

WIND TUNNEL TEST MATRIX DESIGN USING CFD BASED DOE

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ABSTRACT

In this study, wind tunnel test matrix of a newly developed Air-to-Ground missile is designed utilizing the Design of Experiment method based on Computational Fluid Dynamics simulations for various combinations of input parameters that are angle of attack, side-slip angle, Mach number, and the tail fin deflections. First of all, contributions of input parameters to each output (aerodynamic forces and moments) and the correlation between the inputs are identified. Then, separate Response Surface Model for each output that span the whole design space is generated to investigate the design deeper. Finally, optimal interval values for each input parameters are determined for the wind tunnel test matrix. As a result, number of runs in the test matrix was significantly reduced. For all the above purposes, the commercial software ANSYS Fluent and ESTECO's modeFRONTIER are used.

NOMENCLATURE

C_L	lift coefficient
C_D	drag coefficient
C_Y	side force coefficient
C_l	rolling moment coefficient
C_m	pitching moment coefficient
C_n	yawing moment coefficient
M_∞	free-stream Mach number
α	angle of attack
β	angle of side-slip
$\delta_{n, n=1-4}$	deflection angle of n^{th} fin
δ_a	aileron deflection angle
δ_e	elevator deflection angle
δ_r	rudder deflection angle

INTRODUCTION

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Continuous advances in numerical methods and computation power have made Computational Fluid Dynamics (CFD) more reliable and cheaper than ever before. This situation has increased the importance of CFD simulations during an air vehicle design, especially during the preliminary design phase that is needed to assess alternative configurations. However, CFD has not reached to a point yet that can replace the whole role of wind tunnel testing in aerospace industry. This is mostly because design phase may allow only some hundreds of CFD simulations due to time limitation. On the other hand, once the design reaches to design detail phase, the wind tunnel testing becomes more suitable for large number of simulations to generate an aerodynamic database and to validate the final design. At this point, CFD may still play an important role to optimize wind tunnel test matrix while keeping the required number of CFD simulations low.

One of the methods that CFD can be used for deciding wind tunnel test matrix is the Design of Experiment (DoE) method [Fisher, 1920]. The DoE method, as its name suggests, was first proposed to maximize the knowledge gained from experimental data. Prior to this method, experiments were performed to test one factor at a time. By using this technique, many evaluations were needed to get sufficient output. But, the DoE method considers changes in more than one variable at a time. Doing this, one might identify which inputs are more important or which have correlation with each other. The main advantage of the DoE approach is that it eliminates redundant observations so reduces the time and effort required.

In this study, wind tunnel test matrix of a newly developed Air-to-Ground missile is designed utilizing the DoE method based on CFD simulations for various combinations of input parameters that are angle of attack, angle of side-slip, Mach number, and the deflection angles of tail fins. First of all, contributions of input parameters to each output (aerodynamic forces and moments) and the correlation between the inputs are identified. Then, separate Response Surface Models (RSM) for each output that spans the whole design space is generated to investigate the design deeper. Finally, optimal internal values for each input parameters are determined for the wind tunnel test matrix. As a result, number of runs in the test matrix was significantly reduced.

PROBLEM DESCRIPTION

The initial step of describing the problem considered should be to define proper inputs and expected outputs. There are several input parameters that affect the aerodynamic response of the geometry considered for this study. Among them, Mach number, angle of attack, angle of side slip, and four angles corresponding to fin deflections are the most important ones and their upper and lower limits are given in (Table 1). The output variables for this study are the coefficients of aerodynamic forces and moments on the configuration. Since the missile is symmetric in longitudinal axis, side-slip angle is bounded with the positive values. For negative side-slip angles, outputs are obtained from the results of positive cases using the conversion table shown in (Table 2).

Table 1: Input Variables and Their Lower and Upper Bounds

Variable	Lower Bound	Upper Bound
α (degs.)	-12	12
β (degs.)	0	9
δ_1 (degs.)	-15	15
δ_2 (degs.)	-15	15
δ_3 (degs.)	-15	15
δ_4 (degs.)	-15	15
M_∞	0.30	0.95

Table 2: Conversion Table for Beta Symmetry

β	δ_1	δ_2	δ_3	δ_4	C_L	C_D	C_Y	C_I	C_m	C_n
$+\beta$	$+\delta_1$	$+\delta_2$	$+\delta_3$	$+\delta_4$	$+C_L$	$+C_D$	$+C_Y$	$+C_I$	$+C_m$	$+C_n$
$-\beta$	$-\delta_4$	$-\delta_3$	$-\delta_2$	$-\delta_1$	$+C_L$	$+C_D$	$-C_Y$	$-C_I$	$+C_m$	$-C_n$

METHOD

DoE with CFD

The main purpose of the DoE method is to collect enough information on the relation between the system inputs and outputs. One might start with choosing random points over the design space to evaluate for this purpose. But, it would be waste of time to perform experiments for points that are too close to each other. In fact, the chosen points should be uniformly distributed as much as possible. It is an easy task for a one dimensional problem, but, when the problem is multi-dimensional, it becomes complicated. Fortunately, there are various models presented in the literature for this purpose, such as Sobol [Sobolprime and Levitan, 1999] or Uniform Latin Hypercube [McKay, Conover and Beckman, 1979] methods to name some. These methods are called as Space Filling Algorithms and they differ only on the definition of uniformity in point distribution. modeFRONTIER Software presents the possibility of choosing from one of the most known methods. In this study, the Uniform Latin Hypercube method is chosen for its superior properties [McKay, Conover and Beckman, 1979] such as fast convergence rate and higher flexibility compared to other methods.

Another critical decision when performing DoE is the number of points to distribute across design space. The number of sampling points should be high enough to collect necessary information about the system and the number changes depending on the problem considered. This is due to the reason that as the dimension (number of input variables) of a problem increases, the volume of the design space increases much faster and the sampling data becomes sparse. To collect meaningful data, the number of sampling points should be increased with the dimension of the problem. As a general rule of thumb, this is at least 5 to 10 times of the number of input variables. To see if the chosen number of points is enough, the parameter called Collinearity Index [Riccoa, Rigonia and Turcoa, 2013] should be checked. In case of two or more input parameters are linearly correlated, their collinearity indices become much higher than 1. However, this situation can also occur when the number or the distribution of sampling points are bad, or even the inputs are not correlated. In order to check the fidelity of collinearity, number of sample points can be increased and if the indices are still higher than 1, correlation is justified. If the collinearity indices are close to 1, then the sampling can be considered as just fine. In (Table 3, 4, and 5), the collinearity indices of the input variables for all the outputs are given for 30, 50, and 70 sampling points that are distributed using Uniform Latin Hypercube method. As one can observe from these tables, the collinearity indices are low and as the number of sampling points increase from 30 to 70, they get closer to 1.

