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IN-FLIGHT MODAL IDENTIFICATION OF AN AIRCRAFT USING OPERATIONAL MODAL ANALYSIS

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ABSTRACT

Operational Modal Analysis is a methodology used to determine the modal properties of a dynamic system by using structural dynamic response measurements obtained in-service conditions without measuring excitation forces. In this paper, modal characteristics of an aircraft have been investigated by using Operational Modal Analysis. The structural dynamic response data has been collected from a flight test by 12 accelerometers, 6 on each wing. A suitable maneuver interval has been chosen for response data in order to satisfy fundamental assumptions of Operational Modal Analysis. Wing-related natural frequencies, mode shapes and corresponding damping ratios have been found.

INTRODUCTION

The dynamic characterization of an aircraft plays an important role in the design process and both numerical simulations and experiments should be performed simultaneously in order to predict the dynamic behavior of aircraft during the operative conditions. These dynamic behaviors are dependent on the modal parameters of the system which are natural frequencies, mode shapes and damping ratios. The system modal parameters representing the signature of the dynamic system, can be used to track the vibration related problems and for the aeroelastic instability of flying aircraft.

In this paper, Operational Modal Analysis, in short OMA, has been performed to determine the modal parameters of an aircraft in operative conditions. The modal parameters have been estimated by using only the dynamic response data taken in-flight, without any need for excitation forces. The unknown excitation forces, which are atmospheric turbulence and vibration coming from engine, should be such that all modes of the system of interest are excited [Brincker et al., 2015]. According to recent researches, Operational Modal Analysis approach for in-flight case is suitable and widespread for determining the aircraft's dynamic

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characteristics [Neu et al., 2015; Follador et al., 2016; Coppotelli et al., 2013; Marulo et al., 2005; Danowsky et al., 2015].

The advantage of Operational Modal Analysis is that it allows to estimate the modal parameters in the effective working condition of the aircraft which means it yields high accuracy and good reliability in the flight tests. In addition, researches show that the dynamical behavior of a structure may change in its operational environment, especially when the input excitation differs from those applied during the laboratory testing and when the structural behavior exhibits some degree of nonlinearities [Marulo, 2005].

METHOD

Operational Modal Analysis can be performed by using frequency or time domain techniques. Examples of time domain techniques are the Autoregressive Moving Average Method (ARMAV) and the Stochastic Subspace Identification (SSI). The frequency domain methods are the Basic Frequency Domain Method (BFD) and the Frequency Domain Decomposition Method (FDD). FDD method allows identification of natural frequencies, corresponding mode shapes and damping ratios from the dynamic system in frequency domain.

Frequency – Domain Decomposition (FDD)

The FDD method has been widely used for OMA and it is based on singular value decomposition of the cross spectral density matrix with the advantage that allows all information in one single plot; that is the plot of singular values of the cross spectral density matrix. In addition, FDD method allows to determine the noise level of the modes that are to be observed.

Theoretical Background [Brincker, 2000; Rainieri et al., 2007]

Considering the response $\boldsymbol{y}(\boldsymbol{t})$ in modal coordinates,

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{a_1 q_1}(\boldsymbol{t}) + \boldsymbol{a_2 q_2}(\boldsymbol{t}) + \ \dots \ = \boldsymbol{Aq}(\boldsymbol{t})$$

Where $\boldsymbol{A}$ is the matrix of mode shapes and $\boldsymbol{q}(\boldsymbol{t})$ is the column vector of modal coordinates. With the definition of auto correlation function matrix $\boldsymbol{R_y}(\boldsymbol{\tau})$,

$$\begin{aligned}\boldsymbol{R_y}(\boldsymbol{\tau}) &= \boldsymbol{E}[\boldsymbol{y}(\boldsymbol{t})\boldsymbol{y^T}(\boldsymbol{t}+\boldsymbol{\tau})] \\ &= \boldsymbol{AE}[\boldsymbol{q}(\boldsymbol{t})\boldsymbol{q^T}(\boldsymbol{t}+\boldsymbol{\tau})]\boldsymbol{A^T} \\ &= \boldsymbol{AR_q}(\boldsymbol{\tau})\boldsymbol{A^T}\end{aligned}$$

Where $\boldsymbol{R_q}(\boldsymbol{\tau})$ is the auto correlation matrix of modal coordinates. Taking the Fourier transform of both sides,

$$\boldsymbol{G_y}(\boldsymbol{f}) = \boldsymbol{AG_q}(\boldsymbol{f})\boldsymbol{A^T}$$

The cross spectral density matrix $\boldsymbol{G_y}(\boldsymbol{f})$ is obtained. Since $\boldsymbol{G_y}(\boldsymbol{f})$ is Hermitian and mode shape matrix $\boldsymbol{A}$ contains complexity, the final form of above equation becomes,

$$\boldsymbol{G_y}(\boldsymbol{f}) = \boldsymbol{A}[\boldsymbol{g_n}^2(\boldsymbol{f})]\boldsymbol{A^H}$$

where $g_n{}^2(f)$ is the diagonal matrix including auto spectral densities of $G_q(f)$. This kind of decomposition may be done by taking Singular Value Decomposition (SVD) of the $G_y(f)$ matrix. After decomposition, $G_y(f)$ takes the form of,

$$G_y(f) = USU^H = U[s_n{}^2]U^H$$

This technique reduces the $G_y(f)$ matrix to a more simplified form for each frequency line. The peak amplitudes of $s_n{}^2$ are associated to dominant natural modes.

