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CONSTRAINED MODEL PREDICTIVE CONTROL FOR SPACECRAFT RENDEZVOUS

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ABSTRACT

Rendezvous and docking of a pair of low Earth orbit spacecraft problem is addressed. Equations for nonlinear orbital motion of both spacecrafts are derived and a simulation code for this motion is developed. To control relative motion, linearized Hill-Clohessy-Wiltshire (HCW) equations are used in chaser-target spacecraft configuration. All authority is given to the chaser spacecraft, and the target is kept passive. The HCW equations are linearized assuming a circular orbit. Model Predictive Control (MPC) strategy is applied with constraints. Simulation results are given and discussed. A parametric study is also performed to obtain the proper prediction horizon as well as weighting matrices to be used in the simulations.

INTRODUCTION

The objective of rendezvous problem between two spacecrafts is to reach a prescribed relative configuration in each other's proximity. In the literature, it is generally studied that one vehicle (chaser) is to be actively controlled, and the other (target) is kept passive like in rendezvous with a space-station or a Mars Sample Return capture scenario [Regnier et al., 2005]. The same configuration is used in this study as well. Satellite formation flying is basically concerned with the relative motion of the chaser with respect to the target, most commonly expressed in the rotating Hill reference frame. Although the actual relative dynamics of the chaser are nonlinear, and non-periodic, it is possible to derive a set of linearized ordinary differential equations which approximate the full dynamics, and have periodic solutions. These are commonly known as the Hill-Clohessy-Wiltshire (HCW) equations [Hill, 1878; Clohessy and Wiltshire, 1960]. These equations are also referred as Hill's equations or the Clohessy-Wiltshire (CW) equations in the literature.

Autonomy is a key technology in the spacecraft operations. Round-trip communication delays are too long to react to unmodeled perturbations or critical situations. This is especially more pronounced for missions around distant bodies such as Moon or Mars [Fehse, 2003].

Model Predictive Control well suits to aerospace problems due to its re-planning nature, i.e., the explicit consideration of the system dynamics and constraint-handling ability. It is allowing fuel efficient, feasible plans to be determined autonomously and online.

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Also, the disturbances in space environment such as air drag, solar pressure, and earth oblateness (J_2) effects can be compansated by MPC feedback [Di Cairano et al., 2012]. This makes it possible to use HCW model in which these effects are neglected.

The purpose of this paper is to investigate the relative motion control problem of the rendezvous and docking of a pair of spacecraft in Low Earth Orbit (LEO). In particular, to investigate the applicability of the constrained MPC approach and its effectiveness in rendezvous and docking of orbiting spacecraft.

This paper's organization can be summarized as follows. In the following section, equations of motion of orbital relative motion is given. Both nonlinear models, and linear models are introduced. Then, the Model Predictive Control (MPC) method with constraints that are needed for spacecraft rendezvous is explained. Simulations with different tuning parameters and initial conditions are given and discussed next. Finally, conclusions are given.

METHOD

Orbital Relative Motion Model

Many mathematical models for spacecraft rendezvous may be found in the literature [Carter, 1998]. The choice depends on the parameters of scenario. In simulations it is more realistic to use general nonlinear equations of the relative motion between chaser spacecraft and passive target vehicle. These equations may be written for circular orbits with some assumptions [Wie, 1998]:

$$\begin{aligned}
\ddot{x} &= 2n\dot{y} + n^2(R+x) - \mu \frac{R+x}{\left[(R+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + u_x \\
\ddot{y} &= -2n\dot{x} + n^2 y - \mu \frac{y}{\left[(R+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + u_y \\
\ddot{z} &= -\mu \frac{z}{\left[(R+x)^2 + y^2 + z^2\right]^{\frac{3}{2}}} + u_z
\end{aligned} \tag{1}$$

Where x, y, and z are the components of the chaser spacecraft position relative to the target in the local vertical, local horizontal (LVLH) frame. In this frame, the x direction is radial, y is along-track, and z completes the orthogonal set. Figure 1 shows the corresponding LVLH frame.