Table 3: Collinearity Indices for 30 Initial Design Points

	C_L	C_D	C_Y	C_I	C_m	C_n
α	1.2442	1.0045	1.0189	1.0169	1.0402	1.0253
β	1.0982	1.1730	1.1081	1.1051	1.0679	1.1082
M_∞	1.2924	1.0398	1.0521	1.0515	1.0728	1.0559
δ_1	1.1021	1.1880	1.1859	1.1784	1.1285	1.1813
δ_2	1.0597	1.1470	1.0185	1.0238	1.0228	1.0195
δ_3	1.0043	1.0899	1.0253	1.0187	1.0244	1.0230

δ_4	1.1230	1.0440	1.1114	1.1125	1.0999	1.1140
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Table 4: Collinearity Indices for 50 Initial Design Points

	C_L	C_D	C_Y	C_l	C_m	C_n
α	1.0587	1.0542	1.0141	1.0104	1.0112	1.0178
β	1.0394	1.0688	1.0365	1.0323	1.0308	1.0382
M_∞	1.1021	1.0318	1.0147	1.0150	1.0157	1.0145
δ_1	1.0590	1.0185	1.0532	1.0493	1.0476	1.0525
δ_2	1.0079	1.0189	1.0031	1.0027	1.0032	1.0033
δ_3	1.0048	1.0333	1.0223	1.0212	1.0262	1.0232
δ_4	1.0486	1.0320	1.0238	1.0257	1.0255	1.0228

Table 5: Collinearity Indices for 70 Initial Design Points

	C_L	C_D	C_Y	C_l	C_m	C_n
α	1.0321	1.0719	1.0021	1.0001	1.0025	1.0050
β	1.0185	1.1069	1.0009	1.0001	1.0034	1.0018
M_∞	1.0962	1.0177	1.0006	1.0002	1.0012	1.0016
δ_1	1.0353	1.0245	1.0003	1.0003	1.0005	1.0009
δ_2	1.0078	1.0183	1.0010	1.0002	1.0004	1.0023
δ_3	1.0022	1.0302	1.0003	1.0002	1.0023	1.0003
δ_4	1.0221	1.0202	1.0004	1.0003	1.0028	1.0008

Screening Analyses

An important step of the DoE method is called Screening Analyses. Once the proper distribution of sampling points is satisfied, effect of the inputs on the outputs and the interactional effects can be identified using the Screening Analyses. For this purpose, the parameter called Contribution Index [Riccoa, Rigonia and Turcoa, 2013] is used. This parameter defines the contribution percentage of each input parameter to the variance of an output variable. The contribution may be obtained independently for each input or interactional contributions that are second order can be obtained using modeFRONTIER.

In (Table 6), the interactional effects in terms of contribution indices are given for the present problem and the values above 1% are highlighted. As one can observe, the contributions for all the outputs are almost due to main effects and interactional effects are low. The largest interactional effects are between angle of attack, side-slip angle and Mach number and the others are around or less than 1%. Some other interpretations can be made based on the numbers given in (Table 6). For instance, the lift coefficient of the present geometry changes almost only with angle of attack (~99% contribution) and Mach number is mainly effective on the drag coefficient. It is also noticeable that force coefficients are affected by fewer inputs compared to the moment outputs. Such interpretations are helpful to design the wind tunnel test matrix.

Table 6: Contribution Indices

	C_L	C_D	C_Y	C_I	C_m	C_n
α	0.98668	0.73560	0.01695	0.04224	0.46907	0.00785
β	0.00194	-0.00040	0.70072	0.45891	0.00506	0.12705
M_∞	-0.00066	0.21798	0.01163	0.01655	-0.00062	0.01383
δ_1	0.00282	0.00213	0.08255	0.10627	0.12989	0.25167
δ_2	0.00302	-0.00005	0.04718	0.06230	0.08676	0.21219
δ_3	0.00272	0.00545	0.02946	0.09592	0.11704	0.11597
δ_4	0.00243	0.00215	0.04111	0.08060	0.13199	0.16301
$\alpha * \beta$	-0.00046	0.00165	0.03400	0.04651	0.00087	0.01895
$\alpha * M_\infty$	-0.00018	0.00600	0.00214	0.02497	0.03978	0.00141
$\alpha * \delta_1$	0.00056	0.00548	0.00049	0.00642	0.00029	0.00119
$\alpha * \delta_2$	0.00019	0.01095	0.00009	0.00043	0.01314	0.00108
$\alpha * \delta_3$	0.00077	0.00234	0.00468	-0.00201	0.00372	0.01638
$\alpha * \delta_4$	-0.00035	0.00525	0.00589	0.00628	0.00015	0.01548
$\beta * M_\infty$	0.00224	-0.00383	0.01377	0.01074	0.00178	0.01071
$\beta * \delta_1$	-0.00080	0.00265	-0.00058	0.00009	-0.00229	0.00416
$\beta * \delta_2$	0.00054	0.00201	0.00021	0.00930	0.00334	0.00070
$\beta * \delta_3$	0.00035	0.00004	0.00094	-0.00133	0.00018	0.00400
$\beta * \delta_4$	-0.00004	-0.00017	-0.00091	0.00090	0.00161	-0.00089
$M_\infty * \delta_1$	0.00016	0.00149	0.00087	-0.00117	0.00030	0.00459
$M_\infty * \delta_2$	0.00012	-0.00036	0.00403	0.00102	0.00066	0.01610
$M_\infty * \delta_3$	-0.00036	0.00024	0.00089	0.01109	0.00095	0.00869
$M_\infty * \delta_4$	-0.00042	0.00258	0.00351	0.01424	-0.00235	0.00615
$\delta_1 * \delta_2$	-0.00150	0.00104	0.00003	-0.00101	-0.00241	-0.00138
$\delta_1 * \delta_3$	-0.00022	0.00090	0.00008	0.00201	-0.00086	0.00087
$\delta_1 * \delta_4$	0.00016	-0.00088	0.00045	0.00266	-0.00024	0.00001
$\delta_2 * \delta_3$	-0.00005	-0.00047	0.00018	0.00373	0.00116	0.00012
$\delta_2 * \delta_4$	-0.00027	0.00016	0.00053	0.00010	0.00009	0.00124
$\delta_3 * \delta_4$	0.00061	0.00008	-0.00090	0.00222	0.00094	-0.00115

