KALMAN FILTER BASED LOOSELY-COUPLED GPS/INS INTEGRATION FOR **BOEING-747 AIRCRAFT MODEL**

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ABSTRACT

Inertial Navigation Systems (INS's) require some type of aiding and updating in the long term to remain valid. In many applications for purpose of INS's error compensation its integration by GPS is preferred. A Kalman filter algoritm may be used for integration of INS and GPS. The Kalman filter forms estimates of the errors in position, velocity and attitude, as well as inertial sensor biases, scale-factor errors and misalignments.

In this study the Kalman filter based loosely-coupled integrated INS/GPS is designed and investigated on the Boeing-747 Aircraft Model.

INTRODUCTION

Inertial navigation uses gyroscopes and accelerometers to maintain an estimate of the position, velocity, attitude, and attitude rates of the vehicle in or on which the INS is carried, which could be a spacecraft, missile, aircraft, surface ship, submarine, or land vehicle [5].

After compensation for sensor errors and gravity, the accelerometers outputs are integrated once and twice to obtain velocity and position, respectively. The velocity errors are excited by accelerometer errors (primarily bias and scale factor) and imprecision in knowing local gravity, and the attitude errors are significant due to gyro precession.

Dominant INS errors are caused by imperfect knowledge of initial conditions (for example, those existing after alignment) and by error propagation in time. The nine, nonlinear differential navigation equations - three from the fundamental equation of navigation, three from integrating velocity to get position, and three from the equation for direction cosine matrix rate of change - can be perturbed by a wide variety of error sources, not only those resulting from incorrect initial conditions [2]. The actual differential equations for INS operation are nonlinear, but the error equations are valid for linearized versions of these differential equations; hence, the requirement for the errors themselves to remain small, otherwise a linear analysis is not valid.

The performance of an INS is characterized by a time-dependent drift in the accuracy of the position estimates it provides. The rate at which navigation errors grow over long periods of time is governed predominantly by the accuracy of the inertial alignment, imprefections in the

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inertial sensors that the system uses and the dynamics of the trajectory followed by the host vehicle. Whilst improved accuracy can be achieved through the use of more accurate sensors, there are limits to the performance that can reasonably be achieved before the cost of the inertial system becomes prohibitively large.

For many vehicles requiring a navigation capability, there are two basic but conflicting requirements to be considered by the designer, namely those of achieving high accuracy and low cost. Many works [5,9,11] examine the scope satisfying these demanding requirements by using integrated navigation systems, in which inertial navigation systems are used in conjunction with other navigation aids. The variety of modern navigation aids now available is extensive and, coupled with advances in estimation processing techniques and high-speed computer processors, have resulted in greater application of integrated navigation systems in recent years.

To compensate inertial sensors errors, INS can be aided with information obtained from external sensors. For this purpose integrated navigation systems with INS may be used. In such way aided inertial system, one or more of the inertial navigation system output signals are compared with independent measurements of identical quantities derived from an external source. Corrections to the inertial navigation system are then derived as functions of these measurement differences. By judicious combination of this information, it is possible to achieve more accurate navigation than would be achieved using the inertial system in isolation. Navigation aiding of this type may be provided by baro or/and radar altimeters, Doppler radar, airspeed indicators, GPS, magnetic sensors etc. [10-11]. Such sensors may be used to provide attitude, velocity or position updates, any of which may be used to improve the performance quality of the inertial navigation system.

Compensation of output errors of inertial sensors can be performed via calibration procedure. Calibration is the process of observing the gyroscope outputs with known inputs and using that data to fit the unknown parameters of mathematical models for the outputs (including errors) as functions of the known inputs. This relationship is inverted for error compensation (i.e., determining the true inputs as functions of the corrupted outputs) [2].

Integrated GPS/INS applications effectively perform sensor error model calibration "on the fly" using sensor error models, sensor data redundancy, and a Kalman filter. A Kalman filter algorithm may be used for the integration of different measurement data with inertial measurements. It may be possible to allow some minor relaxation in pre-flight alignment accuracy and in the precision of the inertial sensors. Such techniques can be extended to achieve a measure of sensor calibration as part of the aiding process. The Kalman filter forms estimates of the errors in position, velocity and attitude, as well as inertial sensor biases, scale-factor errors and misalignments.