Assuming the input is white noise, the general decomposition of spectral density matrix becomes,

$$G_y(w) = \sum_{n=1}^{N} \left(\frac{a_n \gamma_n^T}{-i\omega - \lambda_n} + \frac{a_n{}^* \gamma_n^H}{-i\omega - \lambda_n{}^*} + \frac{\gamma_n a_n{}^T}{-i\omega - \lambda_n} + \frac{\gamma_n{}^* a_n^H}{-i\omega - \lambda_n{}^*} \right)$$

where nominators are the residues of the system output power spectral density, containing mode shapes a_n and modal participation vectors γ_n. λ_n are the poles of the system. It is known that residues are weighted averages of mode shape vectors and for a given frequency, only a few modes contribute significantly to the residue. Thus, output spectral density matrix can be approximated as,

$$G_y(\omega) \cong \sum_{n=1}^{N} \left(\frac{c_n{}^2 a_n{}^* a_n{}^H}{-i\omega - \lambda_n{}^*} + \frac{c_n{}^2 a_n a_n{}^T}{i\omega - \lambda_n} \right)$$

Where $c_n{}^2$ is proportional to the inner product between the mode shape over the input spectral density and inversely proportional to natural frequency, damping and modal mass. Turning back to SVD equation for discrete frequencies $\omega = \omega_i$,

$$G_y(\omega_i) = U_i S_i U_i^H$$

Near a peak corresponding to the kth mode in the spectrum, only the kth mode is dominant, and the PSD matrix approximates to a rank one matrix as,

$$G_y(\omega_i) = s_i u_{i1} u_{i1}{}^H \qquad \omega_i \to \omega_k$$

Then the first singular vector at the kth resonance is an estimate of the kth mode shape,

$$\emptyset_k = u_{r1}$$

And singular value ω_k is the corresponding natural frequency of kth mode. The important thing is that the resulting mode shape vectors are complex which implies damped mode shapes. To find the undamped normal modes, a conversion is made. This conversion method rotates the complex mode shape to show maximum real values [Bartkowicz et al., 2011; Rainieri et al, 2014].

Not only natural frequencies and mode shapes can be estimated by this method, an additional technique called Enhanced Frequency Domain Decomposition (EFDD) also allows to estimate the damping ratios of the system. To find damping ratios, modal coherence function [Brincker et al., 2015] of each mode is estimated to identify SDOF frequency regions and outside of these frequency regions are excluded with the help of bandpass filter. The resulting Spectral Bell functions are inverted to time domain by Inverse Fast Fourier Transform (IFFT) to estimate the autocorrelation of the SDOF modes. Auto and cross-correlation functions between response signals take the same form as the impulse response functions of the system excited by a white noise [Vivo et al., 2014]. Thus, once identified the correlation function, the damping ratio is determined by fitting $e^{-\xi\omega_n t}$ function to decaying sinusoids [Brincker et al., 2001].

Accelerometer Installation and Flight Data Selection

The accelerometers are placed along the wing symmetrically before flight test as shown in Figure 1. From the whole flight test data, only a straight cruise maneuver interval is selected which lasts 300 seconds and considered as Gaussian as proved in histogram graph in Figure 2. A further indicator for the consistency of acquired time data is given by Spectrogram plot shown in Figure 3. Spectrogram graph gives a general idea about the stationarity of time data.

Since structural dynamic responses are Gaussian and stationary, by using the assumption that if input is stationary, so does output [Lalanne, 2009]; it may be assumed that the force excitation is also stationary, which then satisfies stationary input assumption of FDD technique.

