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STATIC ANALYSIS OF ARBITRARILY LAMINATED COMPOSITE CANTILEVER PANELS WITH CHEBYSHEV COLLOCATION METHOD

İlke Algül[1], Hasan Kurtaran[2] and Ahmet Sinan Öktem[3]
Gebze Technical University
Gebze/Kocaeli, Turkey

ABSTRACT

This work presents the static analysis of moderately thick, arbitrarily laminated composite cantilever plate and shells. First order shear deformation theory in association with an extension of linear strain-displacement relationships is used to consider the transverse shear effect through the thickness direction. Equilibrium equations for laminated composite shells are obtained using the virtual work principle. The displacement fields in the governing equilibrium equations are expanded with fast converging finite double Chebyshev series. Several arbitrarily laminated composite plate and panel problems with cantilever type of boundary condition are solved numerically by using Chebyshev Collocation Method (CCM). Parametric studies such as effects of lamination stacking, angle of orientation and radius effect are investigated and its accuracy is ascertained with a commercial finite element software (ANSYS).

INTRODUCTION

The usage of fiber reinforced laminated composite plates and shells is highly demanded in structural applications especially in aerospace areas. Boeing 787 Dreamliner can be given as the most recent example produced transport jet, which was manufactured using plastic materials reinforced with carbon fiber. Composite materials have attracted significant attentions due to their specific properties such as high strength-to-weight and stiffness-to-weight ratios, corrosion resistance, longer fatigue life, stealth characteristics and most importantly tailoring of these structures for desired usage area. The anisotropic behavior and bending-stretching coupling of structure may create difficulties for the analyses of composite shells. Therefore understanding the behavior of these structures is very important to enable safe and economical designs.

Several books can be found to understand the mechanical behavior of laminated composite shell structures. Many researches have been conducted on the static analysis of arbitrarily laminated composite shells. A brief summary of the literature including static analysis of

[1] Research assistant and PhD Candidate in a Department of Mechanical Engineering, Gebze Technical University, Email: ilkees@gtu.edu.tr
[2] Prof. in a Department of Mechanical Engineering, Adana Science and Technology University, Email: hasan@adanabtu.edu.tr
[3] Associate Prof. in a in a Department of Mechanical Engineering, Gebze Technical University, Email: sinan.oktem@gtu.edu.tr

arbitrarily laminated composite plate and panels is given in following. [Rango, Belloma and Nallim, 2012] surveyed the static analysis of thick, arbitrarily laminated composite plates with different materials, boundary conditions, fiber orientation angles and span to thickness ratios by using Ritz method. [Nik and Tahani, 2009] examined bending analysis of rectangular laminated plates with arbitrary lamination and boundary conditions by employing Extended Kantorovich Method. [Vel and Batra, 2004] analyzed rectangular thick arbitrarily laminated plates subjected to a sinusoidal load on top surface with different sets of edge boundary conditions e.g. two opposite edges simply supported and the other two edge subjected eight different conditions or all four edges clamped by using generalized Eshelby-Stroh formalism. [Chaudhuri and Abu-Arja, 1989] carried out arbitrarily laminated anisotropic cylindrical shells subjected to axially varying internal pressure under the framework of the constant-shear-angle theory or the first-order shear-deformation theory (FSDT) with different arbitrary boundary conditions.

Literature review indicated that static analysis of arbitrarily laminated composite plate and panels with cantilever type of boundary conditions are limited. Therefore, the primary aim of this study is to fill this gap using Chebyshev Collocation Method. Effects of length-to-thickness ratio, angle of orientation and radius effect of the static behavior of the panels are investigated.

SOLUTIONS OF EQUATIONS OF MOTION

Panel (length *a*, width *b*) is shown in Fig.1. The points, which have equal distances to the two inclined surfaces, are known to be middle surface. *x*, *y* and *z* stated the curvilinear coordinate system.

