WAVE NUMBER IN TURBULENCE AND ITS DISCRETE NATURE

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ABSTRACT

The content of this paper is about the definition of wave number frequently encountered in turbulence studies. At the onset the paper reminds the conventional definition and makes some remarks that forced the author to seek different approaches. The proposed definition dates from a 1980 paper of the author.
In view of some additional findings, the same definition is brought back. The method of calculation of wave number is explained and ten test cases are reported in view of the proposed definition. The numerical results give the possibility to consider the proposed definition to be an acceptable one (at least this is the thought of the author).
The proposed definition and the results thus obtained are interpreted in physical terms with discrete nature of turbulence. The physics is mostly the subject of the "INTERPRETATİON" section. In "Conclusion" the author suggests that the approach to turbulent flow from a different avenue may be a complementary tool in turbulence studies. The paper contains eleven Tables to present the numerical results of ten Test Cases and ten Figures to show the associated one dimensional spectra with respect to wave number.

BACKGROUND

The rather conventional definition of wave number in turbulence studies can be expressed as:

(1) $$k = \frac{\omega}{\overline{U}}$$

where, **"k"** and "ω" are respectively the wave number and frequency of fluctuating part "u" of the instantaneous velocity "U". "$\overline{U}$" is the temporal mean velocity. The relation (1) is the consequence of G. I. Taylor's "frozen-turbulence" concept which accepts the time variation of a turbulence quantity at a specified time "t" and a position "**s**" to be the one that occurred "Δt" time before at a point "s-Δs" with:

(2) $$\Delta s = \overline{U}\Delta t \,.$$

If "G" is the symbol of a turbulence quantity, this concept is expressed as:

$$\frac{DG}{Dt} = 0$$

In practice all length scales are converted to time scales (or vice versa) through the relation (2), including wave number and the frequency: namely (1).

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As noted by G.I. Taylor himself, this concept is applicable in cases where the turbulence intensity is small. This limitation corresponds to the applicability of (2) rather in larger wave number region of energy spectrum. If wave lengths are considered to be commensurate with eddy sizes, the relation (2) gives a poor image for the zone of larger eddies or smaller frequency. For example, in a pipe of say 10cm diameter, the larger eddy sizes at frequencies of 1 or 2c/s and at a mean velocity of 10m/s become 10m and 5m respectively according to (2). So, large eddies do not correspond to the intrinsic dimensions of the domain in which they are generated. In addition, an eddy size does not have a one to one correspondence with the turbulence characteristics of the flow neither in terms of dimensions, nor in terms of the local structure of turbulence. In other words, for a given mean velocity, the eddy sizes are the same whatever the specifics (say geometry, dimensions, spectral characteristics, etc.) of the turbulence domain. It is clear that this particular "universality characteristic" is a property that requires the existence of "universal equilibrium range" which lies in the higher wave number range of energy spectrum.
When this kind of questions are raised, it sees as if people are satisfied (though mildly) with the representation of high wave number range with the help of (2). On the other end it is possible to seek other avenues which may help to connect wave number to frequency so to represent the totality of wave number range of the spectrum; further than this, it may be quite rewarding.

A DEFINITION OF WAVE NUMBER AND ITS IMPLEMENTATION

Definition: Turbulence energy is thought to be carried by "discrete material packages of finite life span". Hence the energy varies in an oscillatory way and intermittently. This package emits an energy signal (or print) in the form of a wavy pattern of finite extent. This description leads to a different relation between "u, k and ω", which is proposed as:

"the fluctuating part of the instantaneous velocity
is at the same time the energy transport velocity of the package" i.e:

(3)* $$u = \frac{d\omega}{dk} \; .$$

This definition accepts the fluctuating velocity to be the group velocity of the finite wavy pattern of the print of the energy of the package.

Spectrum: Calculation of wave number in a turbulent flow must bear the intrinsic properties of the turbulence of the domain in which it is generated. The part of this information regarding the spectral characteristics is supplied by the energy spectrum. Therefore, the spectrum G(ω) is a critical element of the case investigated and must somehow be known.

Probability density function, PDF: In order to use (3), a further relation is needed between "u" and "ω". The required relation is obtained from:

(4) $$\int_{-\infty}^{\infty} P(y)u^2 dy = u'^2 \int_0^{\infty} G(\omega) d\omega$$

P(y) and "u' "are respectively the PDF and "rms" of the velocity fluctuations. "y" is dimensionless instantaneous velocity. P(y) is the same for instantaneous andfluctuating velocity. G(ω) is the spectrum value at "ω" of the turbulence energy divided by "u'^2

C.Çıray. "On Wave Number Definition". METU Journal of Pure and Applied Sciences. Vol.13. No:3.pp: 431-488. December 1980 (this issue of the Journal appeared in 1982).

