9. ANKARA INTERNATIONAL AEROSPACE CONFERENCE
20-22 September 2017 - METU, Ankara TURKEY
AIAC-2017-052

RELIABILITY ANALYSIS OF TACTICAL UNMANNED AERIAL VEHICLE (UAV)

Yılmaz KOÇ[1], Turkish Aerospace Industries, Inc. Ankara, Turkey
Bariş SÜRÜCÜ[2], Middle East Technical University Ankara, Turkey
Nafiz ALEMDAROĞLU[3] Atılım University Ankara, Turkey

ABSTRACT

To be competitive in the market, it is very important to design cost effective and reliable products. For this purpose, it is necessary to consider reliability as an integral part of the design procedure. Therefore, reliability which is a design parameter that affects cost and safety of a system should be taken into consideration in early phases since it is very difficult to change design at the later phases. Due to these causes the importance of reliability in manufacturing fields, especially in aviation field, is increasing day by day.

In this paper; reliability prediction analysis is introduced for METU Tactical UAV. Reliability prediction for systems and items in Tactical UAV is calculated by using two different approaches namely; first one is that failure characteristics of items are considered as a constant and thus exponential distribution is applied. Secondly, simulated time to failure data having characteristics of Weibull distribution is used and probability distribution best representing the data is determined to show how predicted reliability changes if it is assumed to be exponentially distributed. For the simulation study, graphical approaches namely Quantile-Quantile plotting and probability-probability plotting have been conducted to determine the distribution model. Three-parameter Weibull distribution is considered to model simulated data; its unknown parameters have been estimated with MLE and for which Goodness of Fit Tests have been applied. In this study, effect of assumption on distribution model to represent the simulated data has been emphasized.

Key words: Mission Reliability, Exponential Distribution, Weibull Distribution, UAV

INTRODUCTION

Reliability is defined as the probability that an item will perform its intended function for a specified interval under stated conditions or the duration or probability of failure-free performance under stated conditions according to MIL-HDBK-338B. The main objective of this study is to examine the mission reliability of METU Tactical Unmanned Aerial Vehicle (UAV) by using different technique and distribution models based on simulation study. Mission reliability of Tactical UAV is assessed according to the some specified mission profiles which are determined based on the operational scenarios of the UAV. During the simulation study, effects of assumptions such as determination of distribution models and selection of methods to be applied to predict mission reliability of UAV are mentioned.

[1] Safety and Reliability Engineer, Email: yikoc@tai.com.tr
[2] Professor in Department of Statistics, Email: sbaris@metu.edu.tr
[3] Professor in Department of Airframe and Powerplant Maintenance, Email: nafiz.alemdaroğlu@atilim.edu.tr

Definitions

Definitions for Reliability, Failure, Failure Rate, Mean Time Between Failures (MTBF), Mean Time To Failure (MTTF) defined in MIL-HDBK-338B [Department of Defense, 2007] and MIL-STD-785B [Department of Defense, 1998] were given.

RELIABILITY MATHEMATICS

Reliability engineering is a discipline that is heavily dependent on mathematical probabilities and statistics to measure and analyze data and draw inferences about present and future performance of items or and/or systems [Tiku, 1981]. Mathematical modeling of items and/or systems failures is a key parameter to obtain any idea about the performance while items and/or systems operate.

Distributions

Different probability distribution models such as Weibull, Normal, Exponential, Binomial, etc. are used to model the different periods of the “bathtub” curve mathematically. In this paper, both exponential distribution and Weibull distribution are taken into account.

<u>Exponential Distribution:</u> The exponential distribution models the randomly occurring failures during the normal life (useful life) period of the “bathtub” curve where constant failure rate characteristic of items are obtained. On other rate function in the exponential distribution is assumed not to be changed over time. The words, failure exponential distribution has an advantage over other statistical (probability) distributions in that it is described totally by the single parameter λ, it has considerably wide applicability compared to other distributions. Mathematical formulation for exponential distribution is given as follows:

The cumulative distribution function F(t) is defined as the probability in a random trial that the random variable is not greater than t. F(t) is also called unreliability function and it gives the percentage of the population has failed for a specific time.

$$F(t) = \int_{-\infty}^{t} f(t)\, dt \tag{1}$$

Where f (t) is called probability density function and it describes the “where” failure occurs over time. Reliability function can be described in terms of the unreliability function since it represents the percentage that item has survived for a specific time. By definition, reliability function formula is;

$$R(t) = 1 - F(t) = \int_{t}^{\infty} f(t)\, dt \tag{2}$$

By differentiating the reliability function;

$$f(t) = -\frac{dR(t)}{dt} \tag{3}$$

Failure rate is the ratio of probability that failure occurs in the interval and it is given by;

$$\lambda(t) = \frac{R(t) - R(t+\Delta t)}{\Delta t\, R(t)} \tag{4}$$

As interval length ∆t approaches zero, limits of the failure rate is called instantaneous failure rate or specifically called “hazard rate”.