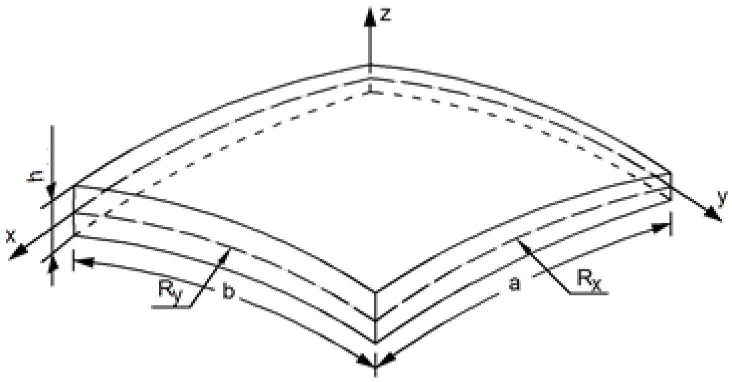

Figure 1*:* Composite panel

The displacement field at general point (*x*, *y* and *z*) of the panel based on first order shear deformation theory may be written as:

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z.\theta_x(x,y) \\
v(x,y,z) &= u_0(x,y) + z.\theta_y(x,y) \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{1}
$$

The strain-displacement correlations for moderately thick doubly curved panels using the displacement field in Eq. (1) from the theory of elasticity in curvilinear coordinates can be written as following [Reddy, 2004]:

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u_0}{\partial x} + z\frac{\partial \theta_x}{\partial x} + \frac{w_0}{R_x} \\
\varepsilon_y &= \frac{\partial v_0}{\partial y} + z\frac{\partial \theta_y}{\partial y} + \frac{w_0}{R_y} \\
\gamma_{xy} &= \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} + z\frac{\partial \theta_x}{\partial y} + z\frac{\partial \theta_y}{\partial x} + \left(\frac{1}{R_y} - \frac{1}{R_x}\right).\left(\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}\right) \\
\gamma_{yz} &= \theta_y + \frac{\partial w_0}{\partial y} - \frac{v_0}{R_y}
\end{aligned}
\tag{2}
$$

$$\gamma_{xz} = \theta_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R_x}$$

Five coupled linear governing differential equations of a moderately thick laminated composite doubly curved panel can be written by using virtual work principle as follows:

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + \frac{Q_{xz}}{R_x} - \frac{\partial M_{xy}}{\partial y}\left(\frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\right) = 0 \\
&\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{Q_{yz}}{R_y} + \frac{\partial M_{xy}}{\partial x}\left(.\frac{1}{2}(\frac{1}{R_y} - \frac{1}{R_x})\right) = 0 \\
&-\frac{N_x}{Rx} - \frac{N_y}{Ry} + \frac{\partial Q_{yz}}{\partial y} + \frac{\partial Q_{xz}}{\partial x} = qw \\
&\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_{xz} = 0 \\
&\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_{yz} = 0
\end{aligned} \tag{3}$$

N_x, N_y, and N_{xy} are the in-plane force resultants, M_x, M_y, and M_{xy} are the in-plane moment resultants and Q_{yz} and Q_{xz} are the transverse shear force resultants.

$$\begin{aligned}
&\left(N_x\,, N_y, N_{xy}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) d_z \\
&\left(M_x\,, M_y, M_{xy}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) z d_z \\
&\left(Q_{yz}, Q_{xz}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} k_s.\left(\tau_{yz}, \tau_{xz}\right) d_z
\end{aligned} \tag{4}$$

The displacements and rotations of middle point and also the loadings are expressed in space domain as a summation of double Chebyshev series of the first kind as following [Fox and Parker, 1968]:

$$\begin{aligned}
&\mathrm{u}_0(x,y) = \sum_0^M \sum_0^N \delta_{mn}.u_{mn}.T_m(x).T_n(y) \\
&\mathrm{v}_0(x,y) = \sum_0^M \sum_0^N \delta_{mn}.v_{mn}.T_m(x).T_n(y) \\
&\mathrm{w}_0(x,y) = \sum_0^M \sum_0^N \delta_{mn}.w_{mn}.T_m(x).T_n(y) \\
&\theta_X(x,y) = \sum_0^M \sum_0^N \delta_{mn}.\theta x_{mn}.T_m(x).T_n(y) \\
&\theta_y(x,y) = \sum_0^M \sum_0^N \delta_{mn}.\theta y_{mn}.T_m(x).T_n(y) \\
&q(x,y) = \sum_0^M \sum_0^N \delta_{mn}.q_{mn}.T_m(x).T_n(y)
\end{aligned} \tag{5}$$