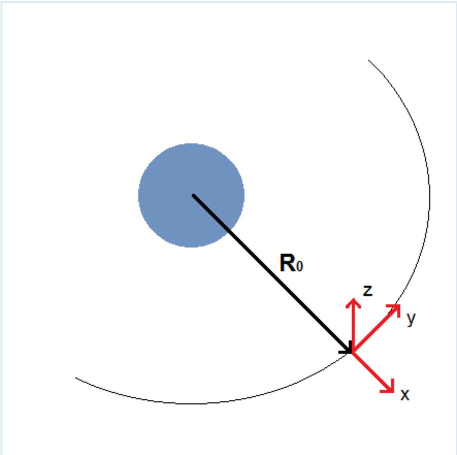

Figure 1: Local Vertical Local Horizontal Frame

In the above equatıons, only two-body gravitational equations of motion with no perturbations are considered. It is also assumed that the target is in a circular orbit about the Earth, and the relative distance between the target and the chaser is much smaller than the target's orbital radius.

The above equations may be linearized about the nominal trajectory to obtain,

$$\begin{aligned} \ddot{x} - 3n^2 x - 2n\dot{y} &= f_x \\ \ddot{y} + 2n\dot{x} &= f_y \\ \ddot{z} + n^2 z &= f_z \end{aligned} \tag{2}$$

Where, mean motion mean angular velocity is, [Seidelmann and Urban, 2013]:

$$n = \sqrt{\frac{\mu}{R_0^3}} \tag{3}$$

In state space representation, the linearized HCW equations may be written as:

$$\dot{\mathrm{X}} = \mathrm{AX} + \mathrm{BU} \tag{4}$$

Where $\mathrm{X} \in \mathbb{R}^6$ is the state vector and $\mathrm{U} \in \mathbb{R}^3$ is the control input vector and,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$X = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \qquad U = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^T$$

Model Predictive Control

To apply the MPC method, the above continuous time system is discretized with sample time T_s.

$$X(k+1) = AX(k) + BU(k) \tag{6}$$

Optimal control input may be found by minimization of a quadratic cost function. Unlike the conventional Linear Quadratic problems which uses a cost function of present states and inputs, MPC input minimizes a cost function constituted of predicted state and input values over a prediction horizon.

$$J(k) = \sum_{i=0}^{N-1} \left[x^T(k+i|k) Q x(k+i|k) + u^T(k+i|k) R u(k+i|k) \right] + x^T(k+N|k) \bar{Q} x(k+N|k) \tag{7}$$

Vectors including N predictions of states and inputs predicted at step k can be defined as,

$$\mathbf{u}(k)=\begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N-1|k) \end{bmatrix} \qquad \mathbf{x}(k)=\begin{bmatrix} x(k+1|k) \\ u(k+2|k) \\ \vdots \\ u(k+N|k) \end{bmatrix} \tag{8}$$

With the predicted states at discrete intervals in time, discrete-time state space model may be written as,

$$\mathbf{x}(k)=\mathcal{M}x(k)+\mathcal{C}\mathbf{u}(k) \tag{9}$$

$$\mathcal{M}=\begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} \qquad \mathcal{C}=\begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}$$

Then the cost function to be minimized (Eqn. 7), may be written in matrix form as [Rossiter, 2013]:

$$J(k)=\mathbf{u}^T(k)H\mathbf{u}(k)+2x^T(k)F^T\mathbf{u}(k)+x^T(k)Gx(k) \tag{10}$$

where,

$$H=\mathcal{C}^TQ\mathcal{C}+R$$
$$G=\mathcal{M}^TQ\mathcal{M}+Q$$
$$F=\mathcal{C}^TQ\mathcal{M}$$

$$Q=\begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & Q & 0 \\ 0 & \cdots & 0 & \overline{Q} \end{bmatrix} \quad \text{and} \quad R=\begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & R & 0 \\ 0 & \cdots & 0 & R \end{bmatrix}$$

For unconstrained case, it is convenient to calculate an explicit solution for optimal input offline by equating gradient of (10) to zero.

$$\mathbf{u}^*(k)=-H^{-1}Fx(k) \tag{11}$$

When there are constraints on inputs and states, optimization problem should be solved in each time step. This requires an online implementation of controller unlike the application of a constant feedback gain. Then, the input that needs to be applied becomes the solution of the optimization problem shown in Eqn. 12.

$$\mathbf{u}^*(k)=\min_{\mathbf{u}} J(k) \tag{12}$$

Optimal solution of Eqn. 12 is a stack vector of future control inputs in control horizon. At each time step, only the first input is applied, i.e., $u^*(k)=1\quad 0\quad 0\quad \cdots\ \mathbf{u}^*(k)$.