$$h(t)=\lim_{\Delta t\to 0}\left[\frac{R(t)-R(t+\Delta t)}{\Delta t\,R(t)}\right]=\frac{1}{R(t)}\left[-\frac{d\,R(t)}{dt}\right] \quad (5)$$

By substituting the differentiated reliability function equation (3) into hazard rate equation; hazard rate equation becomes;

$$h(t)=-\frac{f(t)}{R(t)} \quad (6)$$

Hazard rate or instantaneous failure rate has a significant and fundamental relationship because relationship does not depend on the statistical distribution. Taking the derivatives of both side of the hazard rate equation (6);

$$\frac{dR(t)}{R(t)}=-h(t)\,dt \quad (7)$$

Then,

$$R(t)=\exp-\left(\int_0^t h(t)\,dt\right) \quad (8)$$

For exponential distribution, hazard rate is assumed to be constant and denoted by λ, then

$$R(t)=e^{-\lambda t} \quad (9)$$

Where R is the reliability, t is time that the item is at risk under specified operating conditions, and λ is the failure rate of the item.

Weibull Distribution:

This distribution was introduced first by a physicist, Waloddi Weibull. Three parameter Weibull distributions are characterized by one is the shape parameter β (dimensionless), one is the scale parameter or called as characteristic life η (time) and another one is threshold parameter or namely location parameter γ (time). Probability density function for three parameter Weibull is given by [Barıs, 2009];

$$f(x;\gamma,\eta,\beta)=\frac{\beta}{\eta}\left[\frac{x-\gamma}{\eta}\right]\exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right] \quad (10)$$

Where $x \geq \gamma, \quad \eta,\beta > 0$

Thus, cumulative distribution function of three parameter Weibull is as follows;

$$F(x)=1-e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \quad (11)$$

Then, Reliability function is given as;

$$R(x)=e^{-\left(\frac{x}{\eta}\right)^{\beta}} \quad (12)$$

Graphical Methods

Graphical methods are widely used methods in statistics to assess whether or not time to failure data follows a specific distribution profile. It briefly provides visual ways of analyzing distribution of variables. Probability-Probability (P-P) plot and Quantile-Quantile (Q-Q) plot are commonly used graphical methods to see how well a theoretical distribution models the empirical data.

Probability-Probability (P-P) Plotting:

The probability-probability plot (percent plot or p-p plot) compares the empirical cumulative distribution function of a variable with a specific theoretical cumulative distribution function. In the P-P plot, if data points are close to reference line, it can be said that data follows a specified distribution.

Quantile-Quantile (Q-Q) Plotting:

Quantile-Quantile (Q-Q) Plot is a plot of the percentiles of any specific distribution against the corresponding percentiles of the observed data. Q-Q plots have the basic property that if time to failure data has linear relation with percentiles of any distribution then the corresponding Q-Q plot will still be linear but with possibly changed location and slope. Construction of Q-Q plot bases on the cumulative distribution function of specified probability distributions. Assuming theoretical cumulative distribution function, F(x) then, for Q-Q plotting;

For $i = 1, 2 \ldots n$, $x_{(i)}$ is plotted against $E(Z_{(i)})$, $Z_{(i)} = \left(\frac{x_{(i)} - \mu}{\sigma} \right)$, where μ and σ are,

respectively, location and scale parameters. This is the expected value of the i^th standardized order statistics for a location scale family. Location scale parameter is a family of unvariate probability distributions parameterized by a location parameter and a non-negative scale parameter.

For practical purposes, $E(Z_{(i)}) \approx F^{-1}(i - 0.3/n + 0.4)$.

Cumulative distribution function of two parameter Weibull distribution is given by equation (12). Quantile function which is the inverse of cumulative distribution function is determined as follows;

$$F^{-1}(p) = \left[\frac{-\ln(1-p)}{\eta} \right]^{1/\beta} \quad (13)$$

Parameter Estimation

Once the time to failure data for items is obtained, significant properties of data distribution including the standard deviation, mean etc., can be determined with the application of some methods. In the first instance, graphical methods namely; Probability-Probability (P-P) plotting and Quantile-Quantile (Q-Q) plotting are used to visualize that how time to failure data fits to some statistical distribution. These methods are useful ways of choosing among the types of distribution. After determination of which one of probability distribution is a good representation of time to failure data, task of estimation of the parameters for the probability distribution follows. There exist analytical techniques being taken into consideration to decide parameters of probability distributions i.e Weibull distribution specifically while performing this analysis.

Maximum Likelihood Method:

Maximum likelihood estimation begins with the mathematical expression known as a likelihood function of the sample data. The likelihood of a set of data is the probability of obtaining that particular set of data given the chosen probability model. This expression contains the unknown parameters. Those values of the parameter that maximize the sample likelihood are known as the maximum likelihood estimates.

