

## DUAL FRACTIONAL ORDER FEEDBACK CONTROL SYSTEM APPLIED FOR MISSILE GUIDANCE

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### ABSTRACT

*Dual Fractional Order Proportional Integral Derivative (FOPID) controller system is proposed in order to control a missile using Proportional Navigation Guidance (PNG) system. This dual FOPID controller design is a novel approach to enhance the performance of (PNG) system by minimizing the value of miss distance. The presented controller has 10 adjustable parameters and the tuning process has been done using a genetic algorithm. The need for this controller is justified with the nonlinear nature of the PNG system, to which many alternatives were applied in the literature with less accuracy. The proposed control system proved to display a smaller value for miss distance and less time needed for the defending missile to collide with the attacker. The results have been discussed with a set of simulations.*

### INTRODUCTION

Interception accuracy of a missile is one of the most critical control problem that researchers have been trying to solve. For this purpose, one of the significant performance tests that measures the accuracy of missiles is called miss distance estimation. The term Miss Distance (MD) could be defined as “the minimum distance between a guided flying object and its intended target site during their intersection” [Guo, Qu, Feng and Sheng, 2016]. Oftentimes Proportional Navigation Guidance (PNG) system is used in order to guide the missile and to keep it on the right course during navigation and to track a specific moving target. PNG scheme is stated to be the most commonly preferred method that is used for missile guidance [Weiwen, Xiaogeng and Xiaohong, 2010]. In the literature, there are many researches focusing on designing controllers for the proportional navigation system. In [Erer, Tekin and Özgören, 2016] a bias term has been introduced and integrated into the proportional navigation control system in order to solve the problem of path following and impact angle control against stationary target that is subjected to signal error which is a function of pursuit angle. In [Radhika, Parthasarathy and Kumar, 2016], an estimation technique using Kalman filter with navigation algorithm is presented in order to intercept highly maneuvering target with a small MD. In [Su, Chen and Kebo, 2016], a modified guidance law for calculating the required acceleration demand is designed and tested against a maneuvering target. The simulation results show the effectiveness of the proposed method for calculating the required acceleration

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in order to have a MD near zero. In [Tyan, 2016], a new method for analyzing impact angle is proposed by studying the rotating angle of relative velocity as an analytical solution for calculating the impact angle instead of the traditional method. PID controller is also one of the frequently used approach, in [Byungjun, 2016] a proportional integral derivative controller has been integrated into the proportional navigation guidance in order to achieve a better miss distance accuracy, but according to [Weiwen, Xiaogeng and Xiaohong, 2010], the PN guidance system is considered to be a nonlinear control problem, therefore in order to control a system with nonlinearities, it's difficult to observe a satisfactory performance with integer order PID controller, whereas fractional order PID controller is an alternative approach enabling more adjustable parameters than the classical one. In this research, an adaptive fractional order proportional integral derivative controller for PN guidance system is designed in order to control a missile defending a target by colliding with the attacker. The performance of this controller has been proven by measuring the accuracy of the missile to hit the target which is presented by the value of miss distance, and also by observing the time needed for the missile to hit the target. Simulation results stipulate the superiority of the dual FOPID controller which has much better accuracy compared to the conventional method presented by lower value of miss distance. It is also shown that an improvement on the time needed for the missile to hit the target. The rest of the paper is organized as the following: section 2 includes a summary of fractional order proportional integral derivative (FOPID) controller, section 3 is an explanation of proportional navigation guidance system used in this research and finally section 4 includes the method for designing the proposed controller and the simulation results associated with the system.

## FRACTIONAL ORDER PID CONTROLLER

PID controllers are considered as standard tools in industry for many reasons such as their practicality and the availability of wide range of tuning rules of their parameters. Another type of PID controller has been presented in industry which is fractional order PID controller, this FOPID denoted as  $(PI^\lambda D^\mu)$  is a generalization of the conventional or integer order PID controller that can be described using noninteger order of Laplace variable  $s$  as the following:

$$C(s) = \frac{D(s)}{U(s)} = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \quad (1)$$

where  $\lambda$  is the integration order and  $\mu$  is the order of differentiation, both of which are positive and real numbers. The history of fractional calculus has started in 1695 by a letter from L'Hôpital asking Leibniz about the meaning of a derivative that has a fractional order of  $(1/2)$ . But the presence of fractional calculus has been seen just during the last few decades in automatic control application systems. Using the derivative operator  $D=(d/dt)$ , a generalization could be made by defining the operator  $D^\alpha$  where  $\alpha$  is a non-integer variable that belongs to the real numbers ( $\alpha \in \mathbb{R}$ ), using this definition, the differentiation could be done by using a positive value of  $\alpha$  ( $\alpha > 0$ ), while integration could be done by using negative value of  $\alpha$  ( $\alpha < 0$ ). these operators are called differintegration operators and in literature there are two famous definitions for these operators, these two definitions are made by Riemann–Liouville and by Caputo.

