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EVALUATION OF THE SEMI-EMPIRICAL EQUATIONS OF PRESSURE SWIRL ATOMIZERS

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ABSTRACT

Purpose of this study is to evaluate the accuracy of semi-empirical equations which are describing the operation and performance of pressure swirl atomizers. Correlations that are found in the literature are compared with the experimental data available in the literature. Through this study accuracies of the correlations can be graded on a wider range of relevant parameters and comparisons between the correlations can be done. After presenting the semi-empirical correlations error analyses is conducted on the sets of semi-empirical correlations with two separate methods. In the first analysis, equation sets that delivered the minimum error for each parameter are found and their calculation ranges are analyzed. In second analysis, results are filtered based on the error values and the calculation ranges and through this method an equation set that could be used for all of the parameters is found.

INTRODUCTION

Pressure swirl atomizers are widely used in the industry in applications such as food processing, fire suppression, gas turbines, engines etc. Main advantage of the pressure swirl atomizers is being simple and easiness of the manufacturing. In pressure swirl atomizers, fluid enters the swirl chamber via tangential ports. In the swirl chamber vortex motion is obtained and this motion causes pressure drop at the center of the atomizer which results in the formation of the air core. Convergent section accelerates the fluid at the entrance of the exit orifice. As a result liquid forms a high speed liquid film at the exit of the nozzle which spreads and breaks in to a conical spray. In all of the applications it is aimed to design an atomizer that could produce the droplets with predetermined sizes and amount, at the operating conditions. Many studies are conducted with the purpose of inspecting the internal flow and the spray formed by the pressure swirl atomizers in order to establish relations and analyze the effects of the geometrical and operating parameters. Analysis of the effects of the ambient gas properties, geometry of the atomizer, working liquid properties and pressure difference through the nozzle is done by [Rizk and Lefebvre, 1983]. Later, same authors have analyzed the internal flow characteristics of the pressure swirl atomizers and derived semi-empirical correlations for determining the film thickness at the exit orifice, flow number and discharge coefficient [Rizk and Lefebvre, 1985]. Additionally, authors have evaluated the accuracies of the equations with the experimental work reported in [Kutty et al., 1978]. An experimental study to inspect the film thickness at the exit orifice of the pressure atomizers and to evaluate the semi-empirical correlations for predicting the film thickness is conducted by [Lefebvre and

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Suyari, 1986]. Although results of this study shows a good agreement with experimental data, it should be considered that comparison has done with only 4 experimental points. Inspection of the effects of the physical properties of the working liquid such as viscosity and surface tension is done by [Lefebvre and Wang, 1987]. This study resulted in the derivation of a semi-empirical correlation for predicting the SMD values of the spray. Derivation of the semi-empirical correlation for predicting the spray angle and the discharge coefficient of the pressure swirl atomizers is done by [Babu et al., 1982]. Additionally, in their work authors have made comparisons with the experimental data of [Kutty et al., 1978] and [Kutty, 1974]. Results showed that developed correlation has a high accuracy and can be used for making reliable predictions. A correlation for determining the SMD values of the spray is conducted by the [Xiao and Huang, 2014]. Authors have also made experimental study to evaluate the accuracy of the derived equation. Results shows that developed equation can predict the SMD values within ±%20.

Literature survey has shown that the researchers determined the accuracies of their own semi-empirical correlations based on their own experimental studies. This kind of analysis hampers to making comparisons between the correlations and limits the range of the assessment. Additionally, in those works, measured values of the parameters are used. To give an example, when comparing the equations for calculating the film thickness with the measured values, [Lefebvre and Suyari, 1986] have used the measured values of the pressure difference instead of calculation it through semi-empirical correlations. Considering the fact that in the design process of pressure swirl atomizers pressure difference is usually not known, and in fact is one of the most important parameters that has to be calculated, it makes the analysis impractical. Therefore more detailed study is required to examine these correlations. In this paper a calculation method is presented in order to evaluate the semi-empirical equations as a set. In this method only required inputs are mass flow rate, liquid properties and dimensions of the atomizer and rest of the parameters will be calculated from the semi-empirical equations. Additionally, through the presented calculation method, optimization of the atomizer geometry can be done by calculating required pressure difference, internal flow properties and spray characteristics of the atomizer.

In the next section experimental data that is available in the literature is presented. Later, semi-empirical equations are presented, the developed calculation methodology is explained and finally accuracy analysis of the correlations and conclusion are given.

EXPERIMENTAL DATA

As mentioned earlier, the experimental data used for evaluating the accuracy of the equation sets is taken from the previous studies that is available in the literature. This data comes from the combination of 17 different works and considering the length of this study is not given here in detail. When all of the experimental data are combined, total of 427 points are acquired. In total, the validation data includes 226 measurements of spray angle, 113 measurements of film thickness, 30 measurements of exit velocity, 136 measurements of SMD and 43 measurements of breakup length. Fig. 1 shows the covered ranges of the experimental studies by comparing the minimum and maximum points for the bulk Reynolds number and bulk Weber number. It can be seen that experimental data used for the validation covers a wide range of data and the evaluation of the equations could be done for various operating and geometric conditions.

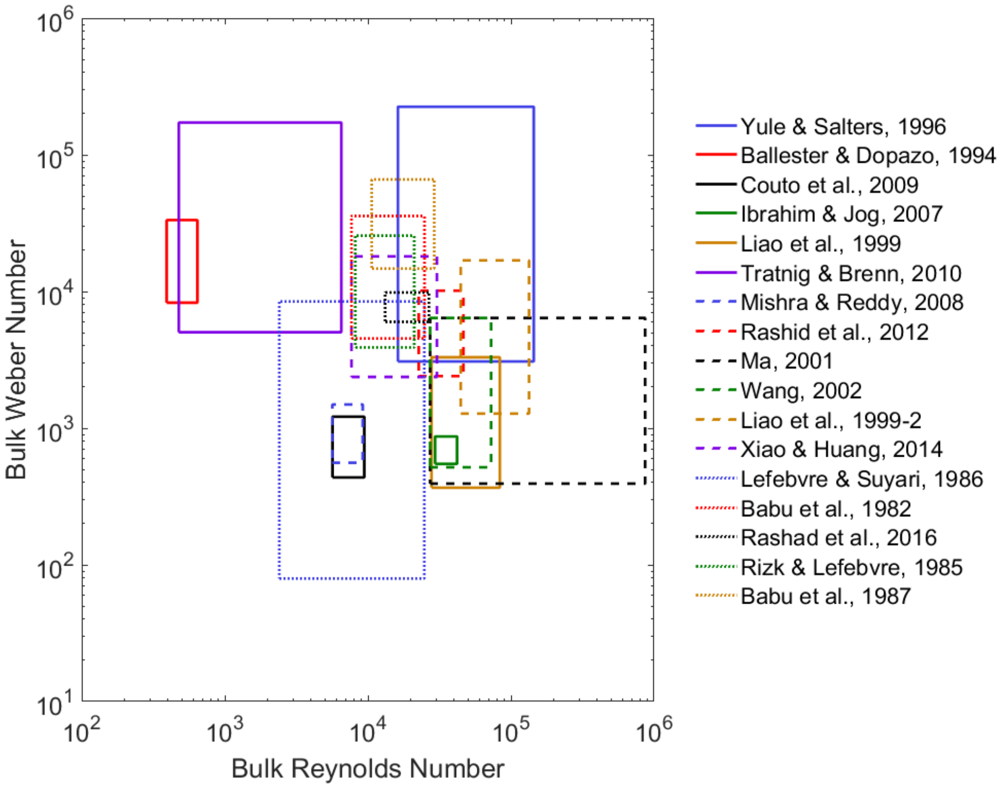


Figure 1: Reynolds and Weber Numbers of the Experimental Studies

SEMI-EMPIRICAL EQUATIONS

In this section the semi-empirical correlations that are found from the literature is given with explanation of the theories used derivation process.

Equations for Predicting the Discharge Coefficient

Discharge coefficient is defined as the ratio of the actual flow rate to the flow rate calculated by assuming the inviscid flow (Eq. 1). In other words it shows the effect of the viscous losses on the mass flow rate at a given pressure difference. As these losses increases the values of the discharge coefficient reduces. If C_D of an atomizer is known it can be used for calculating the required supply pressure in order to achieve desired mass flow rate. Equations for predicting the discharge coefficient that are found from the literature are given below.

$$C_D = \frac{\dot{m}_L}{A_0 \times (2 \times \rho_L \times \Delta P_L)^{0.5}} \tag{1}$$

$$C_D^2 = 0.0616 \times \frac{A_P}{D_s \times d_0} \times \frac{D_s}{d_0} \tag{Cd-1}$$

$$C_D^2 = 0.225 \times \frac{A_P}{D_s \times d_0} \tag{Cd-2}$$

$$C_D = 0.35 \times \left(\frac{A_P}{D_s \times d_0}\right)^{0.5} \times \left(\frac{D_s}{d_0}\right)^{0.25} \tag{Cd-3}$$

$$C_D = 0.45 \times \left(\frac{d_0 \times \rho_L \times U}{\mu_L}\right)^{-0.02} \times \left(\frac{l_0}{d_0}\right)^{-0.03} \times \left(\frac{A_P}{D_s \times d_0}\right)^{0.52} \times \left(\frac{D_s}{d_0}\right)^{0.23} \times \left(\frac{L_s}{D_s}\right)^{0.05} \tag{Cd-4}$$

$$C_D = 1.03 \times \left(\frac{D_p}{d_0}\right)^{0.616} \times \left(\frac{A_P}{D_s \times d_0}\right)^{-0.021} \times \left(\frac{D_s}{d_0}\right)^{0.203} \times \left(\frac{L_s}{d_0}\right)^{-0.314} \tag{Cd-5}$$

$$C_D = 0.466 \times \left(\frac{d_0 \times \rho_L \times U_0}{\mu_L}\right)^{-0.027} \times \left(\frac{l_0}{d_0}\right)^{0.229} \times \left(\frac{A_P}{D_s \times d_0}\right)^{0.517} \times \left(\frac{D_s}{d_0}\right)^{0.187} \times \left(\frac{L_s}{D_s}\right)^{0.091} \quad \text{(Cd-6)}$$

$$C_D = 1 - \frac{2}{\pi} \times tan^{-1}\left(2.13 \times \left\{\frac{(4/\pi) \times K + 1.2}{[(4/\pi) \times K + 1]^2 - 1}\right\} \times e^{-0.12 \times \left(D_S/\sqrt{A_P}\right)}\right) \quad \text{(Cd-7)}$$

$$C_D = 1.323 \times 10^{-3} \times \left(\frac{A_P}{D_s \times d_0}\right)^{0.29} \times {d_0}^{-0.82} \times {\Delta P_L}^{0.03} \quad \text{(Cd-8)}$$

$$C_D = 1.335 \times 10^{-3} \times \left(\frac{A_P}{D_s \times d_0}\right)^{0.29} \times {d_0}^{-0.82} \times {\Delta P_L}^{0.07} \quad \text{(Cd-9)}$$