Maximum likelihood estimation addresses all the limitations of probability plots and provides more precise parametric fits than graphical estimation.

$$L = \prod_{i=1}^{n} f\, x_i(x_i, \theta) \tag{14}$$

$$\frac{d \log L}{d\theta} = 0$$

Likelihood function for two parameters Weibull distribution; and its β derivative and η derivative that maximizes the function, are given as follows [Balakrishnan, 2008];

$$L(x_1, ..., x_n ; \beta, \eta) = \prod_{i=1}^{n} \left(\frac{\beta}{\eta}\right) \left(\frac{x_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{x_i}{\eta}\right)^{\beta}} \tag{15a}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln x_i - \frac{1}{\eta} \sum_{i=1}^{n} x_i^{\beta} \ln x_i = 0 \tag{15b}$$

$$\frac{\partial \ln L}{\partial \eta} = -\frac{n}{\eta} + \frac{1}{\eta^2} \sum_{i=1}^{n} x_i^{\beta} = 0 \tag{15c}$$

Firstly, scale and shape parameter are estimated with respect to equations above. Then, in order to determine threshold parameter; following likelihood function for estimation of threshold parameter is used;

$$\frac{\partial \ln L}{\partial \gamma} = -\frac{(\beta - 1)}{\eta} \sum_{i=1}^{n} \left(\frac{x_i - \gamma}{\eta}\right)^{-1} + \frac{\beta}{\eta} \sum_{i=1}^{n} \left(\frac{x_i - \gamma}{\eta}\right)^{\beta-1} \tag{15d}$$

Where, $1 \le i \le n$

Reliability Modeling

Main object of the reliability modeling is to provide a mathematical picture which is a representation of relationships between items, equipment comprising the system. While performing the reliability modeling; system, system elements and environmental conditions in which systems are expected to operate, should be defined in detail. Systems are modeled via using a tool called Reliability Block Diagram (RBD). “A Reliability Block Diagram is a method of representing, in a single and visual way, the reliability relationships between the system and items in the system” [Reliability: A Practitioner Guide, 2003”].

Series configuration is the most commonly used and simplest configuration in RBDs. Series configuration means that any one of failure in the block results in a system failure. In other words, successful operation of a system depends on success of all items under system. The reliability of a system with items of system in series cannot be greater than the reliability of the least reliable component 0. Reliability model for series configuration is given in Figure 1 below.

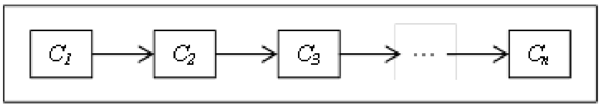

Figure 1: Series Configuration

If it is assumed that items under system are mutually independent. When calculating reliability for mutually independent events, probabilities of events are multiplied. The reliability of the system is given as follows,

$$R_{system} = R_1 \times R_2 \times R_3 ..... \times R_N \qquad (16)$$

OVERVIEW OF METU TACTICAL UAV AND ITS SYSTEM RELIABILITY

METU Tactical UAV

Middle East Technical University (METU) Tactical UAV shown in Figure 2 has been designed to perform reconnaissance and observation operations and the role of METU Tactical UAV is completely non-lethal. It has been designed to perform reconnaissance operations for a certain period of time in a certain range of diameter to get information with Gimbaled Day/Night IR Camera System and Hyperspectral Camera System. It has been designed and first prototype was constructed by Aerospace Engineering Department of METU with the financial support of State Planning Organization in 2005.

Figure 2: METU Tactical UAV

Mission Profile

In order to perform reliability modelling, mission profile analysis including flight phases and their related times for METU Tactical UAV is provided in this section. Figure 3 below indicates the mission profile that has been set for METU Tactical UAV to fly.

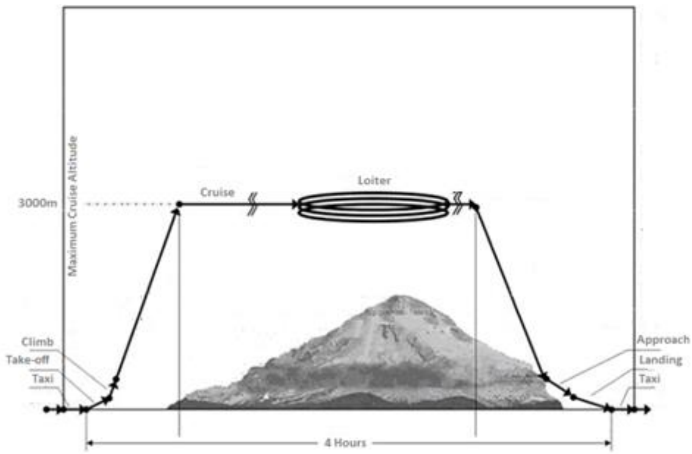

Figure 3: Mission Profile

Table 1 below gives information about the flight phases of the Tactical UAV and approximated duration related to each flight phase. Definitions of flight phases have also been added in the Table. Tactical UAV flies with maximum endurance up to 4 hours; thus it will spend almost 3.4 hours of total flight time for reconnaissance missions. At worst case, Tactical UAV is expected to perform reconnaissance and observation missions at altitude of 3000 m which is the highest cruise altitude of it.

