EQUIVALENT MODEL OF A SANDWICH BEAM WITH RAMP REGION UNDER FOUR POINT BENDING VIA GENETIC ALGORITHM

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ABSTRACT

In this study, an optimization method is introduced to increase the accuracy of a two-dimensional laminated sandwich beam model with a ramp region under 4-point bending loading. The laminated sandwich beam corresponds to a simplified model of a real sandwich structure with a honeycomb core. First, a sandwich panel with actual honeycomb geometry including the ramp region and its equivalent two-dimensional finite element model are developed. The sandwich beam with actual core geometry is considered as a reference model and analytical formulations available in the literature are used for the creation of a simplified two-dimensional model. The face sheets are assumed to be perfectly bonded to the core. The equivalent elastic constants are used as design variables and genetic algorithm optimization method is used to minimize the modeling errors. The optimization process involves finite element solver coupled with genetic algorithm optimization tool. The ramp region of the sandwich panel was also included by modeling it with three steps of different heights. The results show that there is a loss of accuracy when modeling a sandwich beam with two-dimensional conventional shell elements. However, an optimization method can provide considerable improvement to it. Therefore, two-dimensional layered shell modelling approach can accurately represent a sandwich beam under bending dominant loading as long as an optimization process is involved.

INTRODUCTION

Sandwich beams are one of the most common structures used in aviation because of their advantageous stiffness and weight saving features. A typical sandwich structural element consists of two faces, a core and adhesive layers which bond face sheets to the core.

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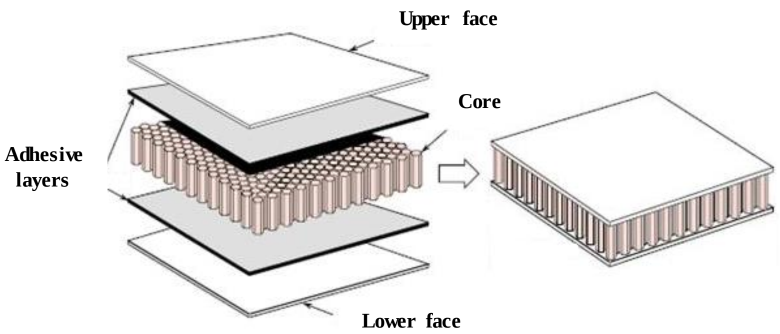


Figure 1: Sandwich Panel [http://www.stressebook.com/honeycomb-sandwich-panels, 2016]

Design principle of a sandwich beam resembles to an I-beam. When an I-beam is loaded in bending, one of the faces goes under tension and the other one is in compression while the web mainly carries shear forces. This load flow path is similar to that of a sandwich beam; however, the web of the I-section is replaced by a core material shown in Figure 2. Therefore, in a typical sandwich structure it is assumed that the face sheets are mainly responsible for in-plane loads while the core is to carry most of the out-of-plane forces. Since most of the aerospace structures are subject to out of plane loads causing bending in a component, accurate design and analysis of core region become crucial.

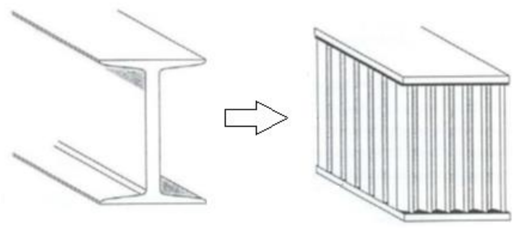
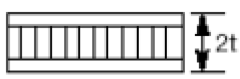
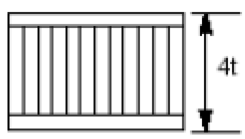


Figure 2: Similarity of Load Carrying Mechanisms of an I-Beam and a Sandwich Beam [Bitzer, 1997]

A sandwich structure is also advantageous when it is compared with conventional solid metal sheet in terms of bending stiffness, strength and weight. A comparison scheme is provided in Table 1.

Table 1: Sandwich Structure Characteristics [Composite Materials Handbook Volume 6, 2013]

	Solid metal sheet	Sandwich Construction	Thicker Sandwich
			
Relative bending stiffness	100	700 7 times more rigid	3700 37 times more rigid
Relative bending strength	100	350 3.5 times as strong	925 9.25 times as strong
Relative weight	100	103 3% increase in weight	106 6% increase in weight

There are three different regions in a sandwich panel as shown in Figure 3. The first region is the monolithic edge region where two faces are bonded together without a core. The second region is the ramp region where two faces are divided by a ramping up core thickness. The third region is the sandwich region where two face sheets are separated by a core of full material.

