9th ANKARA INTERNATIONAL AEROSPACE CONFERENCE **AIAC-2017-060**
20-22 September 2017 - METU, Ankara TURKEY

STATE-SPACE REPRESENTATION OF FLAPPING WINGS IN HOVER

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ABSTRACT

The state-space representation of unsteady wing aerodynamics is developed and implemented for the lift generation at high frequency flapping to hover. This way very fast solution for the response of the lifting surface is determined via solution of a first order ordinary differential equation which governs the time dependent behavior of the state variable. The state variable, in general, is identified as the characteristic flow parameter suitable for representing the unsteadiness of the flow; such as the separation point movement for an airfoil or vortex breakdown location on a delta wing at high angles of attack. For more complex flows, however, the value of the Duhamel integral of the arbitrary unsteady motion becomes an intelligent choice as the state variable. The quasi steady circulation with the leading edge vortex at high AoA is employed as the loading to the Duhamel integral together with the Wagner function to determine the timewise unsteady lift change. Since the frequencies considered are quite high, the contribution of the apparent mass term to the unsteady lift is also considered. The formulation developed here is succesfully implemented for evaluating the unsteady lift generated by the fruit fly in hover.

INTRODUCTION

The studies of flapping wing aerodynamics have become quite popular since it provides quiet and efficient performance of MAV's, as opposed to the engine powered fixed wings, for the applications. For practical applications the prediction of unsteady aerodynamic forces and moments, the experimental and the numerical methods reqiure vast amount of wind tunnel hours as well as huge computational efforts even with todays High Performence Computers. On the other hand, for the real time applications fast predictions are necessary for flight dynamics and control purposes. The state-space representation of unsteady aerodynamic applications provides fast results for the determination of the lift with wing flapping. The state variable is identified either formally or it is based on a well defined physical meaning [Goman and Khrabrov, 1994], [Gulcat, 2011 and 2016] and most recently [Uhlig and Selig, 2017], and its time dependence is calculated with numerical solution of a first order differential equation. It is also applied for morphing wings of finite span [Reich et.al., 2011] and [Izraelevitz et.al., 2017]. For the formal way; the Wagner function which gives the response of the lifting surface to the unit excitation is employed either with Duhemal integral [Taha et.al, 2014] or with Laplace transform of the function [Leishman, 2000]. The Wagner function in terms of the reduced time is given in [Bisblinghoff, et.al, 1995] for airfoils and for the finite wings of different aspect ratios. For the latter, the location of the separation point of an airfoil is chosen as the state varible for 2-D cases and the vortex burst point is considered for delta wings. In both cases, relaxation time parameters and the static aerodynamic behavior of the lifting surfaces must be known.

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Here, the Wagner function approach is chosen to cover the wide range of angle of attacks to represent the backward and forward motion of the wing in hover. The time relaxation is automatically handled within the Wagner function which behaves as the kernel of the Duhamel integral, wherein the reduced time is no longer based on a constant free stream. Therefore, the exponents of the Wagner function must be modified using the time integration of the harmonically varying freestream.

As the application of the developed method, the lift generated by flapping of fruit fly wings, in hover, is determined. The static lift dependence on AoA of the fruit fly given in [Berman and Wang, 2007] and [Taha et.al, 2014] is utilized for determining the contribution of the quasi steady circulation as well as its contribution to the Duhemal Integral.