Table 1: Flight Phases of METU Tactical UAV Flight and Their Durations

Flight Phases	Definitions	Duration (hr)
Start/Warm-up	UAV on the ground with engine running (Engine starting to Idle condition).	0.0500
Taxi	UAV is moving under the power of its engine on runways, with guidance provided by the ground personal.	0.1000
Take-off	Starts after taxi is complete. Generally, Engine of UAV runs at full power.	0.0500
Climb	Starts after takeoff and ends when intended cruising altitude is reached.	0.1000
Cruise + Loiter	Starts when UAV levels at intended cruising altitude and ends when UAV begins descent with intention to land. Loiter is assumed to be performed in mid-flight.	3.4000
Descend	Starts when UAV begins descent with intention to land.	0.1000
Approach	Starts at the end of the descent phase and ends when landing begins.	0.0500
Landing	Starts at the end of descent, and continues while the UAV contacts the ground, and until the UAV has been brought to a low speed under control.	0.0500
Taxi and OFF	UAV is moving under the power of its engine on runways, with guidance provided by the ground personal. Finally, UAV becomes stationary and engine is shutdown.	0.1000
Total Flight Time		4.000

Systems of METU Tactical UAV

During the design process, system to be used in the Tactical UAV has been decided by design team. Some of items in systems have been designed by design team whereas; some of them which have been already used by similar products are selected. In this study only Electrical System of METU Tactical UAV is taken into consideration that has two lithium polymer battery, 5V DC-DC Converter, 12V DC-DC Converter, one junction box and 6 connectors.

Table 2: Items and Components of Electrical System

Items/Components in Electrical System	Quantity
Lithium Polymer Battery	2
5V DC-DC Converter	1
12V DC-DC Converter	1
Junction Box	1
Connector	6
Cables	N/A

Component Reliability of Electrical System

Electrical System reliability (MTBF) data will be provided according to similar approaches applied in aerospace industry. MTBFs of Electrical System are calculated based on assumption that failure rate is constant over time and given in Table 3 below. NPRD is the Non-Electronic Reliability Data which is widely used database in Aerospace Industry and AUC denoted for environment of Airborne Uninhabited Cargo aircrafts.

Table 3: Electrical System Reliability Data

Equipment	Quantity	MTBF	MTBF (AUC) (hours)
Lithium Polymer Battery	2	2860 (NPRD-95-AUC)	2860
5V DC-DC Converter	1	5200 (NPRD-95-AUC)	5200
12V DC-DC Converter	1	5200 (NPRD-95-AUC)	5200
Junction Box	1	1572 (NPRD-95-AUC)	1572
Connector	6	253000 (NPRD-95-AUC)	253000

RBD of electrical system which is totally series system and it is given in Figure 4 below.

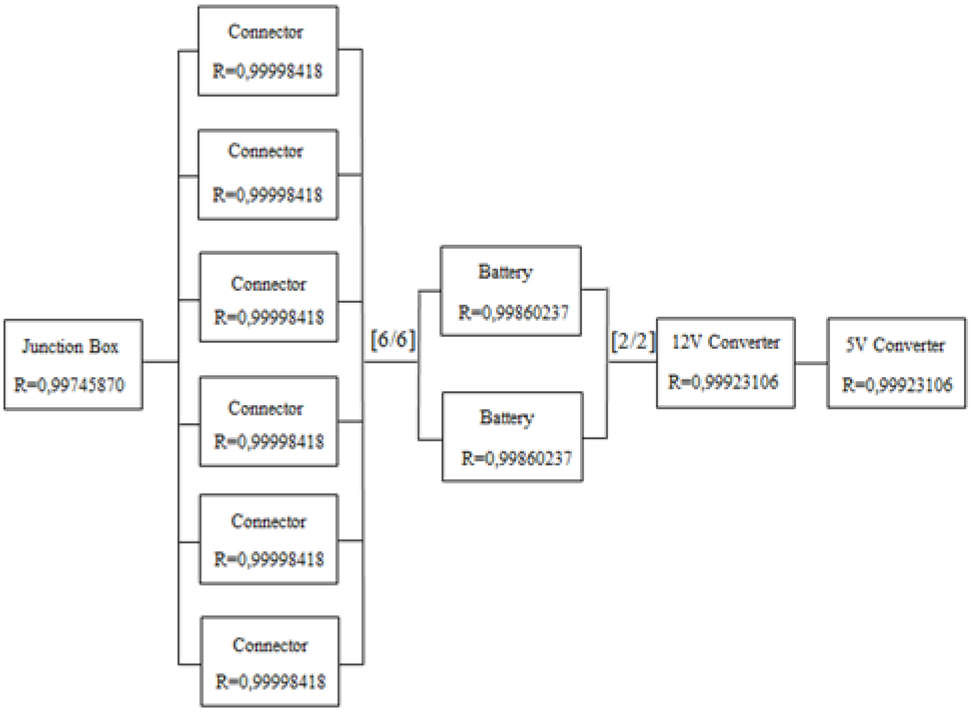

Figure 4: RBD of Electrical System

By using equations (16) and (9), reliability of electrical system is calculated as follows:

$R_{Electrical\ System} = 0.9930492214$

SIMULATION STUDY AND RELIABILITY COMPARISONS

Simulation study is conducted according to created time to failure data for both exponential distribution and Weibull distribution into consideration and predicted reliability of METU tactical UAV with two different approaches are compared.