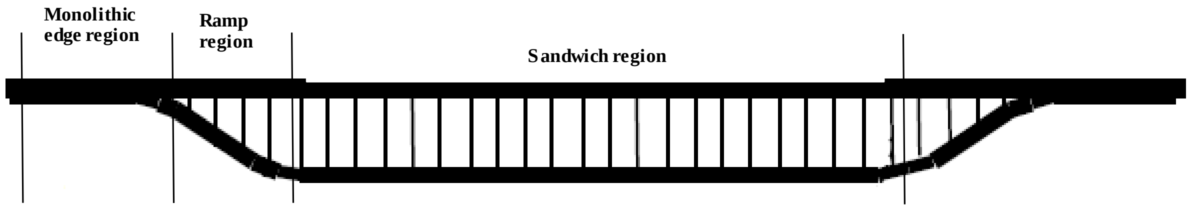


Figure 3: Sandwich Panel Region [Composite Materials Handbook Volume 6, 2013]

There are different shapes and forms of a core material which can be used in an aviation structure depending on the dimensions and complexity of a component. In this study only hexagonal shape is considered leading to modeling difficulties. There are several ways to overcome these difficulties.

Firstly, the complex hexagonal geometry of the honeycomb core can be replaced by an equivalent solid material as shown in Figure 4. In the literature, there are formulas which allows creation of an equivalent solid model representing the porous hexagonal core. These formulas are functions of geometric, physical and material properties of a honeycomb cell.

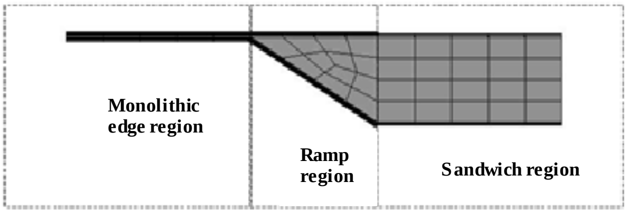


Figure 4: Sandwich Panel with Equivalent Solid Core

Secondly, modeling of ramp region can also be simplified. The ramp region is the region where core height continuously changes according to ramp angle. The ramp region is designed in order to overcome abruptly changing thickness which may result in stress concentrations and undesired high interlaminar shear stresses developed at the ramp start and the ramp end shown in Figure 5.

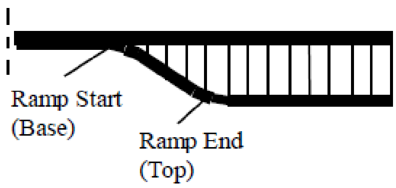


Figure 5: Ramp Start and Ramp End [Composite Materials Handbook Volume 6, 2013]

In order to overcome complexity of the continuously changing height of the ramp region, the ramp can be represented as several steps of different heights, see Figure 6. However, three-dimensional solid modeling of a sandwich beam can still be tedious and costly.

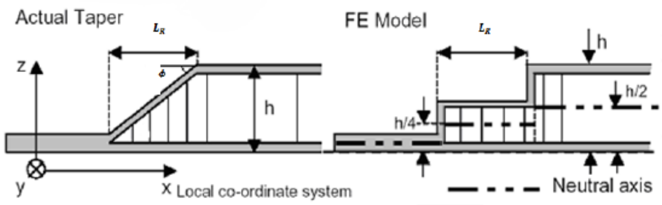


Figure 6: Ramp Region Finite Element Model Idealization Approach

The third simplification method of modeling a sandwich beam is to represent the whole structure with layered shell approach as shown in Figure 7.

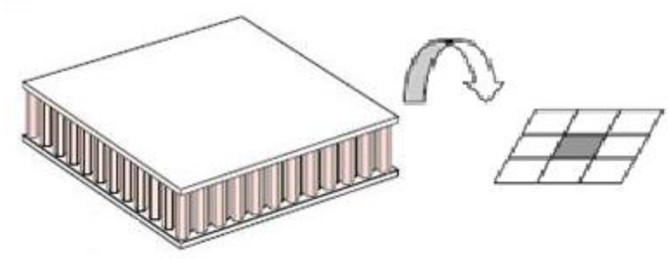


Figure 7: Layered Shell Approach [http://www.stressebook.com/honeycomb-sandwich-panels, 2016]

In this approach, the face sheets and the core are considered as layers of a composite laminate and the thickness of the beam is represented by thicknesses of layers. Therefore, this approach also makes it possible to define ramp region in several steps. Figure 8 shows ramp region idealization using layered shell approach.