METHOD

The lift generated with flapping wing is formulated with the state-space concept based on the Wagner function which gives the response of the lifting surface to an arbitrary motion as

$$l(s)=\rho U^2 bC_l(s)=\rho b\pi(\ddot{h}+U\dot{\alpha})-2\rho U b\pi\left[w(b/2,s)\Phi(0)+\int_0^s w(b/2,\sigma)\frac{d\Phi(s-\sigma)}{d\sigma}d\sigma\right] \tag{1}$$

Where,b is the half chord, *w(b/2,s)* is the downwash at the quarter chord and $\Phi(s)=1-a_1e^{-b_1s}-a_2e^{-b_2s}$ is the Wagner function. For varying free stream, the reduced time reads as $s=\frac{1}{b}\int_0^t U(\tau)d\tau$, whereas for the constant free stream it is *s=Ut/b.*

The downwash value for a pitching wing strip whose equation is given with $z_a(x,t)=-\alpha(t)(x-a)$, where a is the location of the pitch axis, is obtained as

$$w(b/2,t)=\frac{\partial z_a}{\partial t}+U\frac{\partial z_a}{\partial x}=-\dot{\alpha}(t)\left[b/2-a\right]-U\alpha(t) \tag{2}$$

The quasi steady circulation created by the strip is then given with

$$\Gamma_{qs}(t)=bU(t)C_{ls}(\alpha(t))+\pi b(b/2-a)\dot{\alpha}(t) \tag{3}$$

Here, C_{ls} is the lift coefficient given for even at large angles of attack which permits the occurence of leading edge vortex.

The circulatory lift of the strip now becomes

$$l_c(t)=\rho U(t)\,\Gamma_c=\rho U(t)\left[\Gamma_{qs}(t)\,\Phi(0)-\int_0^t\Gamma_{qs}(\tau)\,\frac{d\,\Phi(t-\tau)}{d\tau}d\tau\right] \tag{4}$$

For varying free stream the dervative of the Wagner function reads as

$$\frac{d\,\Phi(t-\tau)}{d\tau}=-a_i\frac{b_i}{b}U(\tau)e^{-\frac{b_i}{b}\int_\varsigma^t U(\varsigma)d\tau}\;,\quad i=1,2 \tag{5}$$

With the above's equation, (5), the unsteady circulation becomes

$$\Gamma_c(t)=(1-a_1-a_2)\Gamma_{qs}(t)+x_i(t) \tag{6}$$

Where,

$$x_i(t)=\int_0^t\Gamma_{qs}(\tau)\;a_i\frac{b_i}{b}U(\tau)e^{-\frac{b_i}{b}\int_\varsigma^t U(\varsigma)d\tau}d\tau$$

The derivative of this equatiıon, with respect to time t, applying the Leibnitz rule, gives

$$\dot{x}_i(t)=\frac{b_i}{b}U(\tau)\;(-x_i(t)+a_i\,\Gamma_{qs}(t)),\;\; i=1,2 \tag{7}$$

Here, $x_i=x_i(t)$ is identified as the state-space variable for determining the unsteady lift generated by the flapping wing via solving (7), which is an ODE with $x_i(0)=0$. The circulatory lift then reads,

$$l_c(t) = \rho U(\tau)\left[(1-a_1-a_2)\Gamma_{qs}(t) + x_1(t) + x_2(t)\right] \quad (8)$$

The non-circulatory lift, on the other hand, is calculated as the apparent mass term as folllows:

$$l_{nc}(t) = -\pi \rho b^2 a_z(t) \cos\eta(t) \quad (9)$$

Where, $a_z(t)$ is the vertical acceleration and $\eta(t)$ is pitching angle. The acceleration , here, has two components: i) Acceleration due to freestream speed change only i.e. $-r\ddot{\varphi}\sin\eta$, and ii) acceleration due to angle of attack change, i.e. $-r\dot{\varphi}\dot{\eta}\cos\eta$. Hence, the acceleration for the apparent mass term becomes $a_z = -(r\ddot{\varphi}\sin\eta + r\dot{\varphi}\dot{\eta}\cos\eta)$. The sectional lift coefficient, with (8) and (9), is defined as follows

$$C_l(t) = \frac{l_c(t) + l_{nc}(t)}{\rho {U_m}^2 b_m} \quad (10)$$

Here, U_m is the maximum free stream velocity at the midspan and b_m is the half chord at the midspan.