Eq. Cd-1 is derived by [Carlisle, 1955] by experimentally proving that the ratio of the diameter of the swirl chamber to the diameter of the exit orifice has effect on the discharge coefficient. Eq. Cd-2 is derived by [Taylor, 1950] by analyzing the internal flow of the pressure swirl atomizer with the assumption of inviscid flow. Eq. Cd-3 is the result of the work done by [Rizk and Lefebvre, 1985] on the analysis of the experimental work of the [Kutty et al., 1978]. Eq. Cd-4 is obtained by [Jones, 1982] through the experimental investigation done to analyze the effects of the liquid properties, geometrical parameters and operating conditions on the large scale pressure swirl atomizers. Eq. Cd-5 is also obtained by the experimental work done on the large scale atomizers [Jain et al., 2014]. Authors have stated that their correlation predicts the discharge coefficient within 8%. Eq. Cd-6 is obtained by taking Eq. Cd-4 as basis and adjusting the exponents and the equation constant [Benjamin et al., 1998]. According to the authors, the correlation coefficient is 98.7 %. Eq. Cd-7 is obtained by curve fitting done on the measured values of the discharge coefficient at different atomizers [Tanasawa and Kobayasi, 1955]. Eq. Cd-8 and Eq. Cd-9 are the results of the experimental research done by [Ballester and Dopazo, 1994]. Both of the equations are obtained by multiple regression techniques. Eq. Cd-8 is obtained from all of the experimental data whereas Eq. Cd-9 is obtained from the experimental data corresponding to atomizers having discharge orifice diameters smaller than 0.8 mm.

Equations for Predicting the Flow Number

Flow number is basically the dimensional form of the discharge coefficient. Both parameters are used to determine the required pressure difference to achieve the given flow rate (Eq. 2). Equations for predicting the flow number that are found from the literature are given below:

$$\mathrm{FN} = \frac{\dot{m}_L}{(\rho_L \times \Delta P_L)^{0.5}} \quad (2)$$

$$FN = 0.0308 \times \frac{A_P^{0.5} \times d_0}{D_S^{0.45}} \quad \text{(FN-1)}$$

$$FN = 0.395 \times \frac{A_P^{0.5} \times d_0^{1.25}}{D_S^{0.25}} \quad \text{(FN-2)}$$

$$FN = 0.371 \times \frac{A_P^{0.5} \times d_0^{1.25}}{D_S^{0.25}} \quad \text{(FN-3)}$$

Eq. FN-1 and Eq. FN-2 are based on the experimental data of the [Kutty et al., 1978] and derived by [Rizk and Lefebvre, 1985]. In Eq. FN-1 unit of the area of inlet ports is given as meter square and the unit of the diameter of exit orifice and the diameter of the swirl chamber is meter. Eq. FN-2 is the dimensionally correct form of the Eq. FN-1 but according to authors it has a lower accuracy. It should also be noted that these equations are generated for atomizers having three inlets and the coefficients have to be altered for geometries having more or less than three inlets. Eq. FN-3 is acquired by [Benjamin et al., 1998]. Authors have used the Eq. FN-2 as a basis and altered the coefficient based on their experimental work. It should be noted that in this work atomizers having four tangential inlets are used.

Equations for Predicting the Film Thickness

In pressure swirl atomizers the formation of air core occurs and as a result fluid forms a thin film inside the exit orifice. It has been acknowledged that the thickness of this film has a strong influence on the droplet sizes of the atomizer [Lefebvre, 1983]. Additionally, size of the air core and the thickness of the film t is related with each other with the ratio X (Eq. 3). The developed semi-empirical correlations for predicting the film thickness are mainly based on analysis of the internal flow of the pressure swirl atomizers with the assumption of inviscid flow or analysis done on the experimental data. Equations for predicting the film thickness are given below:

$$X = \frac{(d_0 - 2 \times t)^2}{d_0^2} \tag{3}$$

$$X = 1 - 0.03265 \times \left(\frac{FN \times \sqrt{\rho_L}}{K_v \times d_0^2 \times \cos\theta}\right) \tag{4}$$

$$C_D = 1.17 \times \left[\frac{(1-X)^3}{1+X}\right]^{0.5} \tag{5}$$

$$\frac{(1-X)^3}{1+X} = 0.09 \times \frac{A_P}{D_s \times d_0} \times \left(\frac{D_s}{d_0}\right)^{0.5} \tag{6}$$

$$t = \frac{0.00805 \times \sqrt{\rho_L} \times FN}{d_0 \times \cos\theta} \tag{7}$$

$$t = 2.7 \times \left[\frac{d_0 \times FN \times \mu_L}{\rho_L^{0.5} \times \Delta P^{0.5}}\right]^{0.25} \tag{8}$$

$$t = 3.66 \times \left[\frac{d_0 \times FN \times \mu_L}{\rho_L^{0.5} \times \Delta P^{0.5}}\right]^{0.25} \tag{9}$$

$$\frac{(1-X)^3}{X^2} \times \frac{\pi}{32} = \left(\frac{A_P}{D_s \times d_0}\right)^2 \tag{10}$$

$$t^2 = \frac{1560 \times FN \times \mu_L}{\rho_L^{0.5} \times d_0 \times \Delta P^{0.5}} \times \frac{1+X}{(1-X)^2} \tag{11}$$

$$C_D = K_v \times \left[\frac{(1-X)^3}{1+X}\right]^{0.5} \tag{12}$$

$$t = \frac{d_0 - \left[d_0^2 - \left(4 \times \dot{m}_L / (\pi \times \rho_L \times U_0)\right)\right]^{0.5}}{2} \tag{13}$$

$$t = 1.44 \times d_0 \times \left[\frac{\dot{m}_L \times \mu_L}{\rho_L \times \Delta P_L \times d_0^3}\right]^{0.25} \times \left(\frac{l_0}{d_0}\right)^{0.6} \tag{14}$$

Eq. 4 is obtained by [Rizk and Lefebvre, 1987] by reshaping the expression (Eq. 7) derived by the [Simmons and Harding, 1980] with the introduction of the velocity coefficient. According to the authors introducing the velocity coefficient allows to consider the pressure losses that occurs in various parts of the atomizer. Eq. 5 is the obtained by [Giffen and Muraszew, 1953] to calculate the discharge coefficient from the measured values of the film thickness. However, in this work it will be used for calculating the film thickness. A modified version of the Eq. 5 is obtained by [Rizk and Lefebvre, 1985], Eq. 12, in which instead of the equation constant they have introduced the velocity coefficient. Eq. 6 is the result of the theoretical study done by the [Lefebvre and Suyari, 1986]. In this work authors have assumed that discharge coefficient will remained constant at Reynolds number higher than 3000. Combining this assumption and the Eq. Cd-3 and Eq. 5 resulted in the Eq. 6. As mentioned above Eq. 7 is obtained by the

[Simmons and Harding, 1980]. Eq. 8 is derived by the [Rizk and Lefebvre, 1985] through combining the experimental data and theoretical investigation on the internal flow characteristics of pressure swirl atomizers including viscous effects. Later, same authors have simplified this equation in a way that it still represents all the fundamental features and acquired Eq. 9. In the following work, calculated film thickness values from this equation is compared with the measured values. It is seen that the constant 3.66 is too large and a better fit is achieved with the constant 2.7. By altering the constant equality shown in Eq. 10 is obtained [Lefebvre and Suyari, 1986]. Analysis of [Giffen and Muraszew, 1953] on the internal flow of pressure swirl atomizers with the assumption of non-viscous fluid flow has led to Eq. 11. Eq. 13 is acquired by modifying Eq. 11 through simplified analytical treatment of internal flow characteristics of pressure-swirl atomizers. Eq. 14 is obtained by the [Kim et al., 2010] by reshaping the Eq. 10, such that it includes the exit orifice length and adjusting the constant according to the experimental study.