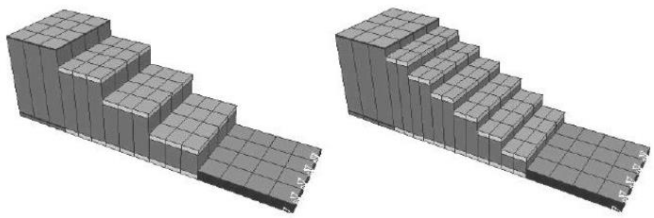


Figure 8: Layered Shell Approach Ramp Region Idealization (Shell Thicknesses are Displayed)

These simplification methods substantially facilitate the finite element modeling and analysis of a sandwich beam with monolithic edge and ramp regions. However, simplification methods are likely to lead inaccurate results.

The first reason of inaccurate results is due to the representation of whole sandwich beam of certain thickness in full two-dimensional environment. At this point, selection of element type and formulation become important to obtain accurate results. In Abaqus 6.14 documentation [Abaqus Manual 6.14, 2017] the following is stated about laminated composite shells: When a plate laying in x-y plane is subject to bending and shear in x-z plane with no membrane loading, it is assumed that there are no gradients of any function in y direction resulting in any slice through x-z plane is the same. In addition, it is assumed that $M_{yy} = M_{xy} = 0$ which means the bending in x-z sections do not induce any moments or twists in y-z sections. Therefore, it is only the Poisson effect that is responsible for inducing bending in different out of plane directions. On the other hand, Abaqus 6.14 documentation [Abaqus Manual 6.14, 2017] provides a benchmark study to compare performance of its conventional shell and solid elements under bending. It is shown that conventional shell elements can behave as accurate as solid elements or continuum shell elements depending on the span of a sandwich beam which is defined as its length to thickness ratio. It is suggested to use conventional shell elements if this ratio is greater than 10. Therefore, considering the sandwich part to be analyzed in this study, it is acceptable to use layered shell approach.

The second reason of inaccuracy is that the analytical formulas employed to homogenize the core properties are developed without considering the effects of face sheets and their development are subject to assumptions. This is the main source of inaccuracy that this study is focused on. The aim of this study is to overcome this type of error by optimizing the material input properties using genetic algorithms. Therefore, in this study layered shell approach with an optimization method will be used in order to accurately represent a sandwich beam in two-dimensions.

METHOD

Actual Model

Actual model shown in Figure 9 is the quarter model of the real structure. It has 15 cells in ribbon direction and 5 cells in the width direction. It is made of aluminum, AL5052. The length of the sandwich beam is 143 mm while the width is 47.63 mm. The cell size is 9.53 mm (3/8 inch) and the cell wall thickness is 0.1016 mm (0.004 inch). The ramp angle is 30 and the face sheet thickness is 0.52 mm. This honeycomb is selected from Hexweb Honeycomb Catalog [Hexcel, 2015]. Thus, this metallic honeycomb core is available to use of aerospace industry.

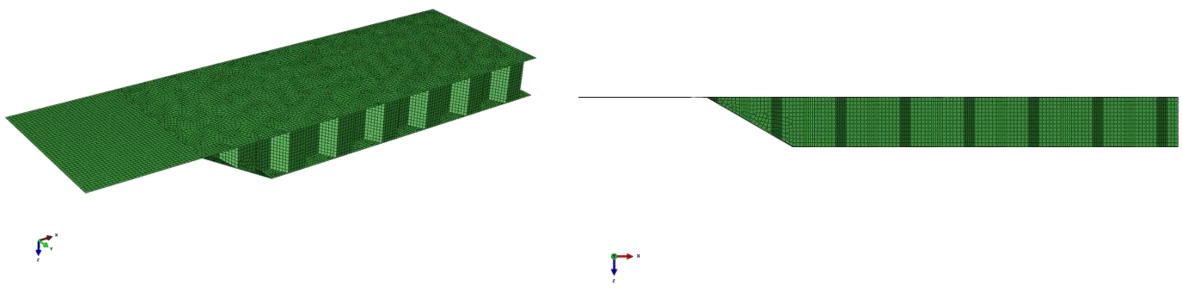


Figure 9: Actual Model

Quarter of the sandwich beam is modeled with symmetric boundary conditions applied XZ and YZ planes. In Figure 10, it is shown that translation of the beam is constrained in the z-direction on left hand side and 0.1 mm downward displacement is applied on the right-hand side of the structure.

