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ANALYSIS, DESIGN, AND TEST OF THE BASE ISOLATION SYSTEMS FOR SPACECRAFT TESTING SYSTEMS

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ABSTRACT

The vibration isolation for vibration induced testing systems, such as, vibration testing and acoustic testing systems are crucially important for massive spacecraft testing systems. In this study, the base isolation systems for mechanical testing systems for spacecraft are analyzed using resiliently mounted rigid body dynamics. The general equation of motion is based 3-D rigid-body with 6-DOF generalized coordinates. The mass, stiffness and damping matrices are derived using rigid body dynamics with small deformation assumption. The model is compared with test results of real spacecraft vibration testing system. The results can be used for design and analysis of general base isolation systems.

INTRODUCTION

Main spacecraft mechanical tests causing vibrations during tests are vibration and acoustic tests. The shakers and chambers have to isolated from the surrounding due to high vibration levels. The vibration testing systems are used to simulate the vibrations caused by launchers and phases of launching operations and determine fundamental vibration frequencies of the spacecraft and subsystems. The test systems can be large and massive because it houses the spacecraft and testing systems. For example, a vibration testing system which can test a spacecraft with launch mass up to 5 tons, is around 350 tons. Similarly, the acoustic reverberation chamber is around 1000 metric tons. The test systems have to be isolated at a certain level so that vibrations do not cause any interference with other systems and satisfy acceptable level working conditions such as maximum 80 dB noise level.

There are numerous studies on vibration isolation systems (Harris and Crede, 1991, ASHRAE 1999). Analytical model for isolation systems can be represented by rigid body supported by elastic springs and dampers. A rigid body is a 6 DOF system with 3 translations and 3 rotations. Static and dynamic behavior of an elastically supported rigid body can be modelled as a 6-DOF system with elastic supports. If the structure unsymmetrical and irregularly supported, the system becomes coupled and analytical solution for static and dynamic analysis becomes cumbersome. In some past studies, the governing equations for isolation systems were derived using dynamic/static equilibrium equations for each direction of 3-D. [Harris and Crede, 1961]. Direct matrix methods are another approach and widely used in formulation of governing equation [Smollen, 1966]. Alternative derivations using rigid body dynamics can be

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illustrative and innovative way for the mechanics literature. This idea is starting point of this study. In the following sections, the base isolation systems for mechanical testing systems for spacecraft are analyzed using resiliently mounted rigid body dynamics. The general equation of motion is based 3-D rigid-body with 6-DOF generalized coordinates. The mass, stiffness and damping matrices are derived using rigid body dynamics with small deformation assumption. The model is compared with test results of real spacecraft vibration testing system. The results can be used for design and analysis of general base isolation systems.

THEORETICAL MODEL FOR ANALYSIS OF BASE ISOLATION SYSTEMS

Consider an arbitrarily shaped rigid body attached to the rigid ground via mutually perpendicular elastic springs with dampers at some connection points. A Cartesian coordinate system o-xyz is selected as a fixed inertial coordinate system and its origin is assigned as the center of gravity of the whole system including the block and testing system on it. The model and related coordinate systems are shown in the Figure 1. The ground plane selected as x-y plane and consequently, the z-axis becomes perpendicular to the ground plane.

The rigid body motion in 3-D is represented with a 6-DOF system which are three translations along and three rotations around three perpendicular directions. They also represent generalized coordinates in dynamic and static analysis of rigid bodies. The rotations around the x and y axes are usually termed rocking and rotation around the z axis is named yawing. The local body moving coordinate system coincides with the inertial coordinate system when the system is at rest.