Generation of RSMs

Once the DoE step is completed, results of the CFD simulations performed for sampling points can be used to generate RSMs to approximate the outputs across the whole design space. This task was performed as the subject of a previous study [Çetiner, Yagiz, Güzel, Özgür, and Koc, 2016]. Here, only some results are presented. In (Fig. 1), one can find the validation results of the RSMs for 3 forces and 3 moments, respectively. In these figures, the computed and predicted results of 30 randomly selected points are compared. In (Table 7), the differences between the computed and predicted results are presented in terms of mean normalized error. As one can observe, the maximum error is obtained for the roll moment and it is less than 4%. In the following figures (from Fig. 2 to 4), more detailed comparison of the

computed and predicted results are given at 3 randomly selected points for a sweep in angle of attack. Overall, the agreement is satisfactory considering the effort spent otherwise would be needed for a whole CFD based database.

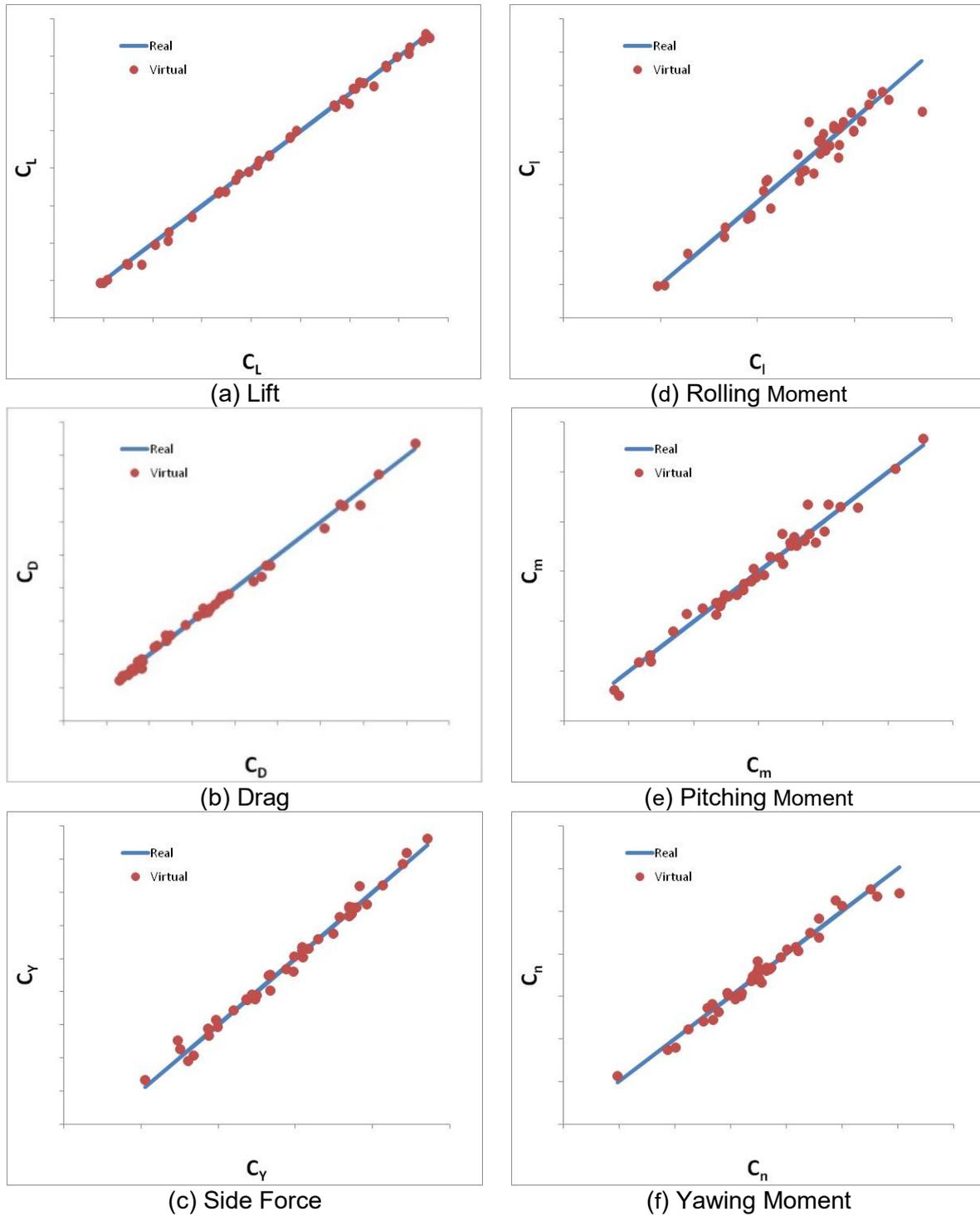


Figure 1: Validation Results of Force and Moment Coefficient RSMs

Table 7: Mean Normalized Errors for the RSM

Mean Normalized Errors (%)	
C_L	1.39
C_D	1.52
C_Y	1.88
C_l	3.76
C_m	3.59
C_n	2.94

