IMPLEMENTATION AND COMPARISON OF DIFFERENT POSITION SOLUTIONS OF GPS RECEIVER IN SATELLITES

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ABSTRACT

Global Positioning System (GPS) receiver can solve and find its position by using pseudo-ranges between receiver and satellites. By using non-linear least squares method, receiver position can be obtained instantaneously. The accuracy of the receiver position obviously depends on the observed pseudo-ranges and how well their error components are modelled. Pseudo-ranges can be obtained from 2 different types of signal information, C/A code phase and carrier phase. However, these pseudo-ranges have error terms which are ionosphere effect error, multipath error, satellite clock error, receiver clock error, troposphere effect error. Since this receiver is designed to use in low earth orbit satellites, multipath error and troposphere error will be ignored. Therefore, the largest error component arises from ionospheric delay effect on the signal. This ionospheric effect can be eliminated by using ionosphere free equations with dual frequency (L1-L2 or L1-L5) observations.

Therefore, in this paper, the position of a GPS receiver will be calculated by using C/A code phase and carrier phase and then ionospheric effect will be eliminated by using dual frequency observations. At the end, the accuracy of 4 different types of solution will be compared.

INTRODUCTION

Global Positioning System (GPS)

The whole GPS positioning contains 3 different task which are cascaded to each other which are acquisition, tracking and positioning. Acquisition task refers to search and find satellite signals and then extract satellite's orbit information (ephemeris). Tracking task refers to being locked to the satellite signal and tracking the changes in signal quantity and quality. The final task is positioning of the receiver by using ephemeris information and receiver information.[ESA]

Through ephemeris messages, GPS satellites tell the receiver their keplerian orbit parameters and their clock parameters to calculate satellites' clock correction. In other words, GPS satellites broadcast their position (X^k , Y^k, Z^k) and their atomic clock time (t^k) where k is the k^{th} satellite. When receiver detects the signal, it tags the signal with a timestamp with its clock (t). So, although receiver and satellite has asynchronous clock, the time difference between signal transmission and signal reception is $\tau = t - t^k$. Since signal propagates with speed of light ($c = 3 \times 10^8$), the most rough pseudo-range between receiver and satellite will be $P = \tau * c$.

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Code Phase Pseudo-range Observation Equation

GPS code pseudorange mode is :

$$R_i^k(t_r,t_e)=\rho_i^k(t_r,t_e)-(\delta t_r-\delta t_k)c+\delta_{ion}+\delta_{trop}+\delta_{tide}+\delta_{rel}+\varepsilon \tag{1}$$

where

R is the observed pseudorange,

t_e denotes the GPS signal emission time of the satellite k,

t_r denotes the GPS signal reception time of the receiver i,

c is the speed of light,

subscript i and superscript k denote the receiver and satellite,

δ t_r and δ t_k are the clock errors of the receiver and satellite at the times t_r and t_e, respectively.

The terms δ_{ion}, δ_{trop}, δ_{tide} and δ_{rel} denote the ionospheric, tropospheric, tidal, and relativistic effects, respectively. The multipath effect is omitted here. The remaining error is denoted as ε.

$$q^k = \sqrt{(X^k-X)^2+(Y^k-Y)^2+(Z^k-Z)^2} \tag{2}$$

Since the geometric range is nonlinear as given Equation 2, linearized observation equation is (detailed analysis of linearization can be found in appendix) :

$$l_k=\frac{-1}{\rho_i^k(t_r,t_e)}\begin{pmatrix}x_k-x_{i0} & y_k-y_{i0} & z_k-z_{i0}\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta y\\ \Delta z\end{pmatrix}-\Delta t+\nu_k \tag{1}$$

where

l_k is the observed minus computed pseudorange,

v_k is the residual,

vector $(\Delta x\ \Delta y\ \Delta z)^T$ is the difference between the coordinate vector $(x_i\ y_i\ z_i)^T$ and the initial coordinate vector $(x_{i0}\ y_{i0}\ z_{i0})^T$,

Δt is the receiver clock error in meters.