M and N are the number of Chebyshev series terms and $T_m(x), T_m(x)$ are the Chebyshev polynomials. u_{mn}, v_{mn}, w_{mn}, $\theta_{x_{mn}}$, $\theta_{y_{mn}}$ and q_{mn} are the unknown coefficients for displacements and loading to be determined. δ_{mn} shows a constant value [Upadhyay, Pandey and Shukla, 2011].

The governing differential equations in (3) are written in terms of displacement and rotations. Equation of motion for doubly curved panel can be shortly written in matrix form as

$$KU = F \tag{6}$$

where *K* and *F* denote stiffness matrix and external force vector, respectively. U denote the displacement vectors. To employ the CCM technique, the panel is divided into (M+1)(N+1) grid points. The total number of unknown coefficients in terms of unknown displacement values is 5(M+1)(N+1). The number of equations written for governing equilibrium equations at internal grid points is 5(M-1)(N-1). The number of equations written for boundary conditions is 10(M+1)+10(N-1) equations. It can be seen that the total number of equations is equal to the total number of unknown coefficients. Eq. (6) is solved successively to find unknown displacement values.

NUMERICAL EXAMPLES

Equilibrium equations of composite panels of square plan-form (a = b) were obtained using virtual work principle. Plate is a special form of panel with a radius R_x= R_y=∞. Based on the theoretical formulation explained in the previous sections, a commercial computer program using Matlab is developed to solve these derived equations by using CCM. In the all examples, each lamina has same thickness, and total thickness is h=25 mm. Uniformly distributed transverse load q_0=-1 Pa is applied to the top of the structure. The displacements, w^* are computed at the middle of the free edge which is parallel to clamped edge and stresses, σ_x^* are computed at the middle of the clamped edge of the panel for all cases. The following orthotropic material properties of Boron-Epoxy composite material in the principal material coordinate system are assumed: E_1=204 GPa, E_2=18.5 GPa, G_{12}= G_{13}= G_{23}=5.59 GPa, ρ=2100 kg/m^3, v_{12}=0.23.

The cantilever type (CFFF) of boundary conditions are prescribed at the edges as shown below:

1. Clamped (C):

$$x\text{=0,y=0..b} \quad u_0 = v_0 = w_0 = \theta_x = \theta_y = 0 \tag{7a}$$

2. Free (F):

$$\begin{aligned} &\text{x=0..a and y=0,b} \quad N_y = N_{xy} = Q_y = M_y = M_{xy} = 0 \\ &\text{x=a, y=0,b} \quad N_x = N_{xy} = Q_x = M_x = M_{xy} = 0 \end{aligned} \tag{7b}$$

The following normalized quantities are defined:

$$\mathrm{w}^* = \frac{10E_2h^3}{q_0a^2}w, \qquad \sigma_x^* = \frac{h^2}{q_0a^3}\sigma_x \tag{8}$$

The convergence of the non-dimensional displacement, w* and stress, σ_x* results with the proposed solution method is compared with the finite element results in Table 1 by increasing series terms for the case of arbitrarily laminated [30^0/-30^0/30^0] plate and panel (R/a=10) structures with cantilever type of boundary condition in order to verify the accuracy of the solution. The convergence of the displacement and stress results occurs with 9x9 and 7x7 series terms, respectively which is enough to obtain close results for the static analysis. The discrepancy between the converged results and finite element results can be attributed to the fact that force and moment type boundary conditions at the respected edges as well as displacement boundary conditions are certainly utilized.