Equations for Predicting the Spray Angle

The term spray angle refers to the theoretical coverage area of the spray and it is a key performance parameter. Equations for predicting the spray angle that are found from the literature survey are given below:

$$2 \times \theta_m = 6 \times K^{-0.15} \times \left(\frac{\Delta P_L \times d_0^2 \times \rho_L}{\mu_L^2}\right)^{0.11} \tag{Ang-1}$$

$$cos\theta = \sqrt{\frac{12 \times \dot{m}_L \times \mu_L \times (A/_B)}{\pi \times \rho_L \times \Delta P_L \times t^2 \times (1 - X)}} \quad A/_B = 400 \tag{Ang-2}$$

$$cos\,\theta = \sqrt{\frac{1 - X}{1 + X}} \tag{Ang-3}$$

$$sin\theta = \frac{(\pi/_2) \times C_D}{K \times (1 + \sqrt{X})} \tag{Ang-4}$$

$$\theta_m = 9.75 \times K^{-0.237} \times \left(\frac{\Delta P_L \times d_0^2 \times \rho_L}{\mu_L^2}\right)^{0.0647} \tag{Ang-5}$$

$$\theta = 90 - tan^{-1}\left[\frac{4}{\pi} \times K \times \left(1.37 + 26.9 \times e^{-11.1 \times (\sqrt{A_i}/D_s)}\right)\right] \tag{Ang-6}$$

$$\theta = 8.078 \times K^{-0.39} \times d_0^{1.13} \times \mu_L^{-0.9} \times \Delta P_L^{0.39} \tag{Ang-7}$$

$$\theta = 0.10985 \times K^{-0.39} \times d_0^{0.63} \times \mu_L^{-0.91} \times \Delta P_L^{0.42} \tag{Ang-8}$$

$$\theta = tan^{-1}\left[\frac{\pi}{4} \times (1 - X) \times \frac{K_\theta}{B}\right] \tag{Ang-9}$$

Eq. Ang-1 is obtained by [Rizk and Lefebvre, 1987] and is based on the experimental research and the assumption of the atomizer constant, K, is the main parameter that effects the spray angle. The subscript m shows that it is the mean spray angle measured at the near nozzle region. Eq. Ang-2 and Eq. Ang-3 are derived in the work of the [Rizk and Lefebvre, 1985] which is based on the theoretical analysis of the internal flow of the pressure swirl atomizers. The constant in the Eq. Ang-2 is obtained from the experimental data. Eq. Ang-3 is the result of analysis done on the exit velocity of the liquid and using the Eq. 5. Eq. Ang-4 is obtained by analyzing the non-viscous flow inside the atomizer [Giffen and Muraszew, 1953]. Eq. Ang-5 is derived by from the experimental work done on the large scale atomizers working at low pressure values and adjusting Eq. Ang-1 according to the this data [Benjamin et al., 1998]. Similar to the Eq. Ang-1 this equation is derived for the mean spray angle values. Eq. Ang-6 is obtained from the theoretical work of the [Tanasawa and Kobayasi, 1955]. Eq. Ang-7 and

Eq. Ang-8 is obtained by [Ballester and Dopazo, 1994] through the experimental research. Eq. Ang-7 is obtained by the analysis done on the all of the experimental data points and Eq. Ang-8 is obtained by the analysis done on the atomizers having exit orifice diameters smaller than 0.8 mm. Eq. Ang-9 is obtained by the combination of theoretical and experimental analysis done on the vortex motion created inside the swirl chamber [Babu et al., 1987]. In this equation K_θ is the correlation factor (Eq. 15 & Eq. 16) and the B is the nozzle parameter (Eq.17).

For $\Delta P_L \geq 2.76$ MPa

$$K_\theta = 0.00812 \times \frac{A_p^{0.034048}}{A_s^{0.24579} \times A_0^{0.17548}} \tag{15}$$

For $\Delta P_L < 2.76$ MPa

$$K_\theta = 0.0831 \times \frac{A_p^{0.34873}}{A_s^{0.32742} \times A_0^{0.26326}} \tag{16}$$

$$B = K \times \left(\frac{D_s}{d_0}\right)^{1-n} \tag{17}$$

Where, for $\Delta P_L \geq 2.76$ MPa

$$n = 17.57 \times \frac{A_0^{0.1396} \times A_p^{0.2336}}{A_s^{0.1775}} \tag{18}$$

For $\Delta P_L < 2.76$ MPa

$$n = 28 \times \frac{A_0^{0.14176} \times A_p^{0.27033}}{A_s^{0.17634}} \tag{19}$$

Equations for Predicting the Velocity Coefficient

Velocity coefficient is defined as the ratio of the actual exit velocity to the theoretical velocity and it represents the effects of the pressure losses. Equations for velocity coefficient are given below:

$$K_v = 0.00367 \times K^{0.29} \times \left(\frac{\Delta P_L \times \rho_L}{\mu_L^2}\right)^{0.2} \tag{Kv-1}$$

$$K_v = \frac{C_D}{(1 - X) \times \cos \theta_m} \tag{Kv-2}$$

$$K_v = 0.692 \times K^{0.22} \times \left(\frac{\Delta P_L \times \rho_L}{\mu_L}\right)^{0.0092} \tag{Kv-3}$$

Eq. Kv-1 is obtained from the analyzing the results of the experimental work of the [Kutty et al., 1978] by the [Rizk and Lefebvre, 1987]. According to the researchers this equation shows satisfactory results for wide range of atomizers. Eq. Kv-2 is obtained through the theoretical analysis done on the internal flow of the atomizers [Rizk and Lefebvre, 1985]. Eq. Kv-3 is derived by [Benjamin et al., 1998] by taking Eq. Kv-1 as a basis and changing the coefficients according to their experimental data on the large scale pressure swirl atomizers. Authors states that this equation has correlation coefficient of 83.7 %.

Equations for Predicting the Exit Velocity

Exit velocity is an important parameter for determining the spray properties. It is known that atomization is achieved by the interaction of liquid sheet with the ambient. As the relative velocity between the liquid sheet and the ambient increases, disintegration of the sheet occurs rapidly. In addition to this, the penetration length of the droplets are determined by the exit velocity. Equations for predicting the exit velocity are given below.

$$U = \sqrt{\frac{2 \times \Delta P_L \times C_D}{\rho_L}} \tag{U-1}$$

$$U = \frac{\Delta P_L \times t^2 \times \cos\theta}{3 \times \mu_L \times d_0 \times \left({A}/{B} \right)} \quad {A}/{B} = 400 \tag{U-2}$$

$$U = K_v \times \left[\frac{2 \times \Delta P_L}{\rho_L} \right]^{0.5} \tag{U-3}$$

Eq. U-1 is derived from the definition of the discharge coefficient. It should be noted that in this equation formation of the air core is neglected and liquid flow area is taken as the area of the exit orifice. Eq. U-2 is derived by [Rizk and Lefebvre, 1985] through the analysis of the inviscid internal flow inside pressure swirl atomizers and the equation constant is determined from the experimental evaluation. Eq. U-3 is derived from the definition of the velocity coefficient. In other words it is the analytical equation for calculating the velocity coefficient.

Equations for Predicting the SMD

When liquid exits the atomizer, it disintegrates in to smaller droplets due to hydrodynamic and aerodynamic processes. The resultant spray contains droplets having wide range of diameters. Sauter Mean Diameter (SMD) of the spray is the most widely used quantity to characterize the size distribution of the spray. Equations for calculating the SMD is given below.

$$SMD = 4.52 \times \left(\frac{\sigma \times \mu_l^2}{\rho_a \times \Delta P_L^2} \right)^{0.25} \times (t \times \cos\theta)^{0.25} + 0.39 \times \left(\frac{\sigma \times \rho_l}{\rho_a \times \Delta P_L} \right)^{0.25} \times (t \times \cos\theta)^{0.75} \tag{SMD-1}$$

$$SMD = 1.072 \times \left(\frac{\sigma \times \mu_l^2}{\rho_a \times \Delta P_L^2} \right)^{0.054} \times (t \times \cos\theta)^{0.838} + 0.2146 \times \left(\frac{\sigma \times \rho_l}{\rho_a \times \Delta P_L} \right)^{0.0523} \times (t \times \cos\theta)^{0.9477} \tag{SMD-2}$$

$$SMD = 2.25 \times \sigma^{0.25} \times \mu_L^{0.25} \times \dot{m}_L^{\ 0.25} \times \Delta P_L^{-0.5} \times \rho_a^{-0.25} \tag{SMD-3}$$

$$SMD = 7.3 \times \sigma^{0.6} \times \mu_L^{0.2} \times \dot{m}_L^{\ 0.25} \times \Delta P_L^{-0.4} \times \rho_L^{-0.2} \tag{SMD-4}$$

$$SMD = 4.4 \times \sigma^{0.6} \times \mu_L^{0.16} \times \dot{m}_L^{\ 0.22} \times \Delta P_L^{-0.43} \times \rho_L^{-0.16} \tag{SMD-5}$$

$$SMD = 0.436 \times \mu_L^{0.55} \times \Delta P_L^{-0.74} \times d_0^{-0.05} \times A_p^{-0.24} \tag{SMD-6}$$

$$SMD = 1.89 \times 0.9615 \times \cos\theta \times \left(\frac{t^4 \times \sigma^2}{U^4 \times \rho_a \times \rho_L} \right)^{1/6} \times \left[1 + 2.6 \times \mu_L \times \cos\theta \times \left(\frac{t^2 \times \rho_a^4 \times U^7}{72 \times \rho_L^2 \times \sigma^5} \right)^{1/3} \right]^{0.2} \tag{SMD-7}$$

$$SMD = 22 \times \left[\frac{K \times d_0 \times \left(1 + \sqrt{X}\right)}{1 - X} \right]^{0.5} \times \left[\frac{t^{1.17} \times (d_0 - t)^{0.67}}{\dot{m}_L^{\ 0.67}} \right] \times \left(\frac{\sigma^2}{\rho_a \times \rho_L} \right)^{1/6} \tag{SMD-8}$$

$$SMD = 0.952 \times d_0 \times K^{0.871} \times \left(\frac{L_s}{d_0} \right)^{1.834} \times Re^{-0.31} \tag{SMD-9}$$