$$D^\alpha y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)} \left(\frac{d}{dt}\right)^r \int_0^t \frac{y(\xi)}{(t-\xi)^{\alpha+1-r}} d\xi \quad (2)$$

$$D^\alpha y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)} \int_0^t \frac{y^{(r)}(\xi)}{(t-\xi)^{\alpha+1-r}} d\xi \quad (3)$$

where  $r-1 \leq \alpha < r$  and  $r$  is an integer. It is also possible to find transfer function in fractional order using Laplace transform  $L(D^\beta) = s^\beta$  where  $L$  is the Laplace transform and  $s$  is the Laplace variable [Efe, 2011].

Using FOPID has many advantages over integer order PID for the following reasons:

FOPID has better performance and robust stability characteristics over the integer order PID [Pradhan, Patra and Pati, 2016]. FOPID has better robustness against uncertainties and gain variations in the plant model, it also has better rejection capability for load disturbances that can be produced by the plant model. Further, FOPID has better ability to handle the noises affecting the system [Edet and Katebi, 2016]. In systems which have time delay, it was proven that fractional order PID controller has better performance results than integer order PID controller.

Mostly, in order to control a non-linear system using integer order PID controller, the system is linearized at different operating points, and then for each point a dedicated PID controller is designed, whereas one fractional order PID controller is sufficient in most cases for non-linear systems [Shah and Agashe, 2016].

### PROPORTIONAL NAVIGATION GUIDANCE SYSTEM

In order to intercept a target, the normal acceleration value ( $a_m$ ) should be found. The acceleration ( $a_m$ ), which is normal to the direction of the missile that keeps the missile on the line of sight (LOS), is proportional to the measured deviation of the missile from the LOS.

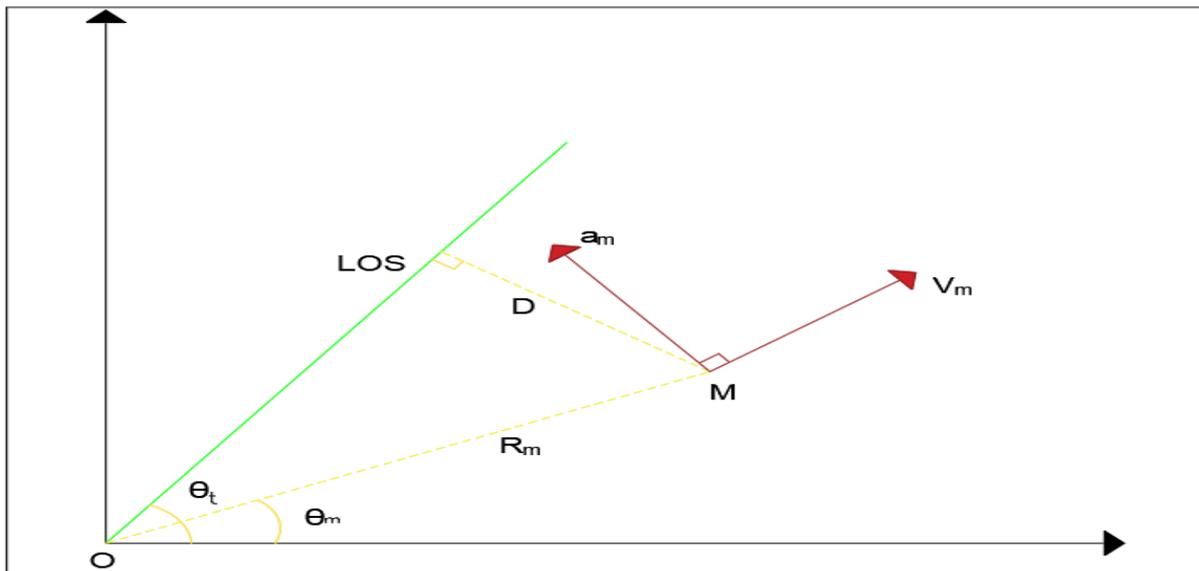


Figure 1: Kinematic model for target and missile positioned away from LOS

$$a_m = KD \quad (4)$$

$$a_m = KR_m \sin(\theta_t - \theta_m) \quad (5)$$

where  $K$  is a constant,  $D$  is the distance between the missile and the LOS,  $\theta_m$  is the angle between the reference and the missile,  $\theta_t$  is the angle between the reference and the target.

