

## A STUDY ON ENERGY TRAPPING PARAMETERS OF A MULTISTABLE ELASTIC BEAM

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### ABSTRACT

*Energy trapping materials are being widely used in personnel protection, packaging of special products, aircraft and land vehicles and in many other industries as impact absorber. Conventional energy trapping materials absorb impact energy by undergoing plastic deformation and thus cannot be reused. A multistable elastic tilted beam can lock in strain energy when exposed to an impact and can fully recover while unloading. This fully reversible cycle enables consistently usage of the system many times. In this paper, factors effecting energy-trapping capacity of an elastic tilted beam are determined by using finite element method (FEM). A 2D planar beam is modeled with the commercial FE program ABAQUS. Geometrical and topological parameters defining the model are varied and many FE runs are conducted to determine the energy trapping capacity of the beam while keeping the volume of the beam constant. Optimum beam dimensions, tilt angle and geometry are found. The objective of this study is to determine the parameters of the tilted beam yielding to maximum energy trapping while using the same amount of material.*

### INTRODUCTION

The primary objective of energy absorbers is to keep the reactive force below a threshold which will cause damage or injury [Ali et al., 2006]. In each specific case, factors leading to need of an energy absorber should be well defined and an appropriate energy absorber should be chosen to prevent failure. For this reason, there is a growing interest on energy absorbing systems to develop more appropriate and cost effective energy absorbers.

The idea of using multistable elastic beams for energy trapping purposes enables reusability of the system. Thus, multistable energy absorbers provide cost advantage over conventional energy absorbing systems. Although the amount of energy absorbed by elastic energy absorbers are low compared to plastic energy absorbers, there are many ongoing studies to find new ways of absorbing energy by undergoing elastic deformation. Some examples of these studies are summarized as follows: [Restrepo et al., 2015] designed a phase transforming

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cellular structure and studied effects of its unit cell geometry on the mechanical behavior of the structure. The structure design is shown in Figure 1. [Correa et al., 2015] designed a negative stiffness honeycomb (NHS) made of nylon 11. The design is tested under compressive loads and its energy absorbing behavior is examined. Energy absorption results of NSH are compared with results of conventional honeycombs. [Kidambi et al., 2016] inspired from energy absorbing behavior of sarcomeres (Cross-bridges in muscles, they are responsible for a significant portion of the elastic energy stored in muscles) and studied bistable constituents of energy absorbing and trapping structures to develop advanced adaptive structures and materials. [Chen et al., 2017] designed different reversible and deployable structures to determine the system's load bearing capacity and predict its activated geometry. Designs are varied between flat and curved configurations and length of bistable material changed. Examples of structure designs are shown in Figure 2.