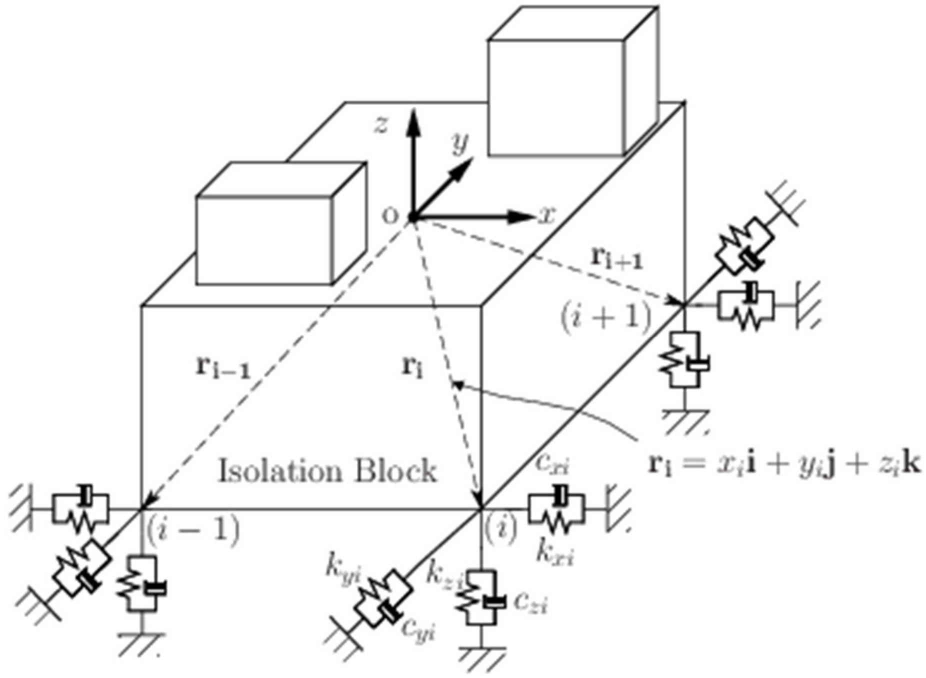


Figure 1: Typical base isolation system

The body has *N*-number of support points and is subjected to a dynamic load causing small translations and rotations. In that case, order of rotations becomes mathematically unimportant. When the center of mass is taken as origin, position of a support at the point-*n* on the body can be represented by a position vector $\mathbf{r}_n = x_n\mathbf{i} + y_n\mathbf{j} + z_n\mathbf{k}$ with respect to the origin at rest. Displacement of the point-*n* $\mathbf{u}_n = u_n\mathbf{i} + v_n\mathbf{j} + w_n\mathbf{k}$ is due to rigid body translation $\mathbf{u}_o = u_o\mathbf{i} + v_o\mathbf{j} + w_o\mathbf{k}$ and rotation $\boldsymbol{\theta} = \theta_x\mathbf{i} + \theta_y\mathbf{j} + \theta_z\mathbf{k}$ of the body around the center of mass.

Using rigid body motion displacement concept, total displacement of the point-*n* can be written in matrix form of

$$\mathbf{u}_n = \mathbf{I}\mathbf{u}_o - \mathbf{R}_n\boldsymbol{\theta} = \begin{bmatrix} u_n \\ v_n \\ w_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_o \\ v_o \\ w_o \end{bmatrix} - \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix}\begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} \tag{1}$$

where $\mathbf{I}$ and $\mathbf{R}_n$ are identity matrix and skew-symmetric matrix form of the position vector $\mathbf{r}_n$ respectively [Angeles, 2002]. The right-side of the Eq. (1) can be cast into one matrix and can be written as

$$\mathbf{u}_n = \mathbf{I}\mathbf{u}_o - \mathbf{R}_n\boldsymbol{\theta} = \begin{bmatrix} \mathbf{I} & \mathbf{R}_n^{\mathbf{T}} \end{bmatrix}\mathbf{q} = \begin{bmatrix} 1 & 0 & 0 & 0 & z_n & -y_n \\ 0 & 1 & 0 & -z_n & 0 & x_n \\ 0 & 0 & 1 & y_n & -x_n & 0 \end{bmatrix}_i \begin{bmatrix} \mathbf{u}_o \\ - \\ \boldsymbol{\theta} \end{bmatrix} \tag{2}$$

where $\mathbf{q}^T = \begin{bmatrix} u_o & v_o & w_o & \theta_x & \theta_y & \theta_z \end{bmatrix}^T$ are generalized coordinates of the system. The forces are induced to springs when the base isolation system is subjected to an external excitation. Assuming that the spring system at every support has mutually placed springs in all global three directions. The rotational stiffness of springs at the supports is assumed to be relatively small and they are neglected. The linear elastic stiffness matrix for support becomes

$$\mathbf{k}_n = \begin{bmatrix} k_{nx} & 0 & 0 \\ 0 & k_{ny} & 0 \\ 0 & 0 & k_{nz} \end{bmatrix} \tag{3}$$