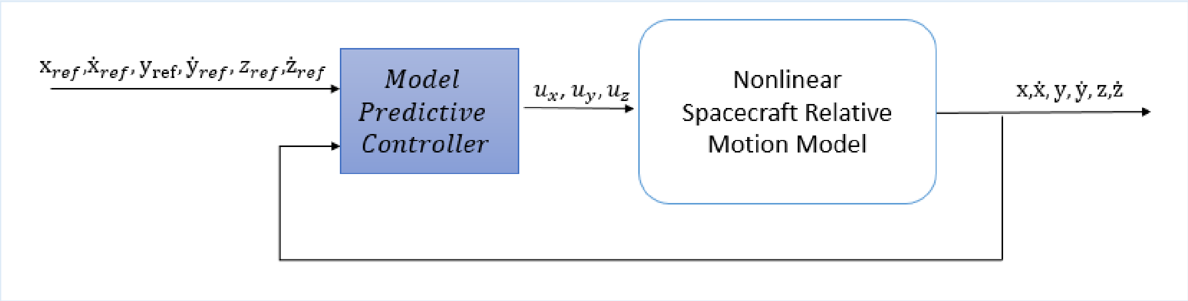

Figure 2: Block diagram of controlled system

Addition of Constraints

Since the last term of summation in Eqn. 10 depends on the current and known states, it may be omitted from the optimization problem. Then, the optimization problem in Eqn. 10 may be rewritten as,

$$J(k) = \frac{1}{2}\mathbf{u}^T(k)H_{quad}\mathbf{u}(k) + f_{quad}^T\mathbf{u}(k) \tag{13}$$

The problem in Eqn. 13 constitutes a quadratic programming problem and existing solvers can be used for constrained optimization in this problem. In this work, Matlab's '*quadprog'* solver is used with linear constraints.

First constraint is on control inputs. Available thrust is limited due to actuator capacities. Instead of saturating the control inputs, this limitation is considered as a constraint, and controller's awareness of this limitation is maintained.

$$u_{\min} < u < u_{\max} \tag{14}$$

For cooperative targets it is necessary to stay in the Line of Sight of the sensors of the target. For uncooperative targets there is no such a limitation; however, it is essential to approach to the target through its spinning/tumbling axis. This can be implemented by creating a linear cone constraint, $A_{cone}x < b_{cone}$, that chaser should be kept inside during the approach to target. Since states in z direction are decoupled, for the sake of simplicity relative motion in orbital xy plane is controlled. Cone constraint is considered as the cone's projection on xy plane with 45° conical half angle. For example for a target performing radial approach starting from (150, 30, 0) m, constraints that prevent overshoot and provide desired approach path is $y < x$ and $-y < x$. These constraints are represented as,

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} x < \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

With increasing number of constraints on states, same structure may be used by addition of new rows to the constraint matrix of Eqn. 15.

Since the optimization variable is the control input, the states and constraints on them should be represented in terms of optimization variable. This may, for example, be written as,

$$G_0\mathbf{u}(k) \le w_0 + E_0 x(k)$$

$$G_0 = \begin{bmatrix} 0 & 0 & ... & 0 \\ A_x B & 0 & ... & 0 \\ A_x AB & A_x B & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_f A^{N-1} B & A_f A^{N-2} B & ... & A_f B \end{bmatrix} \quad E_0 = \begin{bmatrix} -A_x \\ -A_x A \\ -A_x A^2 \\ \vdots \\ -A_f A^N \end{bmatrix} \quad w_0 = \begin{bmatrix} b_x \\ b_x \\ b_x \\ \vdots \\ b_f \end{bmatrix} \tag{16}$$

where, $A_{cone} = A_x = A_f$ and an example of a state constraint matrix is given in Eqn. 15. Right hand side of the inequality is computed in each time step with the knowledge of current state values and associated constraint matrix become constant.