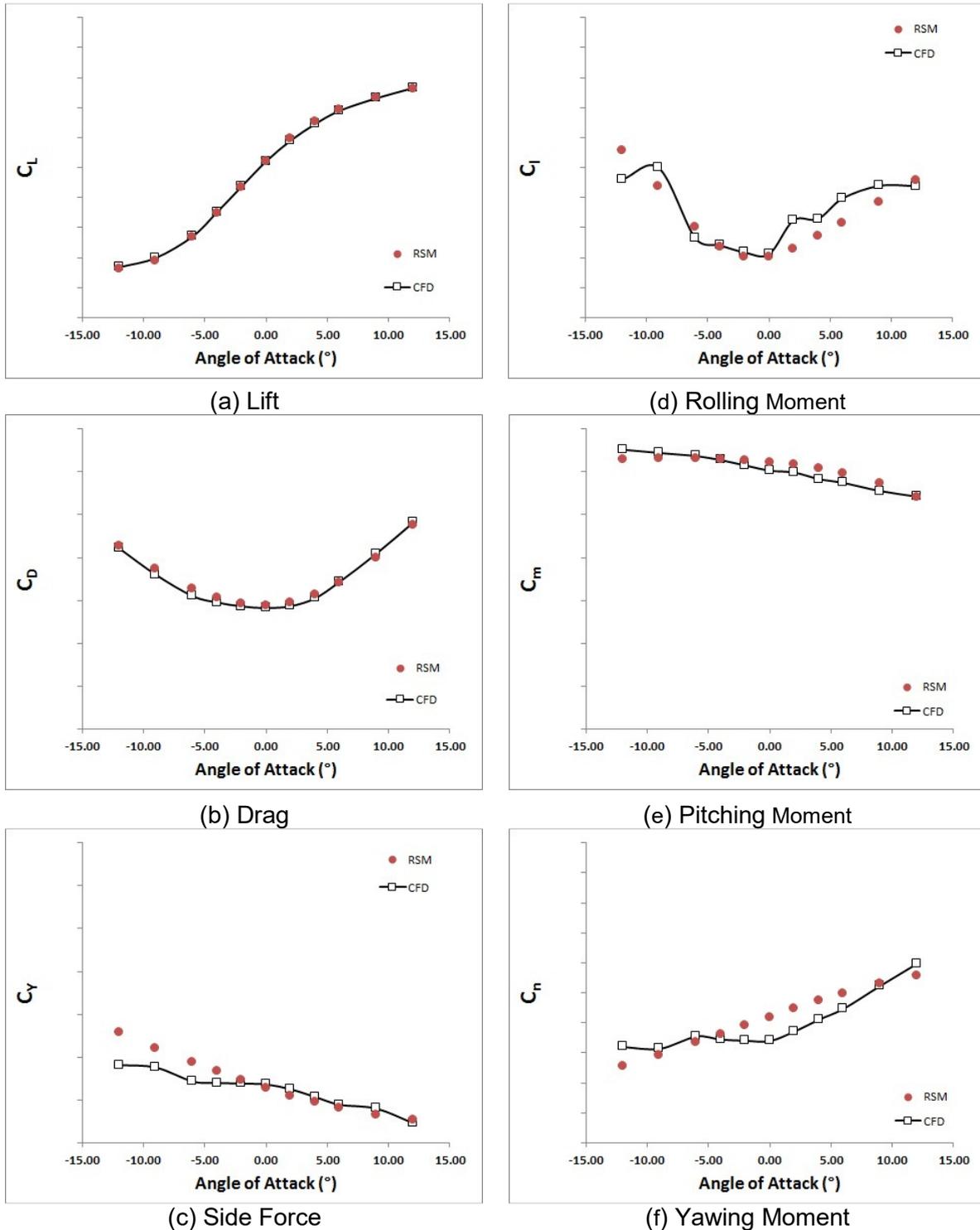
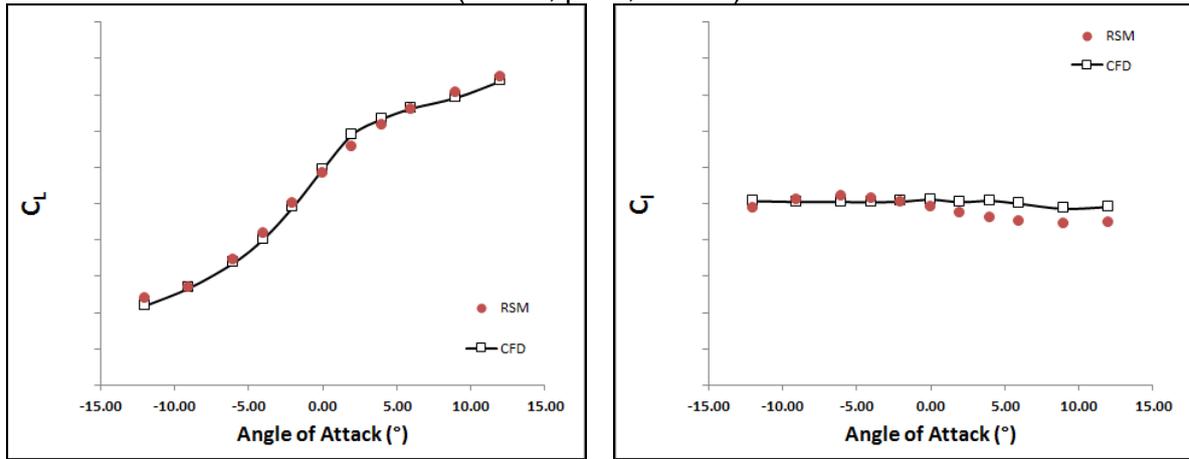


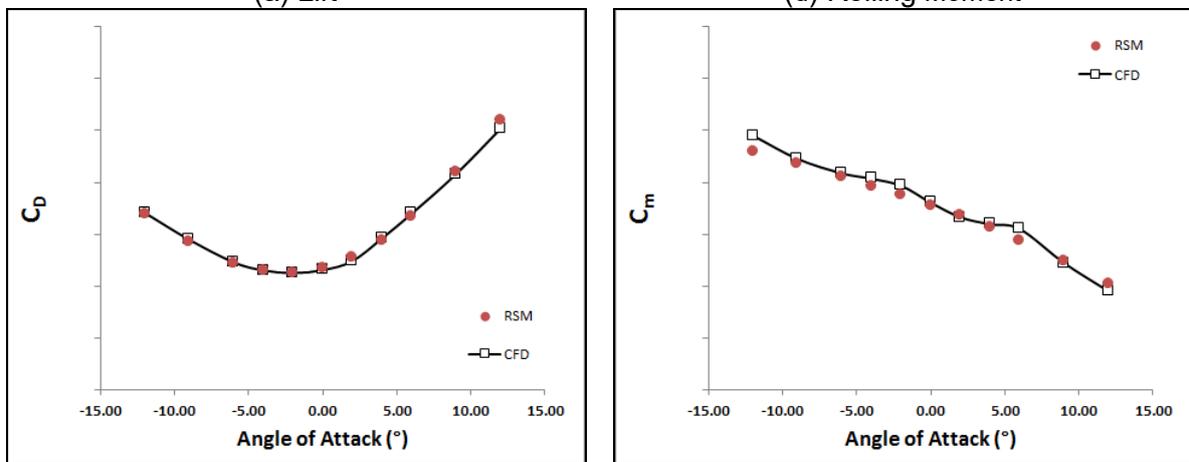
Figure 2: Comparison of CFD and RSM Results of Force and Moment Coefficients