Equation-3 can be written in a more generic form as in vectorized form :

$$l_k=\begin{pmatrix}a_{k1} & a_{k2} & a_{k3} & -1\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta y\\ \Delta z\\ \Delta t\end{pmatrix}+\nu_k \tag{4}$$

Equation-4 can be written in a matrix form :

$$L = AX + V \tag{5}$$

where

L is called the observation vector,
X is the unknown vector,
A is the coefficient matrix,
V is the residual vector.

Carrier Phase Observation Equation

Carrier phase measurement is the distance measure between satellite and receiver which is expressed with carrier phases. Position calculation can be done so precisely by using carrier phase, but the number of whole cycles between satellite and receiver is not measurable (called integer ambiguity). However, this integer ambiguity can be eliminated by using 2 consecutive samples if there is no cycle slip in the signal.

Carrier phase pseudo-range observation equation as given below:

$$\lambda\Phi_i^k(t_\mathrm{r},t_\mathrm{e})=\rho_i^k(t_\mathrm{r},t_\mathrm{e})-(\delta t_\mathrm{r}-\delta t_\mathrm{k})c+\lambda N_i^k-\delta_\mathrm{ion}+\delta_\mathrm{trop}+\delta_\mathrm{tide}+\delta_\mathrm{rel}+\varepsilon \tag{6}$$

where

λΦ is the observed phase in length,
Φ is the phase in cycle,
wave length is denoted as λ,
N_i^k is the ambiguity related to receiver *i* and satellite *k*,
Except for the ambiguity term and the sign difference of the term of ionospheric effect, other terms are the same as that of the pseudorange discussed at code phase pseudorange observation equation.

Equation-6 can be written in a vectorised form which is so similar to Equation-4 :

$$l_k = \begin{pmatrix} a_{k1} & a_{k2} & a_{k3} & -1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta t \end{pmatrix} + \Delta N_i^k + v_k \tag{7}$$

Equation-7 can be written in a matrix form:

$$L = AX + EN + V \tag{8}$$

where

L is called the observation vector,

X is the unknown vector of coordinates and clock error,
A is the *X* related coefficient matrix,
E is an identity matrix of order *K*
K is the number of observed satellites,
N is the unknown vector of ambiguity parameters,
V is the residual vector.

If there is no cycle slip between consecutive samples, then ambiguity does not change. Therefore, if two consecutive samples are extracted from each other, ambiguities cancel out and remaining equation is without ambiguity. Resultant equation is:

$$L = AX + V \tag{9}$$

where
L is called the observation vector difference between 2 consecutive samples,
X is the unknown vector of last sample,
A is the coefficient matrix difference between 2 consecutive samples,
V is the residual vector difference between 2 consecutive samples.

Ionosphere-Free Code Observation Equation with Dual Frequency

Since ionosphere changes the path of the signal, the reception time of the signal will be later than expected .Therefore, there will be ionospheric error on pseudorange measurement.

The ionospheric error can be eliminated by using 2 different signal with different frequencies since ionosphere effect totally depends on frequency inversely [Borre et al., 2007].

Code observation equation for the first frequency is the same as with equation-5.

$$L_1 = AX + V_1 \tag{10}$$

Code observation equation for the second frequency is also same as with equation-5.

$$L_2 = AX + V_2 \tag{11}$$

Ionosphere-free combination of equation-9 and equation-10 can be formed:

$$\frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = AX + V \tag{12}$$

where
f_1 represents first frequency,
f_2 represents second frequency.

Ionosphere-Free Phase Observation Equation with Dual Frequency

Phase observation equation for the first frequency is the same as with equation-8.

$$L_1 = AX + V_1 \tag{13}$$

Phase observation equation for the second frequency is also same as with equation-8.

$$L_2 = AX + V_2 \tag{14}$$

Ionosphere-free combination of equation-12 and equation-13 can be formed:

$$\frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = AX + EN_c + V \tag{15}$$

where f_1 represents first frequency, f_2 represents second frequency and

$$N_c = \frac{f_1^2}{f_1^2 - f_2^2} N_1 - \frac{f_2^2}{f_1^2 - f_2^2} N_2 \tag{16}$$

Ionosphere-Free Phase-Code Combined Observation Equation with Dual Frequency

Phase observation equation which is so similar to equation-8 is given below.

$$L_p = A_{11} X_1 + A_{12} N + V_p \tag{17}$$

Code observation equation which is equal to equation-5 is given below.

$$L_c = A_{11} X_1 + V_c \tag{18}$$

In equation-11 and equation-12,

index p and c denote the phase and code related variables,
X_1 is the vector of the coordinate and receiver clock error,
N is the ambiguity vector,
P is the weight matrix,
V is the residual vector.