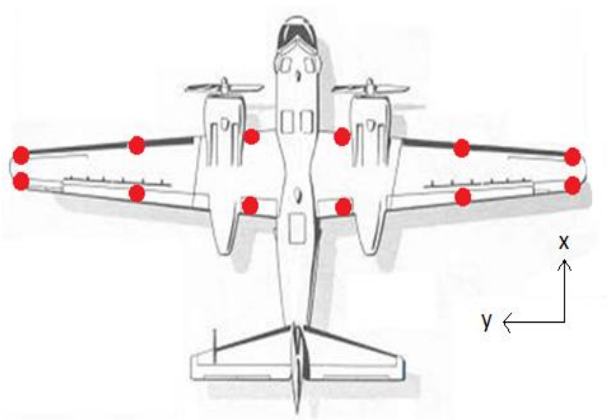


Figure 1 Accelerometer Locations

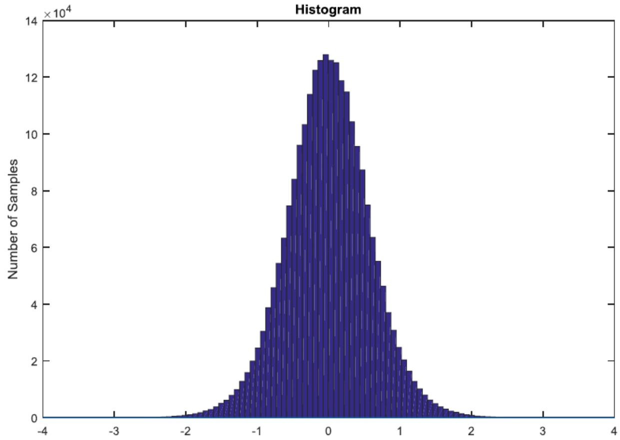


Figure 2 Histogram of Response Data

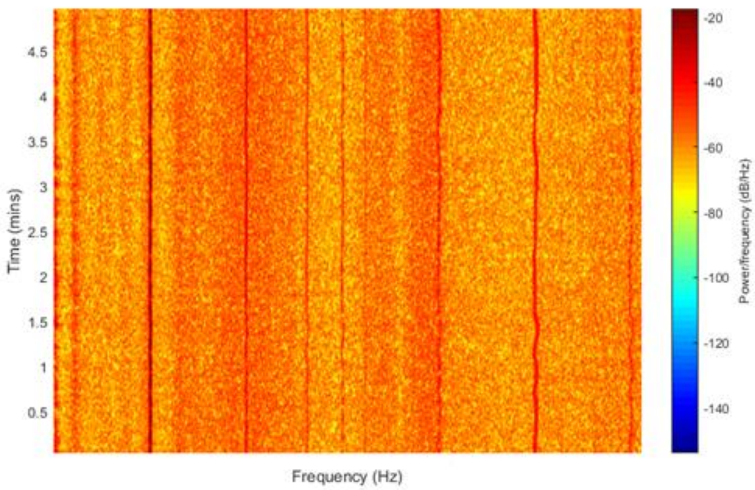


Figure 3 Spectrogram Graph

Results and Discussion

The singular value plots are shown in Figure 4. Singular Value Plots allow to determine the modes that are undiscernible as they appear on spectral density functions. If only one mode is dominating at a frequency, then there is only one dominating singular value at that particular frequency. In case of closely-coupled modes, peaks will also appear in the lower singular value plots as well.

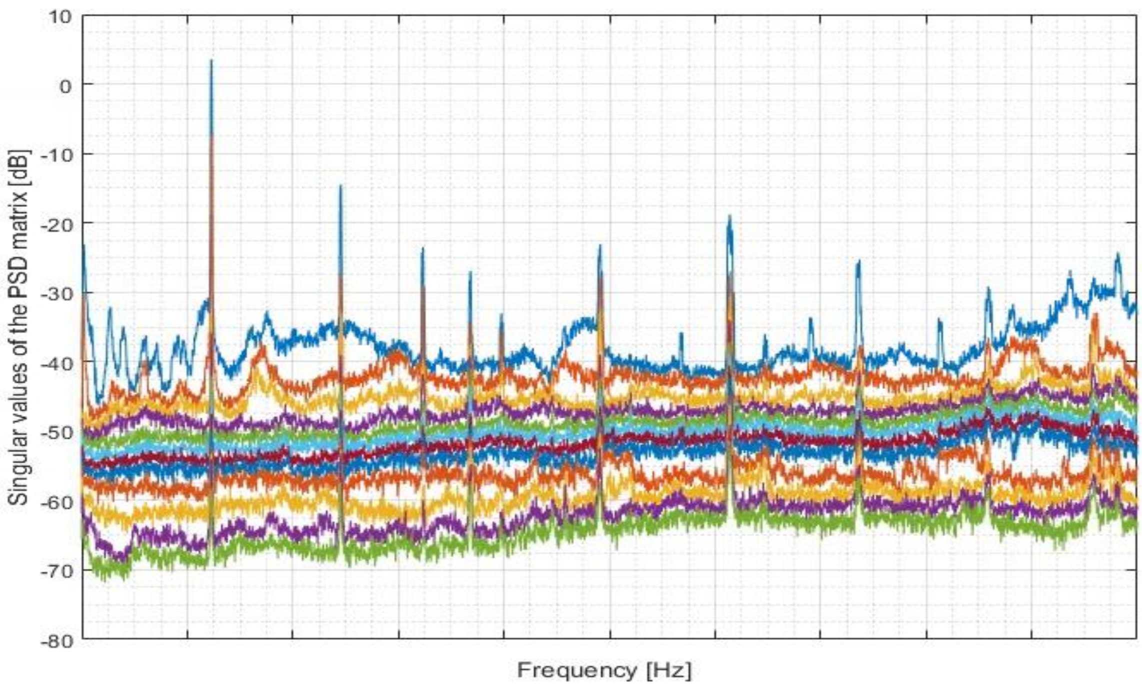
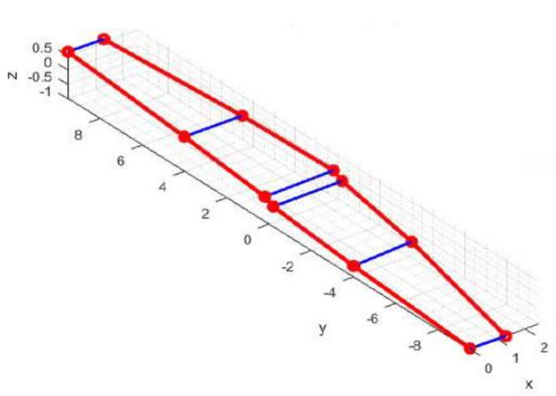
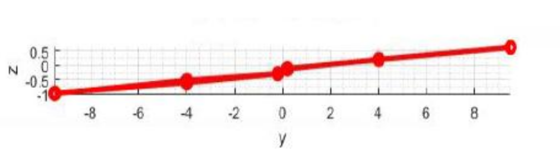