In many systems, the position of the target and missile are calculated with respect to a tracker such as radar system as shown in Figure 2, the kinematic equations for such system can be derived as follows:

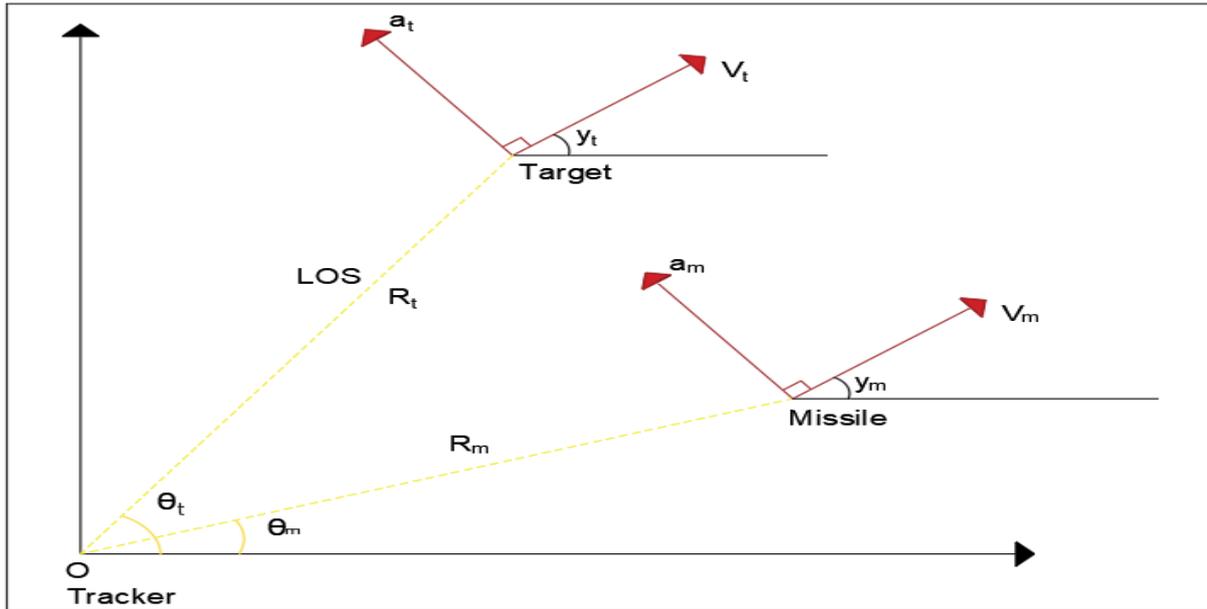


Figure 2: Kinematic model for missile and target with tracker

Missile

$$\left(\frac{dR_m}{dt}\right) = V_m \cos(\gamma_m - \theta_m) \tag{6}$$

$$R_m \left(\frac{d\theta_m}{dt}\right) = V_m \sin(\gamma_m - \theta_m) \tag{7}$$

$$V_m \left(\frac{d\gamma_m}{dt}\right) = a_m \tag{8}$$

where  $R_m$  is the distance crossed by the missile,  $\theta_m$  is the angle between the reference and the missile,  $V_m$  is the speed of the missile,  $\gamma_m$  is the angle between the reference and the speed direction of the missile,  $a_m$  is the acceleration normal to the speed direction of the missile.

Target

$$\left(\frac{dR_t}{dt}\right) = V_t \cos(\gamma_t - \theta_t) \tag{9}$$

$$R_t \left(\frac{d\theta_t}{dt}\right) = V_t \sin(\gamma_t - \theta_t) \tag{10}$$

$$V_t \left(\frac{d\gamma_t}{dt}\right) = a_t \tag{11}$$

where  $R_t$  is the distance crossed by the target,  $\theta_t$  is the angle between the reference and the target,  $V_t$  is the speed of the target,  $\gamma_t$  is the angle between the reference and the speed direction of the target,  $a_t$  is the acceleration normal to the speed direction of the target.

