations process, surrogate models are decided to be used for function evaluations of the optimization to reduce the analysis time. From the literature it is seen that surrogate models based on RBF approximation with multiquadric basis functions yield satisfactory results for VS composite laminate optimizations. [Nik, et.al. 2014]

Radial basis function approach constructs a linear space which depends on the position relative to the known data points according to an arbitrary distance measure [Broomhead and Lowe 1988]. The basis functions, which are generally nonlinear, are introduced as $\psi(\|x - x_i\|)$, where ψ is different basis functions depending on the problem, $\|x - x_i\|$ is the Euclidian distance of the two sample data points. As stated earlier, the radial basis function approach creates linear space between the basis functions. Approximation of a function by the radial basis functions can be written as,

$$\tilde{y}(x) = \sum_{i=1}^{n} w_i \, \psi(\|x - x_i\|) \tag{4}$$

where $\tilde{y}$ is the approximate value of the objective function in an optimization problem and w_i is the weight of the basis function evaluated by fitting the model to the training data. This results in a linear system of equations given by,

$$y = \psi w \tag{5}$$

where y is the vector of function values at the training data, w is the vector of basis function weights and ψ is a matrix, which consists of Euclidian distance of each training data with respect to one another, also called Gramian matrix. Matrix ψ **is** defined by,

$$\psi = \begin{bmatrix} \psi(x_1.x_1) & \psi(x_1.x_2) & ... & \psi(x_1.x_n) \\ \psi(x_2.x_1) & \psi(x_2.x_2) & ... & \psi(x_2.x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \psi(x_n.x_1) & \psi(x_n.x_2) & ... & \psi(x_n.x_n) \end{bmatrix} \tag{6}$$

where *n* is the number of sample data used for interpolation. Once Eqn. (5) is solved for w, the weights of the approximate function are obtained. The size of the weight vector obtained is same as the number of sample points. This a strict interpolation in which the approximate equation satisfies the sampling points exactly. Having obtained the weights, one can then solve Eqn. (4) for the value of the approximate function for any arbitrary point different from the sampling points.

For RBF approximations, multiquadric basis functions are generally used,

$$\psi = \sqrt{r^2 + s^2} \tag{7}$$

where *r* is the Euclidian distance between points and *s* is the width parameter which is chosen a value in the interval [0,1]. Design variable values are also scaled to fit into the [0,1] interval [Nik, et.al. 2014].

For the case studies, surrogate models which simulate the compliance and buckling response of the composite laminates are developed. Surrogate models are developed by performing function evaluations at the sampling points. Large set of sample points are used to ensure the accurate performance of the surrogate model, and while selecting the sample points, Latin Hypercube Sampling is used instead of random sampling for better coverage. Weights of the basis functions are calculated from the sample points by FEA. It is seen that

surrogate models have very good performance in determining the optimum parameters of the reference fiber paths for both case studies.

RESULTS

Open hole composite plates used for the two case studies. Same geometry is investigated for different responses in each case study. A rectangular composite plate with a central hole is generated for the case studies. The plate dimensions are 1x0.5x0.008 m. The geometry of the open hole composite plate and the finite element mesh generated is given in Figure 2. Quadrilateral shell elements with drilling degrees of freedom (CQUADR) are used in the finite element mesh for the in-plane bending case for its better performance for in-plane bending problems. Quadrilateral shell elements without drilling degrees of freedom (CQUAD4) elements are used for the buckling case study. In the finite element model of the open-hole composite plate, there are 580 finite elements. For both case studies, a mid-plane symmetric laminate with 4 equal thickness layers are created with a stacking sequence of $[+\theta/-\theta]_s$. The composite ply mechanical properties and the stacking of the plate, for both of the problems, are given in Table 1.