The errors of INS are increased with passing of the time so some type of aiding is required to compensate errors. GPS integration is mostly used to minimize INS's error in many applications.

In this study the Kalman filter based loosely-coupled integrated INS/GPS is designed and investigated on the Boeing-747 Aircraft Model.

Error Model of INS

In the study, the error model of INS is obtained from 9 parameters which are errors of position, velocities and attitudes with respect to coordinates in x, y and z directions.

The INS model error state vector is given in a below:

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} & \boldsymbol{Z} & \boldsymbol{V}_{x} & \boldsymbol{V}_{y} & \boldsymbol{V}_{z} & \boldsymbol{\psi}_{x} & \boldsymbol{\psi}_{y} & \boldsymbol{\psi}_{z} \end{bmatrix}^{T}$$

The position errors of INS are : [X, Y, Z] The velocity errors of INS are : $[V_x, V_y, V_z]$ The attitude errors of INS are : $[\psi_x, \psi_y, \psi_z]$

The differential equations of INS errors are found from difference between true and estimated values. The equations are shown in the following [11]:

$$\Delta \dot{x}_{INS} = \Delta V_x + \left(\frac{V_x}{R}\tan\left(\varphi\right)\right) \Delta_y + \frac{V_x}{R}\Delta_x$$
(1)

$$\Delta \dot{y}_{INS} = \Delta V_y - \left(\frac{V_x}{R}\tan(\varphi)\right) \Delta_x + \frac{V_x}{R} \Delta_x$$
(2)

$$\Delta \dot{z}_{INS} = \Delta V_z - \left(\frac{V_y}{R} \tan\left(\varphi\right)\right) \Delta_x + \frac{V_x}{R} \Delta_x$$
(3)

$$\Delta \dot{V}_{xINS} = -\frac{g}{R} \Delta_x + \left(2\omega_{ie}\sin(\varphi) + \frac{V_x}{R}\tan(\varphi)\right) \Delta V_y + (2\omega_{ie}\cos(\varphi) + \frac{V_x}{R}) \Delta V_z + a_z \psi_y - a_y \psi_z + \nabla a_x$$
(4)

$$\Delta \dot{V}_{yINS} = -\frac{g}{R} \Delta_y - \left(2\omega_{ie}\sin\left(\varphi\right) + \frac{V_x}{R}\tan\left(\varphi\right)\right) \Delta V_x + \frac{V_y}{R} \Delta V_z + a_x \psi_z - a_z \psi_x + \nabla a_y$$
(5)

$$\Delta \dot{V}_{zINS} = 2\frac{g}{R}\Delta_z - \frac{V_y}{R}\Delta V_y - (2\omega_{ie}\cos(\varphi) + \frac{V_x}{R})\Delta V_x + a_y\psi_x - a_x\psi_y + \nabla a_z$$
(6)

$$\dot{\psi}_{xINS} = \left(\omega_{ie}\sin\left(\varphi\right) + \frac{V_x}{R}\tan\left(\varphi\right)\right)\psi_y + \left(\omega_{ie}\cos\left(\varphi\right) + \frac{V_x}{R}\right)\psi_z + \varepsilon_x$$
(7)

$$\dot{\psi}_{yINS} = -\left(\omega_{ie}\sin\left(\varphi\right) + \frac{V_x}{R}\tan\left(\varphi\right)\right)\psi_x + \frac{V_y}{R}\psi_z + \varepsilon_y$$
(8)

$$\dot{\psi}_{zINS} = -\frac{V_y}{R}\psi_y - \left(\omega_{ie}\cos\left(\varphi\right) + \frac{V_x}{R}\right)\psi_x + \varepsilon_z \tag{9}$$

The error model of INS can be expressed in a discrete matrix form in a below:

	$1 + \frac{V_x}{R}$	$\frac{V_x \tan \phi}{R}$	0	1	0	0	0	0	0		
	$\frac{-V_x \tan \phi + V_x}{R}$	1	0	0	1	0	0	0	0		
$\begin{bmatrix} X(k+1) \\ Y(k+1) \\ Z(k+1) \\ V_x(k+1) \\ V_y(k+1) \\ V_z(k+1) \end{bmatrix} =$	$\frac{-V_y \tan \phi + V_x}{R}$	0	1	0	0	1	0	0	0		
	$\frac{-g}{R}$	0	0	1	$\frac{2\omega_{ie}\sin\phi}{\frac{V_x\tan\phi}{R}}$	$\frac{2\omega_{ie}\cos\phi}{\frac{V_x\tan\phi}{R}}$	0	a_z	$-a_y$	$\begin{array}{c} X(k) \\ Y(k) \\ Z(k) \end{array}$	0 0 0
	0	$\frac{-g}{R}$	0	$\frac{-2\omega_{ie}\sin\phi}{-\frac{V_x\tan\phi}{R}}$	1	$\frac{V_y}{R}$	$-a_z$	0	<i>a</i> _{<i>x</i>}	$\begin{array}{c c} V_x(k) \\ x & V_y(k) \\ V_z(k) \end{array} + $	$= \begin{vmatrix} w_{Vx}(k) \\ w_{Vy}(k) \\ w_{Vz}(k) \end{vmatrix}$
$\psi_{x}(k+1)$ $\psi_{y}(k+1)$ $\psi_{z}(k+1)$	0	0	$\frac{2g}{R}$	$-2\omega_{ie}\cos\phi$ $-\frac{V_x}{R}$	$-\frac{V_y}{R}$	1	a _y	$-a_x$	0	$ \begin{array}{c} \psi_x(k) \\ \psi_y(k) \\ \psi_z(k) \end{array} $	$ \begin{array}{c} w_{\psi x}(k) \\ w_{\psi y}(k) \\ w_{\psi z}(k) \end{array} $
	0	0	0	0	0	0	1	$-(\omega_{ie}\sin\phi+\frac{V_x}{R})$	$\omega_{ie}\cos\phi + \frac{V_x}{R})$		
	0	0	0	0	0	0	$-(\omega_{ie}\sin\phi+\frac{V_x}{R})$	1	$\frac{V_y}{R}$		
	0	0	0	0	0	0	$-(\omega_{ie}\sin\phi+\frac{V_x}{R})$	$-\frac{V_y}{R}$	1		

In the INS error model the the parameter of Vx, Vy and Vz are taken from directly aircraft steady-level flight and the acceleration for each axis is zero. The gravitational accelaration (g) which is taken 32.174 ft/s^2 . The symbol of R is the radius of the earth is 20.900.000 feet (6 378.1 km) and ω_{i_e} is 0.0000728.

The position, velocity and attitude errors of INS can be measured by using Kalman Filtering method. The observation vectors are determined for 9 different state (position, velocity, attitude).

The mathematical model of INS errors is used to determine real values of errors. The system model is where ϕ is the transfer matrix of INS error model and w is composed by noises.

$$x(k+1) = \phi(k+1,k)x(k) + w(k)$$
(10)

The observation vector of positions, velocities and attitudes are defined in equations as following;

$$z_{INS}(k) = H_{INS}x(k) + v_{INS}(k) + \lambda_{INS}(k)$$
(11)

$$z_{GPS}(k) = H_{GPS}x(k) + v_{GPS}(k)$$
(12)

 $z_{INS}(k)$ is the vector of INS measurements; H_{INS} is the measurement matrix of INS; $v_{INS}(k)$ is the INS measurement noise vector, $\lambda(k)$ is the INS bias errors process, $z_{GPS}(k)$ is the GPS measurements vector; H_{GPS} is the GPS measurement matrix; $v_{GPS}(k)$ is the GPS measurement noise vector. Assume that random vectors $w(k) v_{INS}(k)$ and $v_{GPS}(k)$ are a Gaussian white noise.