Applications

The method developed here is applied to the fruit flies hover calculations. The data for the fruit fly wing flapping is given in [Taha et.al, 2014]. The span of the wing is 2.02 mm, the aspect ratio is 3, half chord is 0.34mm and the static lift coefficient is $C_l = 1.833\sin 2\alpha$. The Wagner function is given by [Bisblinghoff, et.al, 1995]. The preliminary calculations are performed for the half span of the wing whose pitching motion for one period of forward and backward sweeps as shown in Figure 1.

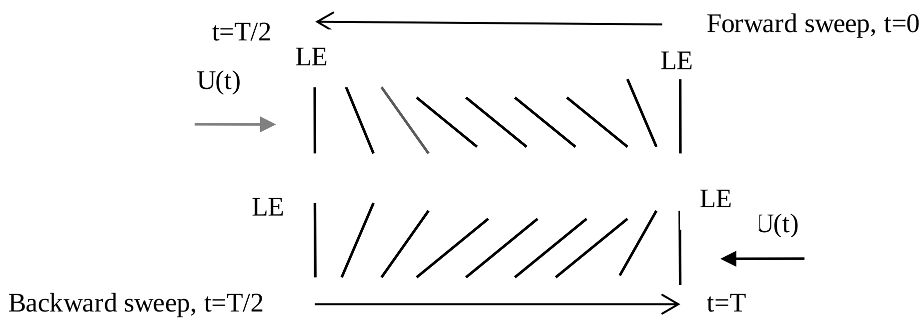

Figure 1. Angle of attack change, 90º-40º-90º during forward and backward sweeps.

Shown in Figure2 is the simplified top and side wiev of the wing-body combination of the fruit fly.

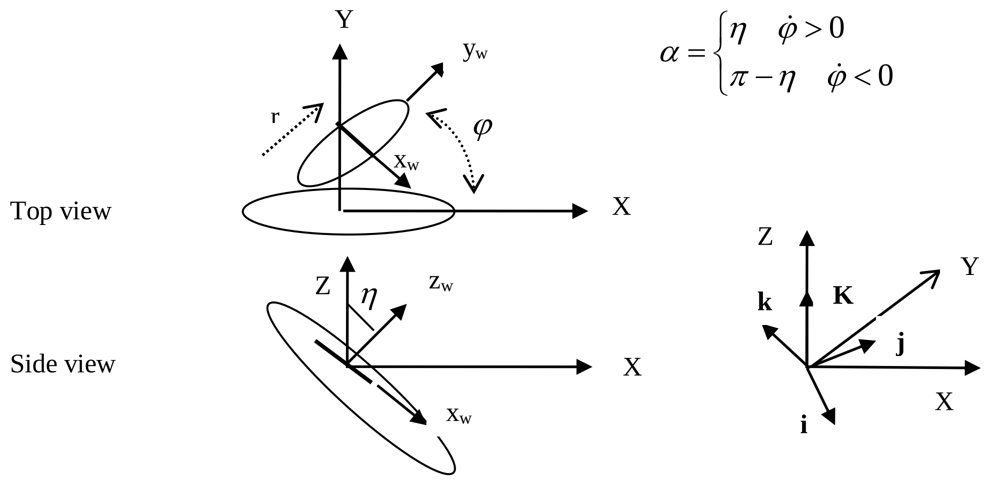

Figure 2. Top and side views of the body and the wing (in side view only a cross section of the wing is shown).