Table 1 - Ply mechanical properties and stacking sequence

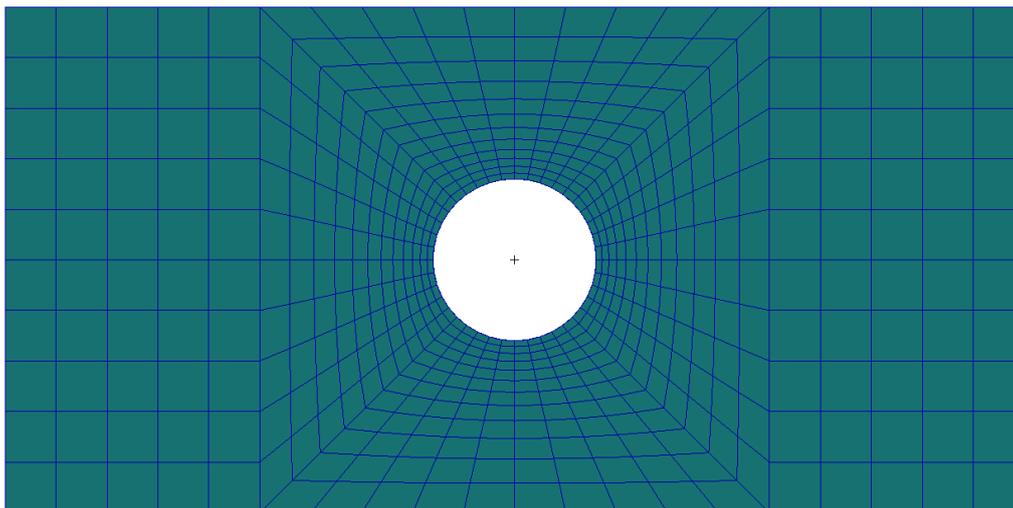| $E_{11} = 213.74\ MPa$	$E_{22} = 18.62\ MPa$	$E_{12} = 5.171\ MPa$	$v = 0.28$
Ply #	Orientation	Thickness (mm)	
1	$+\theta$	2	
2	$-\theta$	2	
3	$-\theta$	2	
4	$+\theta$	2	

Figure 2 – Analysis geometry and the finite element mesh

For the in-plane bending case, the objective function is the minimum total strain energy of the plate. Minimization of the total strain energy implies maximization of the stiffness of the open-hole composite plate. For the buckling case study, first eigenvalue of the plate is the objective function. To increase the buckling load, one needs the eigenvalue to be maximized. Thus, bending problem is a minimization problem and buckling problem is a maximization problem.

For the in-plane bending load case, composite plate is cantilevered at the left edge ($u = v = w = R_x = R_y = R_z = 0$), and a 500N total load is applied in the downward direction along the upper edge of the plate, as shown in Figure 3.
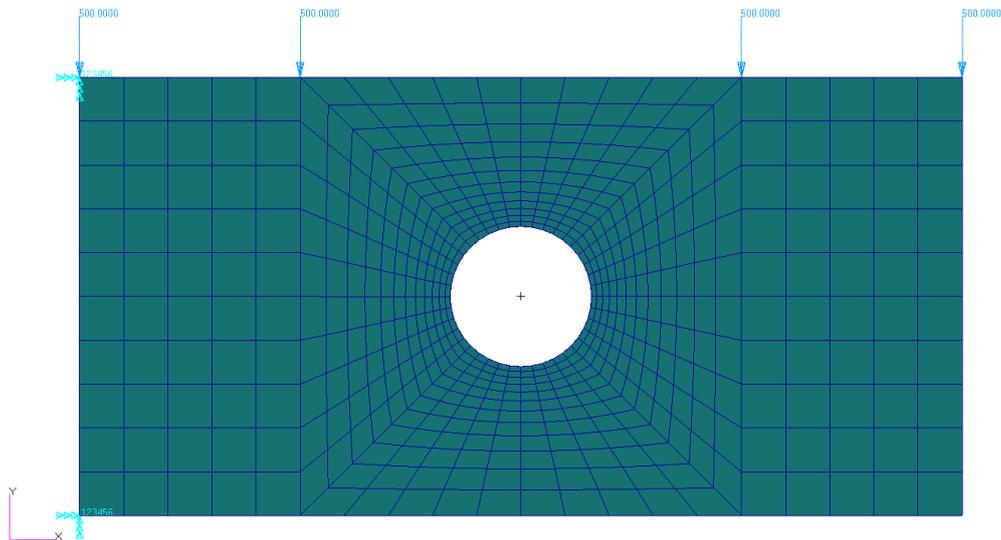
Figure 3 - Load and boundary condition for the bending problem