Reliability Estimation

For the purpose of simulation study, time to failure data for each item of electrical system is created for both exponential distribution and Weibull distribution. Created time to failure data for both exponential distribution and Weibull distribution are given in Table 4. Then, suitable failure distribution will be identified based on a three-step process including identification of possible distributions, estimation of parameters for identified distribution and application of goodness-of-fit tests.

Table 4: Time to failure data for items under electrical system

Lithium Polymer Battery		5V DC-DC Converter	
Exponential Data	Weibull Data	Exponential Data	Weibull Data
285	342	770	609
497	854	1177	1003
817	1114	1229	1335
959	1190	1621	1497
1171	1277	2455	2420
1207	1371	2539	2482

1756	1392	3018	2774
1852	1605	3525	2976
1991	1998	3577	3957
2102	2611	3678	4653
2436	2631	4467	4717
3103	2944	4687	5282
3179	2935	5943	5308
3733	3297	6679	5693
4092	3298	6931	5942
4404	3787	8316	7823
4454	4559	8850	8535
5251	5896	9418	8933
6363	6960	11539	12411
7554	7134	13670	15644

12V DC-DC Converter		Junction Box		Connector	
Exponential Data	Weibull Data	Exponential Data	Weibull Data	Exponential Data	Weibull Data
770	609	196	217	31951	54024
1177	1003	486	755	35680	84421
1229	1335	504	760	48538	85271
1621	1497	514	771	49767	88309
2455	2420	642	846	51520	92295
2539	2482	833	1019	57619	106236
3018	2774	853	1077	59518	114860
3525	2976	964	1173	77537	128145
3577	3957	1115	1227	85158	168795
3678	4653	1277	1286	91250	170727
4467	4717	1293	1399	100371	207018
4687	5282	1495	1431	150709	210560
5943	5308	1548	1451	270886	229198
6679	5693	1806	1711	282454	280665
6931	5942	2173	1791	358128	317908
8316	7823	2338	2028	442389	411351
8850	8535	2566	2115	595399	440900
9418	8933	3281	2363	684150	554837
11539	12411	3377	3436	737368	591792
13670	15644	4170	4593	850064	722678

In order to identify candidate distribution, graphical method is basically used. Q-Q plots are constructed for each data set to show that exponential assumption fails when original data comes from Weibull Distribution and also Weibull assumption fails when original data comes from exponential distribution.

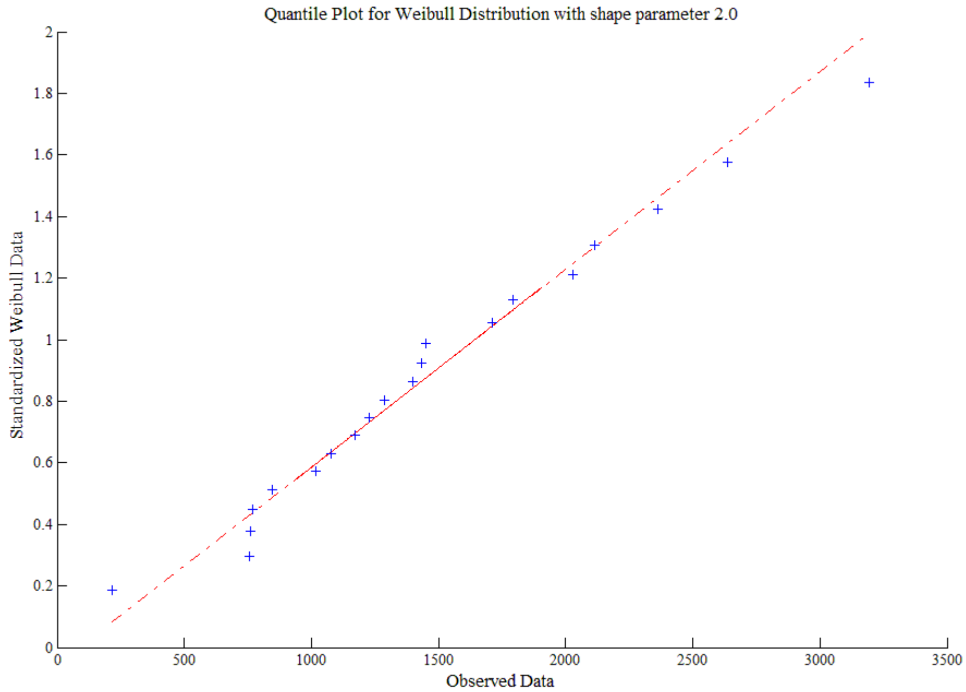

Figure 5: Connector quantile plot for Weibull distribution with shape parameter of 2.0 by using Weibull observed data

Figure 5 shows quantile plot for Weibull distribution with shape parameter of 2.0 by using Weibull observed data for connector of electrical system. Quantile plotting for weibull distribution shape parameter of 1.5 and exponential distribution are also performed and provided in Figure 6 and Figure 7 respectively.