	a/h=20, CFFF, q=-1 Pa, [30^0/-30^0/30^0]			
	w*		σ_x*	
Theory	**R/a= 10**	**Plate**	**R/a= 10**	**Plate**
Present (7x7)	-0.8232	-0.8555	6.5737	6.2332
Present (8x8)	-0.9029	-0.9640	3.6723	4.0058
Present (9x9)	-0.8994	-0.8856	-1.2011	5.0796
FEM	-0.8949	-0.9323	6.4440	5.9785
Error= $\left\lvert \frac{\Pr esent - ANSYS}{\Pr esent} \right\rvert 100$	%5.0	%5.66	%1.97	%4.09

Table 1. Convergence term of non-dimensional tip displacements and stresses (x=0, y=b/2) of composite moderately thick plate and spherical panel

Figs. 2-3 present the variations of non-dimensional displacements, w* and stresses, σ_x* of arbitrarily laminated [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] plate and spherical (R= R_x= R_y) panel of square planform for varying length-to-thickness ratios, *a*/*h*. It is seen that the difference of the non-dimensional displacements, w* between the plate and spherical panel (*R*/*a*=10) results progressively increases for each [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] layer orientations as the length-to-thickness ratio, *a*/*h*, increases from moderately thick to thin range (*a*/*h*>20), but the difference is almost the same for the range of *a*/*h*≤20. The magnitude of non-dimensional displacements, w* of plate and panel with [60^0/-60^0/60^0] layer orientation are relatively higher than the [30^0/-30^0/30^0], [45^0/-45^0/45^0]] layer orientations for the length-to-thickness ratio, *a*/*h* >20 as demonstrated in Fig.2. However, the non-dimensional stress, σ_x* results of the arbitrarily laminated [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] plate and spherical panel (*R*/*a*=10) are almost same for the entire range of length-to-thickness ratios, a/h. The non-dimensional stress values, σ_x*, of plate and panel (*R*/*a*=10) structures for the all investigated lamination orientations shown in Fig. 3 decreases as the length-to-thickness ratio, *a*/*h*, increases from thick to thin range.

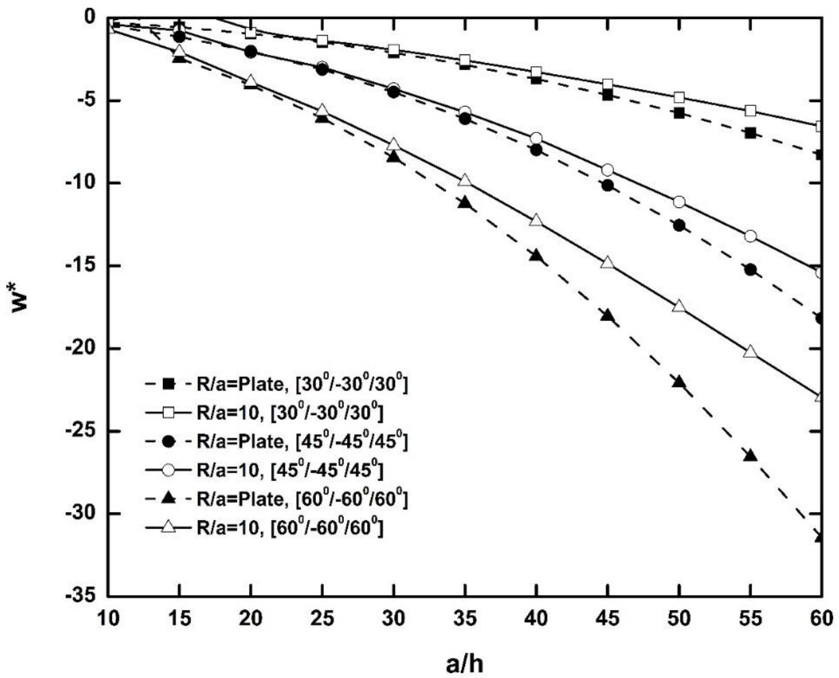

Figure 2. The non-dimensional displacements, w* with a/h ratio, of cantilever composite [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] plate and spherical panel.