For the buckling load case, at the left edge all translational and drilling degrees of freedoms are fixed ($u = v = w = R_z = 0, R_x \neq R_y \neq 0$), and all the other edges are fixed in two the translational degrees of freedom except for the longitudinal degree of freedom ($v = w = R_z = 0$, $u \neq R_y \neq R_z \neq 0$) which is the loading direction, as shown in Figure 4. The boundary condition applied for the buckling load case corresponds to the simply supported boundary condition. A total load of 500N is applied at the right edge as the compressive force.
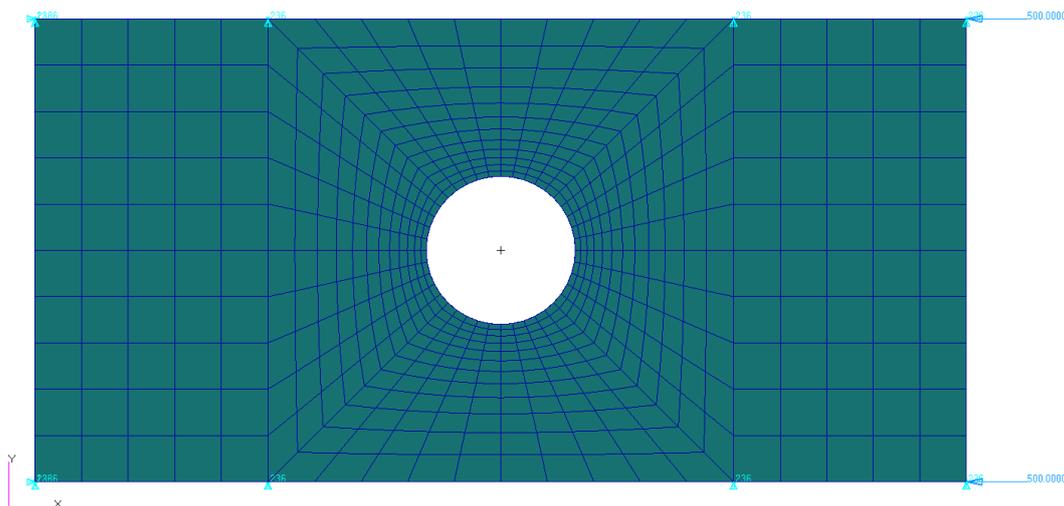
Figure 4 - Load and boundary conditions for the buckling problem

The definitions of the parameters in the results table are presented in Table 2.

The parameters used in both approximation and optimization stages are decided based on the past experiences.

To find out whether the VS composite laminate has an advantage over the conventional straight fiber composite laminate, for each case 2 optimizations are performed one for the VS composite and one for the straight fiber composite laminate.

Table 2 - Definitions of parameters used in the results table

Parameter	Definition
Population	Number of individuals used in the optimization
# of train. Data	Number of training data for the surrogate model
# of ctrl. pnts.	Number of control points for the error calculations of the surrogate model
Width P.	Width parameter; 's' value for the surrogate model
Tolerance	Termination criterion tolerance
Type of term	Type of termination used for the optimization
Rsq	R-squared error calculated for the established surrogate model for the optimization
RMSE	Root mean square error calculated for the established surrogate model for the optimization
S.Average	Average of the surrogate model responses at sample points to compare with RMSE
RUN	Distinct run numbers for the optimization
Max. iter #	Maximum iterations used until termination of the optimization process
Best Obj. Val.	Best objective function value achieved by the optimization with the surrogate model
FEAvalue	Finite element analysis solution using the optimum variable values
FEA Error	Error between the surrogate model response and the finite element analysis response which is calculated by the difference between the surrogate model and finite element solutions divided by the finite element result
Best Var.	Optimum values of the design variables reached in the optimization
Approx. T.	Time passed during the surrogate model generation with the error calculations
Optim. T.	Total optimization time of the 10 individual optimizations
Total Time	Total time of the analysis. Summation of Approx. T. and Optim. T.

Minimization of Compliance of the Cantilevered Rectangular Plate with Central Hole under In-plane Bending Load

For the straight fiber composite laminate, surrogate model parameters and model errors are given in Table 3.