($M=0.6, \beta=6^\circ, \delta_e=15^\circ$)



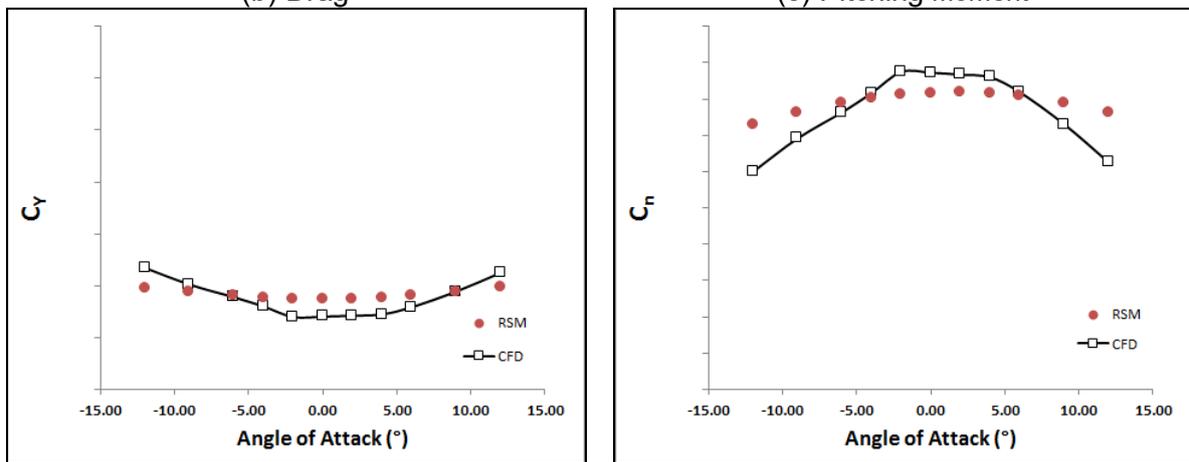
(a) Lift

(d) Rolling Moment



(b) Drag

(e) Pitching Moment



(c) Side Force

(f) Yawing Moment

Figure 3: Comparison of CFD and RSM Results of Force and Moment Coefficients ($M=0.75, \beta=0^\circ, \delta_r=5^\circ$)

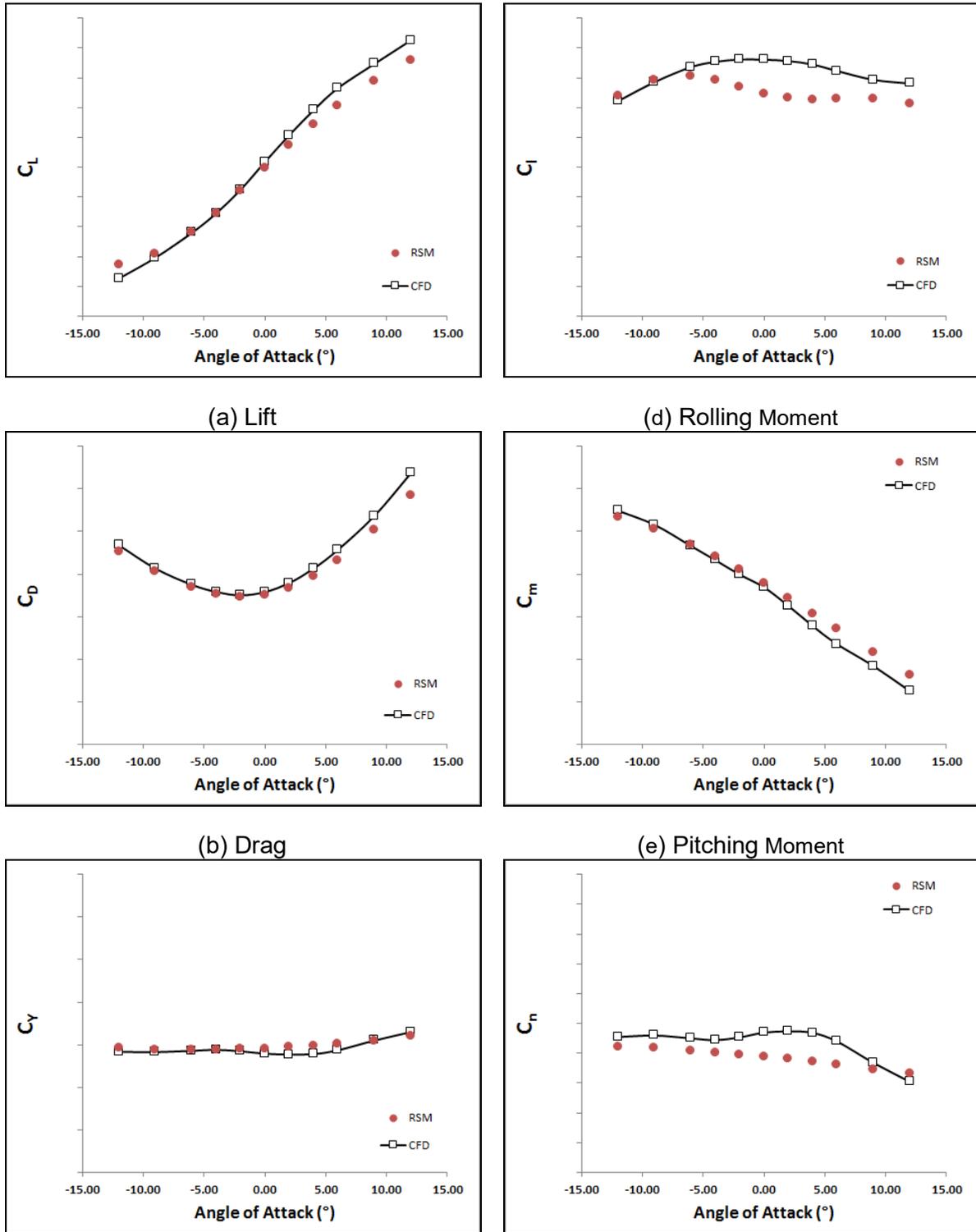


Figure 4: Comparison of CFD and RSM Results of Force and Moment Coefficients ($M=0.95$, $\beta=0^\circ$, $\delta_a=10^\circ$)