The following equalities show the relation between natural variables and their dimensionless counterparts.

$$U = \overline{U} + u = \overline{U}\left(1+\frac{u}{\overline{U}}\right) = \overline{U}\left(1+\frac{u'}{\overline{U}}\frac{u}{u'}\right) = \overline{U}(1+Ix)$$

$$\text{where: } x = \frac{u}{u'} \; ; \qquad I = \frac{u'}{\overline{U}} \quad \text{and} \qquad y = \frac{U}{\overline{U}} = 1+Ix$$

It is proposed to apply the relation (4) within intervals “-u - Δu” to “-u” and “u” to “u + Δu“:

(5) $$\int_{-u-\Delta u}^{-u} u^2 P_L(u)du + \int_{u}^{u+\Delta u} u^2 P_R u)du = u'^2 \int_{\omega}^{\omega+\Delta\omega} G(\omega)d\omega$$

P_L and P_R represent the left and right branches of the PDF if it is skew. Therefore for a skew PDF: $P_L(-x_i) \neq P_R(x_i)$ and $\frac{dP_L(x_i)}{dx} > 0$ and is different in magnitude from $\frac{dP_R(x_i)}{dx} < 0$.

In case the PDF is symmetrical: $P_L(-x_i) = P_R(x_i)$ and : $-\frac{dP_L(x_i)}{dx} = \frac{dP_R(x_i)}{dx}$

Two meanings can be attrıbuted to (5):

a) “+u” and “–u” associated with “U” occur at the same frequency but at different probabilities,

b) “u” and “ω” have one tone correspondence.

With: $$u_{i+1} = u_i + \Delta u_{i+1} = u'\left(x_i + \Delta x_{i+1}\right)$$

Discrete form of (5), in terms of non-dimensional quantities becomes:

(6) $$C_1 \Delta x_{i+1}^2 + C_2 \Delta x_{i+1} = \Delta G\left(\omega_i, \omega_{i+1}\right)$$ where:

(6a) $$C_1 = x_i\left[P_L(x_i) + P_R(x_i)\right] + \frac{1}{2}x_i^2\left[-\left.\frac{dP_L}{dx}\right|_{-x_i} + \left.\frac{dP_R}{dx}\right|_{x_i}\right]$$ and:

(6b) $$C_2 = x_i^2\left[P_L(-x_i) + P_R(x_i)\right]$$

In the derivation of (6) terms containing “Δx_i^3“and higher powers of “Δx_i“ are omitted. The solution of equation (6) yields the relation between “u” and “ω”, which in turn gives the possibility to obtain the connection of “k” to “u” or “ω” through the definition expressed as (3).

The PDF. It is clear that in order to reach the solution the PDF must be available. The PDF chosen in this study is a kind of Maxwell relation in the form of:

(7) $$P(y) = P(1)y^n \exp\left\{A^n\left(1 - y^n\right)\right\}$$

It has to be reminded that y =1+Ix as expressed above. The constants “A” and “P(1)” are derived from the conditions:

(8) $$\int_{-\infty}^{\infty} P(y)dy = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} P(y)y^2 dy = 1+I^2$$

At this stage, it is worthwhile to introduce the “κ‘th” moment “μ^{κ}“of P(y) given in (7). It reads:

(9) $$\mu^{\kappa} = \frac{\left[\Gamma\left(\frac{n+1}{n}\right)\right]^{\kappa-1} . \Gamma\left(\frac{n+\kappa+1}{n}\right)}{\left[\Gamma\left(\frac{n+2}{n}\right)\right]^{2}}$$

$\Gamma(n)$ is the Gamma function and conditions appearing in (8) yıeld:

(10) $$A = \frac{\Gamma\left(\frac{n+2}{n}\right)}{\Gamma\left(\frac{n+1}{n}\right)} \quad \text{and} \quad P(1) = I\frac{nA^{n+1}}{\Gamma\left(\frac{n+1}{n}\right)}\exp\left\{-A^{n}\right\}$$

In addition, from the second relation of (8), with the help of (9), one can write:

(11) $$\mu^2 = \frac{\Gamma\left(\frac{n+1}{n}\right) . \Gamma\left(\frac{n+3}{n}\right)}{\left[\Gamma\left(\frac{n+2}{n}\right)\right]^{2}} = 1 + I^2$$

Procedure: In the light of the developments explained above, the procedure of calculation is as follows:
It is accepted that G(ω),”$\overline{U}$ ”, “u’ “(therefore “I”) are known and P(y) is expressed as in (7). Since “I “is known, (11) yields “n” (See the Table I). Hence P(y) is determined completely. With successive solutions of (6), one can obtain “u (ω)” in numerical form. In conjunction with wave number definition (3) of the wave number, “k” is found for any “u” or for any “ω”.