Table **4** gives the optimization results for the open-hole straight fiber composite laminate . It should be noted that for the in-plane bending load case, the objective is the minimization of the total strain energy of the composite laminate with the open hole.

Table 3 – In-Plane Bending Case Study/ Straight fiber composite laminate / Surrogate model parameters and model errors

Population	# of train. Data	# of ctrl. pnts.	Width P.	Initial Velocity
20	20	20	0.2	Random
Tolerance	**Type of Term.**	**Rsquare**	**RMSE**	**S.average**
0.01	Position	0,9999	0,0342	3,4890

Table 4 - In-Plane Bending Case Study / Straight fiber optimization results

RUN	1	2	3	4	5	6	7	8	9	10
Max. iter #	323	308	272	305	291	304	306	349	335	294
Best Obj. Val.	1,33370	1,33370	1,33370	1,33370	1,33370	1,33370	1,33370	1,33370	1,33370	1,33370
FEAvalue	1,33464	1,33464	1,33464	1,33464	1,33464	1,33464	1,33464	1,33464	1,33464	1,33464
FEA Error	0,00071	0,00071	0,00071	0,00071	0,00071	0,00071	0,00071	0,00071	0,00071	0,00071
Best Var.	19,8899	19,8899	19,8899	19,8899	19,8899	19,8899	19,89	19,89	19,8899	19,8899
Approx. T.	≈ 0,159 hours									
Optim. T.	≈ 0,083 hours									
Total Time	≈ 0,242 hours									

For the straight fiber laminate, the optimum stacking sequence is obtained as $[+19.89°/-19.89°]_s$. Minimum compliance value is 1.3337 from the surrogate model and 1.3346 from the finite element analysis corresponding to the optimum variable value. As

Table **4** shows, the total time spent for the surrogate model generation and 10 individual optimizations is less than 15 minutes.

For the VS composite laminate, surrogate model parameters and model errors are given in Table 5**.**

Table 6 gives the optimization results for the open-hole VS composite laminate.

Table 5 - In-Plane Bending Case Study/ VS composite laminate / Surrogate model parameters and model errors

Population	# of train. Data	# of ctrl. pnts.	Width P.	Initial Velocity
20	100	20	0.1	Random
Tolerance	Type pf Term.	Rsquare	RMSE	S.average
0.01	Position	0,9967	0,0394	4,0007

Table 6 - In-Plane Bending Case Study / VS optimization results

RUN		1	2	3	4	5	6	7	8	9	10
Max. iter #		376	372	363	364	361	386	345	385	338	361
Best Obj. Val.		1,33059	1,33059	1,33059	1,33059	1,33059	1,33059	1,33059	1,33059	1,33059	1,33059
FEAvalue		1,33171	1,33171	1,33171	1,33171	1,33171	1,33171	1,33171	1,33171	1,33171	1,33171
FEA Error		0,00084	0,00084	0,00084	0,00084	0,00084	0,00084	0,00084	0,00084	0,00084	0,00084
Best Var.	T0	21,32	21,32	21,32	21,32	21,32	21,32	21,32	21,32	21,32	21,32
	T1	18,01	18,01	18,01	18,01	18,01	18,01	18,01	18,01	18,01	18,01
Approx. T.		≈ 0,527 hours									
Optim. T.		≈ 0,372 hours									
Total Time		≈ 0,899 hours									

For the optimization of the VS composite laminate, the total time spent for the surrogate model generation and 10 individual optimizations is less than an hour. The optimum design variable values for the reference fiber path of the VS fiber composite laminate are obtained as $T_0 = 21.32°$, $T_1 = 18.01°$. To show the optimization process a sample objective function change is shown in Figure **5**. It is seen that performance of the surrogate model is satisfactory when the best objective function values calculated by the surrogate model and the finite element analysis are compared. On the other hand, VS open-hole composite plate did not show significant improvement over the straight fiber composite plate. Based on the best