NON-LINEAR LEAST SQUARE SOLUTION

Since pseudo-range observation is not equal to real geometric range, the error between observed range and geometric range should be minimized. In literature, it is common to use least square optimization to minimize error. Least square has different types like linear least square, nonlinear least square and weighted non-linear least square. If the model is nonlinear, then nonlinear least square is used to give a solution. In nonlinear least square method, model is linearized around the solution point and then standard least square is applied iteratively. Since all code and phase observation equations given above have already been linearized, standard linear least square solution procedure will be held. [Heij, C., de Boer, P., Franses, P. H., Kloek, T., & van Dijk, H. K., 2004]

The general form of standard least square is

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{19}$$

It can realized that equation-18 has the same form all code and phase observation equations given above. Therefore, they can be used directly to calculate a position solution.

Least square tries to find x which minimizes

$$(\boldsymbol{Ax} - \boldsymbol{b})^2 = \boldsymbol{0} \tag{20}$$

If there are more observations than the number of unknowns, then least square solution can be found.
The solution of least square is

$$\boldsymbol{x} = (\boldsymbol{A^T A})^{-1} \boldsymbol{A^T b} \tag{21}$$

EXPERIMENTS

In this paper, nonlinear least square method is applied to code observation, time differential phase observation, ionosphere-free code observation and ionosphere-free phase – code combined observation equation. Observation and navigation data which are used in the experiments are obtained from low earth orbit satellites as in RINEX format [Rinex File Format]. Therefore, solutions are calculated in Earth Centered Earth Fixed (ECEF) coordinate system and compared with Standard Product #3 files (SP3) [Standard Product]. Therefore, errors which will be mentioned in the result part will in meters in ECEF coordinate system.

RESULTS

Code Observation Solution

Nonlinear least square method is applied to code observation equation and results are obtained. Error – sample graphs in ECEF coordinate for each axis can be seen below.