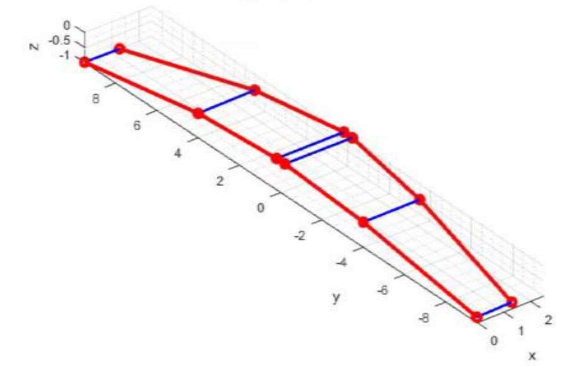
Figure 4 Singular Value Plots

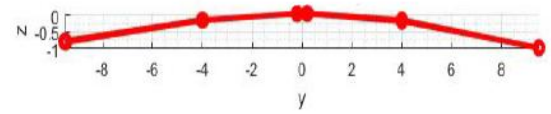
The complex eigenvectors are estimated according to the peaks in 1st singular values and shown as mesh-like view in Table 1. These eigenvectors are converted to real modes and then unity normalized for graphical representation.

Table 1 Mode Shapes

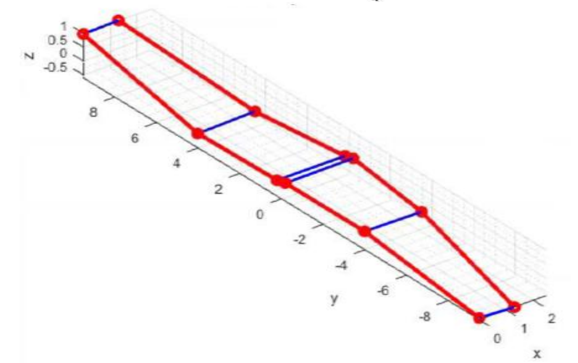
Roll (Rigid Body Mode)	
	