The forces due to displacement and rotation of the system becomes

$$\mathbf{f}_n = \mathbf{k}_n\mathbf{u}_n = \mathbf{k}_n\begin{bmatrix} \mathbf{I} & \mathbf{R}_n^{\mathbf{T}} \end{bmatrix}\mathbf{q} = \begin{bmatrix} \mathbf{k}_n\mathbf{I} & \mathbf{k}_n\mathbf{R}_n^T \end{bmatrix}\mathbf{q} = \begin{bmatrix} \mathbf{K}_{tt,n} & \mathbf{K}_{tr,n} \end{bmatrix}\mathbf{q} \tag{4}$$

where the sub-stiffness matrices $\mathbf{k}_{tt.n} = \mathbf{k}_n\mathbf{I}$ and $\mathbf{k}_{tr.n} = \mathbf{k}_n\mathbf{R}_n^T$ are corresponding to the translation and rotation-translation coupling for the spring system at the point-*n* respectively. Similarly, the moment $\mathbf{t_n}$ caused by the forces on the springs is the cross product of the position vector $\mathbf{r_n}$ and the force $\mathbf{f}_n$ can be written as

$$\mathbf{t}_n = \begin{bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{bmatrix} = \mathbf{r}_n \times \mathbf{f}_n = \mathbf{R}_n\mathbf{f}_n = \mathbf{R}_n\begin{bmatrix} \mathbf{k}_n\mathbf{I} & \mathbf{k}_n\mathbf{R}_n^T \end{bmatrix} = \begin{bmatrix} \mathbf{R}_n\mathbf{k}_n\mathbf{I} & | & \mathbf{R}_n\mathbf{k}_n\mathbf{R}_n^{\mathbf{T}} \end{bmatrix}\mathbf{q} \tag{5}$$

$$\mathbf{t}_n = \begin{bmatrix} \mathbf{K}_{rt,n} & \mathbf{K}_{rr,n} \end{bmatrix}\mathbf{q} \tag{6}$$

Then stiffness matrix for the support-n can written as

$$\mathbf{F}_n = \mathbf{K}_n\mathbf{q} = \begin{bmatrix} \mathbf{f}_n \\ - \\ \mathbf{t}_n \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{tt,n} & | & \mathbf{K}_{rt,n} \\ - & - & - \\ \mathbf{K}_{tr,n} & | & \mathbf{K}_{rr,n} \end{bmatrix}\mathbf{q} \tag{7}$$

The stiffness matrix can be explicitly written as

$$\mathbf{K}_n = \begin{bmatrix} k_{nx} & 0 & 0 & 0 & z_n k_{nx} & -y_n k_{nx} \\ 0 & k_{ny} & 0 & -z_n k_{ny} & 0 & x_n k_{ny} \\ 0 & 0 & k_{nz} & y_n k_{nz} & -x_n k_{nz} & 0 \\ 0 & -z_n k_{ny} & y_n k_{nz} & y_n^2 k_{nz} + z_n^2 k_{ny} & -x_n y_n k_{nz} & -x_n z_n k_{ny} \\ z_n k_{nx} & 0 & -x_n k_{nz} & -x_n y_n k_{nz} & x_n^2 k_{nz} + z^2 k_{nx} & -y_n z_n k_{nx} \\ -y_n k_{nx} & x_n k_{ny} & 0 & -x_n z_n k_{ny} & -y_n z_n k_{nx} & x_n^2 k_{ny} + y_n^2 k_{nx} \end{bmatrix} \tag{8}$$