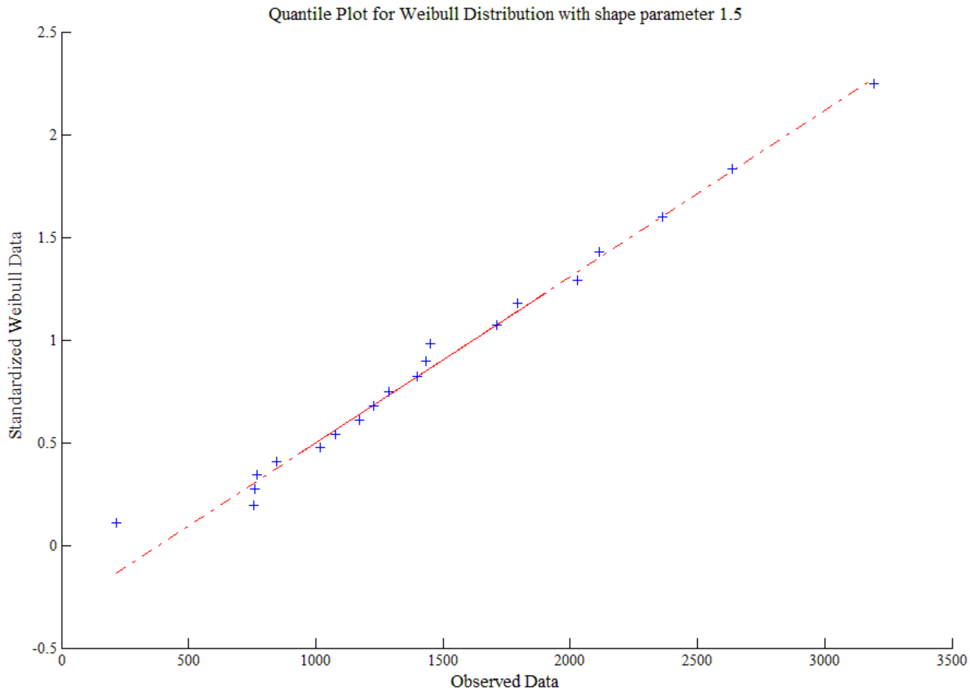

Figure 6: Connector quantile plot for Weibull distribution with shape parameter of 1.5 by using Weibull observed data

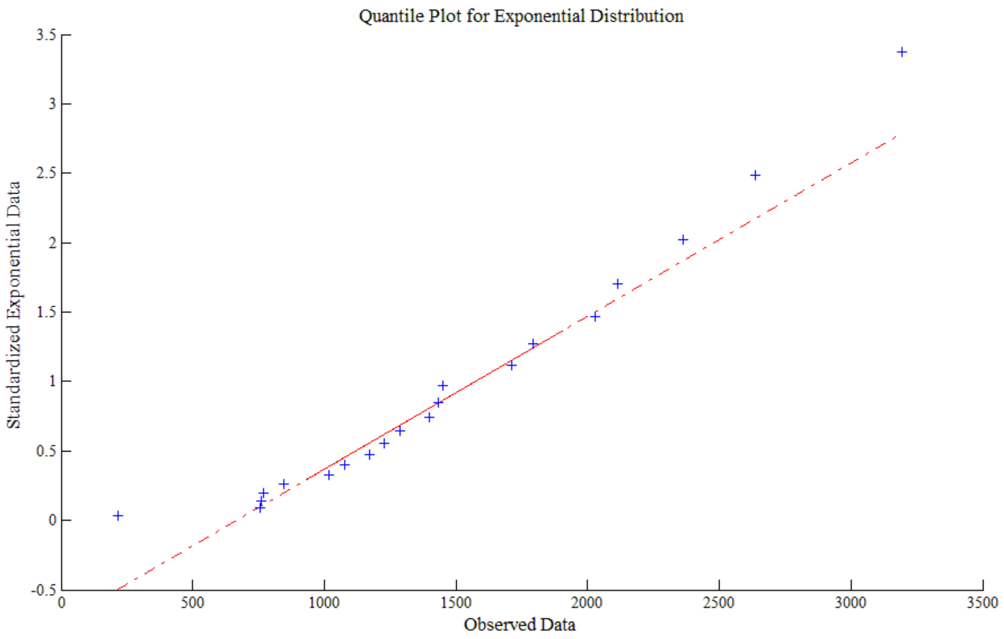

Figure 7: Connector quantile plot for exponential distribution by using Weibull observed data

The three-parameter Weibull distribution is selected based on common usage in engineering, and to minimize the complexity of the data analysis and also goodness-of-fit tests. The most crucial advantage of this distribution is to model bathtub curve i.e. decreasing, constant, and increasing failure rates.

Estimation of Three-parameter Weibull Distribution Parameters

In order to estimate the parameters of three-parameter Weibull distribution, maximum likelihood estimation is used mathematically to fit a line to time to failure data.