Eq. SMD-1 is derived by the [Lefebvre and Wang, 1987] with the assumption of the atomization process occurs in two sub-stages. According to the authors, first stage of the atomization process is highly dependent on the Reynolds number and Weber number. Second stage of the atomization process is mainly dependent on the surface tension and the Reynolds number is not related. Combining these two effects resulted in the Eq. SMD-1. Eq. SMD-2 is derived by [Benjamin et al., 1998] by adjusting the coefficients and the exponents in Eq. SMD-1 according to their experimental data. Eq. SMD-3 is derived by [Lefebvre, 1983] by analyzing the flow inside the exit orifice and its effects on the disintegration of the liquid sheet. The constant is found from the experimental data. Eq. SMD-4 is derived by [Radcliffe, 1960] and it is one of the earliest equations. It is obtained by interpolating the theoretical analysis on the experimental research and additionally is used to produce other equations by other researchers. Eq. SMD-5 is obtained by [Jasuja, 1979] and Eq. SMD-6 is obtained by [Ballester and Cesar, 1996]. Both of these equations are obtained by taking Eq. SMD-4 as basis and altering the exponentials and the equation coefficient according to the conducted experimental research. Eq. SMD-7 is obtained by [Cuoto, 1997] by extending the theory of the [Dombrowski and Johns, 1963] to the pressure swirl atomizers. In this equation the unit of the surface tension is dyne/cm, unit of the density is g/cm^3, unit of the viscosity is centipoise and unit of the exit velocity is cm/s. Eq. SMD-8 is derived by the [Xiao and Huang, 2014] based on the theoretical and experimental analysis done on the break-up process. The first term represents the effect of the geometrical parameters, second term represents effects of the film thickness and the flow rate and the last term represents the effects of the liquid and ambient properties. The constant 22 represents the size of the ligament that is generated from the break-up. This constant is determined from the evaluation of the experimental data and is suggested to be a universal constant. Eq. SMD-9 is obtained through dimensional analysis on the geometrical and operating parameters that effects the droplet diameters and applying multiple regression methods to the experimental data [Jain et al., 2014].

Equations for Predicting the Breakup Length

Break up length is defined as the length of the continuous sheet measured from the nozzle exit to the point of the breakup. It is known that this distance depends upon the atomizer geometry, properties of the liquid and the ambient gas and the relative velocity between the liquid and the ambient gas [Lefebvre, 1989] . Equations for predicting the breakup length is given below.

$$L_b = 100.8 \times d_0 \times W_e^{-0.215} \times \left(\frac{l_0}{d_0}\right)^{0.0424} \times \left(\frac{A_p}{A_0}\right)^{0.395} \quad \text{(Lb-1)}$$

$$L_b = 15.8 \times d_0 \times \left(\frac{\rho_L}{\rho_a}\right)^{0.5} \quad \text{(Lb-2)}$$

$$L_b = 7 \times d_0 \times \left(\frac{\rho_L}{\rho_a}\right)^{0.5} \quad \text{(Lb-3)}$$

$$L_b = 2.59 \times 10^5 \times d_0 \times We_g^{-0.46} \times Re_L^{-0.39} \times \left(\frac{\rho_L}{\rho_a}\right)^{0.2} \quad \text{(Lb-4)}$$

$$L_b = 2.65 \times 10^3 \times d_0 \times We_g^{-0.1} \times Re_L^{-0.3} \times \left(\frac{\rho_L}{\rho_a}\right)^{0.08} \quad \text{(Lb-5)}$$

Eq. Lb-1 is derived by [Kim et al., 2003] through the experimental research and assuming that breakup length is a function of the Weber number and the geometric parameters. It should be noted that this equation does not include the properties of the ambient air which is known to effect the breakup length. Eq. Lb-2 is derived by the [Arai et al., 1987] and is based on the experimental data. Similarly, Eq. Lb-3 is obtained by experimental research at similar conditions [Chehroudi et al., 1985]. Eq. Lb-4 is proposed by [Yule and Filipovic, 1992] through the theoretical investigation and the analysis of the Eq. Lb-2 and Eq. Lb-3. Authors findings have shown that in Eq. Lb-2 and Eq. Lb-3 the effect of the ambient gas density is too high and the effects of the Weber and Reynolds numbers must be considered as well. In the following work, when the comparison between the measured and calculated values of breakup length is done, it is seen that the equation constant and the exponential values need to be altered. As a result Eq. Lb-5 is obtained [Yule and Salters, 1995].

METHODOLOGY

In this section the method used for the assessment of the accuracy of the correlations is explained. In the developed calculation method each parameter is calculated from the semi-empirical equations and these values are used for calculating the parameters at the next step. Flow chart of the calculation method is given in Fig. 2. It should be mentioned that this chart only shows the preferred calculation order and does not fully represents the dependencies of the equations. Additionally, from the equations for predicting the film thickness values it can be seen that some of the correlations includes terms for spray angle and velocity coefficient which are planned to be calculated in the after the calculation of the film thickness. For that correlations, the equations of spray angle and/or equations of velocity coefficient are introduced into them. Resultant correlations comprise more than one equation and this leads to a larger number of equations for film thickness. Table 1 shows all of the film thickness correlations and their equation numbers used in computations.

The calculation is done in 8 steps and in each step, a function is created such that it includes all of the equations for each parameter and each equation is assigned with a binary coefficient i.e., coefficient that can be either 1 or 0. When one of the coefficient at each group becomes 1 the rest of the coefficients in that group will be 0 and in each iteration the next coefficient will become 1 and other will become 0. When it is considered that there is 9 equations for calculating the discharge coefficient, 3 equations for calculating flow number, in other words there is 12 equations for calculating the required pressure difference, 41 equations for calculating the film thickness, 9 equations for calculating the spray cone angle, 3 equations for calculating the velocity coefficient, 3 equations for calculating the exit velocity, 9 equations for calculating SMD and 5 equations for calculating breakup length and number of iterations for each group is equal to the number of the equations in that group, total of 1,793,340 iterations will be done in total and this is also equal to the number of equation sets whose accuracies are evaluated.

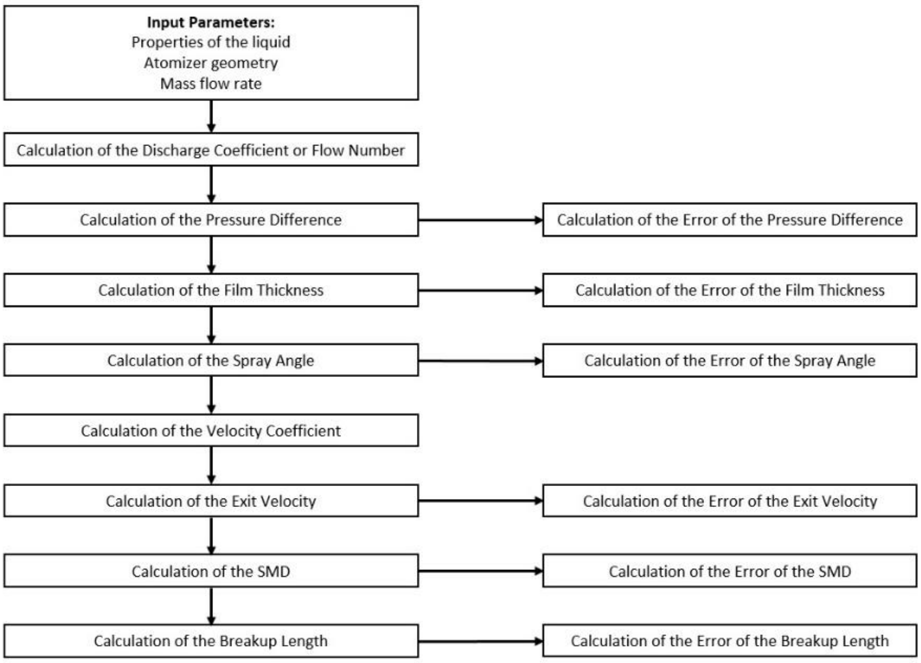


Figure 2: Flowchart of the Calculation Method

Table 1: Correlations for Calculating the Film Thickness

Eq.	Description	Eq.	Description	Eq.	Description
t-1	Obtained by introducing Eq. Ang-1 and Eq. Kv-1 into Eq. 4	t-15	Obtained by introducing Eq. Ang-8 and Eq. Kv-1 into Eq. 4	t-29	Eq. 10
t-2	Obtained by introducing Eq. Ang-1 and Eq. Kv-3 into Eq. 4	t-16	Obtained by introducing Eq. Ang-8 and Eq. Kv-3 into Eq. 4	t-30	Eq. 11
t-3	Obtained by introducing Eq. Ang-2 and Eq. Kv-1 into Eq. 4	t-17	Eq. 5	t-31	Obtained by introducing Eq. Kv-1 into Eq. 12
t-4	Obtained by introducing Eq. Ang-2 and Eq. Kv-3 into Eq. 4	t-18	Eq. 6	t-32	Obtained by introducing Eq. Ang-1 and Eq. Kv-2 into Eq. 12
t-5	Obtained by introducing Eq. Ang-3 and Eq. Kv-1 into Eq. 4	t-19	Obtained by introducing Eq. Ang-1 into Eq. 7	t-33	Obtained by introducing Eq. Ang-2 and Eq. Kv-2 into Eq. 12
t-6	Obtained by introducing Eq. Ang-3 and Eq. Kv-3 into Eq. 4	t-20	Obtained by introducing Eq. Ang-2 into Eq. 7	t-34	Obtained by introducing Eq. Ang-4 and Eq. Kv-2 into Eq. 12
t-7	Obtained by introducing Eq. Ang-4 and Eq. Kv-1 into Eq. 4	t-21	Obtained by introducing Eq. Ang-3 into Eq. 7	t-35	Obtained by introducing Eq. Ang-5 and Eq. Kv-2 into Eq. 12
t-8	Obtained by introducing Eq. Ang-4 and Eq. Kv-3 into Eq. 4	t-22	Obtained by introducing Eq. Ang-4 into Eq. 7	t-36	Obtained by introducing Eq. Ang-6 and Eq. Kv-2 into Eq. 12
t-9	Obtained by introducing Eq. Ang-5 and Eq. Kv-1 into Eq. 4	t-23	Obtained by introducing Eq. Ang-5 into Eq. 7	t-37	Obtained by introducing Eq. Ang-7 and Eq. Kv-2 into Eq. 12
t-10	Obtained by introducing Eq. Ang-5 and Eq. Kv-3 into Eq. 4	t-24	Obtained by introducing Eq. Ang-6 into Eq. 7	t-38	Obtained by introducing Eq. Ang-8 and Eq. Kv-2 into Eq. 12
t-11	Obtained by introducing Eq. Ang-6 and Eq. Kv-1 into Eq. 4	t-25	Obtained by introducing Eq. Ang-7 into Eq. 7	t-39	Obtained by introducing Eq. Kv-3 into Eq. 12
t-12	Obtained by introducing Eq. Ang-6 and Eq. Kv-3 into Eq. 4	t-26	Obtained by introducing Eq. Ang-8 into Eq. 7	t-40	Obtained by introducing Eq. U-1 into Eq. 13
t-13	Obtained by introducing Eq. Ang-7 and Eq. Kv-1 into Eq. 4	t-27	Eq. 8	t-41	Eq. 14
t-14	Obtained by introducing Eq. Ang-7 and Eq. Kv-3 into Eq. 4	t-28	Eq. 9		