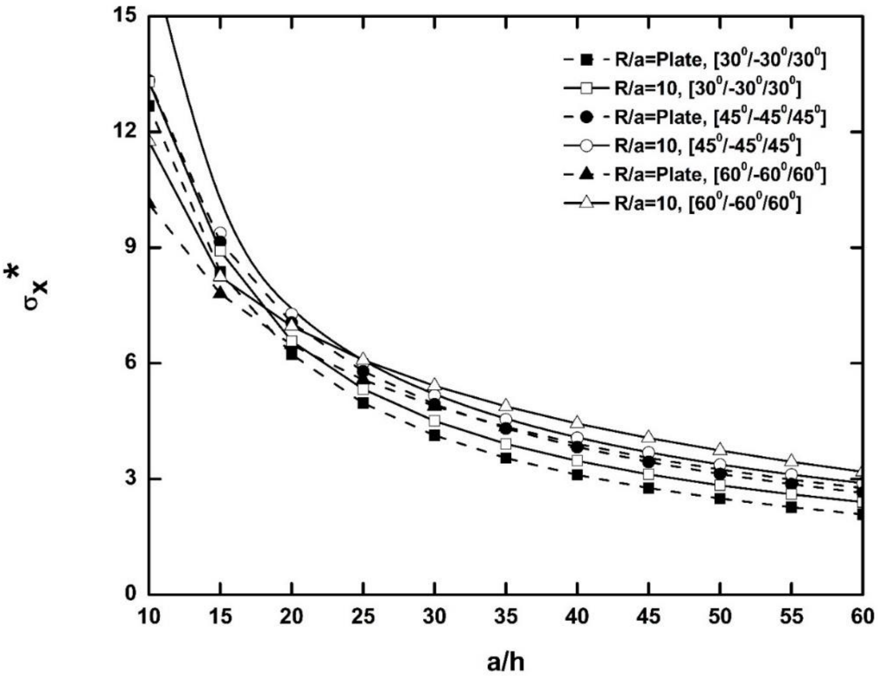

Figure 3. The non-dimensional stresses, σ_x* with a/h ratio, of cantilever composite [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] plate and spherical panel.

Figs. 4-5 present the comparisons of the variations of the non-dimensional displacements, w* and stresses, σ_x* of [θ^0/-θ^0/θ^0] square planform plate for different length-to-thickness ratios, a/h=20, 40, 60 with respect to θ^0 angle variation. The magnitude of non-dimensional displacements, w* of [θ^0/-θ^0/θ^0] square planform composite plate more or less steadily increase with the increase of lamination angle until [70^0/-70^0/70^0] (approx.) while it remains almost constant after [70^0/-70^0/70^0] (approx.) lamination orientation for all length-to-thickness ratio, a/h. However, in contrast to w*, the magnitude of non-dimensional stresses, σ_x*of [θ^0/-θ^0/θ^0] square planform plate changes quite small between 40^0 and 70^0 angles with the increase of lamination angle for the all examined length-to-thickness ratios.