The global stiffness matrix can be assembled by simply summation of all spring contributions

$$\mathbf{F}_G = \mathbf{K}_G\mathbf{q} = \sum_{n=1}^{N}\mathbf{F}_n = \left(\sum_{n=1}^{N}\mathbf{K}_n\right)\mathbf{q} = \begin{bmatrix}\mathbf{f}_G \\ - \\ \mathbf{t}_G\end{bmatrix} = \left(\sum_{n=1}^{N}\begin{bmatrix}\mathbf{K}_{tt,n} & | & \mathbf{K}_{rt,n} \\ - & - & - \\ \mathbf{K}_{tr,n} & | & \mathbf{K}_{rr,n}\end{bmatrix}\right)\mathbf{q} \tag{9}$$

The same analogy can be used to obtain damping matrix as $\mathbf{F}_C = \mathbf{C}\dot{\mathbf{q}}$. Recall that the formulation derived above is based on the assumption that the mass center is the origin of the coordinate system. The mass matrix is independent of the location of supports and it becomes

$$\mathbf{M}_G = \begin{bmatrix}\mathbf{M} & | & \mathbf{0} \\ - & - & - \\ \mathbf{0} & | & \mathbf{J}\end{bmatrix} = \begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & I_{xy} & I_{xz} \\ 0 & 0 & 0 & I_{yx} & I_{yy} & I_{yz} \\ 0 & 0 & 0 & I_{zx} & I_{zy} & I_{zz}\end{bmatrix} \tag{10}$$

where **M** submatrix is mass matrix for the translation and ***J*** is the inertia matrix for the angular movement. The zero submatrices are due to the fact that the center of mass is taken as origin. If the structure is symmetric in mass distribution, then the product inertia values $I_{xy} = I_{yz} = I_{xz} = 0$ become zero and mass matrix becomes diagonal. More general dynamic equilibrium equation including support excitation, such as earthquake, and dynamic loading caused by the equipment on the block can be written as

$$\mathbf{M}_G\ddot{\mathbf{q}}(t) + \mathbf{C}_G\dot{\mathbf{q}}(t) + \mathbf{K}_G\mathbf{q}(t) = -\mathbf{M}_G\ddot{\mathbf{q}}_g(t) + \mathbf{F}_e(t) \tag{11}$$

where the support (or ground) excitation $\ddot{\mathbf{q}}$ vector includes all components of ground accelerations including both translational and rotational components is given as

$$\ddot{\mathbf{q}}_g(t) = \begin{bmatrix}\ddot{\mathbf{u}}_g(t) & \ddot{\boldsymbol{\theta}}_g(t)\end{bmatrix}^T = \begin{bmatrix}\ddot{u}_g(t) & \ddot{v}_g(t) & \ddot{w}_g(t) & | & \ddot{\theta}_{g,x} & (t)\ddot{\theta}_{g,y}(t) & \ddot{\theta}_{g,z}(t)\end{bmatrix}^T \tag{12}$$

and external force vector $\mathbf{F}_e$ is given as $\mathbf{F}_e = \begin{bmatrix}F_x & F_y & F_z & | & T_{g,x} & T_{g,y} & T_{g,z}\end{bmatrix}^T$. Note that both components of support excitation $\ddot{\mathbf{q}}(t)$ and the force vector **F**(t) are functions of time. For undamped free vibration analysis, the damping can be ignored and be written as

$$\mathbf{M}_G\ddot{\mathbf{q}}(t) + \mathbf{K}_\mathbf{G}\mathbf{q}(t) = \mathbf{0} \tag{13}$$

Using the separation of variables method for $\mathbf{q} = \mathbf{Q}e^{-i\omega t}$, the equation for free-vibration analysis reduces to

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]\mathbf{Q} = \mathbf{0} \tag{14}$$