The evaluation of the equation sets is done by calculating their accuracy and their calculation range. Accuracy of the equation sets is the most important criteria in the selection process. Calculation range of the equation set refers to the number of calculated data points for each parameter, divided by the total number of experimental data for that parameter. Since most of the equations are non-linear, it is foreseen that not all of them could be used for the full data range. This parameter is taken into considerations since in the design process the exact working range of the atomizers are not known beforehand and therefore having an equation set that can be used at a wide range is have prime importance.

RESULTS AND ANALYSIS

In this section the results of the calculations are presented. For each equation set, absolute relative error at each experimental data point is calculated and then the RMS of those errors values are calculated. First the presentation of the equation sets with the minimum errors for each parameters is done.

Analysis of Minimum Error and Calculation Range

Errors of the discharge coefficient and flow number correlations are given below. In the figures, y-axis represents the error values, the x-axis represents the number of the calculations and the color bar represents the calculation range. Considering there are 9 equations for predicting the discharge coefficient and 3 equations for predicting the flow number there are only 12 separate error values. The first 9 error values represents the errors of Eq. Cd-1 to Eq. Cd-9 and the last 3 error values represents the errors of Eq. FN-1 to Eq. FN-3. Analysis of the results yields that Eq. Cd-1 has the lowest error value, which is 0.2271. It should be noted that Eq. Cd-4 and Eq. Cd-6 are the only correlations that accounts for the effect of the Reynolds number on the discharge coefficient and those correlations produced higher error values than most of the equations, suggesting that the discharge coefficient can be considered to be independent of the Reynolds number or the proposed functions does not reflect the effect of the Reynolds numbers accurately.

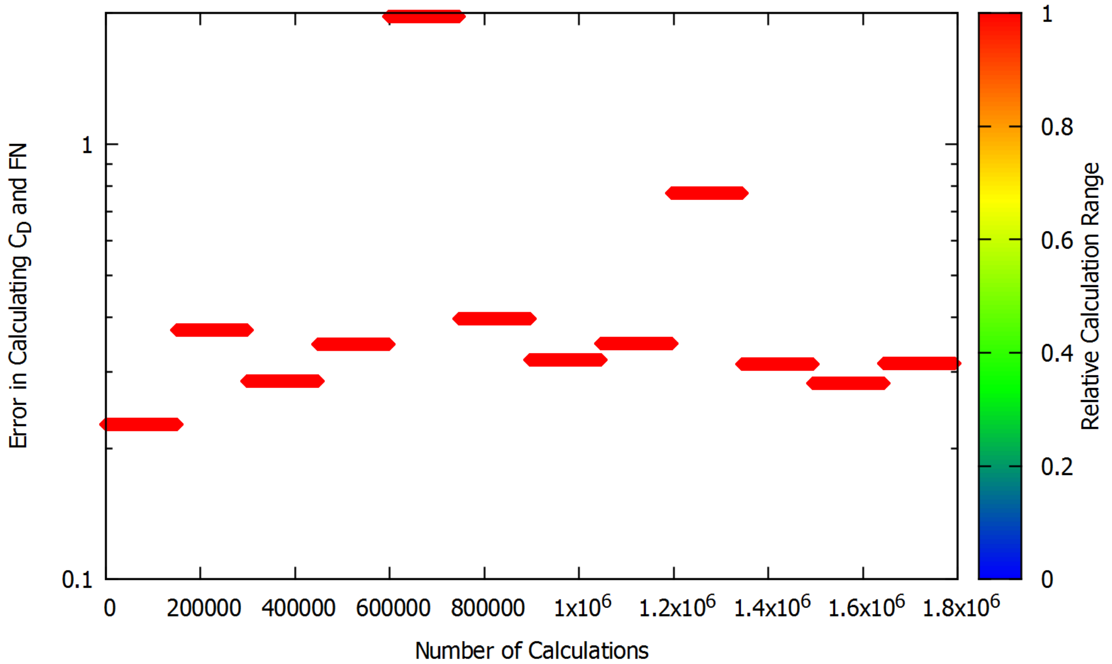


Figure 3: Error Values in Predicting the Discharge Coefficient and Flow Number

As mentioned earlier the prime importance of calculating the discharge coefficient or flow number is that, these parameters allows to relate the geometrical parameters with the required pressure difference for given mass flow rate. Error values in the calculation of the pressure difference with using the discharge coefficient and flow number equations are given in Fig. 4. It can be seen that the error values are higher than the that of the previous calculation. Again, Eq. Cd-1 delivers the minimum error and it is 0.7807.

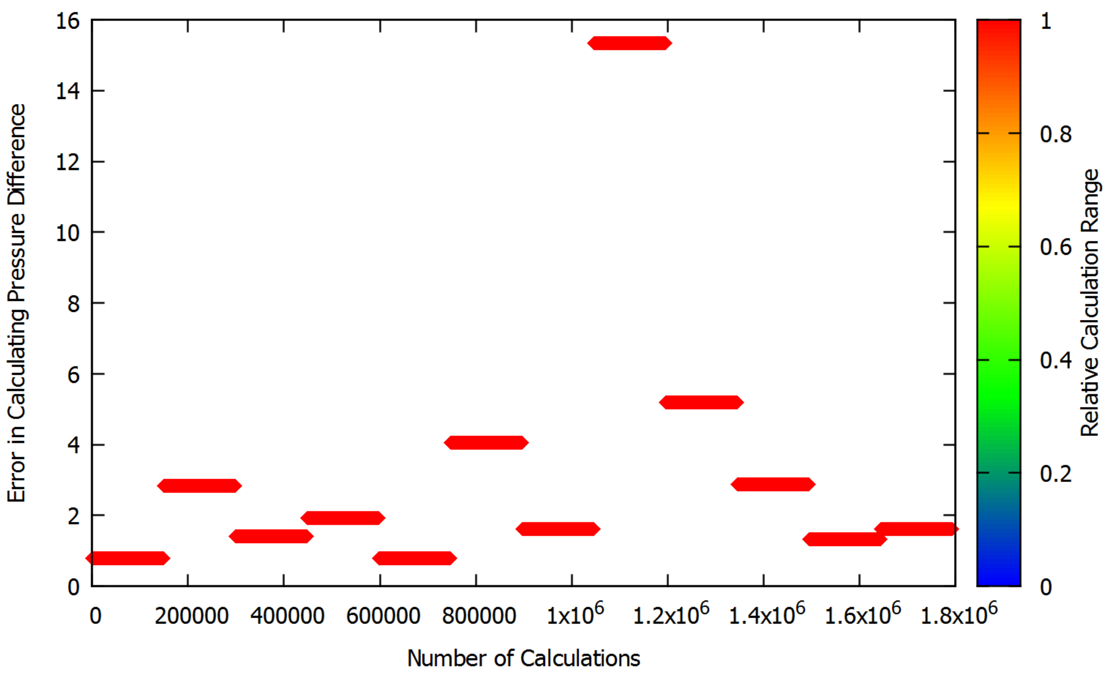


Figure 4: Error Values in Predicting the Pressure Difference

Error values of the film thickness calculation are given Fig. 5. Minimum error for this parameter is found to be 0.215 an it is occurred when Eq. Cd-3 is used for predicting the discharge coefficient and Eq. t-40 is used for predicting the film thickness. Additionally, this equation set can be used for the all of the experimental data range. As an alternative, combination of Eq. Cd-1 and Eq. t-40 can be used. This equation set has the error value of 0.216 and can also be used for the whole data range.