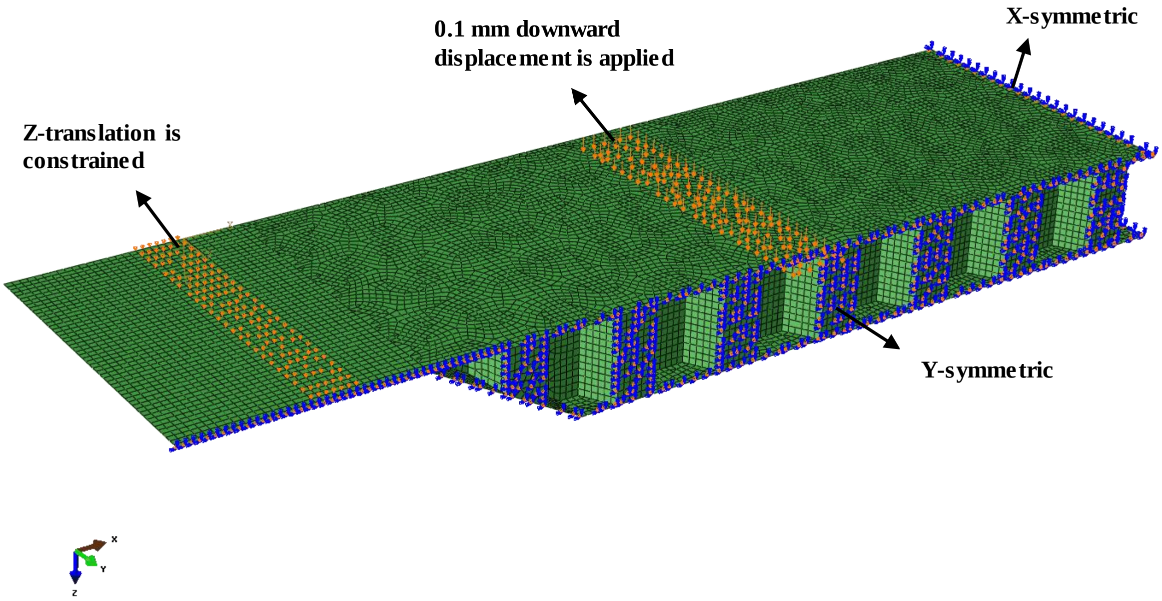


Figure 10: Boundary Conditions

Layered Shell Model

The layered shell modeling approach is 2D representation of a sandwich beam. The plate in Figure 11 is the 2D model of the actual sandwich beam shown in Figure 10. The layered shell model has the same dimensions and boundary conditions with the actual model.

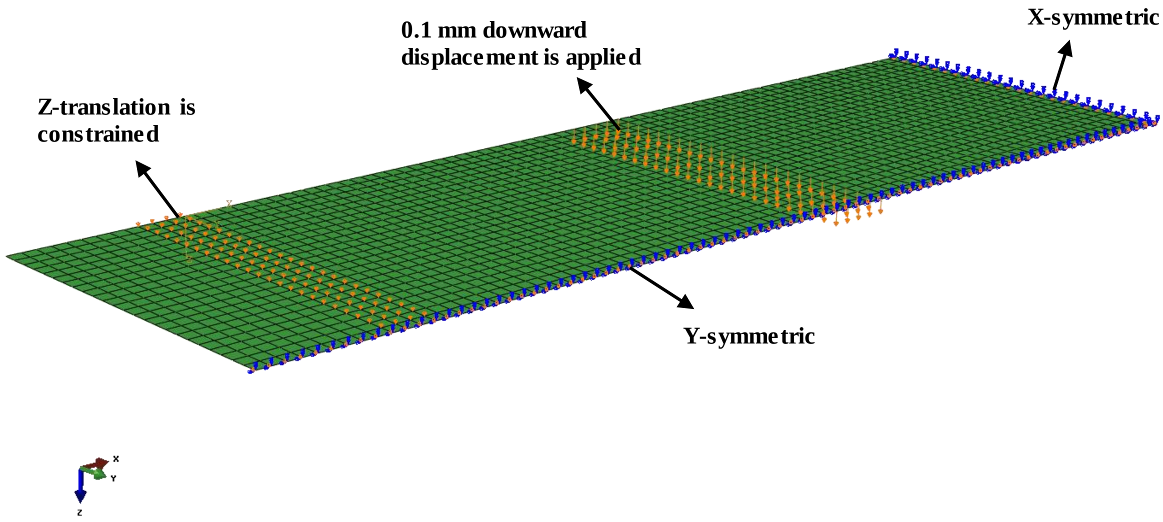


Figure 11: Layered Shell Model

Figure 12 shows the layered shell model with section thicknesses assigned to elements displayed. It can be seen from the picture that the ramp region of the actual sandwich beam is modeled with 3 steps of different heights.