objective function values, it is seen that with the VS composite plate only a 0.2% improvement is achieved over the straight fiber case. The optimized fiber paths are shown in Figure 6. In Figure 6, blue line shows the $+\theta$ orientation, red line shows the $-\theta$ orientation of the fibers. Since the design variables T_0 and T_1 are close to each other, VS behavior cannot be identified easily. Therefore, the fibers are seen as straight fibers in Figure 6. It should be noted that in the optimum reference fiber path has a fiber angle of ±21.3^o at the midpoint of the plate and ±18.0^o the right and left edges of the plate.
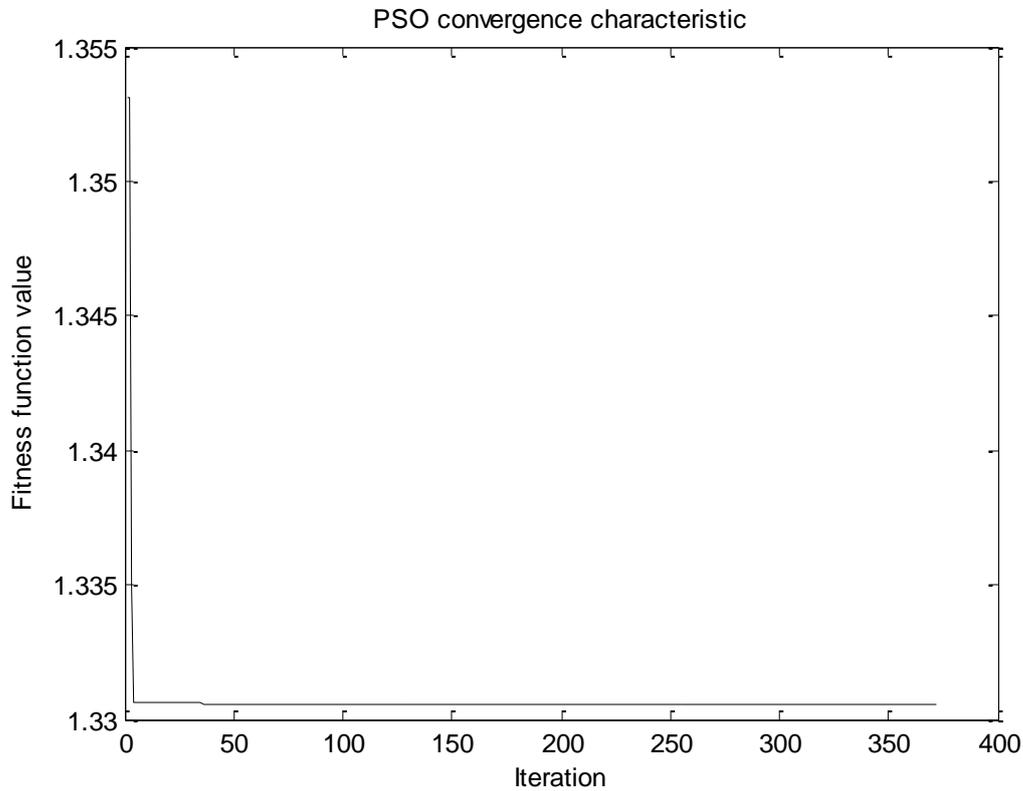

Figure 5 - Best objective function change through iterations for VS compliance problem

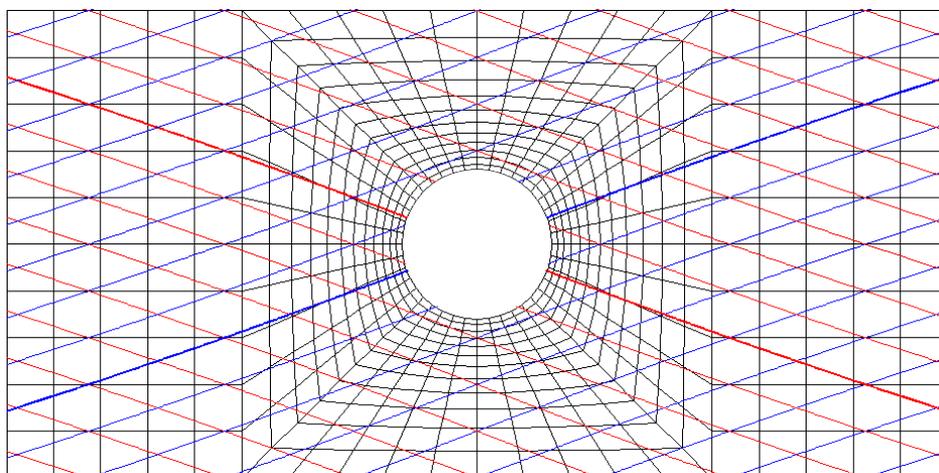Figure 6 - Optimum fiber angles of the VS composite plate for the bending load / Blue line: 1st layer $+\boldsymbol{\theta}$, Red line: 2nd layer $-\boldsymbol{\theta}$