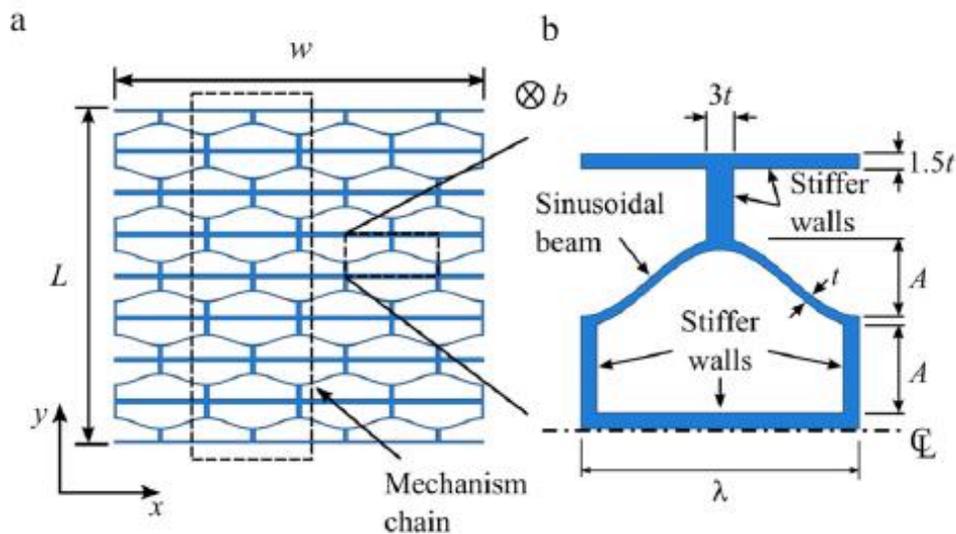


Figure 1: a) A phase transforming cellular structure design. b) A unit cell of the structure in part a [Restrepo et al., 2015]

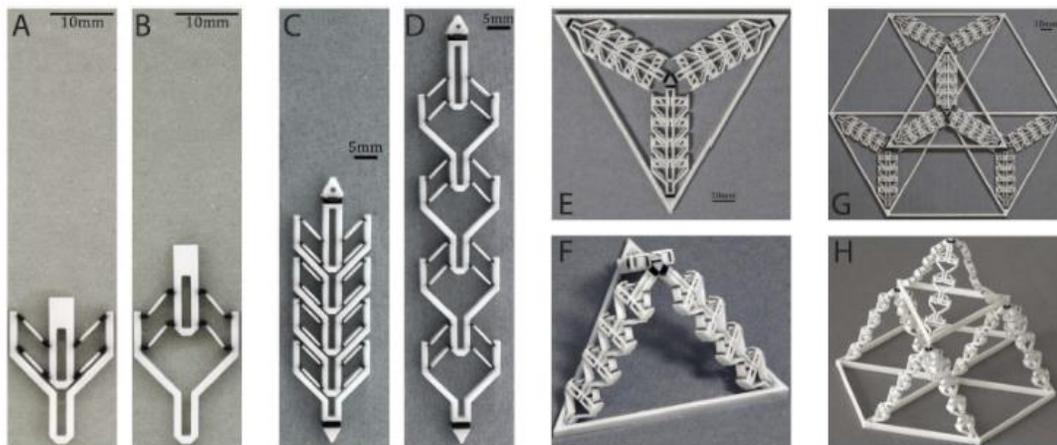


Figure 2: Design examples of reversible and deployable structures [Chen et al., 2017]

Bistability is one popular route towards achieving form or functional changes within a structure [Chen et al., 2017]. It is also necessary for a system to be able to trap energy for energy absorption purposes. [Haghpanah et al., 2016] is defined a phase stability parameter in their study to decide whether a design is bistable or not. Schematic of the defined stability parameter for a specific structure is shown in Figure 3.

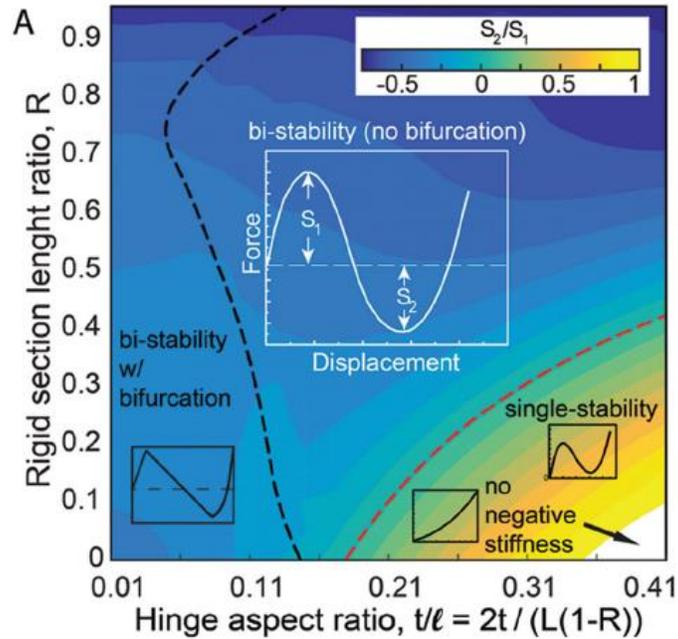


Figure 1: Phase stability parameter  $S_2/S_1$  for a specific structure [Haghpanah et al., 2016]