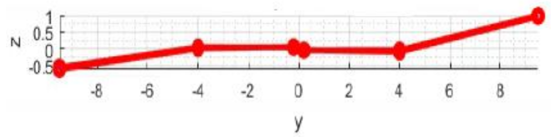
1st Symmetric Wing Bending



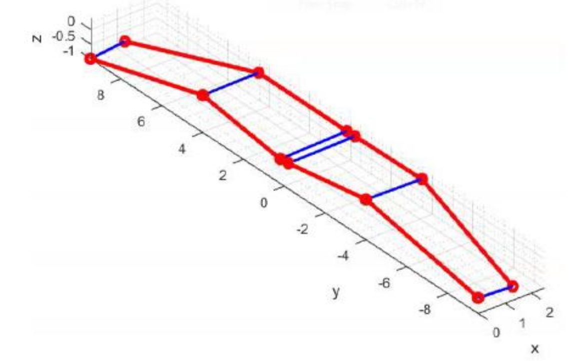


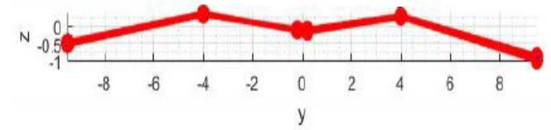
1st Antisymmetric Wing Bending



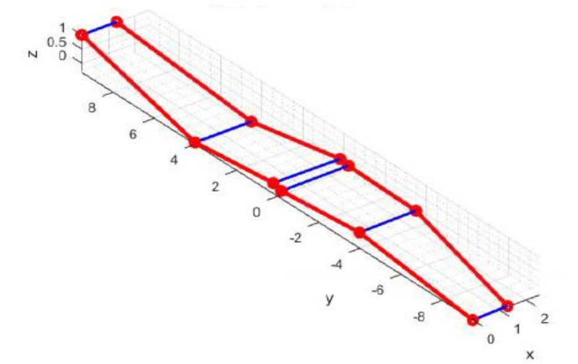


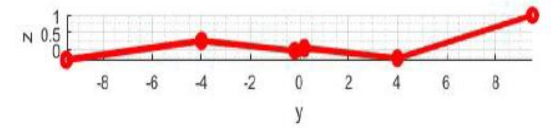
2nd Symmetric Wing Bending



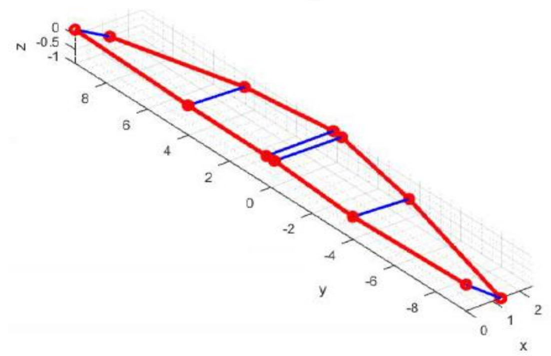


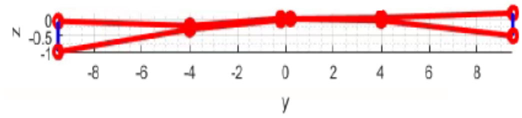
2nd Antisymmetric Wing Bending



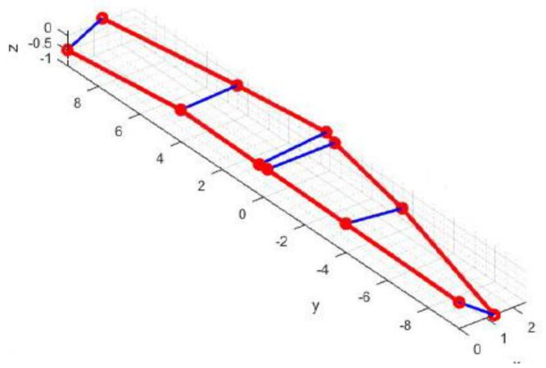


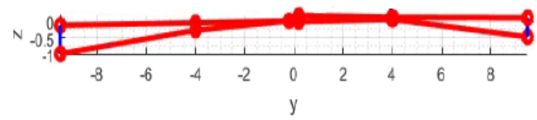
1st Symmetric Wing Torsion





1st Antisymmetric Wing Torsion





Before estimating modal damping ratios, Modal Coherences of each mode is found in order to obtain SDOF bell functions' domain which might be considered as SDOF modal domain. For this case, a priori modal coherence indicator suggested by Brincker et al. is used [Brincker et al., 2007].

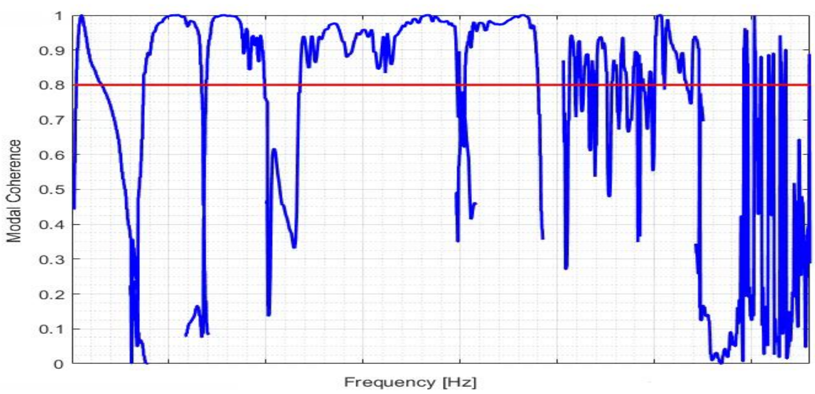


Figure 5 Modal Coherence Functions

Figure 5 shows where the modes can be identified as SDOF so that Auto-Correlation functions can identify Impulse Response Functions of a system excited by a white noise without any noise or beating phenomena [Brincker, 2001]. A margin of 0.8 is chosen in order to apply 8nd order Butterworth Band Pass filter to the time domain response data for each SDOF Modal Domain. It may also be stated that the torsion modes are not excited well enough during the flight test, which results in noisy coherence around torsion frequencies.