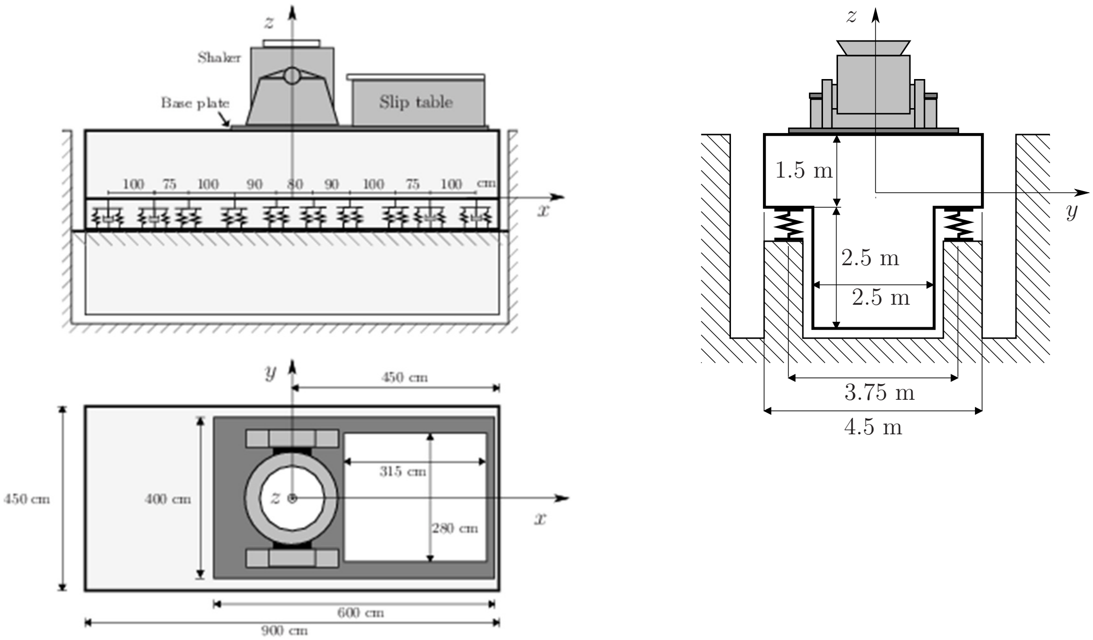
where $\mathbf{Q}^T = \left[U_x, U_y, U_z, \Theta_x, \Theta_y, \Theta_z\right]$ is mode displacement vector. This is an eigen-analysis equation for the system. The system is solved and its 6 eigenvalues and eigen-modes are obtained. For asymmetrically supported body and unevenly distributed mass, eccentricities in one, two or three axes can occur depending upon the mass and stiffness distributions. For full symmetry in all axes, the stiffness matrix becomes diagonal and greatly simplifies the eigen-analysis.

APPLICATION

The vibration isolation system for the application is an isolation system of the actual spacecraft vibration testing system specifically designed for large spacecraft up-to 5 tons. The shakers for the system are large and massive machines and require effective isolated base systems. Test limits of the system impose that the fundamental frequency must be smaller than 3 Hz and vibration isolation must be lower than a certain level. The fundamental frequency requirement comes from the fact that the vibration testing system has to be capable of

executing sine-sweep tests from 5 Hz to 2000 Hz. The second requirement is that the vibration isolation level at the point located 1 meter away from the testing system must be larger than -20 decibel. The total mass of the vibration testing system is around 350,000 kg.(Figure 2)
The inertia block of the isolation system has a shape of a deep beam with thick T-section and is made of reinforced concrete. Details of the block is shown on the Figure 3. The length and height of the block is 9 and 4 meter respectively. The web of the section has a shape of square with a size of 2.5x2.5 m and the flange size is 1.5x4.5 m. Selection of the T-section has two main advantages, first making the top of the block wider and it supports the block from flanges of the section. This shape is the optimal in terms of providing space for the equipment on top of it, and wider support area for the placement of springs and dampers under the block. The mass of the block itself.is 293000 kg.