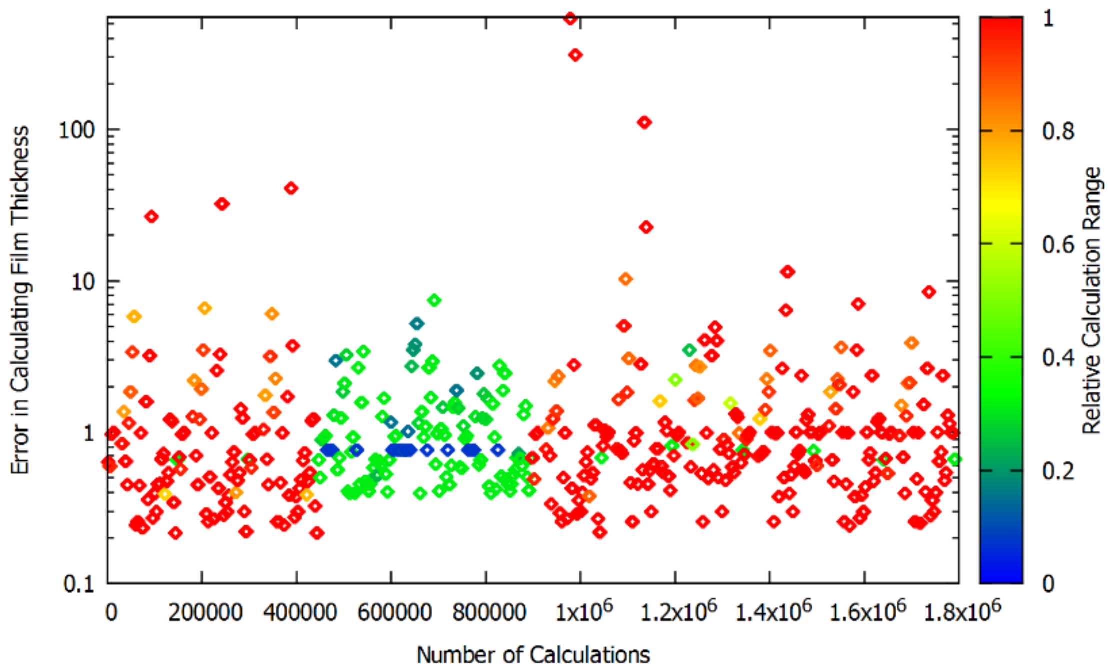


Figure 5: Error Values in Predicting the Film Thickness

Error values of the spray angle prediction is given in Fig. 6. Results shows many repeated error values and it is due to the fact that some of the correlations are only governed by the geometrical parameters and does not account for the operational parameters. As a result altering the equation that is used to calculate the pressure difference does not affect the error value. Additionally, the gaps in the figure is due to the complex numbers that some of the correlations produces. Minimum error value in this calculation is found to be 0.4585 and it is occurred when Eq. Ang-6 is used.

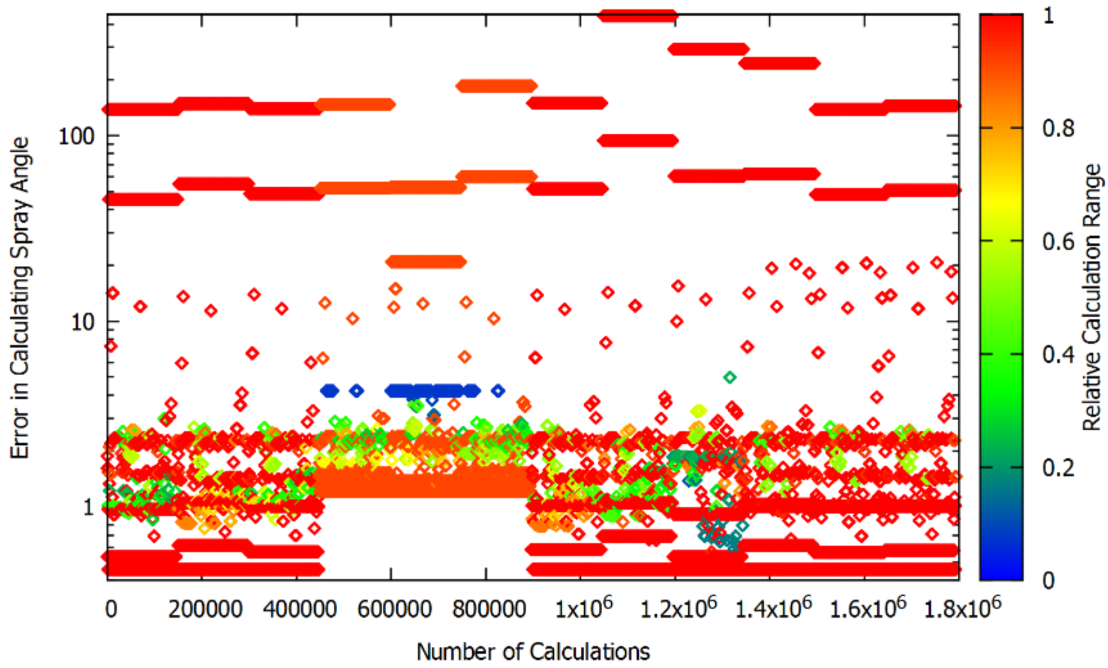


Figure 6: Error Values in Predicting the Spray Angle

Error values of the exit velocity prediction is given in Fig. 7. Lowest error value in this calculation is found to be 0.073 and it is occurred when Eq. Cd-9 is used for predicting the pressure difference, Eq. t-40 is used for predicting the film thickness, Eq. Ang-5 is used for predicting the spray angle, Eq. Kv-2 is used for predicting the velocity coefficient and Eq. U-3 is used for predicting the exit velocity. As an alternative instead of Eq. t-40, Eq. t-21 can be used in the abovementioned equation set and in this case the error value becomes 0.078.

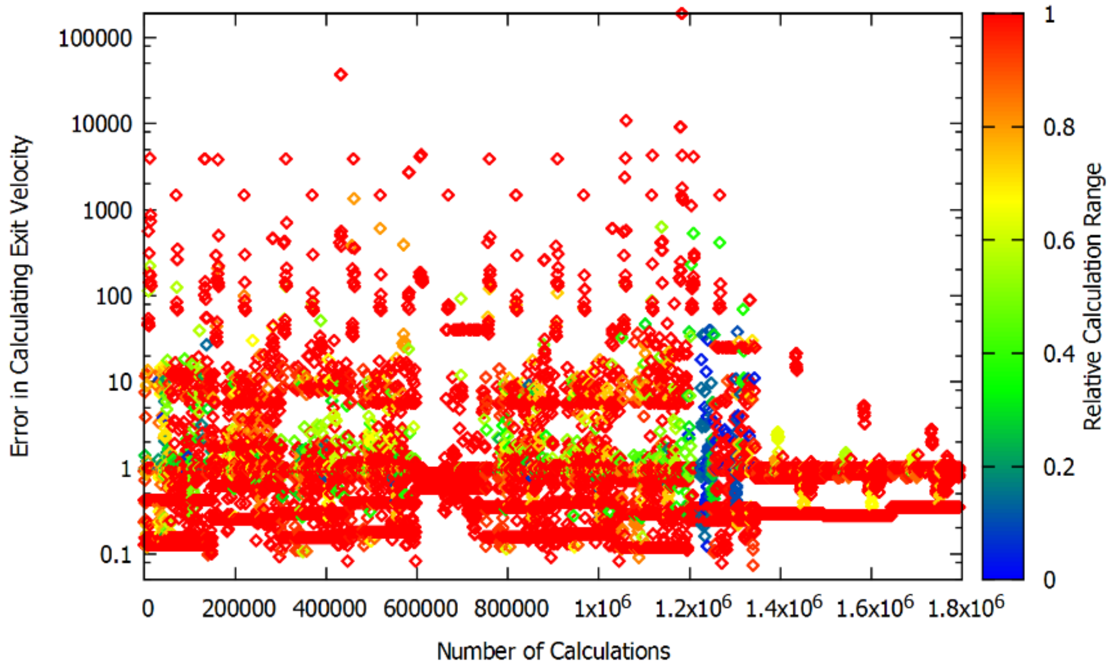


Figure 7: Error Values in Predicting the Exit Velocity

Error values of the SMD prediction is given in Fig. 8. It can be seen that results includes many repeated error values which are caused by the correlation SMD-3 to SMD-6. Since these correlations only includes pressure difference as the variable as long as the equation for predicting the pressure difference is not altered, error value remains constant. Lowest error value in this calculation is 0.169 and is occurred when Eq. Cd-5 is wielded for predicting the pressure difference, Eq. t-1 is wielded for predicting the film thickness, Eq. Ang-6 is wielded for predicting the spray angle and Eq. SMD-1 is wielded for predicting the SMD values. Correlations for predicting the exit velocity or velocity coefficient is not mentioned due to Eq. SMD-1 does not include a term for them. Even though the above equation set governs the minimum error value when its calculation range is analyzed it is found to be 0.24. In order to achieve the minimum error while having a higher calculation range equation set consisting of Eq. FN-3 for predicting the pressure difference, Eq. t-28 for predicting the film thickness, Eq. Ang-4 for predicting the spray angle and Eq. SMD-1 for predicting the SMD can be used. For this equation set the error value is 0.3 and it can be used for the whole experimental data range.