The auto-correlation function of found modes are given in Figure 6 to Figure 12. By fitting exponential decaying function to maximas, damping ratios are found and indicated in figures.

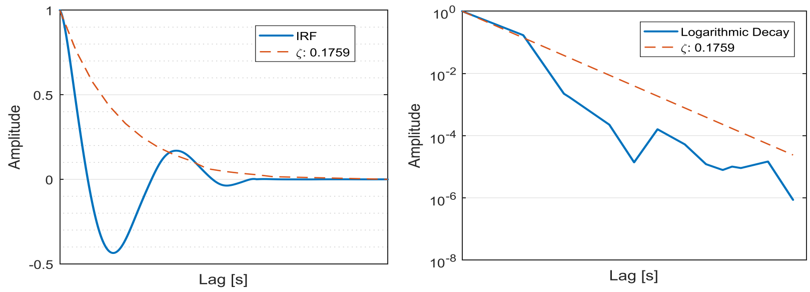


Figure 6 Left: Auto-Correlation of Rigid Body Roll Mode, Right: Logarithmic Decay

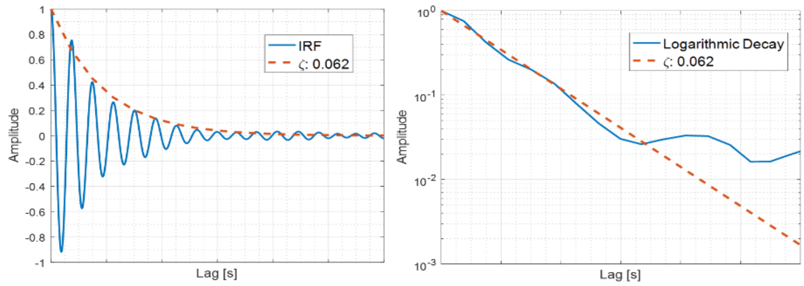


Figure 7 Left: Auto-Correlation of 1st Sym. Bending Mode, Right: Logarithmic Decay

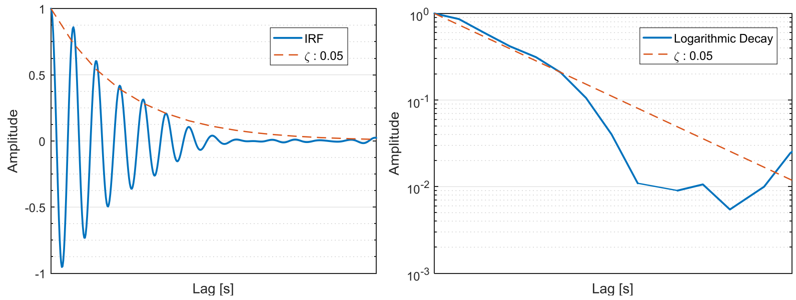


Figure 8 Left: Auto-Correlation of 1st Antisym. Bending Mode, Right: Logarithmic Decay

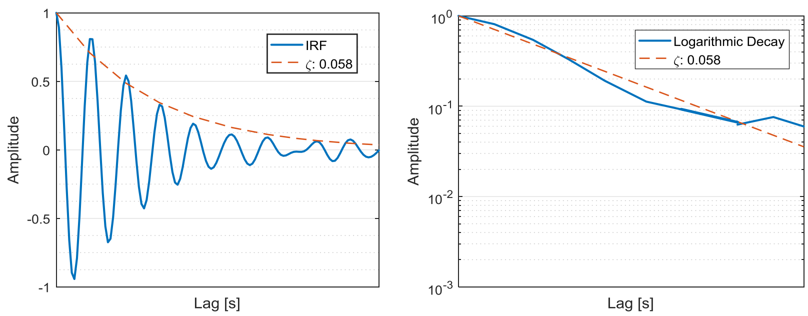


Figure 9 Left: Auto-Correlation of 2nd Sym. Bending Mode, Right: Logarithmic Decay

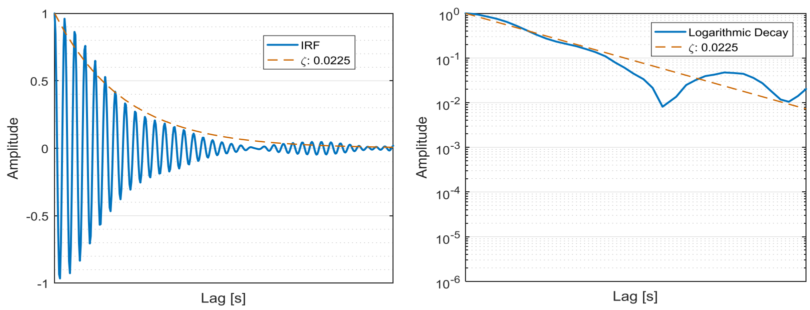


Figure 10 Left: Auto-Correlation of 2nd Antisym. Bending Mode, Right: Logarithmic Decay