Figure 2**:** Spacecraft vibration testing system



a) Side and top views

b) Cross-section of the system

Figure 3**:** Base isolation for spacecraft vibration testing system

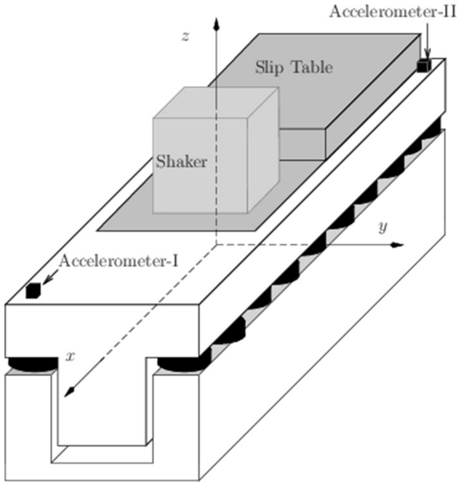


Figure 4: The base isolation system vibration test

The isolation block is supported by 20 equivalent spring systems, except 4 of them with additional viscous dampers, located by pairs along the length of the seismic block i.e. there are 10 lines of spring systems and each line has symmetric 2 spring systems (Figure 4). Each spring system has an assembly of 10 springs placed in line by pair or circular pattern (Figure 5). The spring systems with centrally located dampers are placed around each corner of the block. The stiffness values of each spring system in horizontal and vertical directions are 2470 N/mm and 1350 N/mm respectively. The masses of the spring systems without and with dampers are 230 and 327 kg respectively. The total mass of the springs around 5000 kg and can be neglected both in math and the finite element model since their mass are is less than 1.5 percent of the total mass

The payload on the block consists of the base plate, shaker and horizontal slip table. The heaviest part of the payload is shaker and it is exactly located at the middle of the block so that minimize eccentricity. On the other hand, the slip table, which have relatively smaller mass than the shaker, is located on the negative *x*-side of the block which causes relatively small eccentricity in the *x*-direction. Distance between the mass and stiffness center in vertical direction is also small because the mass center is around the level of the springs and this distance cause a small eccentricity too. Thus, the system has relatively small eccentricities both in *x* and *z* directions causing coupled behavior.

a) Spring system without damper b) Spring system with damper

Figure 5: Spring systems at the supports

Calculation
The math model of the actual test system is based on the method described in the previous sections. The coordinates of the support locations and stiffness of the springs at each direction is substituted into the related equations and mass and static stiffness matrices of the system are obtained and python programming is used to calculate the eigen-frequencies and eigen-vectors. The eigen-frequencies are found and compared with actual test results. The results agree well and shown on the Table 1.

Table 1: The actual test and analytical model comparison

Mode		**Test**	**Math Mod.**	**Test-Math**
No	Shape	f (Hz)	f (Hz)	% Error
1	Pitching	1.27	1.233	2.76
2	Rolling	1.32	1.36	-5.76
3	Rocking	1.36-140	1.42	-2.9
4	Rocking	1.84	1.882	-2.3
5	Rocking	1.97	1.893	-3.9
6	Pit. &Roll.	1.97	2.242	-13.0

CONCLUSION

In this study, the equation of motion for the dynamics analysis of vibration isolation system is derived by using the vector algebra. The static stiffness matrix for the system is derived by using rigid body motion of a massive block undergoing small deformations and supported by elastic mutually placed elastic springs. The system has 6 rigid body degree of freedom as generalized coordinates. The math model is applied to a real vibration isolation system manufactured for vibration testing of spacecraft systems and their subsystems. Unsymmetrical placement of the springs or unsymmetrical mass distribution cause eccentricity between the mass and stiffness center of the system. The eigen-analysis results obtained from math model are compared with the real test results. The results of the math model agree well with the test results. The math model can be used in design, analysis and test of the vibration isolation systems. The study can be extended to variety of related static and dynamic analysis of vibration isolation systems, such as analysis under dynamic excitation and loading.

References

Angeles, Jorge (2002) *Fundamentals of Robotic Mechanical Systems*. Springer-Verlag, New York, p: 95-138, 2002.

ASHRAE (1999) *Vibration Isolation and Control* ASHRAE Handbook, American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) p: 46.37-48*, 1999*

Harris C. M, and Crede. C. E (1961) *Theory of Vibration İsolation* in *Shock and Vibration Handbook Vol.2 McGraw-Hill p: 30.1-57 1961*

Smollen L. E (1966)*, L. E. Generalized Matrix Method for the Design and Analysis of Vibration Isolation Systems,* The Journal of the Acoustical Society of America Vol. 40 p: 195–204. 1966