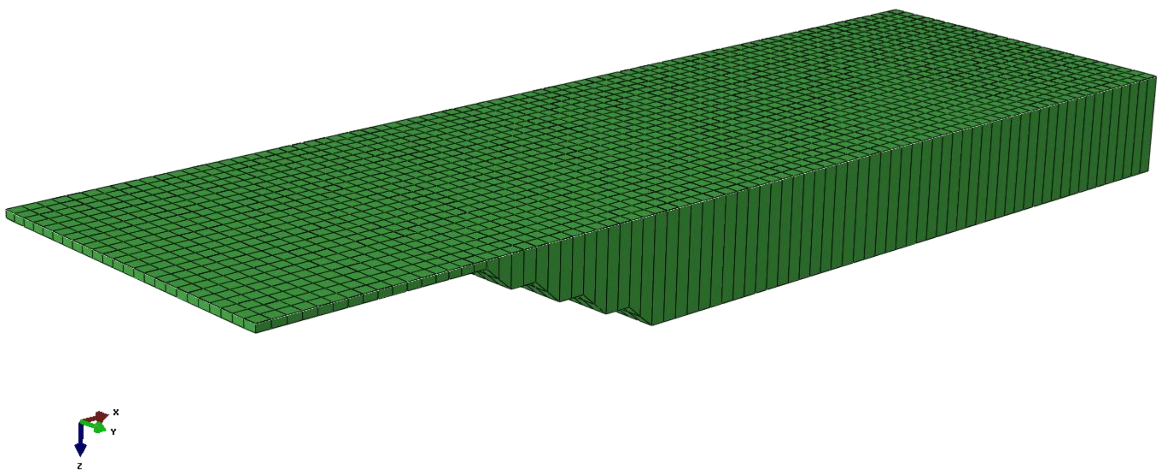


Figure 12: Layered Shell Model (Shell Thicknesses are Displayed)

In the actual model, the core material is isotropic aluminum but for the layered shell model the core material is 2D orthotropic. This conversion is made by means of analytical formulas which are dependent on the geometric properties and the material constants of the actual model such as the cell shape, cell size, cell wall thickness and the Young's modulus and the Poisson ratio. In the literature, there are more than one formulas for the same elastic constants. In this study, the combination of material constants formulas is selected according to study [Aydıncak, 2009] in which a feasible combination was found by comparing different combinations. In this study

[Aydıncak, 2009], it was found that the best result is obtained when the in-plane Young's modulus, the shear modulus and the Poisson ratio are taken from the study of [Masters, 1996]; out-of-plane shear modulus is taken from another study [Grediac, 1993].

Having defined the homogenized core material constants, composite shell sections are defined according to layered shell approach. For instance, the sandwich region is defined in three layers. The first layer is the upper face sheet whose thickness is 0.52 mm and material is AL5052; the second layer is the core with its homogenized core material properties and thickness of 12.7 mm and the last layer is the lower face sheet whose properties are the same with the first layer.

The layered shell model is meshed using conventional four-node reduced integration shell elements (S4R). Abaqus Documentation 6.14 suggests the use of S4R type of elements if thick shell sections are to be modelled. There are total of 5676 nodes and 5504 linear quadrilateral shell elements in the model.

Optimized Layered Shell Model

The optimized layered shell model is also a shell model but its material constants are updated via the genetic algorithm. An optimized model resembles closely to the actual model in terms of its mechanical behavior. Therefore, it enables user to obtain accurate results using layered shell modeling approach.

Genetic Algorithm

Genetic algorithms are commonly used tools to solve discrete, non-differentiable, noisy and global optimization problems. It is first proposed by John Holland in 1962. The genetic algorithm appears as a powerful tool because its search method is population based. Population type search is a method which creates and works with many possible solution points at the same time rather than moving sequentially from one possible solution point to the next. In addition, the genetic algorithm updates its design variables by means of probabilistic rules instead of using deterministic rules. These are the best features of this optimization method since they enable user to avoid locking around a local minimum. Thus, the genetic algorithm is preferred as the optimization method in the current study.