The normal acceleration that is needed to be applied on the missile to keep it on the LOS can be calculated as follows:

$$a_m = (d(R_m(\frac{d(\theta_t - \theta_m)}{dt}))/dt) + (\frac{dR_m}{dt})(\frac{d(\theta_t - \theta_m)}{dt}) \tag{12}$$

Noting that the first term is the distance multiplied by the derivative of the angle, which produces the speed of the missile normal to the distance. Differentiating the speed once yields the normal acceleration, and then we add the second term of the equation, which is the Coriolis effect due to the earth's rotation and the inertia of the missile that is affected by the earth's rotation. By differentiating the first term of the equation, we have

$$a_m = R_m(\frac{d^2(\theta_t - \theta_m)}{dt^2}) + (\frac{dR_m}{dt})(\frac{d(\theta_t - \theta_m)}{dt}) + (\frac{dR_m}{dt})(\frac{d(\theta_t - \theta_m)}{dt}) \tag{13}$$

$$a_m = R_m(\frac{d^2(\theta_t - \theta_m)}{dt^2}) + 2(\frac{dR_m}{dt})(\frac{d(\theta_t - \theta_m)}{dt}) \tag{14}$$

and by substituting  $V_m = \frac{dR_m}{dt}$  into equation (14), we obtain the equation in (15).

$$a_m = 2V_m(\frac{d(\theta_t - \theta_m)}{dt}) + R_m(\frac{d^2(\theta_t - \theta_m)}{dt^2}) \tag{15}$$

## METHOD

### Controller Design and Tuning

In this research, a system of dual Fractional Order Proportional Integral Derivative controller is proposed. Figure 3.1 depicts the conventional PNG system while Figure 3.2 depicts the proposed dual FOPID control system used with the Proportional Navigation Guidance. In the conventional controller, the error signal of missile position is composed by subtracting the missile position from target position, while in the proposed system the error signal is applied to a FOPID control system. This control system consists of two FOPID controllers, one for controlling the missile course by taking signals from the X-axis of the missile position, and the second by using the Y-axis of the missile position. The need for dual controllers is presented due to the different forces that act on the missile in the X and Y axis, such as gravity, which acts only on the Y-axis.