TEST MATRIX DESIGN

As it is stated before, wind tunnel testing is a necessary but a costly task to perform. For this reason, test matrix should be designed carefully to prevent redundant tests. For the problem considered in this study, one may start with considering all combinations of changes in input parameters. Since angle of attack is generally the driving parameter in a wind tunnel test, this

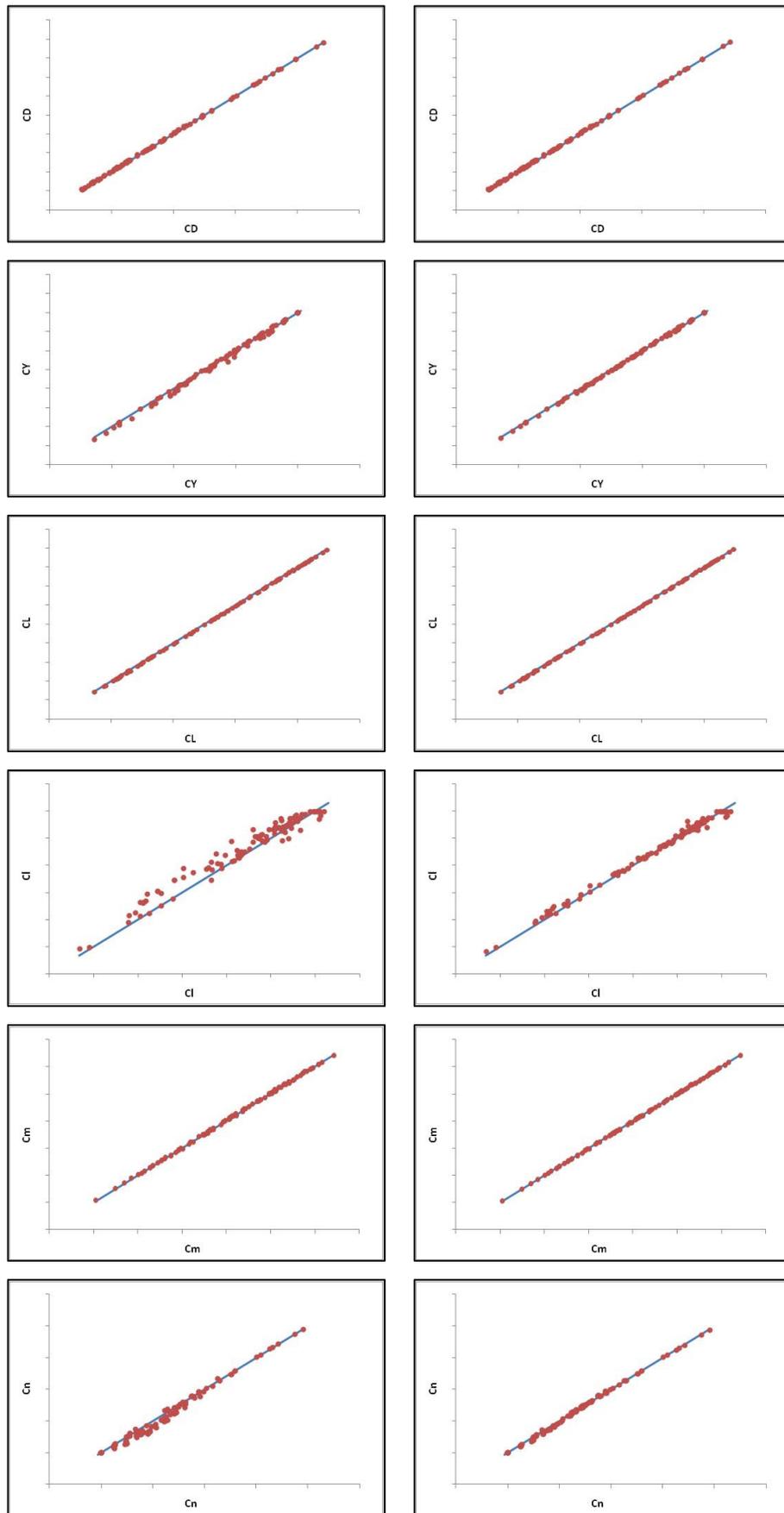
would end up in a total number of wind tunnel runs for any Mach number considered as $(\# \text{side-slip angles}) * (\# \text{fin deflections})^4$. For example, considering 3 side-slip angles and 5 fin deflection angles, the number of runs becomes 1875. Obviously, this is not an affordable number even for this relatively coarse resolution.

The above predicted number of wind tunnel runs can be reduced significantly using the interpretations extracted from the DoE analyses performed before. As can be remembered, the most useful interpretation that has been made is that the interactional effects between the fin deflection angles are negligibly low. This conclusion paves the way of using linearization technique that one may separate the contributions of fin deflections in the resultant forces and moments.

Linearization technique is validated for the present problem with the help of generated RSMs. As the first step, an optimal interval for side-slip angle is searched for assuming angle of attack as the driving parameter. For this purpose, a routine that performs interpolation across the tables of angle of attack, Mach number and side-slip angle variations generated from the RSMs is coded. The fin deflections are taken as zero and excluded from the interpolation assuming that the interactions of fin deflections with the other input parameters are negligible. With the interpolator, aerodynamic forces and moments are interpolated to randomly chosen combinations of input parameters to investigate the deviations from the RSMs.

The first interval tested for side-slip angle variation is 9 degrees. This means 2 separate tables for 0 and 9 degrees of side-slip angle are generated. The comparison of RSM and interpolated data for randomly chosen points are shown in (Fig. 5(a)). As can be seen, almost perfect match is obtained for drag, lift and pitch moment coefficients. This is not surprising because these outputs are almost insensitive to side-slip variations as reported in DoE analyses. On the other hand, the other three outputs show noticeable deviations. The maximum error is computed around 5% that is for the roll moment coefficient. Obviously, a refinement is needed to reduce the error to an acceptable level and, therefore, a third table is generated for side-slip angle of 4.5 degrees. In (Fig. 5(b)), the comparison is given for this case. As expected, the errors that are high before are lowered significantly, to a maximum value of around 1.5%.