In multi-stable systems, snap-through between their stable configurations are required to trap energy in the system. In their study, [Kidambi et al., 2016] observed that selection of stiffness and damping parameters presents an inherent balance between the ability to snap-through and the risk of snap-back that may result in no net energy storage. The snap-back behavior causes the structure to behave like a single stability structure and no energy is being trapped in the structure. For this reason, in a multistable energy trapping system design, possibility of snap-back behavior of the structure should be taken into account.

## METHOD

This study is inspired by an existing study in the literature [Shan et al., 2015]. In the existing study, a structure consisting of many tilted beams as in Figure 4 is designed. The change in energy trapping capacity of the structure is studied for various width/length ratios and tilt angles. In this study, thanks to modularity of the structure in Figure 4, only a single tilted beam is modeled as in Figure 5 by using finite element method. Then the parameters effecting its energy trapping capacity are studied. The whole model's energy trapping capacity can be estimated from the results of one tilted beam.

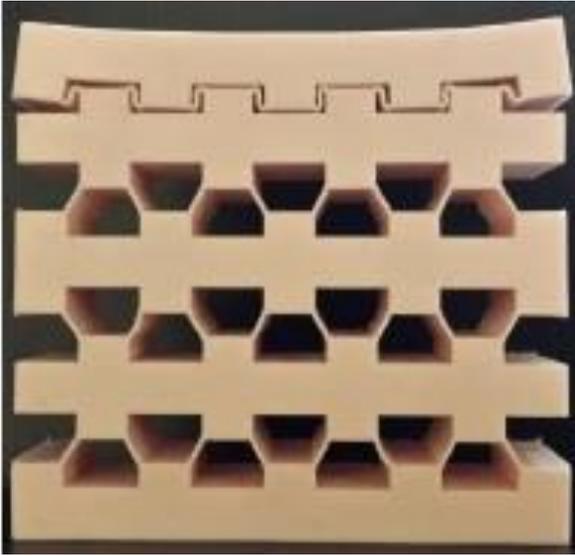


Figure 4: An array of tilted beams [Shan et al.,2015]

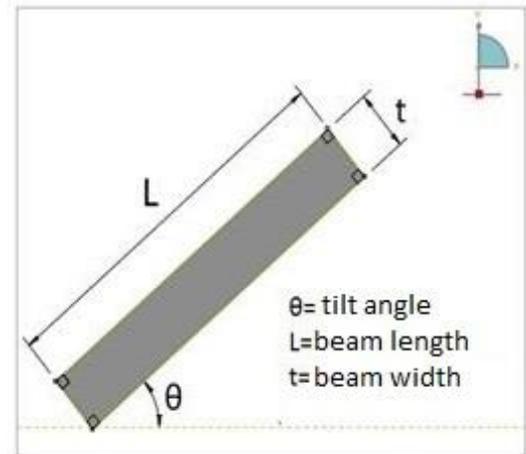


Figure 5: Tilted beam model

First, a mesh convergence study was made for the constructed FE beam model and mesh size of 0.05 mm is decided to be used. In the analyses six node modified, hybrid with linear pressure elements having hourglass control (CPE6MH) are used. The material parameters of modeled beam are defined same as the study in the literature [Shan et al., 2015]. In the existing study, material behavior is defined as hyperelastic Neo-Hookean with coefficients as follows  $C10=0.16$  MPa,  $D1=0.0025$  MPa<sup>-1</sup>. As stated in [Shan et al., 2015], C10 and D1 coefficients are obtained from the tension test experiments of the material “polydimethylsiloxane (PDMS)” which is commonly known as silicone rubber.