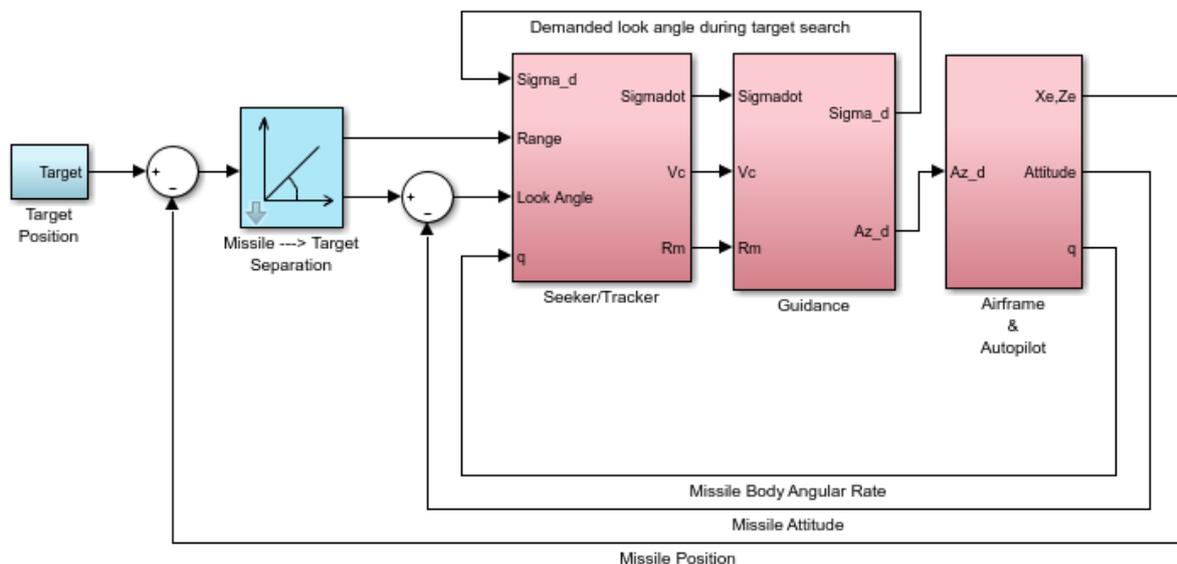


Figure 3.1: Conventional Guidance System

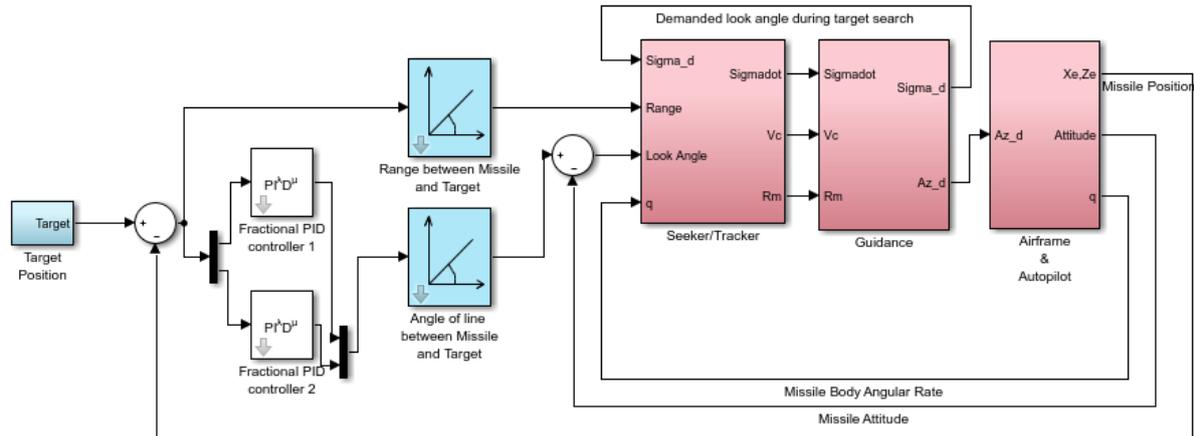


Figure 3.2: Dual Fractional Order PID Guidance System

This FOPID controller has been tuned using a genetic algorithm. Genetic algorithm has been used for tuning the controller, 66 generations have been produced for tuning 10 parameters of the Dual FOPID controllers. The final values for the dual FOPID controllers are shown in Table 1.

Table 1: Tuned Parameters for Dual FOPID Controllers

Parameter	FOPID Controller 1	FOPID Controller 2
$K_i$	0.3856	0.1866
$K_p$	0.6743	0.7392
$K_d$	0.7367	0.2029
$\lambda$	0.6175	0.5748
$\mu$	0.4900	0.9929

### Missile and Target Trajectories

Figure 4.1 shows the flight course using the PNG with conventional method while Figure 4.2 shows the PNG using Dual FOPID controller system. The course of the attacker is the same for both systems during flight time. In these figures we can observe the distances that each attacker crossed before it was intercepted by the defender missile. This shows that Dual FOPID controller took less time to be directed toward the target compared with conventional one which will lead to difference in intersection time between the missile and the target. The simulation process shows that the intersection happened at 3.45 s of simulation time using Dual FOPID controller, while it happened after 3.46 s using Conventional PNG system, this slight improvement in hit time could lead to significant distance due to the high speed of the missile and short simulation time.

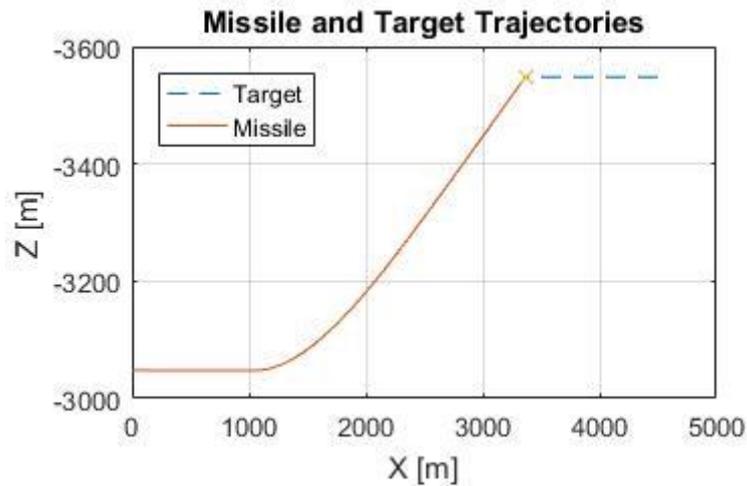


Figure 4.1: Missile and target trajectories using conventional PNG system.