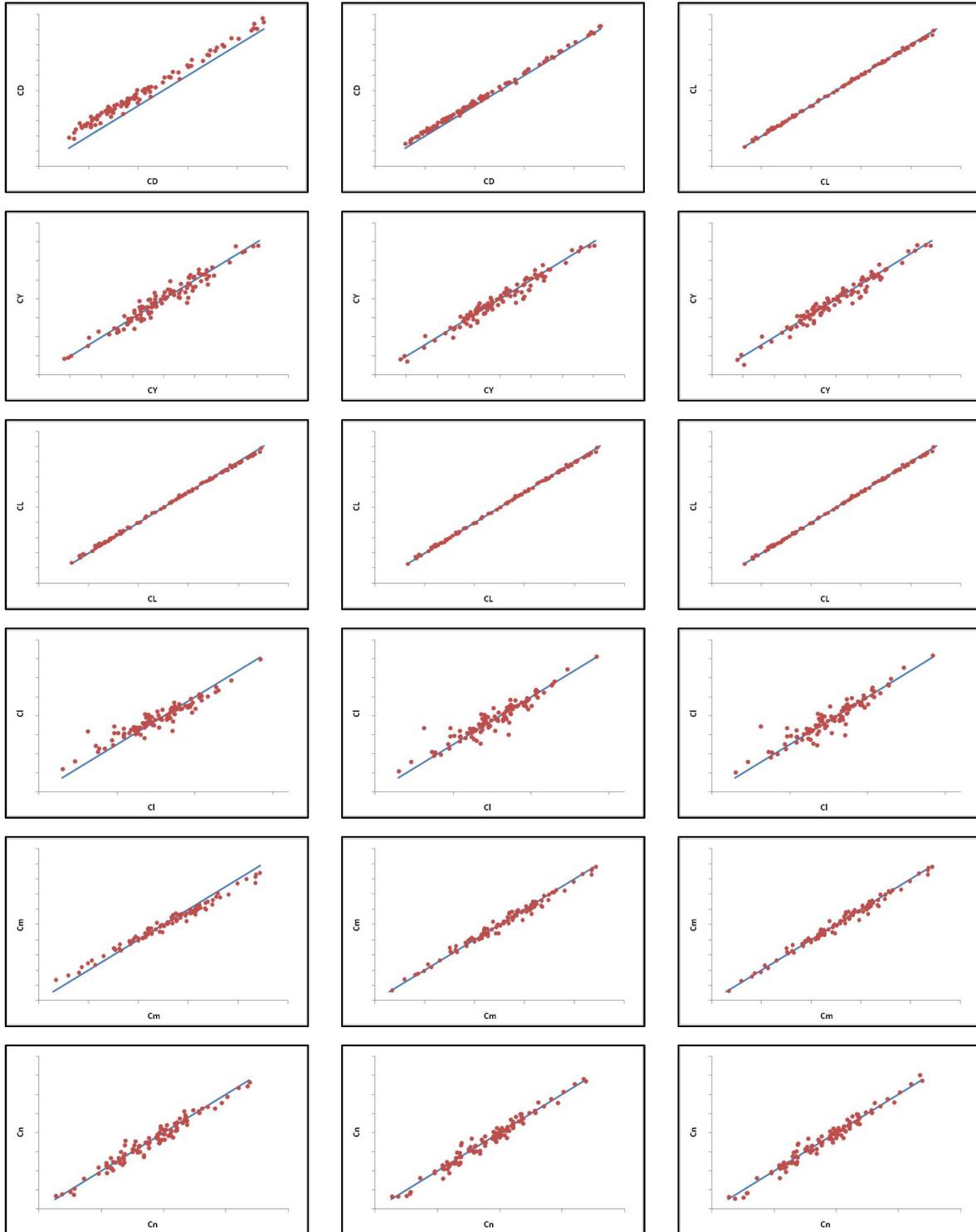
After that, the optimum interval for fin deflection angles is determined. For this purpose, first of all an interval of 15 degrees is selected for each fin deflection at zero side-slip angle and the contributions from each fin are linearly superimposed. Interpolated values are compared with the results from RSMs for each output and the errors are computed. The same process is repeated for the intervals of 7.5 and 5 degrees. The comparison of validation results using these three intervals is given in (Fig. 6.)



(a) $\beta = 0^\circ, 9^\circ$

(b) $\beta = 0^\circ, 4.5^\circ, 9^\circ$

Figure 5: Comparison of validation results for side-slip angles ($\delta_n=0^\circ$)

(a) $\delta = 0^\circ, 15^\circ$ (b) $\delta = 0^\circ, 7.5^\circ, 15^\circ$ (c) $\delta = 0^\circ, 5^\circ, 10^\circ, 15^\circ$ Figure 6: Comparison of validation results for fin deflection angles ($\beta=0^\circ$)

As one can observe from (Fig. 6), as the intervals of fin deflections are reduced, deviation between the approximated and the exact values are decreased. On the other hand, the improvements are not significant as reducing the intervals from 7.5 to 5 degrees except that of the drag coefficient. Nevertheless, the maximum error computed for 100 randomly chosen points is around 4% for 5 degrees interval.

As the next step of linearization procedure, the correlation between the side-slip angle and the fin deflections is investigated assuming no correlation exists between the fin deflection angles themselves. For this purpose, separate tables for side-slip variations at zero fin deflections and for fin deflections variations at zero side-slip for the above determined intervals are generated. These tables are used to superimpose side-slip and fin deflection effects separately.