The sweep variation $\varphi = -75^o \cos\omega t$ and the pitch variation, in terms of arctangent and sine function, of the wing are given in Figure 3. The variable free stream at span r reads as

$$\varphi = \overline{\varphi}\omega r \sin \omega t$$

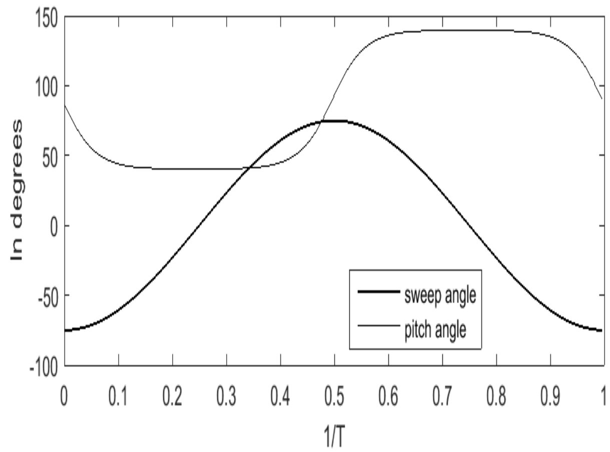

Figure 3. Variation of the sweep angle: $-75^o < \varphi < 75^o$, and the pitch angle $40^o < \eta < 140^o$.

Shown in Figure 4 is the variation of lift and the state variables for period of six of the motion. The contribution of the unsteady circulatory term initially is zero, by time it increases and evantualy it reaches to its constant averaged value. The time averaged sectional lift coefficient reads as

$$C_L = \frac{1}{T}\int_{T}^{2T} C_l(t)\,dt = 0.85 \tag{11}$$

We can find the average lift force generated by the fruit fly in hover and compare it with weight of it given in [Berman and Wang, 2007] as $W = 7.06\mu N$, wherein the frequency of flapping is 268Hz for the optimum solution, hence, the reference speed becomes

$U_m = r\overline{\varphi}\omega = 1.01x10^{-3}x75/180x\pi x268x2x\pi = 2.23m/\sec$.

Using (11), the sectional lift force reads as

$$F_s = \rho U_m^2 b_m C_L = 1.225x2.23^2 x0.34x10^{-3}x0.85 = 1.76x10^{-3}N/m.$$

For two wings with span of 2.02mm the total lifting force can be approxmately calculated as $F = 2x1.76x10^{-3}x2.02x10^{-3} = 7.11\mu N$ which is not even 1% higher than the weight of the fruit fly.

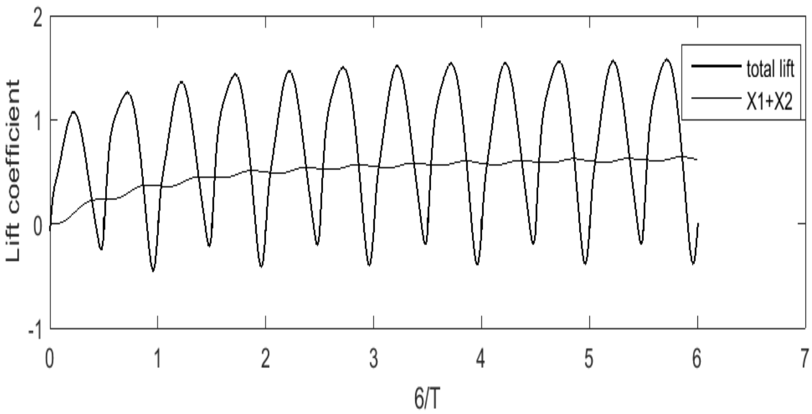

Figure 4. Total lift coefficient, C_l , and state variables variation, X1+X2, for 6 period.

The time averaged lift coefficient is calculated with (11) using one period as shown in Figure 5, which is the full period after the 6th period shown in Figure 4.

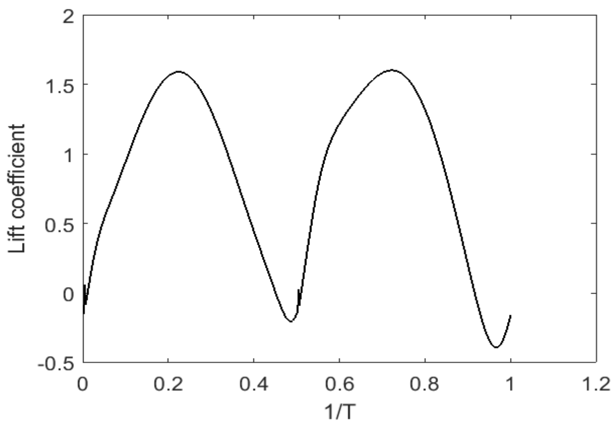

Figure 5. Lift coefficient variation for one period

Here, the sectional lift generated at the midspan of the wing is calculated.