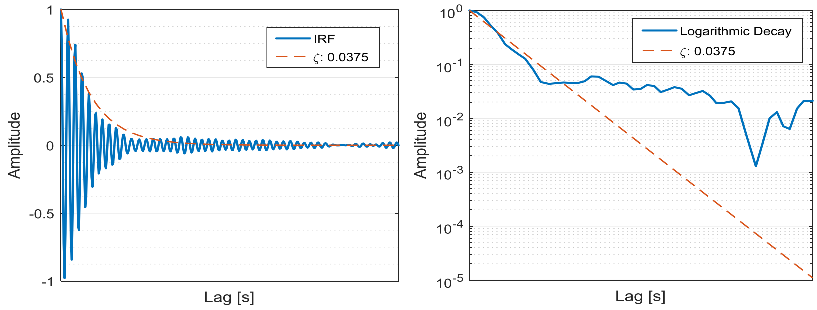


Figure 11 Left: Auto-Correlation of 1st Sym. Torsion Mode, Right: Logarithmic Decay

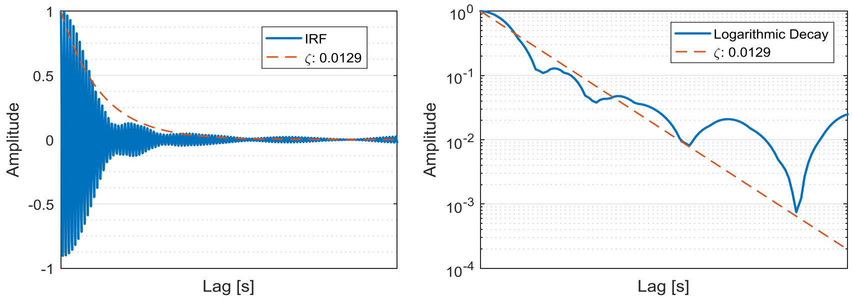


Figure 12 Left: Auto-Correlation of 1st Antisym. Torsion Mode, Right: Logarithmic Decay

It is observed that, some of the auto-correlation functions contain beating after an initial decay, which manifests them as instability in frequency domain. To reduce the beating effect and increase the reliability of damping estimation, two methods can be applied. Firstly, dividing the time record into many segment and applying the same procedure to find damping ratios for each segment would allow to obtain a stabilization graph in terms of time segment and damping as shown in Figure 13. By doing that, even though the effect of beating is still valid, it is possible to determine if damping is stable or not. Figure 13 is an example of a stable damping estimation.

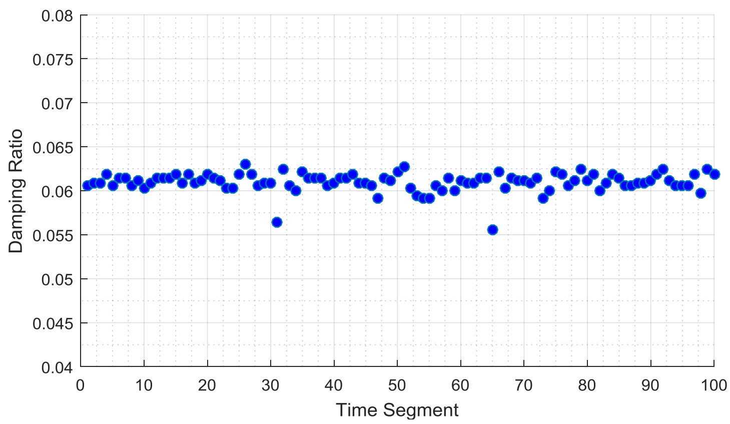


Figure 13 Damping Ratio Estimation for the 1st Sym. Wing Bending Mode

Secondly, the bandwidth of the bandpass filter may be decreased so that resulting SDOF spectral bell does not contain any additional peak or spikes.

To check the orthogonality of resulting mode shapes, Modal Assurance Criterion (MAC) is used [Maia et al., 1998] as in Figure 14. Off-diagonal elements of MAC matrix shows that complex mode shape vectors are uncorrelated with each other, which means found mode shapes satisfy orthogonality condition. To identify only wing-related modes and distract other localized modes, MAC is crucial and an optimization procedure is needed in the case of identifying closely spaced peaks in the singular value plots.