Table 5: Estimated Parameters of Exponential Data for Electrical System, where sample size (n=20)

Electrical System	Maximum Likelihood		
	$\hat{\beta}$	$\hat{\eta}$	$\hat{\gamma}$
Lithium polymer battery	1.476	3165	163.4
5V DC-DC Converter	1.5151	5790.4	373
12V DC-DC Converter	1.5151	5790.4	373
Junction Box	1.5311	1752.3	96.37
Connector	0.9728	249711	32785

Table 6: Estimated Parameters of Weibull Data for Electrical System, where sample size (n=20)

Electrical System	Maximum Likelihood		
	$\hat{\beta}$	$\hat{\eta}$	$\hat{\gamma}$
Lithium polymer battery	1.5588	3195	157.4
5V DC-DC Converter	1.4215	5738.2	362.8
12V DC-DC Converter	1.4215	5738.2	362.8
Junction Box	1.7294	1773.4	61.87
Connector	1.434	280843	21683

Third step is the application of goodness of fit tests. Goodness-of-fit tests are means of examining how well a sample of data agree with assumed distribution as its population. There is a wide literature and study especially on exponential and normal distributions. However, Goodness-of-fit tests for Weibull distribution have been less studied when it is compared to other distributions. Some GOF approaches have been suggested by Mann, Scheuer and Fertig [Mann, 1974] and Tiku and Singh [Tiku, 1981].

According to the assumption that the sample comes from two parameter Weibull distribution, Smith and Bain statistic is on the basis of correlation between expected value of the order statistics and order statistics of the sample [Smith, 1976]. Smith and Bain have provided critical values for the samples containing 8, 20, 40, 60, or 80 observations. Tables for the asymptotic critical values of the Anderson-Darling A^2 statistic and the Cramer-von Mises W^2 statistics for various significance levels have been produced by Stephens [Stephens, 1977].

In order to perform GOF tests for two parameter Weibull distribution for which location parameters are assumed to be zero, the Cramer-von Mises (W^2) test and Anderson-Darling (A^2) test are used at five different significance levels. These two tests are based on the empirical distribution function (EDF) which is a step function and calculated from the sample. EDF is measure of the difference between the EDF and given distribution function and used for testing the fit of the sample to the distribution.

Size of exponentially created data for each item is 20, i.e. T1, T2,…, T20 and let T(1) < T(2) < …. < T(20) be the order statistics; and also supposing that F(T) is the cumulative distribution function of T.

<u>Modified Cramer-von Mises (W^2):</u>
Equation of Cramer-von Mises (W^2);

$$W^2 = n\int_{-\infty}^{\infty}\{F_n(t)-F(t)\}^2 dF(t) \quad (17)$$

where,

$$F_n(t) = \sum_{i=1}^{n} f(t_i) \quad (18)$$

$Z = F(t)$, where $z < Z_i$ (19)

Substituting equation (19) into equation (18), following equation is obtained as

$$Z_n(n)=\begin{cases} i/n & Z < z < Z_{i+1} \\ 1 & Z_{(n)} < z \end{cases} \quad , i = 1,2,...,n-1 \tag{20}$$

$$z_{(0)} = 0 \text{ and } z_{(n+1)} = 1 \tag{21}$$

Then,

$$\begin{aligned} W^2 &= n\int_0^1 \{z_n(n)-z\}^2 dz = n\sum_{i=1}^{n}\int_{z_i}^{z_{(i+1)}}\left[i/n - z\right]^2 dz \\ &= n/3\sum_{i=1}^{n}\left[\left(z_{(i+1)}-1/n\right)^3-\left(z_{(i)}-1/n\right)^3\right]^2 = \sum_{i=1}^{n}\left\{z_i-\frac{2i-1}{2n}\right\}^2+\frac{1}{12n} \end{aligned} \tag{22}$$

<u>Anderson Darling (A^2):</u>
Equation of Anderson Darling (A^2);

$$A^2 = n\int_{-\infty}^{\infty}\{F_n(t)-F(t)\}^2\frac{1}{F(t)(1-F(t))}dF(t) \tag{23}$$

$$\begin{aligned} A^2 &= n\int_0^1 \{z_n(n)-z\}^2\frac{1}{z(1-z)}dz \\ &= -n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln z_{(i)}+\ln\{1-z_{(n+1-i)}\}\right] \end{aligned} \tag{24}$$

Main assumption for analysis to be valid is that distribution is continuous.

Table 7 gives critical values for five different significance levels for both Cramer-von Mises (W^2) and Anderson-Darling (A^2) [Rasha Abdul, 2012].

Table 7: Critical values for Cramer-von Mises (W^2) and Anderson-Darling (A^2)

Sample Size	Test Statistics	Significance Levels				
		0.01	0.05	0.10	0.15	0.20
20	A^2	0.9529	0.7539	0.6439	0.2423	0.2016
	W^2	0.2369	0.2116	0.1999	0.1815	0.1659
100	A^2	0.9556	0.7642	0.6514	0.2600	0.2078
	W^2	0.2429	0.2173	0.2048	0.1881	0.1702

In order to perform simulation study, 10000 random samples of size 20 for each item are generated and for which the goodness-of-fit test for Weibull distribution with the help of the tests W^2 and A^2 have been carried out. Number of acceptance of hypothesis for each significance level value of both W^2 and A^2 test statistics are given in Table 8.