SIMULATION, RESULTS AND DISCUSSION

In the simulations, orbital parameter are chosen as $n = 0.0011\, rad/s$ which implies that spacecrafts are flying in a circular orbit at 500 km of altitude. Sampling period is selected as $T_s = 1.5\sec$. Control inputs are limited to $u = 0.5 m/s^2$.

One of the most important tuning parameters in an optimal control problem is the weight matrices of states and inputs namely Q and R. For parametric studies, the weighting matrices are selected as,

$$Q = \begin{bmatrix} 10^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^{-1} \end{bmatrix} \quad R = \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

Where, α is the relative weight between the matrices. Terminal cost weight matrix $\overline{Q}$ is chosen as the solution of Discrete Algebraic Riccati Equation.

Simulations are carried out for different planning horizons and optimization weight matrices. To evaluate performance of controllers in different cases two performance metrics are used as in Eqn. 18.

$$J_1 = \sum_{k=0}^{t_{docking}} |u_x(k)| + |u_y(k)| \quad \text{and} \quad J_2 = \sum_{k=0}^{t_{docking}} \sqrt{u_x(k)^2 + u_y(k)^2} \tag{18}$$

The first metric indicates the fuel consumption where the second metric is the indication of energy used. Performance metrics for different planning horizons with constant weighting parameter $\alpha = 1$ are shown in Table 1. From the figure it may be observed if distant horizons are selected, the fuel consumption is slightly increased. To obtain feasible solutions, the horizon length should not be too small since it may result erroneous results. On the other hand, since at each step, optimization problem is solved in this planning horizon, too long horizon means higher computational cost. For this reasons horizon length is taken as, $N = 15$. In simulations control and prediction horizons are taken to be same.

Table 1. Performances of Controllers with different planning horizon lengths

	N=15	N=20	N=25	N=30	N=35	N=40
J_1	32.8227	33.3431	33.3444	33.3451	33.3458	33.3463
J_2	26.9994	27.0900	27.0911	27.0916	27.0923	27.0927

For different weighting ratios or parameter α in Eqn. 17, performance metrics are calculated together with the times past between initial condition and docking. Results are shown in Table 2. From these results, best value for α is decided to be 1. Although higher α values yielded smaller fuel and energy metrics, docking times increased as well.

Table 2. Performance of controllers with different weight matrix ratios

	$\alpha=10^{-6}$	$\alpha=10^{-3}$	$\alpha=1$	$\alpha=10^{3}$	$\alpha=10^{6}$
J_1	32.8867	32.2871	22.6378	17.0239	10.3695
J_2	27.0551	26.5276	18.1171	13.5245	8.8741
$t_{docking}$	34.5000	34.5000	34.5000	40.5000	84.0000

Nonlinear simulations are conducted to make a rendezvous with the target starting from initial position (150, 30, 0) m. Due to given initial conditions, only along-track distance and radial distance need to be controlled. The results are presented in Fig. 3 and 4. It may be observed from the figures that the rendezvous with the target spacecraft is realized successfully. In Fig. 3 relative distances and applied control inputs are given.