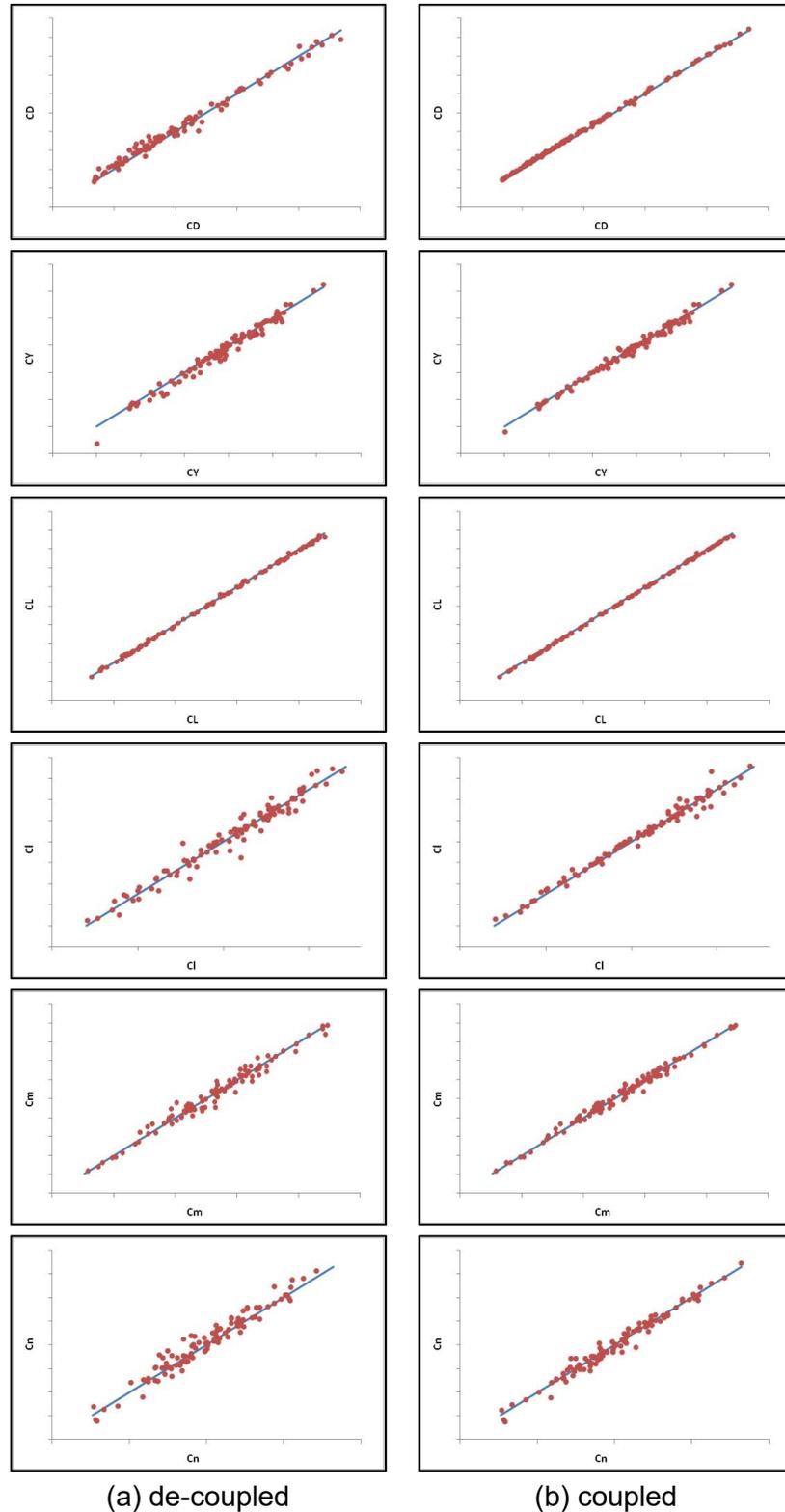


Figure 7: Comparison of validation results for side-slip and fin deflections angles

Additionally, tables for fin deflections at side-slip angles other than zero are also generated to investigate the coupling effect. The validation results for 100 randomly chosen points are presented in (Fig. 7).

The results presented in (Fig.7) suggest some coupling between side-slip and fin deflection angles, because including coupling seems to reduce errors. For this case the maximum error goes from 4% to 2%, however both numbers might be acceptable in engineering point of view. It should also be considered that including this coupling increases the wind tunnel runs from 15 to 51 for a single Mach number for the current case.

RESULTANT TEST MATRIX

Considering the results of above given analysis, the resultant wind tunnel test matrix given in (Table 8) is formed. As given in this table, the total number of wind tunnel runs for a single Mach number is 51. This means significant reduction in run number considering 1875 runs that would be needed if not considering linearization of fin deflections effects.

Table 8: Resultant Wind Tunnel Test Matrix

Polar #	AOS	δ_1	δ_2	δ_3	δ_4
1	0.0	0	0	0	0
2	4.5	0	0	0	0
3	9.0	0	0	0	0
4	0.0	-5	-5	-5	-5
5	0.0	-10	-10	-10	-10
6	0.0	-15	-15	-15	-15
7	4.5	15	15	15	15
8	4.5	10	10	10	10
9	4.5	5	5	5	5
10	4.5	-5	-5	-5	-5
11	4.5	-10	-10	-10	-10
12	4.5	-15	-15	-15	-15
13	9.0	15	15	15	15
14	9.0	10	10	10	10
15	9.0	5	5	5	5
16	9.0	-5	-5	-5	-5
17	9.0	-10	-10	-10	-10
18	9.0	-15	-15	-15	-15
19	0.0	15	15	-15	-15
20	0.0	10	10	-10	-10
21	0.0	5	5	-5	-5
22	0.0	-5	-5	5	5
23	0.0	-10	-10	10	10
24	0.0	-15	-15	15	15
25	4.5	15	15	-15	-15
26	4.5	10	10	-10	-10
27	4.5	5	5	-5	-5
28	4.5	-5	-5	5	5
29	4.5	-10	-10	10	10
30	4.5	-15	-15	15	15
31	9.0	15	15	-15	-15
32	9.0	10	10	-10	-10
33	9.0	5	5	-5	-5
34	9.0	-5	-5	5	5
35	9.0	-10	-10	10	10
36	9.0	-15	-15	15	15
37	0.0	5	-5	-5	5
38	0.0	10	-10	-10	10
39	0.0	15	-15	-15	15
40	4.5	-15	15	15	-15
41	4.5	-10	10	10	-10
42	4.5	-5	5	5	-5
43	4.5	5	-5	-5	5
44	4.5	10	-10	-10	10
45	4.5	15	-15	-15	15
46	9.0	-15	15	15	-15
47	9.0	-10	10	10	-10
48	9.0	-5	5	5	-5
49	9.0	5	-5	-5	5
50	9.0	10	-10	-10	10
51	9.0	15	-15	-15	15

CONCLUSION

In this study, wind tunnel test matrix of a newly developed Air-to-Ground missile is designed utilizing the DoE method based on CFD simulations. First of all, contributions of input parameters to each output (aerodynamic forces and moments) and the correlation between the inputs are identified. Then, separate Response Surface Models (RSMs) for each output that span the whole design space is generated to investigate the design deeper. With help of the generated RSMs, optimal interval values for each input parameters are determined to use in interpolation. In the end, a test matrix of 51 runs for a single Mach number is designed and a significant reduction in number of runs is provided.

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