Finite element analyses are run by applying displacement boundary condition to the modeled beam. Bottom end of the tilted beam is constrained both in vertical and horizontal directions while the upper end is constrained only in horizontal direction. Prescribed vertical displacement is applied to the upper end of the beam. Reaction force ( $F_R$ )-Displacement ( $U$ ) graphs are drawn for each analysis and the energy trapping capacity of the beams are computed from the area under  $F_R$ - $U$  curve. At the end of all analyses, applied vertical displacement is removed to check whether the beam is bistable or not. Bi-stability of the beam is required to trap the strain energy stored in the system; otherwise stored energy will be released upon removal of the displacement boundary condition. Figure 6 shows the path followed in each analysis.

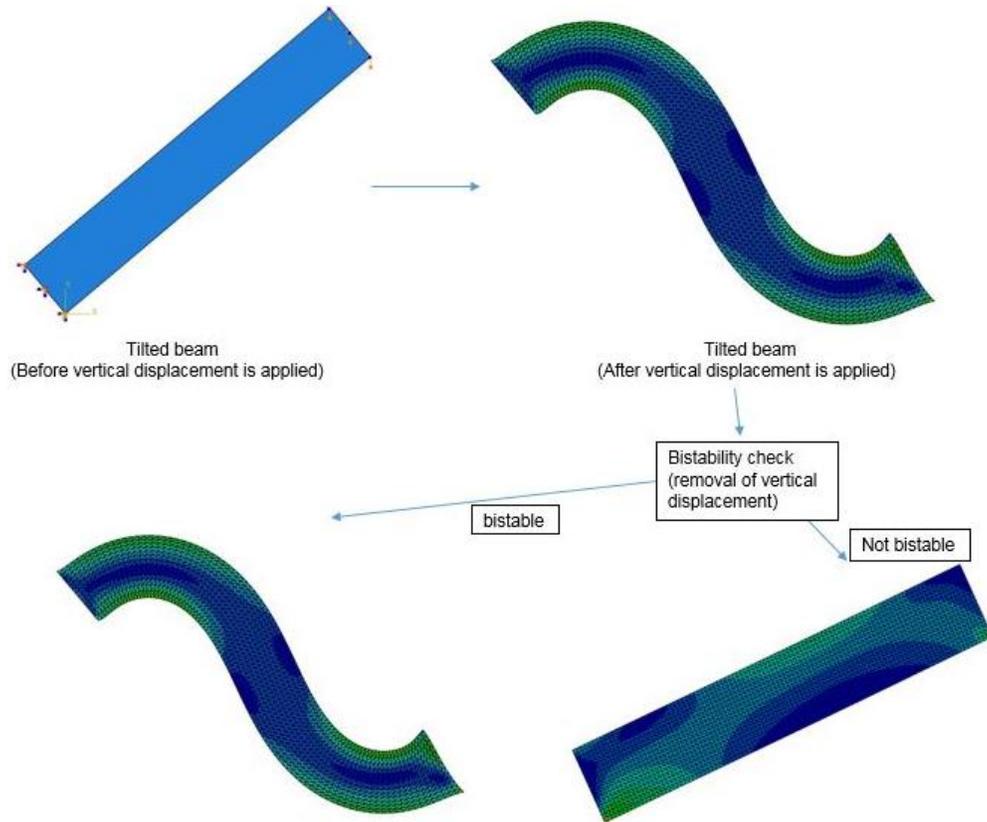


Figure 6: The path followed in each analysis

### Effect of Width/Length Ratio on the Energy Trapping Capacity of the Tilted Beam

A reference width ( $t$ ), length ( $L$ ) and tilt angle ( $\theta$ ) dimensions of the tilted beam are randomly chosen for the first analysis as  $t=0.61$  mm,  $L=5.06$  mm and  $\theta=40^\circ$ . Based on the reference dimensions, volume and tilt angle of the beam are kept constant in all other analyses and energy trapping capacity of the tilted beam is observed for corresponding width and length pairs. Results are presented in Table 1. As can be seen from the table as the beam becomes less slender, i.e., as length/width ratio decreases, energy trapping capacity increases.