For the same geometry optimization under in-plane bending load is also performed for the composite laminate without a hole. In this case, optimum design variables T_0 and T_1 are obtained as 21.13^o and 17.98^o, respectively. At the optimum configuration objective function

value is 1.1914 joules which is a lower value than the best objective function value for the open-hole case. It is noted that the optimum design variables for the no hole case are quite close to the open-hole composite plate with lower strain energy. Hence, it is observed that in this particular case study, the open hole does not alter the reference fiber path in the optimum configuration.

Maximization of the Buckling Load of the Rectangular Plate with the Central Hole

For the straight fiber composite laminate, surrogate model parameters and model errors are given in Table 7.

Table **8** gives the optimization results for the open-hole straight fiber composite laminate. It should be noted that for the particular case study, the objective is the maximization of the buckling load of the composite laminate with the open hole.

Table 7 - Buckling Case Study/ Straight fiber composite laminate / Surrogate model parameters and model errors

Population	# of train. Data	# of ctrl. pnts.	Width P.	Initial Velocity
20	20	20	0.2	Random
Tolerance	Type of Term.	Rsquare	RMSE	S.average
0.01	Position	1,0000	0,0003	0,1674

Table 8 - Buckling Case Study / Straight fiber optimization results

RUN	1	2	3	4	5	6	7	8	9	10
Max. iter #	310	290	351	262	344	296	341	354	327	342
Best Obj. Val.	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498
FEAvalue	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498	0,22498
FEA Error	0,00001	0,00001	0,00001	0,00001	0,00001	0,00001	0,00001	0,00001	0,00001	0,00001
Best Var.	19,8418	19,8418	19,8418	19,8418	19,8418	19,8418	19,84	19,84	19,8418	19,8418
Approx. T.	$\approx 0,107$ hours									
Optim. T.	$\approx 0,072$ hours									
Total Time	$\approx 0,179$ hours									

As can be seen from Table 7, the surrogate model performance is very satisfactory for the compressive load case. The optimum eigenvalue is obtained as 0.22498 and the corresponding stacking sequence of the composite laminate is $[+19.84°/-19.84°]_s$. It should also be noted that the responses of surrogate model and finite element analysis match exactly for this case.

For the VS composite laminate, surrogate model parameters and model errors are given in Table 9.

Table **10** gives the optimization results for the open-hole VS composite laminate.

Table 9 - Buckling Case Study/ VS composite laminate / Surrogate model parameters and model errors

Population	# of train. Data	# of ctrl. pnts.	Width P.	Initial Velocity
20	100	20	0.1	Random
Tolerance	Type pf Term.	Rsquare	RMSE	S.average
0.01	Position	0,9952	0,0017	0,1603

Table 10 - Buckling Case Study / VS optimization results

RUN	1	2	3	4	5	6	7	8	9	10
Max. iter #	372	341	334	358	308	339	390	321	340	340
Best Obj. Val.	0,27463	0,27463	0,27463	0,27463	0,27463	0,27463	0,27463	0,27463	0,27463	0,27463

FEAvalue		0,27860	0,27860	0,27860	0,27860	0,27860	0,27860	0,27860	0,27860	0,27860	0,27860
FEA Error		0,01424	0,01424	0,01424	0,01424	0,01424	0,01424	0,01424	0,01424	0,01424	0,01424
Best Var.	**T0**	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	T1	42,37	42,37	42,37	42,37	42,37	42,37	42,37	42,37	42,37	42,37
Approx. T.	$\approx 0,397$ hours										
Optim. T.	$\approx 0,339$ hours										
Total Time	$\approx 0,736$ hours										