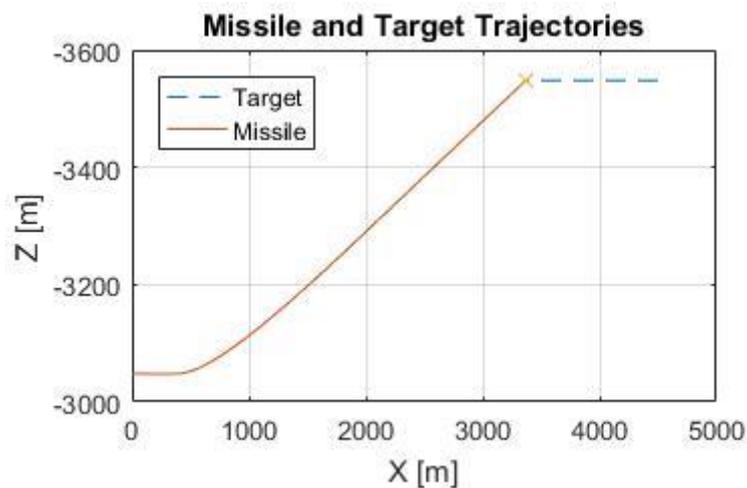


Figure 4.2: Missile and target trajectories using Dual FOPID controller.

Figure 5.1 shows that conventional PNG system took about one second to align the Gimbal Angle (thrust force angle of the missile) with the true look angle (angle of the line between missile and target). Figure 5.2 shows that Gimbal Angle is aligned with True Look Angle most of the time.

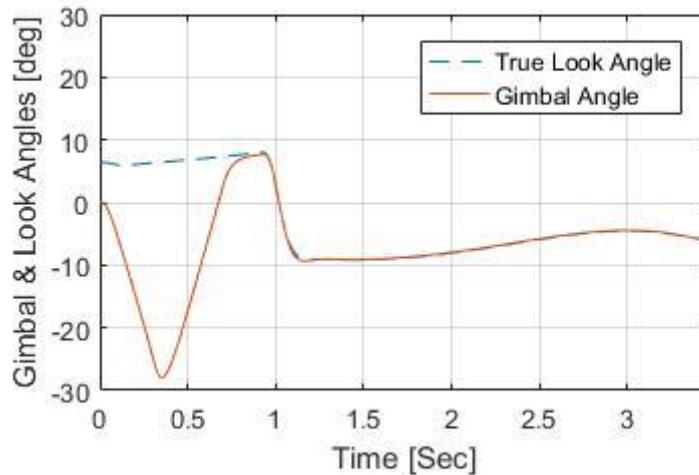


Figure 5.1: Gimbal and look angles using conventional PNG system.

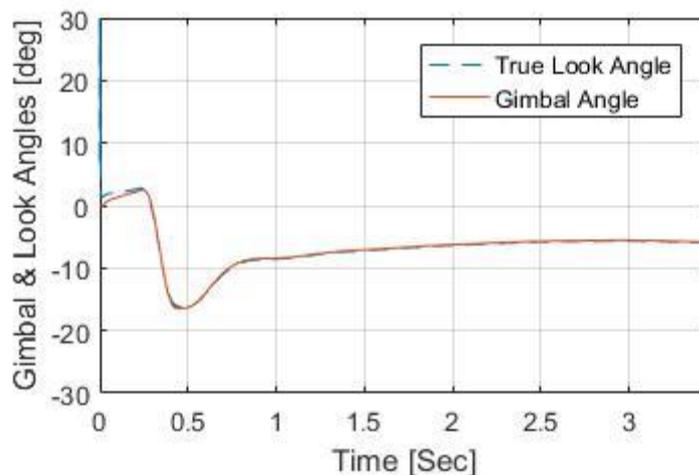


Figure 5.2: Gimbal and look angles using Dual FOPID controller.

### Miss Distance Analysis

The MD value for both systems has been calculated and compared. The measured value of MD showed an impressive improvement for accuracy of PNG using FOPID controller over the conventional one. It has been proved that MD accuracy for FOPID is much better than the conventional PNG controller, with a MD value of 0.0009 for Dual FOPID controller, and a MD value of 0.2682 for conventional PNG controller. This indicates that the Dual FOPID controller system is much better than the conventional one. It is also shown by the measurement results that using Dual FOPID controller is better than using single FOPID controller for such a problem setting, which has difference in the applied forces on each axis.

### CONCLUSIONS

By this research, a novel approach of dual fractional order PID controller has been introduced and integrated into PN guidance system. The need for the dual FOPID setup is due to the

difference in forces applied on the reference axes of the missiles, such as gravity which acts just on one axis. This has been proved by comparing it with a single FOPID controller. The simulation results of the proposed Dual FOPID controller system showed an improvement on the time of impact with the target and a significant improvement on the accuracy of the missile to hit the target compared with the conventional method.

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