### Effect of Tilt Angle on the Energy Trapping Capacity of the Tilted Beam

A length and width pair that leads to maximum energy trapping is determined in previous section. Effect of tilt angle study is also made based on reference beam dimensions. The length and width dimensions of the reference beam are kept constant and only the tilt angle is changed now. The results are presented in Table 2. As it can be seen from the table, the energy trapping capacity of the beam exhibits an increasing trend with the tilt angle.

Table1: Effect of beam's width and length change on its trapped energy capacity for constant volume of the beam.

Change in width of the beam	Change in length of the beam	Loaded Energy to the beam (a) mJ	Energy spent by the beam itself (b) mJ	Trapped Energy (a-b) mJ	Change in trapped energy (%)	Is the beam bistable?
(-) 20 %	(+) 25 %	$2.94 \cdot 10^{-1}$	$4.95 \cdot 10^{-2}$	$2.45 \cdot 10^{-1}$	-56.64 %	YES
(-) 17 %	(+) 20 %	$3.39 \cdot 10^{-1}$	$5.52 \cdot 10^{-2}$	$2.84 \cdot 10^{-1}$	-49.73 %	YES
(-) 13 %	(+) 15 %	$3.92 \cdot 10^{-1}$	$6.13 \cdot 10^{-2}$	$3.31 \cdot 10^{-1}$	-41.42 %	YES
(-) 9 %	(+) 10 %	$4.61 \cdot 10^{-1}$	$6.81 \cdot 10^{-2}$	$3.93 \cdot 10^{-1}$	-30.44 %	YES
(-) 5 %	(+) 5 %	$5.40 \cdot 10^{-1}$	$7.40 \cdot 10^{-2}$	$4.66 \cdot 10^{-1}$	-17.52 %	YES
<i>Reference Beam Dimensions</i> <i>t=0.61 mm, L=5.06 mm, <math>\theta=40^\circ</math></i>		$6.45 \cdot 10^{-1}$	$7.97 \cdot 10^{-2}$	$5.65 \cdot 10^{-1}$	0.00 %	YES
(+) 5 %	(-) 5 %	$7.67 \cdot 10^{-1}$	$8.09 \cdot 10^{-2}$	$6.86 \cdot 10^{-1}$	21.42 %	YES
(+) 11 %	(-) 10 %	$9.18 \cdot 10^{-1}$	$7.72 \cdot 10^{-2}$	$8.41 \cdot 10^{-1}$	48.85 %	YES
(+) 18 %	(-) 15 %	$1.01 \cdot 10^0$	$5.01 \cdot 10^{-2}$	$9.61 \cdot 10^{-1}$	70.09 %	YES

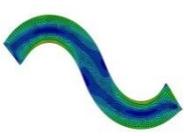
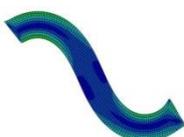
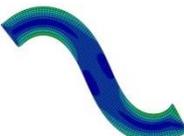
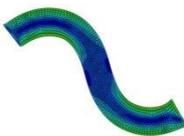
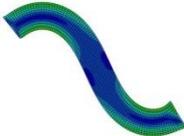
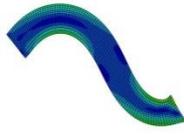
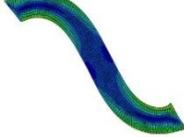
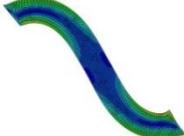
Table 2: Effect of tilt angle on energy trapping capacity of the beam.