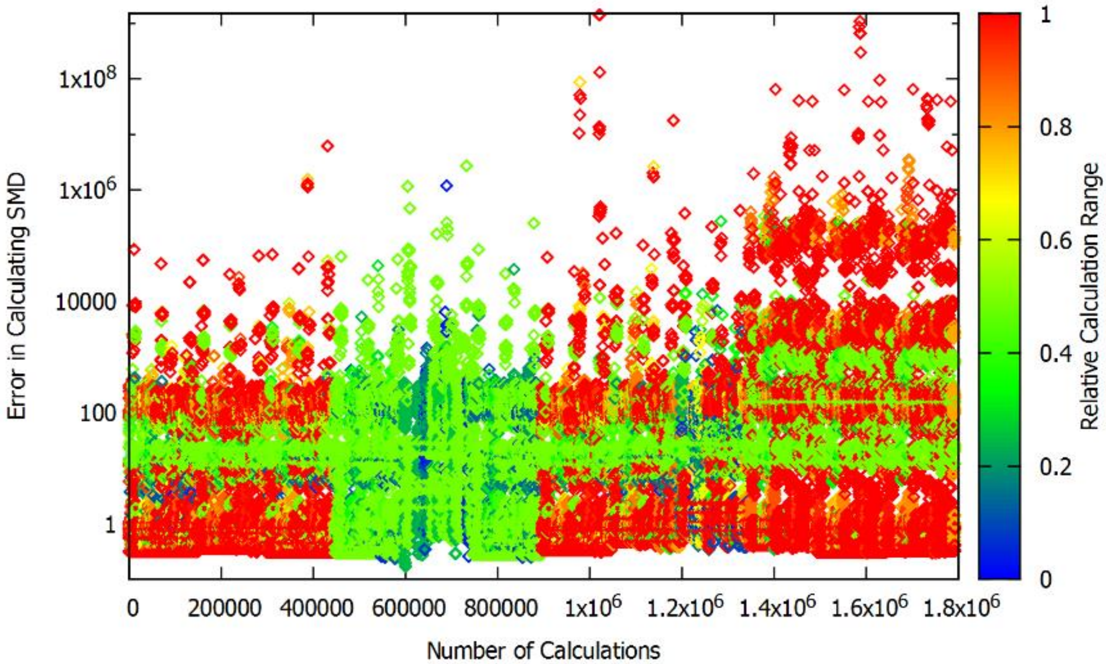


Figure 8: Error Values in Predicting the SMD

Error values of the breakup length calculation are given in Fig. 9. The most important outcome of this calculation is that none of the equation sets can be used for the whole experimental data range and the maximum calculation range is found to be 0.86. Lowest error value for this parameter is found be 0.369 and is encountered when Eq. Cd-3 is wielded for predicting the pressure difference, Eq. t-11 is wielded for predicting the film thickness, Eq. Ang-9 is wielded for predicting the spray angle, Eq. Kv-2 is wielded for predicting the velocity coefficient, Eq. U-3 is wielded for predicting the exit velocity and Eq. Lb-1 is wielded for predicting the breakup length. This equation has the calculation range of 0.86.

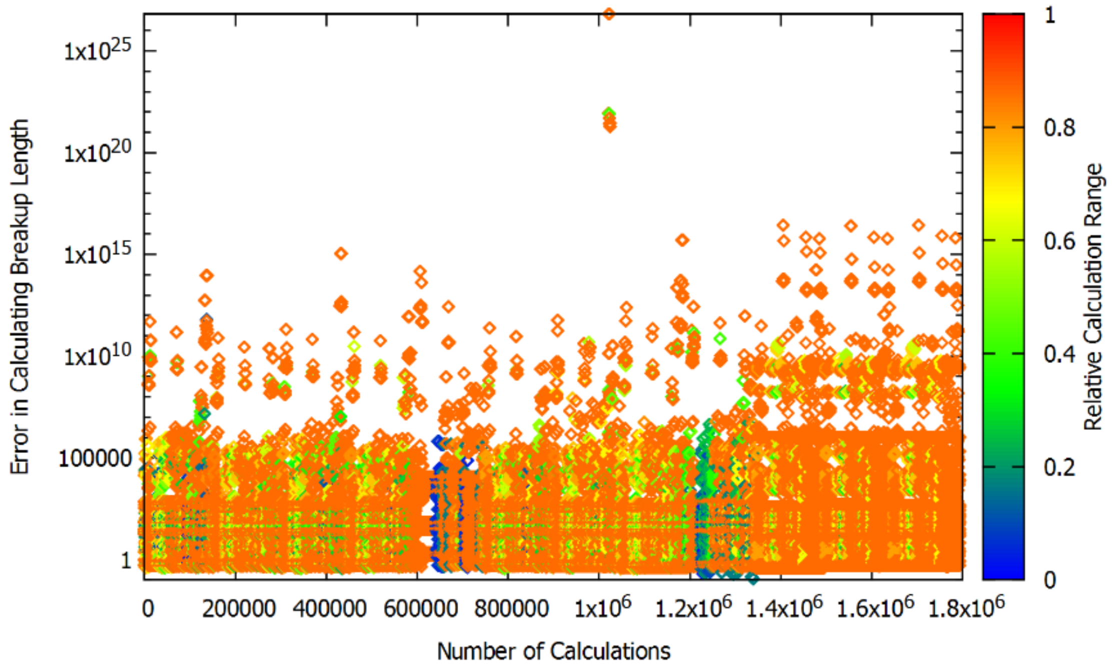


Figure 9: Error Values in Predicting the Breakup Length

An important result of analyzing the minimum errors for each parameter is that there is no unique equation set that could be used for all of the parameters. However, in the design optimization it is more useful to use a single equation set. To obtain such an equation set, cluster analysis on the results is done. It is aimed that, even though no single set could achieve the lowest error possible, equations that produced the low error values most frequently could be obtained and their combination can be used as the final set. For that purpose, the filtering of the results is done such that equation sets that have error values higher than 0.5 for the calculations of film thickness, spray angle, exit velocity, SMD and equation sets that have larger errors than 1 for the calculation of the breakup length and equations sets with calculation ranges lower than the highest possible range are eliminated. Later from the remaining equation sets, the most encountered equations and average errors when these equations are wielded are plotted. In the figure size of the rectangles shows the how many times that equation come upon and the color of the rectangles shows the average error value of the analyzed parameter when that equations are used. Therefore equations with largest area and smallest error value are the most suitable ones.

Results of pressure difference calculation is given in Fig. 10. Inspection of the results shows that Eq. Cd-1, Eq. Cd-3 and Eq. Cd-7 are the most encountered equations and out of these three equations Eq. Cd-1 has the lowest average error value which is 0.78. Therefore Eq. Cd-1 is selected to calculate the pressure difference.

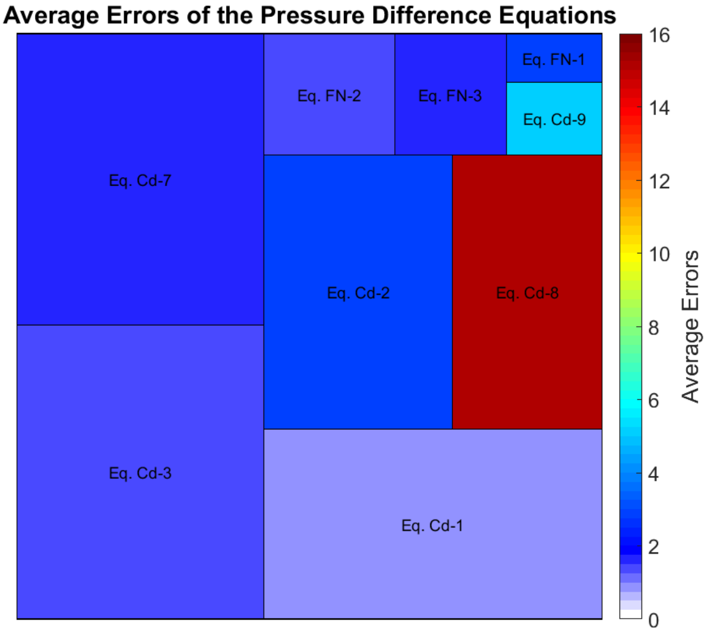


Figure 10: Cluster Analysis of the Pressure Difference Equations

Results of the film thickness calculation is given in Fig. 11. The results shows that Eq. t-18, Eq. t-21, Eq. t-27 and Eq. t-29 are the most frequently encountered correlations. Out of these four correlations Eq. t-21 has the lowest error value which is 0.25 and therefore is selected as the most suitable correlation for predicting the film thickness.

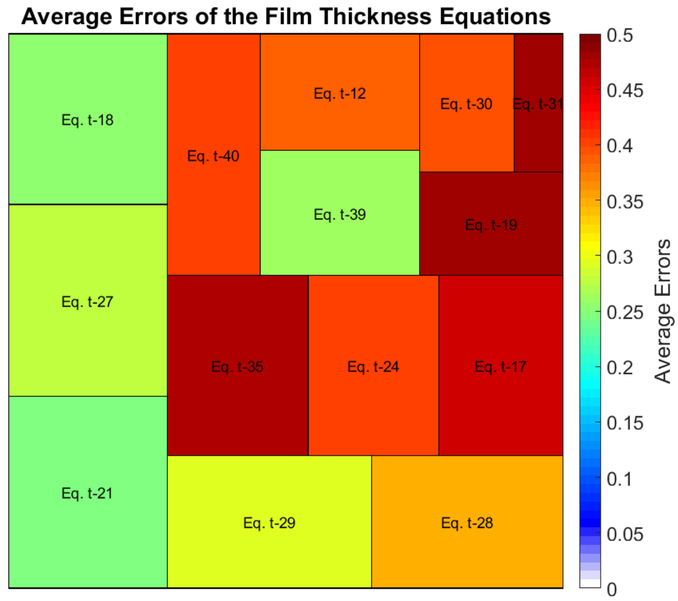


Figure 11: Cluster Analysis of the Film Thickness Equations

Results of the spray angle calculation is provided in Fig. 12. Different than the other parameters only one equation is encountered in the filtered results region which is Eq. Ang-6. It should also be noted that compared to other parameters except the pressure difference, calculation of the spray angle is where the highest errors values are seen.

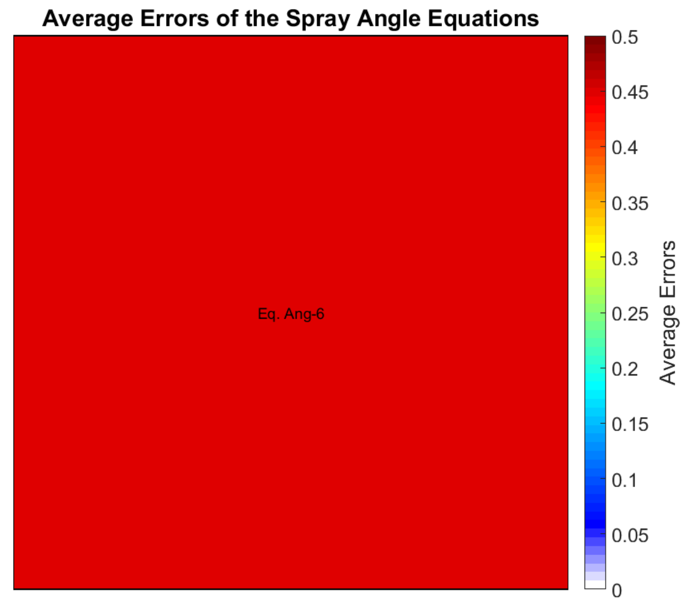


Figure 12: Cluster Analysis of the Spray Angle Equations

Results of the velocity coefficient are provided in Fig. 13. An important feature of the parameter velocity coefficient is that it cannot be measured experimentally and is mainly used for the calculation of the exit velocity. It's analytical definition is derived from the exit velocity. Since, in the analyzed experimental works no researchers provided the velocity coefficient values, in the analysis of the velocity coefficient equations errors of the velocity calculation are taken into consideration. Results shows that Eq. Kv-3 is the most encountered equation however, Eq. Kv-2 is the equation with the lowest error value. Since the difference between the number of occurrences of these equations is considerably lower than the difference between the error values of these equations, Eq. Kv-2 is found to be the most suitable equation.