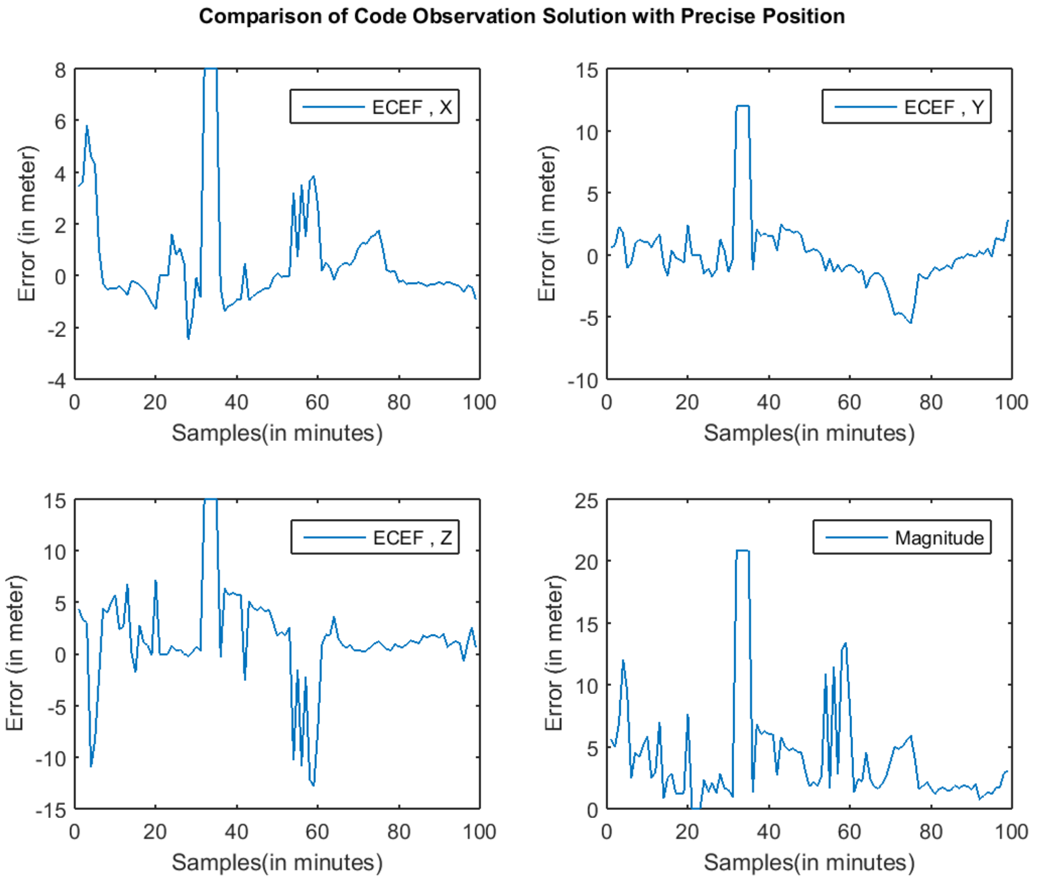

Figure 1. Comparison of Code Observation Solution with Precise Position

Since least square uses only the measurement at the epoch solution is calculated, the errors changes abruptly from one epoch to another. In other words, there is not any model to make errors consistent in consecutive epochs.

Also, since prefiltering is disabled during experiments, there are some spikes in the errors.

Mean of magnitude of error is 4.24 meter and standard deviation is 4.414 meter.

Time Differential Phase Observation Solution

Time differential phase observation model assumes that ambiguity does not change between consecutive epochs if there is no cycle slip between 2 consecutive epochs. Although it is thereotically true, it does not work in practice if there is not a way to detect cycle slip in receiver hardware since cycle slip detection in software works with threshold mechanism. Therefore, if there is a cycle slip in data and software can not detect it, assumption will not exist anymore. However, software solves it as if there is no cycle slip since it can not detect it. Results of time differential phase observation solution is given below.