SOME RESULTS

On the presentation of the results:
The development and the procedure of calculation described in the previous section are checked for ten TEST CASES. Results are presented numerically and graphically.

A: Numerical results are supplied in Table 1 to 11. TABLE1 gives “A”, “μ^2 “, “$I_{CALC.}$” and “μ^3 “ for “n” in the range of 5(5)85. This table is used to determine “n” and other constants of the PDF related to the flow under consideration when I_{ACTUAL} of the flow is known. “n” is chosen as the value nearest to $I_{CALCULATED}$ appearing in this table. The actual range of “n” used in “TABLE 1” to evaluate TEST CASES was, 5(1)85, i.e. much more detailed than the one presented as TABLE 1.
TABLE 2 to 11 sum results of the calculations. Each table contains the specific numerical data of the tested case appearing at the top of the TABLE. The description of physical

conditions of the flow and the acquisition of data is depicted at the end of tabulated values. Frequency, the energy interval of the spectra, non-dimensional value of fluctuating velocity, wave number, wave length and left and right side value of the PDF are given in seven columns and 12 rows for frequencies ranging from 1 to 500kc/s.

B: Spectra of these ten TEST CASES are presented graphically where the calculated wave number is preferred instead of conventional practice of frequency usage.

Some common features of results:
A: Largest wave lengths are seen to be commensurate with the dimensions of the domain where the flow takes place. It is worthwhile to observe in TABLE 9, i.e. TEST Case 8, that the 6mm maximum wave length is almost the bar diameter of the reported grid turbulence.

B: Largest eddies are nearer the wall (larger velocity gradients) in TEST CASES 1, 2, 3 and 4, 5, 6, 7. For TEST CASES 8, 9 and 10 the author has no reached serial measurements for the sake of comparison.

C: Fluctuating velocities increase in magnitude markedly with increasing wave number or decreasing eddy size. The variation in “x “ (non-dimensional “u”) is of one order of magnitude and changes between “0.2 – 0.3” (for smallest wave numbers) to almost a fixed value around “2.3 for largest wave numbers. It is remarked that the probability of occurrence of these high amplitude and high frequency fluctuations is very low as can be noticed from related PDF values shown in the last two columns of tables. See also Figure 11.

D: The shapes of spectra G(k) supplied in Figures “1” to “10” are different than G(ω). Spectra in terms of wave numbers reflect local turbulence structure more than G(ω).

E: Slope of G(k) at high “k” is “-5/3” in each of the test cases as shown in Figures 1 to 10.

SOME INTERPRETATION:
DISCRETE NATURE OF TURBULENCE

A: The numerical results shown in TABLES 2 to 11 for ten TEST CASES are evidences that to conceive the fluctuating part of the instantaneous velocity as the group velocity is a plausible approach to explain the relation of wave number to frequency. It is worth noting that this thin- king gives the possibility to represent the totality of the spectrum as it may be observed in all ten TEST CASES.

B:The item “C” in the Results is rather interesting. In the high range of wave numbe,r the fluctuating components of velocities are high whereas their probability of occurrence is low. The high velocities can be interpreted to correspond to the feedback of turbulence kinetic energy from high wave number range. The low probability may also be thought as the incipient creation of eddies. Altogether in physical terms it suggests bursting phenomenon. But then the question: Does bursting occur with larger or smaller eddies?

C:The group velocity is physically related to a wave packet. But fluctuation velocity is measured to be a portion of the translational velocity of a particle. In this case, a question may seem to be appropriate: how group velocity of the wave packet is at the same time the translational velocity of the associated eddy?
A plausible reminder is offered in quantum theory. Indeed, this theory says that “the group velocity of a wave packet and the velocity of the associated particle are the same*”.

D:Then, it is not beyond reason to conceive turbulence to be composed of a low energy continuum of fluid laden with discrete but energetic (relative to low energy fluid continuum) coherent fluid elements (perhaps eddies),
In such a case, the energy of an eddy is depicted as a wave packet during its life span. At the end of its life span, its material integrates with the continuum part of the flow almost with no energy, till when it may be a part of another eddy. The velocities of the fluid particles that form a coherent structure behave as “pilot waves” with close wave numbers and give rise to wave packet behavior. This description brings to the fore the discrete nature of turbulence.