A typical fruit fly wing is given in Figure 6.The shape of the wing is elliptic and the Wagner function for the elliptical wings with different aspect ratios are provided in [Bisblinghoff, et.al, 1995].

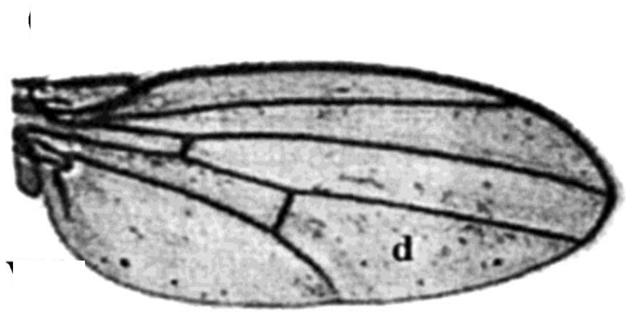

Figure 6. Wing shape of a typical fruit fly.(Dimensions: body to root: 0.20mm, span: 2.02mm, midchord: 0.86mm, S=1.36mm^2)
(www.google.com.tr/search?q=shape+of+the+fruit+fly+wing)

Finite wing: The Wagner function for the elliptical wings are given in [Bisplinghoff, et al, 1996] as follows $\Phi_W(s) = 0.6 - 0.17e^{-o.54s}$. The spanwise integration of the sectional lift given in (8) and (9) with respect to r, from root to tip results in for the total wing

$$\int_{r_1}^{r_1+R}(l_c(t)+l_{nc}(t))dr=\rho\int_{r_1}^{r_1+R}\left\{U(\tau)\left[(0.6-0.17)\Gamma_{qs}(t)+x_1(t)+x_2(t)\right]-\pi\rho b^2 a_z(t)\cos\eta(t)\right\}dr \quad (12)$$

The integrand of (12) can be separated into 3 different components after letting $U(\tau)=\dot{\varphi}r$ as follows: i) two circulatory terms as pitch and the pitch rate, and ii) a non circulatory term. Hence, contributions of each term to the integral results in

Pitch rate: $l_{cr}(t)=\rho I_1\dot{\varphi}\left[0.43\pi\dot{\alpha}(t)/2+x_3(t)\right],\quad I_1=\int_{r_1}^{r_1+R} rb^2 dr$

Pitch: $l_{cp}(t)=\rho I_2\dot{\varphi}\left[0.43C_L(\alpha,t)+x_4(t)\right],\quad I_2=\int_{r_1}^{r_1+R} r^2 b\,dr$

Where, $\dot{x}_3(t)=\dfrac{b_1\bar{r}\dot{\varphi}(t)}{\bar{b}}\left[-x_3(t)+0.17\dot{\alpha}(t)\right]$ *and* $\dot{x}_4(t)=\dfrac{b_1\bar{r}\dot{\varphi}(t)}{\bar{b}}\left[-x_4(t)+0.17\dot{\varphi}(t)C_L(\alpha,t)\right]$

Non-circulatory: $l_{nc}(t)=\pi\rho I_1\left[\ddot{\varphi}(t)\sin\eta(t)+\dot{\varphi}(t)\dot{\eta}(t)\cos\eta(t)\right]\cos\eta(t)$

Here, $\bar{b}$ *and* $\bar{r}$ the reference half chord and span values. The numerical values pertinent to the wing of a fruit fly pictured in Figure 6 is given in Appendix. Using the values for I_1, I_2 and $\bar{b}$ the wing's total lift coefficient reads as

$$C_L(t)=\frac{l_{cr}(t)+l_{cp}(t)+l_{nc}(t)}{\rho U_m^2 S/2} \quad (13)$$

The plot of total lift coefficient together with the 2-D case is shown in Figure 7.