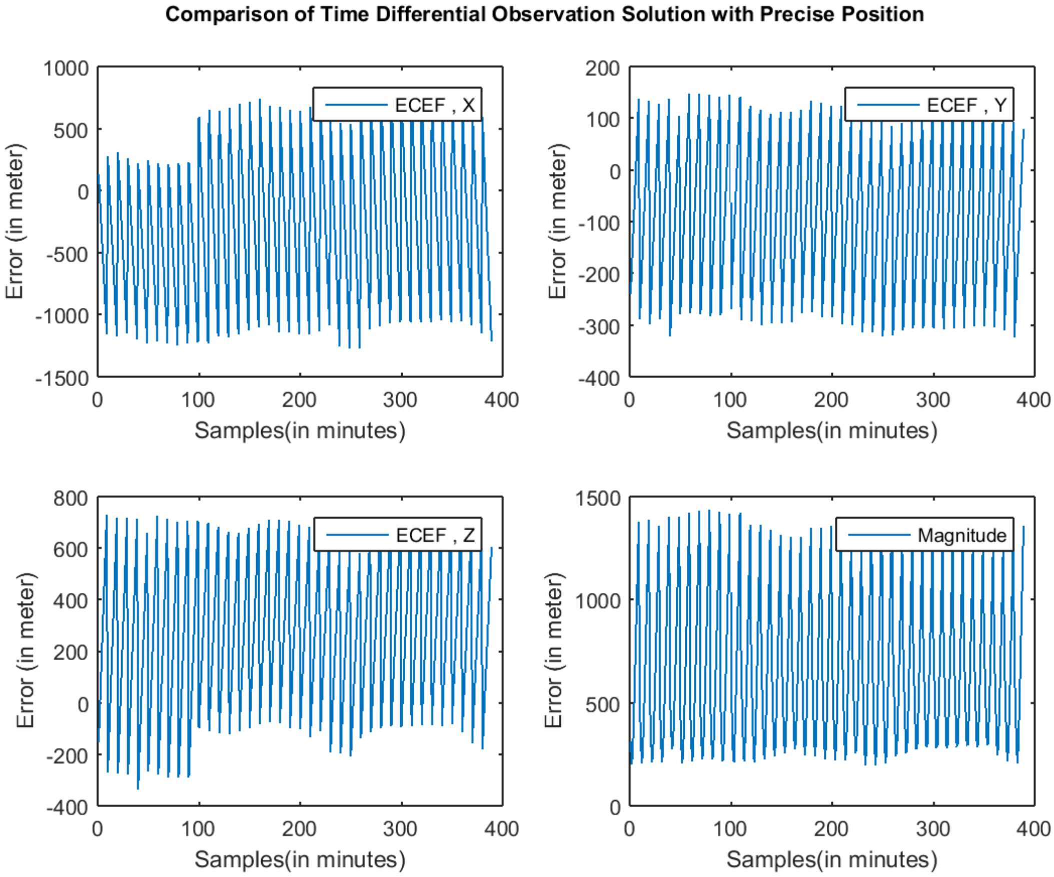

Figure 2.Comparison of Time Differential Phase Observation Solution with Precise Position

Mean of magnitude of error is 669.35 meter and standard deviation is 354.71 meter.

Ionosphere-Free Code Observation Solution

Ionosphere-free code observation equation eliminates almost 99.9% of ionospheric delay. However, it is nosier than the individual codes since measurement is tripled (ex. If L1 and L2 are used) because of combination equation. [Delay, S. H., Carrano, C. S., Groves, K. M., & Doherty, P. H., 2015]

$$Pc = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2}; = \frac{\gamma P_1 - P_2}{\gamma - 1} \qquad \gamma = \left(\frac{77}{60}\right)^2$$

$$\sigma_{Pc} = 3\,\sigma$$

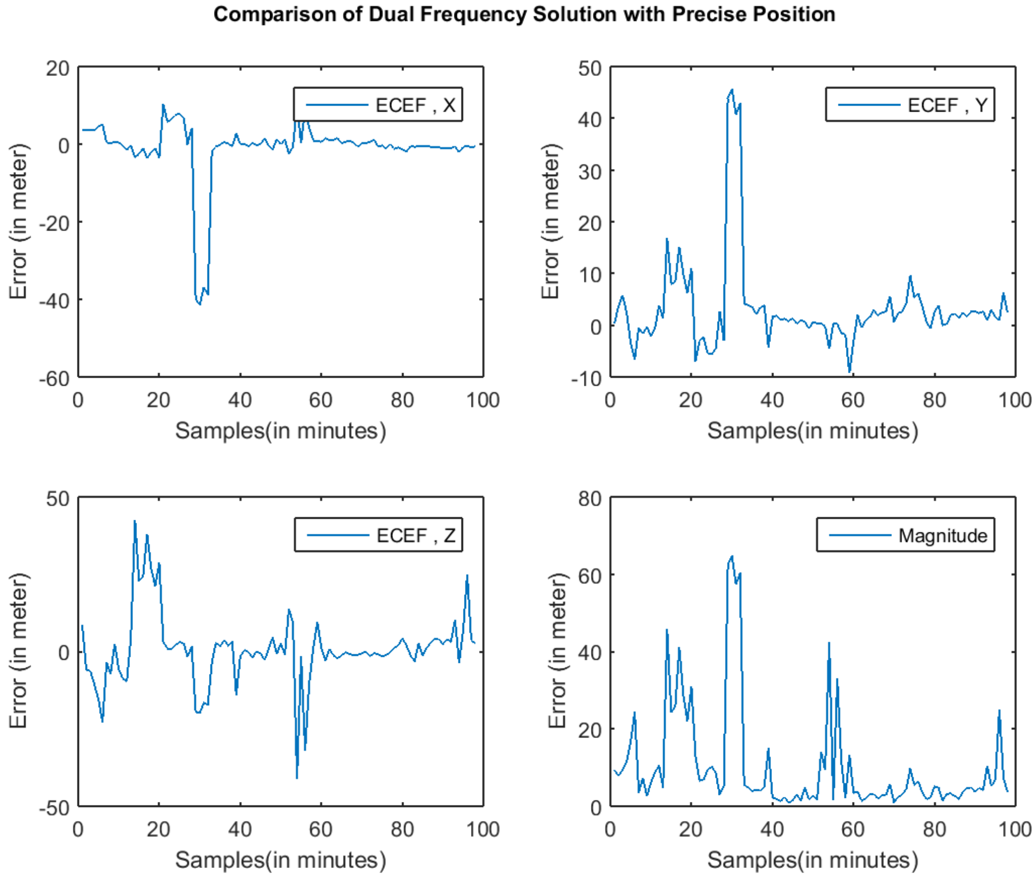

Figure 3.Comparison of Ionosphere-Free Code Observation Solution with Precise Position