The measurement equations is written in matrix form:

$$z(k) = \begin{bmatrix} z_{1}(k) \\ z_{2}(k) \\ z_{3}(k) \\ z_{4}(k) \\ z_{5}(k) \\ z_{6}(k) \\ z_{7}(k) \\ z_{8}(k) \\ z_{9}(k) \end{bmatrix} = \begin{bmatrix} z_{x_{INS}} - z_{x_{GPS}} \\ z_{y_{INS}} - z_{y_{GPS}} \\ z_{z_{INS}} - z_{Vx_{GPS}} \\ z_{Vy_{INS}} - z_{Vy_{GPS}} \\ z_{\psi x_{INS}} - z_{\psi x_{GPS}} \\ z_{\psi y_{INS}} - z_{\psi y_{GPS}} \\ z_{\psi y_{INS}} - z_{\psi y_{GPS}} \\ z_{\psi z_{INS}} - z_{\psi z_{GPS}} \\ z_{\psi z_{INS}} - z_{\psi z_{GPS}} \\ z_{\psi z_{INS}} - z_{\psi z_{GPS}} \end{bmatrix}$$
(13)

The sum of standard deviations of the INS and GPS comprises the covariance matrix of measurement noise R(k) which is given a below:

$$R(k) = diag(\sigma_{x_{INS}}^{2} + \sigma_{x_{GPS}}^{2} - \sigma_{y_{INS}}^{2} + \sigma_{y_{GPS}}^{2} - \sigma_{z_{INS}}^{2} + \sigma_{z_{GPS}}^{2} - \sigma_{y_{X_{INS}}}^{2} + \sigma_{y_{X_{GPS}}}^{2} - \sigma_{y_{y_{INS}}}^{2} + \sigma_{y_{y_{GPS}}}^{2}$$

$$(14)$$

$$\sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{x_{INS}}}^{2} + \sigma_{y_{x_{GPS}}}^{2} - \sigma_{y_{y_{INS}}}^{2} + \sigma_{y_{y_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} + \sigma_{y_{z_{GPS}}}^{2} - \sigma_{y_{z_{INS}}}^{2} - \sigma_$$

Boeing 747 aircraft model

The Boeing 747 aircraft model is as follows :

$$x(k+1) = Ax(k) + Bu(k) + Gw(k)$$
(15)

$$y(k) = H(k)x(k) + v(k)$$
 (16)

Where x is the state vector, A is the system matrix, B is the control distribution matrix, G is the process noise transition matrix, u(k) is the control input vector, y the vector of measurements, H is the measurement distribution matrix, w(k) is the process noise and v(k) is the sensor noise.

The aircraft state variables are:

$$\mathbf{x}(k) = [u(k) \, w(k) \, q(k) \, \theta(k) \, v(k) \, p(k) \, r(k) \, \phi(k) \, \psi(k)]^T$$
(17)

Where u is the forward velocity, w is the normal velocity, q is the pitch rate, θ is the pitch angle, v is the lateral velocity, p is the roll rate, r is the yaw rate, ϕ is the roll angle and ψ is the yaw angle. The units of the angles (θ , ϕ , ψ) in degrees. The units of the angle rates (q, *p*, *r*) is *deg/s*. Finally, the unit of the velocity *v*, *u* and *w* is *nm/hr*.

The control input vector is:

$$u(k) = \left[\delta_E(k) \,\delta_F(k) \,\delta_A(k) \,\delta_R(k) \right] \tag{18}$$

1.0000 0.0000 -0.03220.6730 -0.0001 0.9995 0.9994 0.0010 -0.0010 0 (19)A =-0.00270.0003 0.0012 0.9992 0.9998 Λ 0.0010 0.0008 -0.0046 -0.0030 (20)B =0.0011 0.006 0 -0.0008 -0.006 Ô O

Where δ_E , δ_F , δ_A and δ_R are elevator, flap, aileron and rudder deflections, respectively and the units are radians.

To avoid aircraft model being unstable, a linear quadratic controller is added to the aircraft dynamic model.

Kalman filtering for integrated navigation

The Kalman Filtering is used widely in navigation systems. The Kalman filter minimizes the measurement error such as accelerometers, gyroscopes and GPS sensor errors. To find best value of measured parameters the filter estimates and calculates the state vector of system and integrates systems such as INS and GPS.