Tilt Angle (degree)	Energy loaded to the beam (a) mJ	Energy spent by the beam itself (b) mJ	Trapped energy (a-b) mJ	Change in trapped energy w.r.t. reference beam
30	$3.33 \cdot 10^{-1}$	$3.11 \cdot 10^{-2}$	$3.02 \cdot 10^{-1}$	-46.55 %
35	$4.79 \cdot 10^{-1}$	$5.41 \cdot 10^{-2}$	$4.25 \cdot 10^{-1}$	-24.78 %
40	$6.45 \cdot 10^{-1}$	$7.97 \cdot 10^{-2}$	$5.65 \cdot 10^{-1}$	0.00 % <i>(Reference beam)</i> <i>(t=0.61mm, L=5.06mm and <math>\theta=40^\circ</math>)</i>
45	$8.24 \cdot 10^{-1}$	$1.05 \cdot 10^{-1}$	$7.19 \cdot 10^{-1}$	27.26 %
50	$1.02 \cdot 10^0$	$1.29 \cdot 10^{-1}$	$8.91 \cdot 10^{-1}$	57.70 %

### Effect of Beam Geometry on the Energy Trapping Capacity of the Tilted Beam

Maximum energy trapping parameters such as length, width and tilt angle are determined for the modeled beam in previous two sections. In addition to this, effects of further geometrical changes to the energy trapping capacity are also studied. Here the beam is not restricted to have straight edges. To this end, concave, convex and s-shaped beams are studied and the results are presented in Table 3. As can be seen from the table particular s shape of the beam can increase the energy trapping capacity up to 20%.

Table 3: Effect of geometry on energy trapping capacity of the beam.

Beam geometry	Energy loaded to the beam (a) mJ	Energy spent by the beam itself (b) mJ	Trapped energy (a-b) mJ	Change (%)	Undeformed shape	Deformed Shape
Reversed S_Shape_r5	$4.05 \cdot 10^{-1}$	$5.84 \cdot 10^{-3}$	$3.99 \cdot 10^{-1}$	-29.24%		
3S Shape_r1	$5.22 \cdot 10^{-1}$	$5.17 \cdot 10^{-2}$	$4.70 \cdot 10^{-1}$	-16.63%		
Concave/convex_r15	$5.49 \cdot 10^{-1}$	$2.17 \cdot 10^{-2}$	$5.27 \cdot 10^{-1}$	-6.52%		
<u>Reference Beam</u> ( $t=0.61\text{mm}$ , $L=5.06\text{mm}$ and $\theta=40^\circ$ )	$6.43 \cdot 10^{-1}$	$7.89 \cdot 10^{-2}$	$5.64 \cdot 10^{-1}$	0.00%		
2S_Shape_r5	$6.35 \cdot 10^{-1}$	$6.12 \cdot 10^{-2}$	$5.74 \cdot 10^{-1}$	1.72%		
S_Shape_r20	$6.98 \cdot 10^{-1}$	$9.79 \cdot 10^{-2}$	$6.00 \cdot 10^{-1}$	6.38%		
Convex/concave_r15	$6.19 \cdot 10^{-1}$	$9.83 \cdot 10^{-3}$	$6.09 \cdot 10^{-1}$	7.99%		
S_Shape_r6	$7.63 \cdot 10^{-1}$	$1.24 \cdot 10^{-1}$	$6.39 \cdot 10^{-1}$	13.28%		
S_Shape_r5	$7.76 \cdot 10^{-1}$	$1.04 \cdot 10^{-1}$	$6.72 \cdot 10^{-1}$	19.13%		

## CONCLUSION

A multistable elastic tilted beam that is reusable and can trap impact energy by undergoing elastic deformation is analyzed. Effects of geometrical and shape parameters on the energy trapping capacity are studied. According to gathered results, the energy trapping capacity of the beam increases as tilt angle of the beam increases. Moreover, energy trapping capacity of the beam also increases when width of the beam becomes less slender. When the beam geometry is changed from straight-edged to an “s” shape, the energy trapping capacity of the beam is increased for particular s-geometries. One may find the trapped energy values that are presented in this study very low for an impact absorber system. However, increasing the dimensions of the beam and using many beams in a structure increase trapped energy values.

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