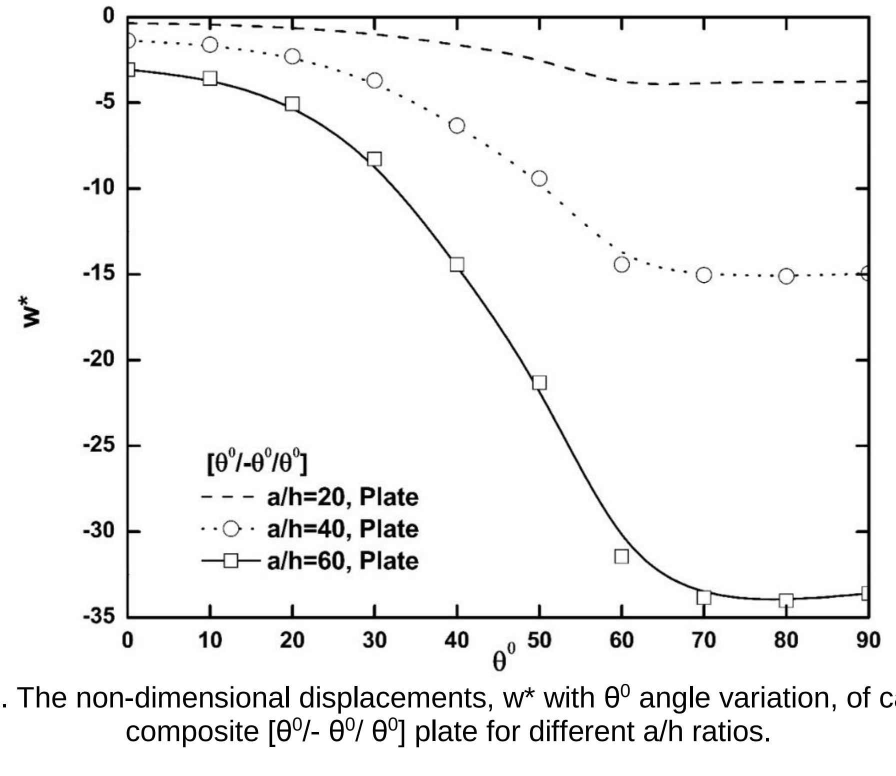

Figure 4. The non-dimensional displacements, w* with θ^0 angle variation, of cantilever composite [θ^0/- θ^0/ θ^0] plate for different a/h ratios.

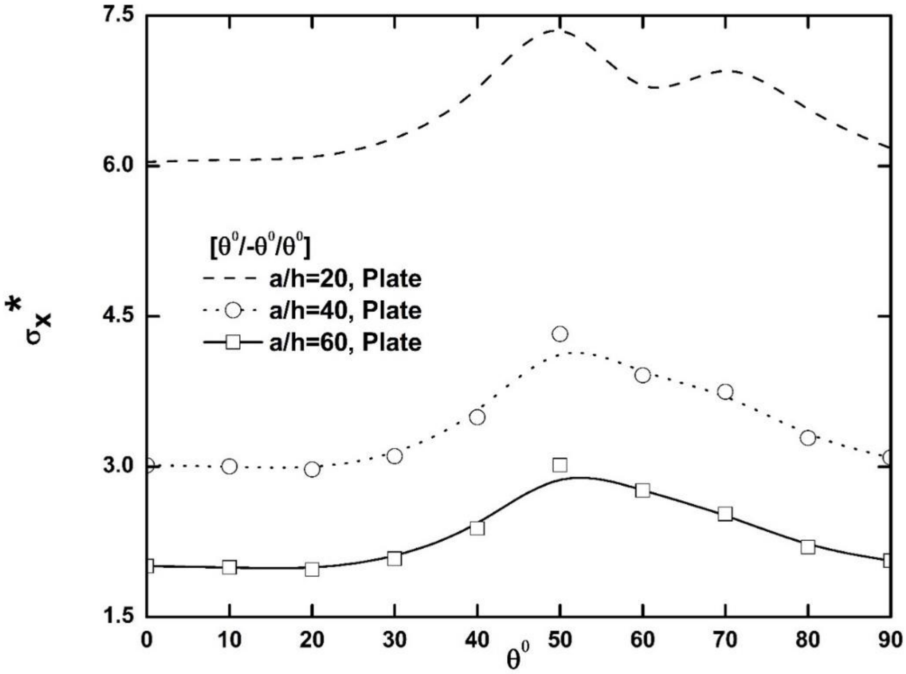

Figure 5. The non-dimensional stresses, σ_x* with θ^0 angle variation, of cantilever composite [θ^0/- θ^0/ θ^0] plate for different a/h ratios.

Figs. 6-7 display the variations of the non-dimensional displacements, w* and stresses, σ_x* of [θ^0/-θ^0/θ^0] square planform spherical panel (*R*/*a*=10) for different length-to-thickness ratios *a/h*=20, 40, 60 with respect to θ^0 angle. The characteristics of non-dimensional displacement, w* and stress, σ_x* of [θ^0/- θ^0/ θ^0] spherical panel structure exhibit almost same behavior as has been earlier examined for the case of plate structure in Figs. 4-5.