Table 8: Critical values for Cramer-von Mises (W^2) and Anderson-Darling (A^2) when n=20

Significance Level	Cramer Von Mises Test W^2	Anderson Darling Test A^2
0.01	0.95	0.95
0.05	0.91	0.90
0.10	0.89	0.85
0.15	0.84	0.75
0.20	0.76	0.69

As a result of the analysis, based on the Cramer-von Mises test statistic, approximately 95 percent of the data are considered to agree with two parameter-Weibull distribution for significance level of 0.01. Thus, for exponentially created time to failure data, two-parameter Weibull distribution is thought to be well fitted distribution when percent of passed data sets for each significance level are considered.

Electrical System Reliability Based on Weibull Distribution

For the purpose of simulation study, time to failure data for each item of electrical system are created for both exponential and weibull data. Items reliability are recalculated by using equation (16) and their estimated parameters i.e. shape (β), characteristic life (η) and Threshold parameter (γ) given in tables 5 and Table 6.

On the contrary to exponential distribution, when Weibull distribution is employed, infant mortality and wear out characteristics of item are taken into account for reliability estimations. Summary of the predicted probabilities of systems;

Reliability of electrical system after simulation study (exponential data);

$$R_{\text{exponential data}} = 0.99945656$$

Reliability of electrical system after simulation study (weibull data);

$$R_{\text{weibull data}} = 0{,}99980388$$

Reliability Comparison

Firstly, reliability of electrical system is calculated based on time to failure data for exponential distribution, namely constant failure rate. By using same data set, Weibull distribution assumption is performed and electrical system reliability will be calculated accordingly. Difference in two approaches is investigated.

Table 9: Reliability Comparison for Electrical System based on exponential data

Exponential Distribution	Weibull Distribution
0.993049214	0.99945656

Table 10: Reliability Comparison for Electrical System based on weibull data

Exponential Distribution	Weibull Data
0.993049214	0.99980388

Firstly, reliability of electrical system has been calculated based on time to failure data for exponential distribution. It is predicted as 0.99305 according to assumption of sample data to be well fitted to exponential distribution, namely constant failure rate. By using same data set, Weibull distribution assumption is performed and electrical system reliability is calculated accordingly. Reliability of electrical system is 0.99946 when the assumption is Weibull distribution. Both results may be considered to be so close when success point of view is into account. However, difference in two approaches is indeed more meaningful with respect to unreliability considerations. It can be concluded from the result that 695 of 100000 UAVs will fail to carry out mission for exponential assumption and on the other hand, 54 of 10000 UAVs with assumption of Weibull distribution will fail.

Secondly, reliability of aircraft is also predicted and calculated when time to failure data having characteristics of Weibull distribution. It is observed that when the time to failure data has the characteristics of Weibull distribution, significant differences exist between both approaches. Unreliability of electrical system is 0.0002 when the original data comes from Weibull distribution. It is 0.007 when exponential assumption is made and failure rate is assumed to be constant.

CONCLUSION

In this paper, a method is developed to show difference between common reliability approaches preferred in the industry and usage of time to failure data obtained from the actual aircraft operational environment. Although there is a lack of time to failure data obtained under actual operational conditions of use and environment, simulation study is performed. Reliability analysis model based on Exponential distribution and Weibull distribution approach give alternatives to the designer to select the convenient approach for a specific situation.

References

Mann, N. R., Schafer, R. E. and Singpurwalla, N. D. (1974). "Methods for statistical analysis of reliability and life data".John Wiley & Sons.

MIL-HDBK-338B Electronic Reliability Design Handbook, Department of Defense USA, Notice 1, Jun 2007.

MIL-STD-785B Reliability Program For Systems And Equipment Development And Production, Department Of Defense USA, Notice 3, July 1998.

N. Balakrishnan, M. Kateri, On the maximum likelihood estimation of parameters of Weibull distribution based on complete and censored data, Statistics & Probability Letters, 2008, pp 2971–2975.

Rasha Abdul Hussein Ali (2012). "Fitting a Two Parameters of Weibull Distribution Using Goodness of Fit Tests". Al- Mustansiriyah J. Sci. Vol. 23, No 6, 2013

Reliability: A Practitioner Guide, 2003, Intellect

Smith, R.M.; Bain, L.J. (1976). "Correlation type goodness-of-fit statistics with censored sampling". Communications in Statistics, Part A-Theory and Methods. 5:119-132.

Stephens, M.A. (1977)."Goodness-of-fit for the logistic distribution".Biometrica, 64, 585-588.

"Successful Strategies for Achieving Reliability Requirements In Weapon Systems Acquisition", James M. Thorne, 2002.

Surucu Barıs, Sazak Hakan S. (2009). "Monitoring Reliability for a three-parameter Weibull distribution". Reliability Engineering and System Safety Vol. 94, 503-508

System Reliability toolkit, reliability Information analysis center, 2005

Tiku, M.L.; Singh, M. (1981)."Testing the two parameter Weibull distribution". communications in Statistics, Part A-Theory and Methods. 10:907-917.