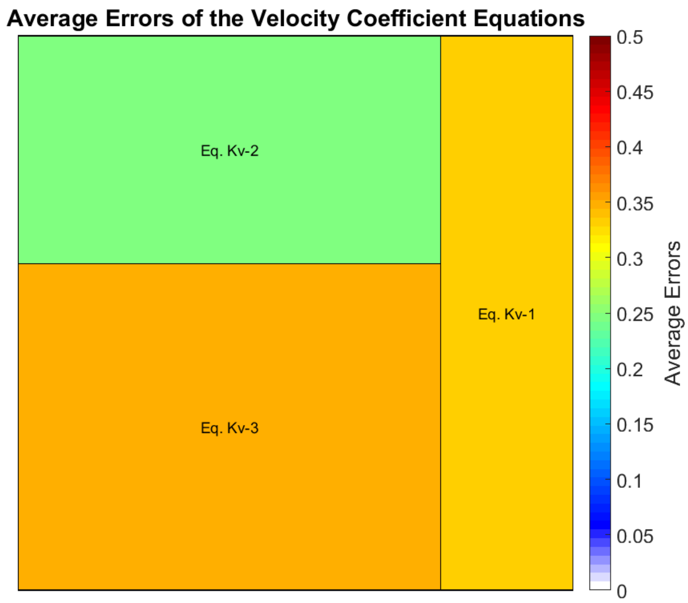


Figure 13: Cluster Analysis of the Velocity Coefficient Equations

Results of analysis done on the exit velocity correlations are given in Fig. 14. Inspection of the shows that Eq. U-1 is the most suitable for predicting the exit velocity since it is both the most encountered equation and the equation with the minimum error value. An important point that should be mentioned is, Eq. U-1 does not include a term for the velocity coefficient therefore, equation found from the previous inspection is not needed.

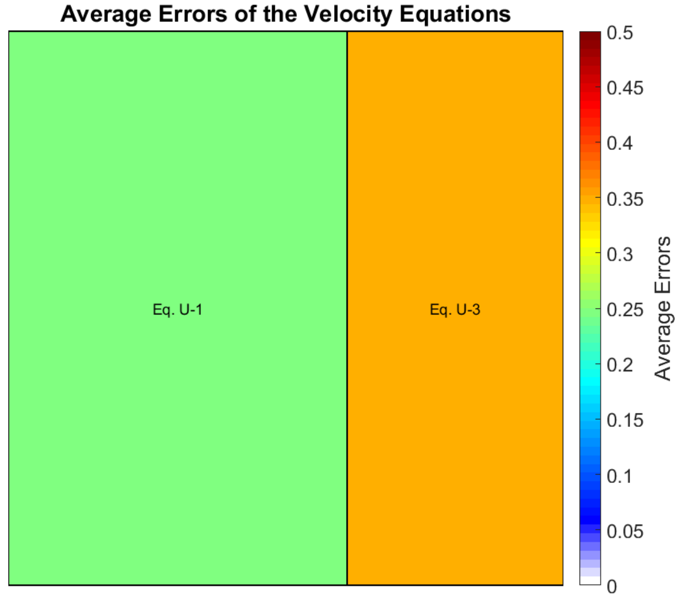


Figure 14: Cluster Analysis of the Exit Velocity Equations

Results of the SMD calculation is given in Fig. 15. Total of four correlation are found in the filtered analysis range. Out of these correlations, Eq. SMD-3 is the one with the minimum error and the came up most frequently and hence it is the selected correlation for predicting the SMD values.

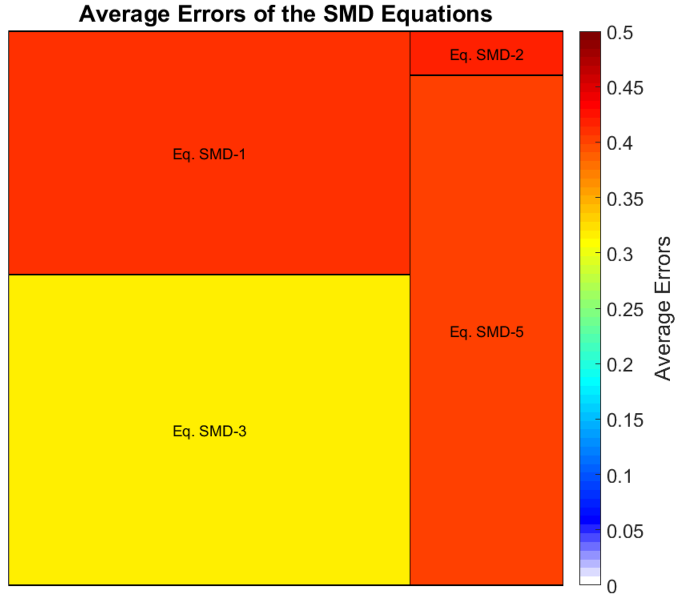
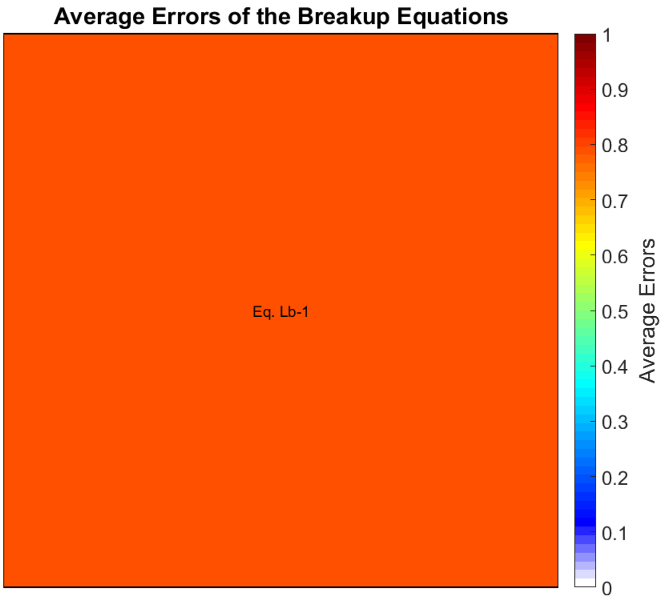


Figure 15: Cluster Analysis of the SMD Equations

Lastly, results of the analysis done on the breakup length equations are given. Similar to the spray angle selection of the most suitable equation is very simple since there is only one equation. Even though this equation, Eq. Lb-1, has a considerably high error value it is the only option for calculating the breakup length.



CONCLUSION

From the two different error analysis different sets of equations are selected to be the most suitable ones for predicting the performance parameters of the pressure swirl atomizers. Minimum error analysis yielded different equation sets for each performance parameters while through the cluster analysis a universal equation set that can be used for all of the parameters is obtained. It can be seen that the universal equation set provides computational simplicity on the other hand, equation sets obtained from the minimum error analysis have slightly lower error values. All of the selected equations are listed in Table 2.

Table 2: Selected Equation Sets from the Minimum Error and Cluster Analysis

Parameter	Equation Sets Selected from Minimum Error Analysis	Equation Sets Selected from Cluster Analysis
Pressure Difference	Eq. Cd-1	Eq. Cd-1
Film Thickness	Eq. Cd-1 + Eq. t-40	Eq. Cd-1 + Eq. t-21
Spray Angle	Eq. Ang-6	Eq. Ang-6
Exit Velocity	Eq. Cd-9 + Eq. t-21 + Eq. Ang-5 + Eq. Kv-2 + Eq. U-3	Eq. Cd-1 + Eq. U-1
SMD	Eq. FN-3 + Eq. t-28 + Eq. Ang-4 + Eq. SMD-1	Eq. Cd-1 + Eq. SMD-3
Breakup Length	Eq. Cd-3 + Eq. t-11 + Eq. Ang-9 + Eq. Kv-2 + Eq. U-3 + Eq. Lb-1	Eq. Cd-1 + Eq. U-1 + Eq. Lb-1

Furthermore, results of this study has revealed that Eq. Cd-1 has the lowest error for calculating the pressure difference. However, error of this equation is very high considering the fact that the pressure difference is the main operation parameter and needed to be calculated more accurately. In addition, Eq. Cd-1 includes only three geometrical parameters. Since discharge coefficient and flow number are the only parameters that links the pressure difference and therefore the rest of the performance parameters with the geometry of the atomizer, an equation that includes all of the geometrical parameters is required for the optimization studies.

Another important observation that should be noted is, despite the fact that error values of semi-empirical equations for calculating the pressure difference are high, other performance parameters can be predict accurately. This is the result of two or more equations damping each other's errors. This outcome emphasizes the importance of the interaction of the equations that are present in an equation set. Therefore, while developing new equations for predicting the performance parameters, it should be kept in mind that this equation will be used with other correlations and the behaviors of other equations should be examined in order to increase the practicality of the developed equation.

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