Mean of magnitude of error is 8.77 meter and standard deviation is 9.59 meter.

Although it eliminates the ionospheric effect, it gives a worse solution than standard least square since combination equation increases the measurement noise.

Ionosphere-Free Phase-Code Combined Observation Solution

In this experiment, firstly ionosphere-free code and phase combination is calculated. Then, these combinations are merged.

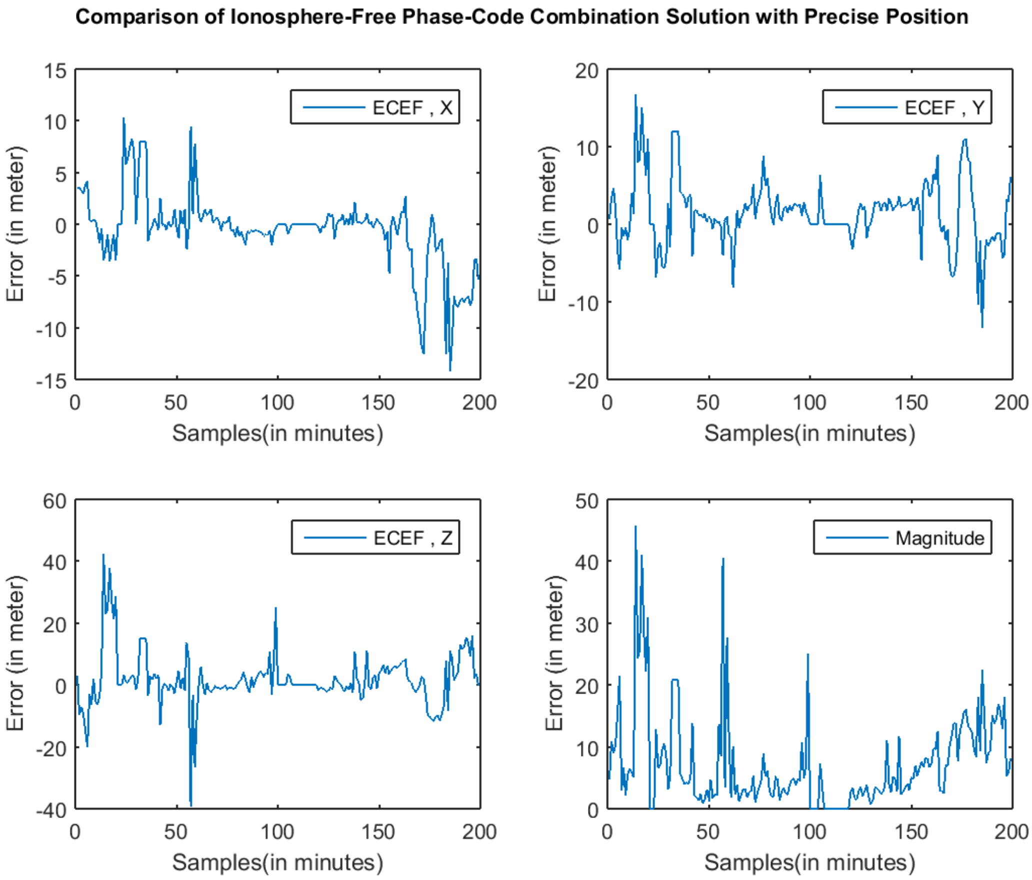

Figure 4.Comparison of Ionosphere-Free Phase-Code Combination Solution with Precise Position

Although ambiguity is resolved in this method by converging the phase measurements to code measurements, this ambiguity resolution is really coarse and adds more noise into the solution. That is why the errors are more spiky than standard least square.