In genetic algorithm, there are several issues to be considered to obtain a convergent, feasible and accurate solution. The first issue is the definition of fitness function which outputs fitness values to the genetic algorithm. In other words, it is objective function which is to be minimized. In this study, the objective function is defined as minimization of the difference of reaction forces between the actual and the layered shell model.

The objective function is given in equation 1 below where RF_{REF} is the reference reaction force obtained from the actual model, RF_{LS} is the reaction force output of the layered shell model.

$$\frac{(RF_{REF} - RF_{LS})^2}{RF_{REF}^2} \qquad \text{Eqn. (1)}$$

Definition of the fitness function is the unique point in a genetic algorithm optimization problem. In this study, the fitness function takes material constants generated by genetic algorithm as input and writes them into Abaqus input file. Then, the input file is sent to Abaqus Standard Solver which outputs reaction forces at the supports in a file. The file is not directly readable by Matlab. To overcome this problem, another tool is implemented into the process. The tool is called Abaqus2Matlab and developed by [Papazafeiropoulos, 2017]. It converts the Abaqus output file into Matlab matrices. After this conversion is completed, Matlab sums the reaction forces at the nodes and compares the result against the reference reaction force obtained from the actual model using equation 1. The value obtained becomes the fitness value of the individual considered for that moment. The algorithm does the same for all the individuals in a population.

The second issue considering the definition of genetic algorithm optimization problem is the number of variables and constraints on them. In this study, possible variables are the 2D orthotropic material parameters of the core. Namely the six elastic material constants $E_1, E_2, \upsilon_{12}, G_{12}, G_{13}, G_{23}$ can be defined as the design variables. The optimization process is expected to provide optimum values for these design variables which can minimize equation 1. However, the honeycomb core is kind of structure which reacts against especially the out-of-plane shear forces, which means it is weak in the in-plane directions and strong in the out-of-plane directions. Therefore, the out-of-plane material constants are the main concern in this study and it is possible to omit G_{12} since its effect on the core is negligible. Nevertheless, E_1, E_2, ν_{12} are still required to be kept as design variables to apply non-linear material stability constraint as defined in the software manual [Abaqus Manual 6.14, 2017] and can be seen in Equation 3. Due to the fact that the material inputs will be constantly changing during the process, upper and lower bounds are also given as constraints to design parameters. The bounds defined can be seen in in equation 2 and the constraint applied is given in the equation 3.

$$[0.1,0.1,0.1,100,100] < [E_1, E_2, \upsilon_{12}, G_{13}, G_{23}] < [0,0,0.90,1000,1000] \qquad \text{Eqn. (2)}$$

$$|\nu_{12}| < \left(\frac{E_1}{E_2}\right)^{1/2} \qquad \text{Eqn. (3)}$$

The third issue is the options of genetic algorithm, which is closely related to convergence of the problem. Population size, fitness scaling, selection, crossover and mutation options can significantly affect the convergence of the problem. The scaling function is a form of conversion function which is responsible for listing raw fitness values obtained from the fitness function in a range which is suitable for the selection function to operate on. The parents which will pass to the next generation will be determined by the selection function based on their position within this range. After selection operation is completed, the crossover function runs and combines two individuals to form a child or another individual that will constitute the next generation i.e., the next iteration. Finally, the mutation function makes it possible for genetic algorithm to search for a broader space by changing the individuals randomly to ensure diversity. Then, the new population is evaluated again by the fitness function and an updated fitness value is obtained which will be checked against the termination criteria. The algorithm runs until one of the termination criteria is satisfied. The simplified flowchart of the optimization algorithm used in this study is shown in Figure 13 below.