It assumes linear discrete dynamic system is taken. Dynamics of system is identified by system's dynamic state equation where measurement equation is used to measure the system. Linear system equations are written in a below;

State equation :

$$x(k+1) = \Phi(k+1,k)x(k) + G(k+1,k)w(k)$$
(21)

Measurement equation

$$z(k) = H(k)x(k) + v(k)$$
(22)

x(k) : N- dimensional state vector of system

 $\Phi(k+1,k)$: N x N transfer matrix of system

G(k+1,k) : N x R dimensional transfer matrix of the system noise

z(k) : S dimensional measurement vector

- H(k) : S x N dimensional measurement matrix of the system
- v(k): S dimensional measurement noise vector with zero mean and correlation matrix;

$$E[v(k)v^{T}(j)] = R(k)\delta(kj)$$

w(k): R dimensional random Gaussian noise vector (system noise) with zero mean and correlation matrix

 $E[w(k)w^{T}(j)]$: correlation matrix, E is static average operator $\delta(kj)$ is Kroneker symbol

$$E[w(k)w^{T}(j)] = Q(k)\delta(kj)$$

$$\delta(kj) = \{1, k = j; 0, k \neq j\}$$

The following equations composes Kalman Filtering algorithm which are used to estimating of state vectors [6]. The initial conditions are $x(0/0) = \overline{x(0)}$ and P(0/0) = P(0).

$$\hat{x}(k/k) = \Phi(k,k-1)\hat{x}(k-1/k-1) + K(k) [y(k) - H(k)\Phi(k,k-1)\hat{x}(k-1/k-1)]$$

= $\hat{x}(k/k-1) + K(k)\Delta(k/k-1)$ (23)

$$\hat{x}(k/k-1) = \Phi(k,k-1)\hat{x}(k-1/k-1)$$
(24)

$$\Delta(k/k-1) = z(k) - H(k)\hat{x}(k/k-1)$$
(25)

$$P(k/k) = P(k/k-1) - K(k)H(k)P(k/k-1) = (I - K(k)H(k))P(k/k-1)$$
(26)

$$P(k/k-1) = \Phi(k,k-1)P(k-1/k-1) \Phi^{T}(k,k-1) + G(k,k-1)Q(k-1)G^{T}(k,k-1)$$
(27)

$$K(k) = P(k/k)H^{T}(k)R^{(-1)}(k) = P(k/k-1)H^{T}(k)[H(k)P(k/k-1)H^{T}(k) + R(k)]^{-1}$$
(28)

In the equations; x(k / k) is the estimation value, I is the unity matrix, K(k) is the gain matrix of filter, $\Delta(k)$ is innovation sequence, P(k/k) is the correlation matrix of estimation error, P(k/k-1) is the correlation matrix of extrapolation error, H(k) is measurement matrix of the system, R(k) is correlation matrix of measurement noise, Q(k) is correlation matrix of system noise, G(k) is transfer matrix of the system noise, the index k/k-1 indicates that one step predicted values; where k/k symbolise the estimate values at time k using all measurements including z(k).

Simulation results

In the study Kalman Filter is used to estimate errors of state variables and simulation results show errors of positions, velocities and attitudes for three axes. The positions, velocities and attitudes error of integrated system are shown in simulation results.

The noise correlation matrix is Q(k) and initial correlation matrix is P(0/0) of Kalman Filter given below:

$$Q(k) = 10I_{9\times9}$$
, $P(0/0) = 10I_{9\times9}$

The simulation results are given in Figs. 1-6. The figures 2, 4 and 6 illustrate difference between model and estimation and they prove success of Kalman Filter algorithm.







Figure 2: Difference between model and estimation for positions error



Figure 3: Estimation of velocity error of INS for X, Y and Z axis



Figure 4: Difference between model and estimation for velocities error

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Figure 5: Estimation of angle error of INS for X, Y and Z axis



Figure 6: Difference between model and estimation for angle error

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CONCLUSION

The performance of an INS is characterized by a time-dependent drift in the accuracy of the position estimates it provides. To compensate inertial sensors errors, INS can be aided with information obtained from external sensors. For this purpose integrated navigation systems with INS may be used. In this study the Kalman Filter is used to integrate loosely-coupled INS/GPS for Boeing-747 aircraft model. The estimation of INS errors of position, velocity and attitude for each axis is performed and investigated. The simulation results show that the integrated INS/GPS navigation system have high accuracy.

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