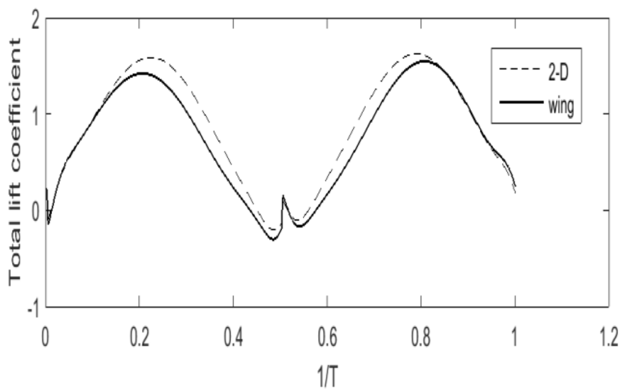

Figure 7. Variation of the total lift coefficient of the wing for one period

The averaged lift coeffcient for the wing using (13) reads as

$$\overline{C}_L=\frac{1}{T}\int_T^{2T}C_L(t)\,dt=0.75 \quad (14)$$

The averged lift force, for both wings considered, then reads as

$$F=2\overline{C}_L\rho U_m^2 S/2=7.11\mu N,$$

with wing flapping frequency *f=240 Hz* it is possible to attain hover for the fruit fly.

CONCLUSIONS

State-space representation for unsteady aerodynamic computations is developed and implemented for fruit fly hover.
First, 2-D approach is considered for a rectangular wing for the sake of simplicity in testing the approach. A bit high frequency of flapping needed to maintain the weight of the fruit fly in balence with the lifting force.
Finally, the elliptical wing shape of the fruit fly is considered with a hub length (distance from the body to the root of the wing). This way, the frequency of the flapping is reduced ten percent in order to maintain hover.

Appendix

The properties of the eliptical wing shape of the fruit fly is given in Figure A. Accordingly, the first and the second moment of inertia for the wing read as

$$I_1/2 = \int_{r_1}^{r_1+R} rb^2(r)dr$$ =eval(int((-(x-1.01-.20)^2/1.01^2+1)*0.43^2*x,.20,2.22))= 0.3013 mm^4

$$I_2/2 = \int_{r_1}^{r_1+R} r^2b(r)dr$$ =eval(int(sqrt(-(x-1.01-.20)^2/1.01^2+1)*0.43^2*x^2,.20,2.22)= 0.5043 mm^4

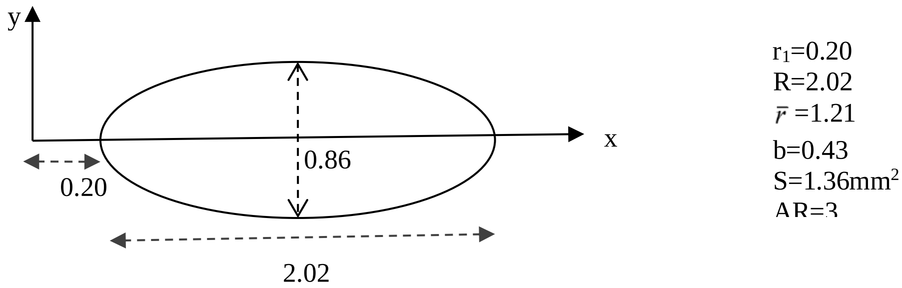

Figure A. Pertinent dimensions, in mm, of the elliptical wing of a fruit fly.

The equation of the ellipse is $$\frac{(r-1.01-0.20)^2}{(1.01)^2} + \frac{b^2}{(0.43)^2} = 1$$

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