Mean of magnitude of error is 7.29 meter and standard deviation is 7.70 meter.

CONCLUSION

As can be seen in Table-1, time differential phase observation does not work at all by implementing cycle slip detection algorithm on software since cycle slip detection algorithm works based on thresholding mechanism and cycle slip occurs so commonly. To overcome this problem and solve ambiguity, lambda method is generally used by using double receiver which has a large distance between them [Inside Gnss]. However, having double receiver which far from each other is not possible on satellite.

Although ionosphere-free equation eliminates the ionospheric delay, it makes solution noisier since combined equation makes noise tripled (if L1 and L2 are used). That is why its solutions are worse than standard least square.

Although ionosphere-free solution is noisier, combination with phase measurements makes solution a little bit better. Although it gives a better solution, it needs much more computation to find a solution. Therefore, it is not so desired on board.

Experiment	Mean of Error (m)	Standard Deviation of Error (m)
code observation	4.24	4.414
time differential phase observation	715.45	823.45
ionosphere-free code observation	8.77	9.59
ionosphere-free phase – code combined observation	7.29	7.70

Table 1.Comparison of Errors in four experiments

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Rinex File Format : https://igscb.jpl.nasa.gov/igscb/data/format/rinex211.txt

Standard Product: ftp://igs.org/pub/data/format/sp3_docu.txt

Appendixes

Code Observation Equation Linearization

Since geometric range is a nonlinear function (Equation 6) with respect to receiver position (X, Y, Z), nonlinear least square will be used in this paper and that equation will be linearized and receiver position will be solved iteratively [He, Y., & Bilgic, A., 2011].

$$g(X,Y,Z) = \sqrt{(X^k - X)^2 + (Y^k - Y)^2 + (Z^k - Z)^2} \tag{2}$$

If Taylor expansion is applied to g(X,Y,Z) around (X_{itr}, Y_{itr}, Z_{itr}) and higher order terms are neglected , the next iteration will be

$$\begin{gathered} g(X_{itr+1}, Y_{itr+1}, Z_{itr+1}) = g(X_{itr}, Y_{itr}, Z_{itr}) \\ + \frac{\partial g(X_{itr}, Y_{itr}, Z_{itr})}{\partial X_{itr}} * \Delta X_{itr} + \frac{\partial g(X_{itr}, Y_{itr}, Z_{itr})}{\partial Y_{itr}} * \Delta Y_{itr} + \frac{\partial g(X_{itr}, Y_{itr}, Z_{itr})}{\partial Z_{itr}} * \Delta Z_{itr} \end{gathered} \tag{3}$$

where

$$\frac{\boldsymbol{\partial g(X_{itr}, Y_{itr}, Z_{itr})}}{\boldsymbol{\partial X_{itr}}} = -\frac{\boldsymbol{X^k - X_{itr}}}{\boldsymbol{q_{itr}{}^k}}$$

$$\frac{\boldsymbol{\partial g(X_{itr}, Y_{itr}, Z_{itr})}}{\boldsymbol{\partial Y_{itr}}} = -\frac{\boldsymbol{Y^k - Y_{itr}}}{\boldsymbol{q_{itr}{}^k}}$$

$$\frac{\boldsymbol{\partial g(X_{itr}, Y_{itr}, Z_{itr})}}{\boldsymbol{\partial Z_{itr}}} = -\frac{\boldsymbol{Z^k - Z_{itr}}}{\boldsymbol{q_{itr}{}^k}}$$

Therefore, linearized code phase pseudo-range observation equation becomes

$$\begin{gathered} P_{itr}{}^k = q_{itr}{}^k - \frac{X^k - X_{itr}}{q_{itr}{}^k} * \Delta X_{itr} + - \frac{Y^k - Y_{itr}}{q_{itr}{}^k} * \Delta Y_{itr} + - \frac{Z^k - Z_{itr}}{q_{itr}{}^k} * \Delta Z_{itr} \\ + c\left(dt_{itr} - dt_{itr}{}^k\right) + T_{itr}{}^k + l_{itr}{}^k + e_{itr}{}^k \end{gathered} \tag{4}$$