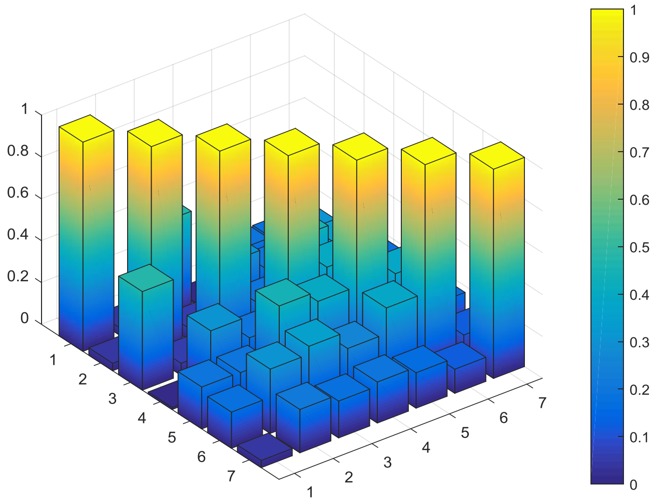


Figure 14 Modal Assurance Criterion (MAC)

CONCLUSION

Operational Modal Analysis have been widely used in civil engineering structures since the integrated test systems, developed for mechanical engineering applications, are not practical and economical to test these large structures in lab environment. Considering the OMA applications, there are only a small portion of studies related with aerospace applications. In this paper, an aircraft's dynamic characteristics have been investigated for in-flight condition without any use of lab environment. The natural frequencies, mode shapes and damping ratios have been found in frequency domain with very low computational time. The mode shapes are logical considering the wing as a cantilever beam fixed from the fuselage. The low coherences of torsion modes indicate they have not been well excited during the straight cruise of the aircraft. Although the number of accelerometers is small, MAC values are good assistants to distinguish real modes from noise modes. In addition, non-parametric damping estimation as it has been done in this work is needed to be verified with parametric methods and stabilization graphs carefully since in-flight dampings may be biased due to noise.

As a future work, Stochastic Subspace Identification technique may be applied in order to make a comparison between two methods and see the validity of a time domain method for in-flight modal identification. Moreover, the number of accelerometers may be increased in order to see the modes in which wings couple with other parts of aircraft.

References

Bartkowics, T. J., and James III G. H. (2011) *Ares I-X In-Flight Modal Identification*, 52nd Structural Dynamics and Materials Conference, Denver, Colorado

Brincker, R., Zhang L., and Andersen, P. (2000) *Modal Identification from Ambient Responses Using Frequency Domain Decomposition*, In Proceedings of IMAC-XVIII. San Antonio, TX, USA.

Brincker, R., Ventura C., and Andersen P. (2001) *Damping Estimation by Frequency Domain Decomposition*, In Proceedings of IMAC-IXX, 2001, pp.698–703.

Brincker, R., Andersen, P. and Jacobsen, N.J. (2007) *Automated frequency domain decomposition for operational modal analysis*, In Proceedings of IMAC-XXIV, 2007.

Brincker, R., and Ventura, C. E. (2015) *Introduction to Operational Modal Analysis*, John Wiley & Sons, Ltd, 2015

Coppotelli, G., Grappasonni, C., Tozzi, D. (2013) *Operational Modal Analysis from Flight Test Data*, In Proceedings of IMAC-XIII, May 2013, Guimaraes, Portugal

Danowsky, B. P. (2015) *Modal identification from Sköll flight tests*, Systems Technology, Inc., Working Paper 1439-12, Hawthorne, California

De Vivo, A., and Brutti C. (2014), *Vega In-Flight Modal Identification with the Operational Modal Analysis Technique*, Journal of Spacecraft and Rockets, Vol51, No. 5, September-October

Follador, R. C., Souza, C. E., Marto, A. G., Silva, R. G. A., Góes, L. C. S. (2016) *Comparison of In-Flight Measured and Computed Aeroelastic Damping: Modal Identification Procedures and Modeling Approaches,* J. Aerosp. Technol. Manag., São José dos Campos, Vol.8, No 2, pp.163-177, Apr.-Jun., 2016

Lalanne, C., (2009) *Random Vibration Volume 3,* 2009, John Wiley & Sons, Inc.

Maia, N. M. M., Silva, J. M. M., He, J., Lieven, N. A. J., Lin, R. M., Skingle, G. W., To, W., and Urgueira, A. P. V. (1998) *Theoretical and Experimental Modal Analysis*, Research Studies Press Ltd., UK, Exeter

Marulo, F., Franco, F., Ficca, A., and Pitera, A. (2005) *Experiences in Using Operational Modal Analysis*, In Proceedings of 1st International Operational Modal Analysis Conference, April, 2005

Neu E., Janser F., Khatibi, A.A., Braun C., and Orifici, A.C. (2015) *Operational Modal Analysis of a Wing Excited by Transonic Flow*, Aerospace Science and Technology 49, p: 73-79, November 2015

Rainieri, C., Fabbrocino, G., Cosenza, E., and Manfredi, G. (2007) *The Operational Modal Analysis for the Identification of Historical Structures: The Tower of the Nations in Naples*, In Proceedings of International Operational Modal Analysis Conference, 2007, Copenhagen, Denmark

Rainieri, C., Fabbrocino, G. (2014) *Operational Modal Analysis of Civil Engineering Structures, An Introduction and Guide for Applications*, Springer Science Business Media, 2014, New York