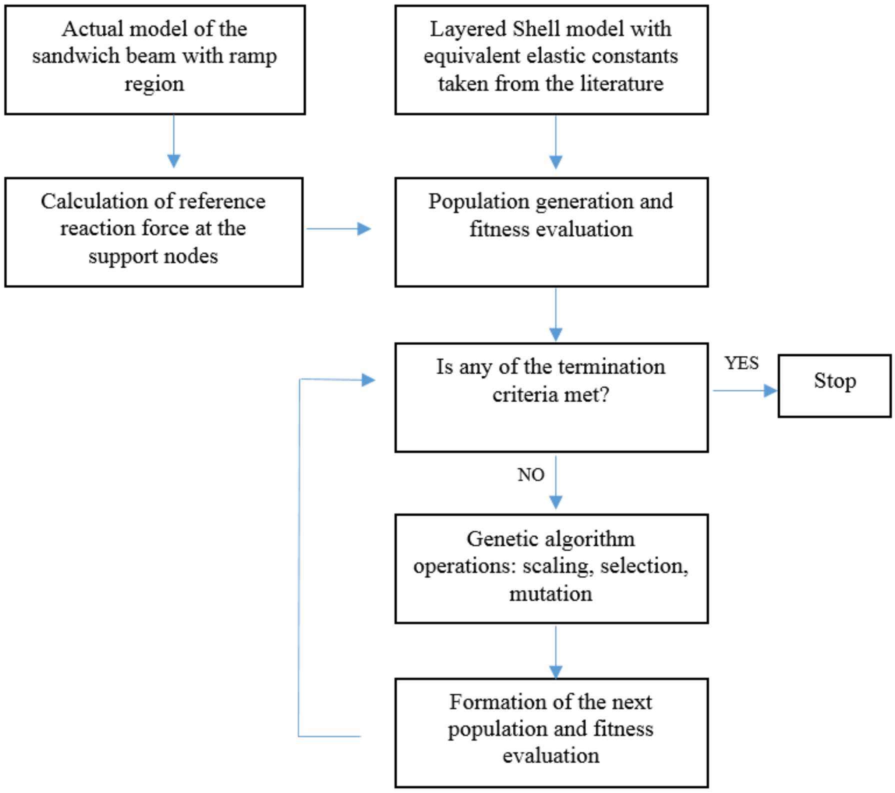


Figure 13: Flowchart of the Algorithm Used

Genetic Algorithm Convergence Study

The options selected in genetic algorithm significantly change the convergence behavior. Therefore, a comparison study was conducted to determine the best combination which can provide fast and accurate results. For the comparison study, a simple plate model is developed whose material is isotropic AL5052 and boundary conditions are the same with the layered shell model. It is shown in Figure 14.

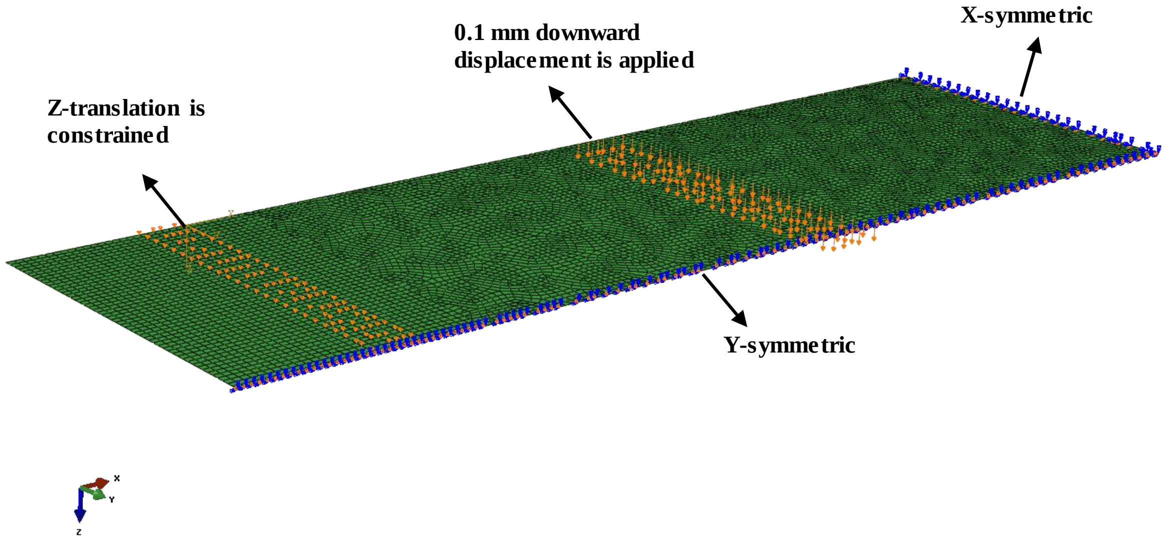


Figure 14: Simplified Model

The comparison study is conducted with reference combination options which are given in Table 2. This selection is based on the study of [Çınar, 2015]. The method of comparison study is changing each of the parameters for every run and comparing the reaction forces with the reference model. The initial material parameters of the model to be optimized are 20% lower than the original AL5052 properties. Objective function defined is the same with the equation 1. Upper and lower bounds are defined are [1000, 0.1] and [100000, 0.49] for the Young's Modulus and the Poisson Ratio, respectively. The best performed combination of options is determined according to resulting value of the objective function and the number of iterations. The results are presented in Table 3.