For the VS open-home laminate under compressive load, the performance of the surrogate model is also satisfactory when the S.average and RMSE results are compared with each other. The optimum design variable values for the reference fiber path of the VS composite laminate are obtained as $T_0 = 0°$, $T_1 = 42.37°$. This result is comparable with the result presented by Olmedo and Gürdal [Olmedo and Gürdal 1993]. Comparing the optimum buckling loads obtained by the straight fiber laminate and VS laminate, it is seen that there is a 22% increase in the buckling load. To show the optimization process a sample objective function change is shown in Figure 7. Figure 8 shows the resultant fiber paths where, blue line shows the $+\theta$ orientation, red line shows the $-\theta$ orientation of the fibers. The optimized fiber paths are shown in Figure 8. For the VS composite laminate with open hole, Figure 8 shows that for the optimum configuration, reference fiber path has a fiber angle of 0° at the midpoint of the plate and $±42.37^o$ at the right and left edges of the plate. It is concluded that 22% increase in the buckling load for the VS composite plate with respect to the straight fiber configuration is a significant increase showing the effectiveness of the variable stiffness concept.

For the same geometry optimization of the buckling load is also performed for the composite laminate without a hole. In this case, optimum design variables T_0 and T_1 are obtained as 0° and 38.05^o, respectively. At the optimum configuration, eigenvalue is 0.3159 which is higher than the best objective function value for the open-hole case, as expected. It is noted that the optimum design variable T_0 for the no hole case is same as the open-hole case. However, when the hole is removed, optimum design variable T_1 is approximately 4° lower than the open-hole case. In this example, the effect of the open-hole on the reference fiber path definition of the variable stiffness composite laminate is clearly seen. It should be noted that T_0 specifies the fiber angle at the mid of the laminate, whereas T_1 specifies the fiber angle at the edges of the laminate. Thus, for the open-hole case, reference fiber path deviates more from the straight fiber composite compared to the no hole case.
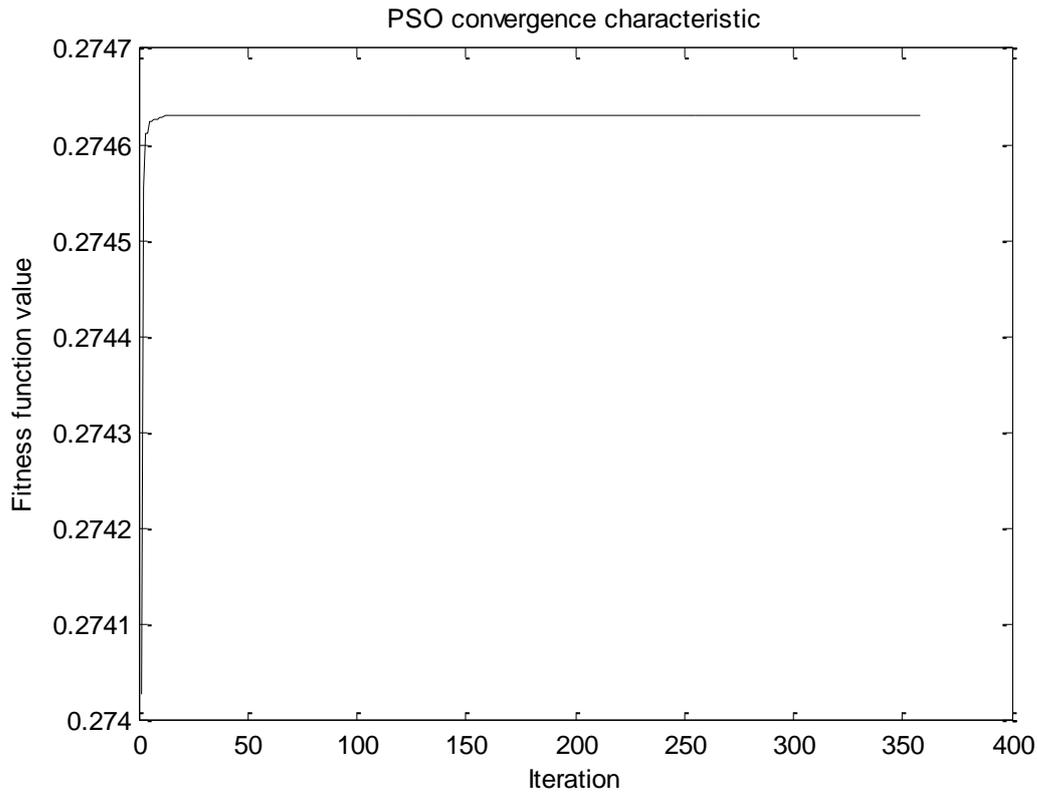
PSO convergence characteristic