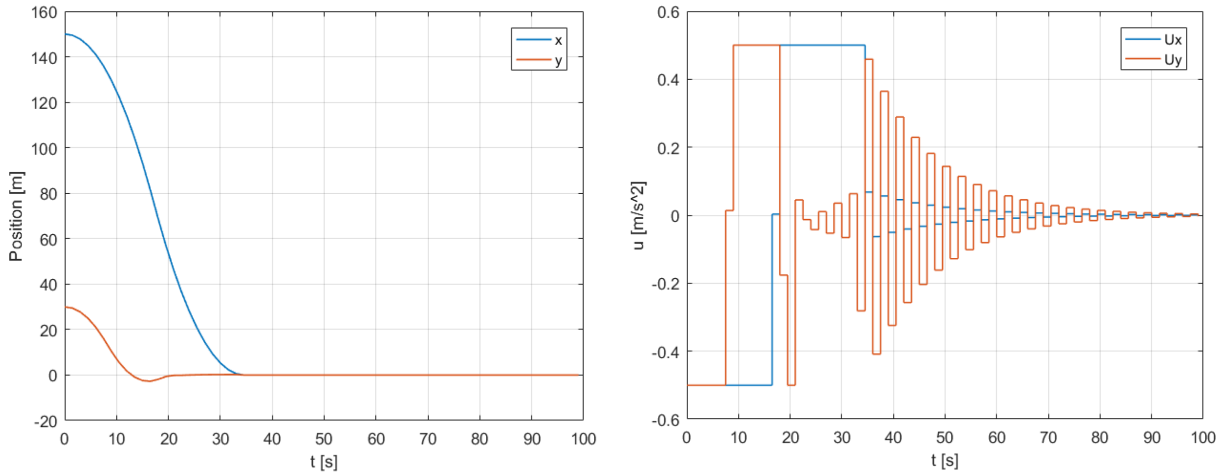

Figure 3: Relative distances in orbital plane and applied control inputs during the rendezvous maneuver of the chaser

The complete rendezvous path is given in Figure 4. Constraints are successfully satisfied and no overshoot or constraint violation is observed. Thus, the chaser approaches the target within the specified approach cone.

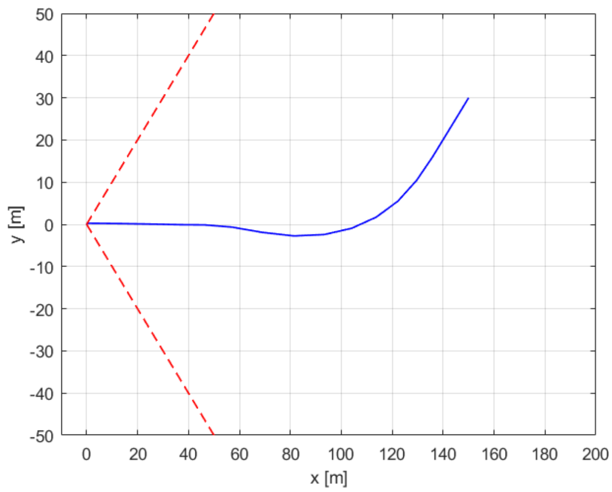

Figure 4: Rendezvous path of Chaser spacecraft

Same simulation parameters are used controlling the chaser satellite starting from different initial positions. In Fig. 5 results are given. Similar success in satisfying constraints and preventing overshoot achieved for all cases.

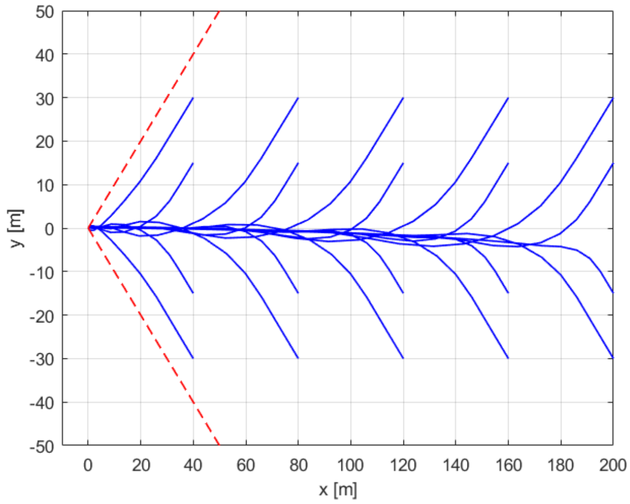

Figure 5: Rendezvous operation starting from different initial positions

CONCLUSION

In this study, spacecraft relative motion control is addressed. The rendezvous problem of the chaser spacecraft is solved using constrained model predictive control method. Constraints on both inputs and states are added to the optimization problem. It is demonstrated that constraints on states keep the chaser within the prescribed cone and help to achieve a safe trajectory. A parametric study is carried out to find the best prediction horizon and relative weight parameter. Results are tabulated and best values for a scenario starting from a fixed initial condition is found. It is observed that unnecessarily large horizons should be avoided because of both computational burden and high fuel consumption.

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