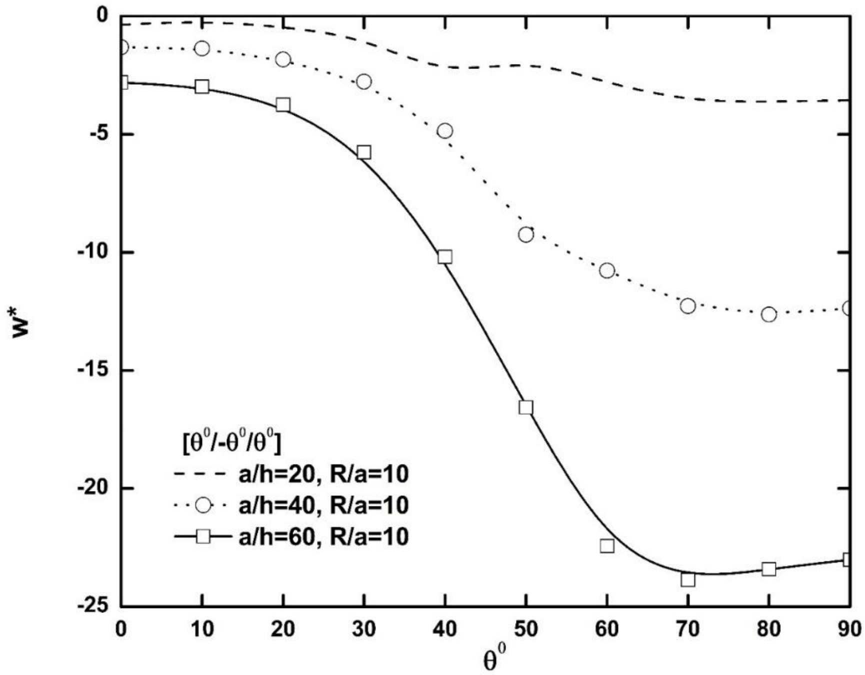

Figure 6. The non-dimensional displacements, w* with θ^0 angle variation, of cantilever composite [θ^0/- θ^0/ θ^0] spherical panel (R/a=10) for different a/h ratios.

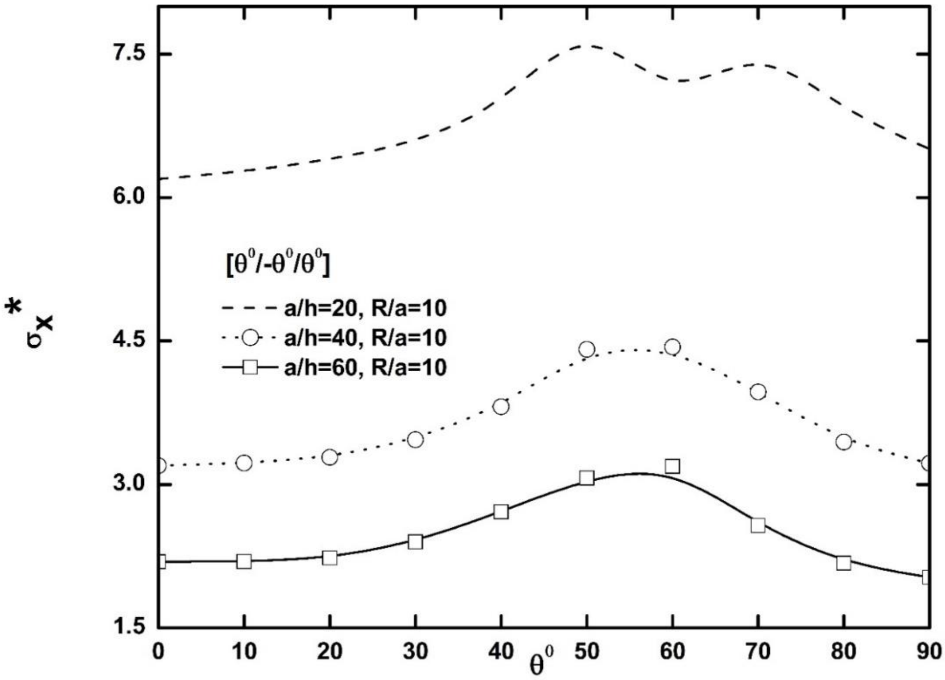

Figure 7. The non-dimensional stresses, σ_x* with θ^0 angle variation, of cantilever composite [θ^0/- θ^0/ θ^0] spherical panel (R/a=10) for different a/h ratios.