Figure 7 - Best objective functions through iterations for VS buckling problem

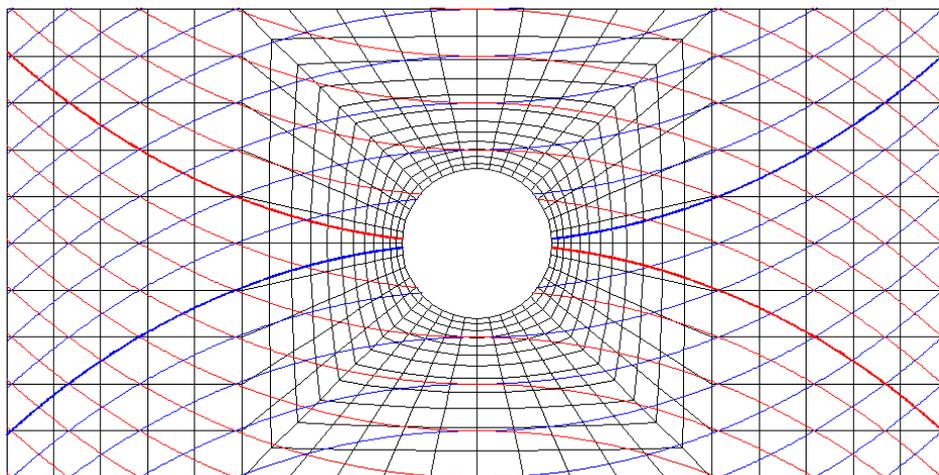Figure 8 - Optimum fiber angles of the VS composite plate for the buckling load case // Blue line: 1st layer $+\theta$, Red line: 2nd layer $-\theta$

CONCLUSION

The effectiveness of the surrogate models on the compliance and buckling load optimization of variable stiffness open hole composite laminates under in-plane bending and compression loads is studied. For both problems surrogate models are established to be used in function evaluations during the optimization process. It is seen that the results obtained by the surrogate models are satisfactory, since the surrogate model errors are small, and the differences between the surrogate model based optimization responses and finite element analysis results with the optimum design variable values are acceptable. With the surrogate model based optimization, the designer can get the optimum values of the design variables with considerably less computational effort.

For the minimum compliance problem studied, it is seen that the VS composite laminate does not present an advantage over the straight fiber composite. However, for the buckling

load optimization problem with the VS composite laminate, at the optimum configuration buckling load has increased 22% compared to the straight fiber case.

From the results obtained from the optimizations performed on the same plate geometry without hole, it can be stated that the VS concept have an effect on the definition of the reference fiber path since the design variable T1 changes by slightly more than 4° compared to the open-hole case for the buckling load maximization case study. For the minimum compliance problem under in-plane bending load, it is seen that the existence of the hole has almost not effect on the optimum reference fiber path definition. However, at the optimum configuration total strain energy in the composite laminate without hole is lower than the open-hole composite laminate.

Based on the examples given in the present study, it can be concluded that depending on the problem type which includes geometry, loading etc. variable stiffness concept may or may not present advantage over the straight fiber composite configuration. Straight fiber configuration may be advantageous in terms of manufacturing cost and maturity of the analysis methods. Therefore, in the design of composite structures, it would be recommended to study the performance of both straight fiber and variable stiffness concepts in an optimization framework to decide on the most appropriate manufacturing method.

Further improvements of the present study can be on improving the surrogate model, such as implementing an iterative RBF Network approximation for the response or establishing an adaptive surrogate model with shrinking boundaries. Moreover, a study can be conducted on the effect of the hole on the optimum reference fiber path definition for different load cases to ascertain how effective the hole is on the optimum configuration.

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