Table 2: Reference Combination of Options

Fitness scaling	Rank
Selection	Remainder
Mutation	Adaptive feasible
Cross over	intermediate

Table 3: Results of Comparison Study

Option changed	Fitness Scaling	Selection	Mutation	Cross over	E	Nu	Obj.Fun	#of Iter.
Ref.	rank	remainder	adaptive feasible	intermediate	70532	0.33	2.97E-09	4
Fitness Scaling	**shift linear**	remainder	adaptive feasible	intermediate	70272	0.33	1.88E-07	6
Selection	rank	**tournament**	adaptive feasible	intermediate	70272	0.33	1.88E-07	6
Mutation	rank	remainder	**uniform**	intermediate	74346	0.22	2.32E-08	10
Cross Over	rank	remainder	adaptive feasible	**two point**	74796	0.19	1.20E-05	10

Table 3 shows that the best performing combination of options is the reference options since its objective function and number of iterations are the lowest compared to other combinations. Therefore, the reference combination of options is used in this study.

Having decided on the best combination of options the effect of having a smaller population size is also investigated. Results provided in Table 4 shows that the smaller population size increases the number of iterations and reduces the accuracy of the solution. In addition to these results the software manual [Matlab Documentation, 2016] also suggests a population size of 50 for the problems with five or less design variables.

Table 4: Effect of Population Size

Option changed	Population size	E	Nu	Obj. Fun	#of Iter.
Ref.	50	70532	0.33	2.97E-09	4
Pop. size	10	79972	0.265	1.40E-08	10

RESULTS AND DISCUSSION

Actual Model and Analytical Equivalent Model

The actual model where the hexagonal cells and ramp region are modelled is compared with the analytical equivalent model whose elastics constants shown in Table 5 are determined using the formulas existing in the literature [Masters, 1996; Grediac 1993].

Table 5: Analytically determined elastic constants of the core material

E1 [MPa]	0.10
E2 [MPa]	0.10
Nu12 [-]	0.90
G12 [MPa]	0.25
G13 [MPa]	469.78
G23 [MPa]	281.89

The reaction force resultant in z-direction for both analytical and actual model provided in Table 6 shows that the analytical equivalent model results in 26% deviation in the reaction force.

Table 6: Resultant reaction forces

	RF [N]
Actual model	44.99
Analytical Equivalent Model	56.81

Optimized Equivalent Model

Main reason why involving optimization process in the equivalent modeling is to overcome 26% of error. To ensure that the algorithm gives truly a global minimum, several runs are given with the same initial elastic constants i.e., the analytical constants. In the first and second cases shown in Table 7 five elastic constants are considered as design variables while in the third and last cases only out-of-plane shear moduli are considered since the sandwich beam is under 4-point bending loading and the core is expected to react against out-of-plane shear forces. Results show great agreement on G13 while the other material parameters are found to be ineffective on the resulting reaction force within pre-defined upper and lower limits for them.

Table 7: Optimized Elastic Constants

Case number	**E1 [MPa]**	**E2 [MPa]**	**Nu12 [-]**	**G13 [MPa]**	**G23 [MPa]**	**Obj. Fun.**
1	36.5	28.08	0.31	240.36	824.77	6.73e-9
2	34.18	47.16	0.38	241.38	609.07	3.08e-8
3	Fixed-0.1	Fixed-0.1	Fixed-0.9	242.45	678.84	1.47e-8
4	Fixed-0.1	Fixed-0.1	Fixed-0.9	242.25	509.99	5.85e-8

Dominating effect of G13 parameter is believed to be due to the nature of the structure and the type of loading. Since the main deformation occur in the XZ-plane and the core ribbon direction is in X-direction, G13 parameter drives the problem. The optimized G13 is found to be around 240 MPa and the analytically found G13 is 470 MPa. The difference between the parameters is 49%.

CONCLUSIONS

The results suggest that layered shell modeling approach can be used to represent a sandwich beam as long as an optimization process is involved. The main reason causing difference in the reaction forces believed to be the assumptions in the derivation of analytical formulas and effect of the face sheets.

The future work of this study will be including G23 in the optimization by changing the ribbon direction. In addition, the effect of the face sheet thickness will also be included.

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