CONCLUSIONS

In this study, arbitrarily laminated composite cantilever plate and spherical panel examples were solved with Chebyshev Collocation Method. The effects of length-to-thickness ratio, orientation angle and radius ratio were studied. The results of Chebyshev collocation method were compared with those of finite element method and close results were observed.
The brief conclusion obtained from the results in the previous section is summarized as follows:
1) 9x9 terms and 7x7 terms can yield sufficiently accurate displacement and stress results, respectively with Chebyshev Collocation Method.

2) The difference of the non-dimensional displacements, w* between the plate and spherical panel (*R*/*a*=10) results progressively increases for each [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] layer orientations as the length-to-thickness ratio, *a*/*h*, increases from moderately thick to thin range (*a*/*h*>20), but the difference is almost the same for the range of *a*/*h*≤20. The magnitude of non-dimensional displacements, w* of plate and panel with [60^0/-60^0/60^0] layer orientation are relatively higher than the [30^0/-30^0/30^0], [45^0/-45^0/45^0]] layer orientations for the length-to-thickness ratio, *a*/*h* >20. However, the non-dimensional stress, σ_x* results of the arbitrarily laminated [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] plate and spherical panel (*R*/*a*=10) are almost same for the entire range of length-to-thickness ratios, a/h. The non-dimensional stress values, σ_x*, of plate and panel (*R*/*a*=10) structures for [30^0/-30^0/30^0], [45^0/-45^0/45^0] and [60^0/-60^0/60^0] layer orientations decreases as the length-to-thickness ratio, *a*/*h*, increases from thick to thin range.
3) The normalized tip deflection of plate and panel (*R*/*a*=10) structures increases considerably slowly with the increase in fiber orientation up to around θ = 20º. After θ= 20º, normalized tip deflection increases significantly until around θ = 70º. And also after θ= 70º, normalized tip deflection slope is changing insignificantly. However, the magnitude of non-dimensional stresses, σ_x*of [θ^0/- θ^0/ θ^0] square planform plate and panel (*R*/*a*=10) changes quite small between 40^0 and 70^0 angles with the increase of lamination angle for the all examined length-to-thickness ratios.

REFERENCES

Abu-Arja, K. R. and Chaudhuri, R. A. (1989) Closed-Form Solutions for Arbitrary Laminated Anisotropic Cylindrical Shells (Tubes) Including Shear Deformation, AIAA Journal, Vol. 27(11), p: 1597-1605,Nov 1989

Fox, L. and Parker S. (1968) Chebyshev Polynomials in Numerical Analysis, Oxford University Press, 1968

Nik, A. M. N. and Tahani M. (2009) Analytical solutions for bending analysis of rectangular laminated plates with arbitrary lamination and boundary conditions, Journal of Mechanical Science and Technology, Vol. 23, p: 2253-2267, August 2009

Rango, R. F., Bellomo, F. J. and Nallim A. (2012) General Ritz Algorithm for Static Analysis of Arbitrary Laminated Composite Plates using First Order Shear Deformation Theory, TJER, Vol. 10(2), p: 1-12, December 2013

Reddy, (2004) J.N., Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, CRC Press, New York, 2004

Upadhyay, A. K., Pandey, R., Shukla K. K. (2011) Nonlinear dynamic response of laminated composite plates subjected to pulse loading, Communications in Nonlinear Science and Numerical Simulation, Vol. 16, p: 4530-44, November 2011

Vel, S. S. and Batra, R. C. (1999) Analytical Solution for Rectangular Thick Laminated Plates Subjected to Arbitrary Boundary Conditions, AIAA Journal, Vol. 37 (11), p: 1464-1473, November 1999