The discrete nature of turbulent flow was perhaps “hinted” for the first time in L. Prandtl’s and G,I. Taylor’s “mixing length theories”. Indeed, both scientists describe a mass of fluid which maintains **its identity** (momentum in the case of L. Prandtl and vorticity in the case of G.I.Taylor) **for a while** corresponding to a length “ℓ_Y” in “y” direction** (perpendicular to the direction of $\overline{U}$).

CONCLUSION

This study shows that wave number of the energy spectrum can be related to frequency if the fluctuating portion of the instantaneous velocity is taken as the group velocity. The results obtained on ten different TEST CASES are believed to warrant the truth of this assertion. From physical point of view, this assertion is explained in terms of “discrete nature of turbulence” with wave packets and discrete coherent structures.
Discrete nature of turbulence, perhaps as described in this paper, can be a promising avenue to study turbulence and turbulent flow..

HISTORICAL NOTE***

Indeed, the history of science poses an interesting analogous problem. At the beginning of 20th century, the spectrum of the energy density of black body radiation “E_b” was represented in terms of wave length “λ” by Rayleigh-Jeans formula:

$$E_b(\lambda) = \frac{8\pi k_B T}{\lambda^4}$$

“k_B“ and “T” are respectively Boltzmann constant and the absolute temperature. This relation was good (or harm- less) for high frequencies. At low frequencies it was severely at fault. Indeed, it was established that $E_b(\lambda) = 0$
as $\lambda \to 0$. The answer came from H. Planck when for the first time he showed the particle behavior of waves as discrete packages of energy. The approach of Planck was good to explain the spectrum for the whole range of wave number. This was the first step towards quantum theory.

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*It is curious to note another point which has a connection to the same theory with an event in the early development period of quantum theory. (*The reader may like to see the short “Historical Note” that follows).*

**In reality this mass of fluid maintains its identity along an inclined path where the component ℓ_X is several times ℓ_Y. Indeed, $\ell_X = \ell_Y\left(1+\frac{\overline{U}}{U_\tau}\right)$ where U_τ is the shear stress velocity.

*** Extracted from: F. R. Incropera. “Introduction to Molecular Structure and Thermodynamics.1974, John Wiley & Sons. ISBN 0-471-42710-1. Chapter2. p.6 onwards.

NOTE: The following Tables (1 to 11) are copied from “On Wave Number Definition in Turbulence” referred in the text, page 2).

TABLE 1

Properties of PDF

n	A	μ^2	$I_{calculated}$	μ^3
5	0.966341	1.04212	0.205236	1.12413
10	0.965121	1.01278	0.113058	1.03745
15	0.972123	1.00613	0.078296	1.01802
20	0.977243	1.00359	0.059929	1.01059
25	0.980875	1.00236	0.048554	1.00697
30	0.983539	1.00167	0.040818	1.00494
35	0.985565	1.00124	0.035211	1.00368
40	0.987153	1.00096	0.030943	1.002844
45	0.988426	1.00076	0.027604	1.00227
50	0.989476	1.00062	0.024936	1.00185
55	0.990351	1.00052	0.022714	1.00154
60	0.991092	1.00043	0.020854	1.00130
65	0.991728	1.00037	0.019286	1.00111
70	0.992279	1.00032	0.017954	1.00096
75	0.992672	1.00028	0.016773	1.00084
80	0.993188	1.00025	0.015747	1.00074
85	0.993567	1.00022	0.014810	1.00066

This table is used to find “n” starting from: $\mu^2 = 1 + I^2$ and “A” .

TABLE 2

Data and Characteristics of TEST CASE 1

$\overline{U}$ =31.5 m/s I_{ACTUAL} = 0.0410 n=30 A= 0.983539

u'=129cm/s $I_{n=30}$=0.0408 P(1) = 0.4078

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		0			0.4078	0.4078
	0.0115					
1		0.3597	0.2800	3.572	0.4525	0.3249
	0.0226					
2		0.5145	0.3914	2.555	0.4494	0.2867
	0.04355					
5		0.6790	0.6363	1.572	0.4272	0.2474
	0.031					
10		0.7655	0.9735	1.027	0.4074	0.2278
	0.0278					
20		0.8326	1.583	0.6317	0.3883	0.2132
	0.0663					
50		0.9637	3.20	0.3115	0.3429	0.1864
	0.0911					
100		1.113	5.55	0.1800	0.2811	0.1587
	0.1700					
200		1.342	9.523	0.1050	0.1804	0.1227
	0.2540					
500		1.648	19.30	0.0518	0.0706	0.0852
	0.106					
100		1.835	33.28	0.0301	0.0312	0.0676
	0.0821					
2000		2.021	58.54	0.0171	0.0109	0.0534
	0.0950					
5000		2.279	126.5	0.0079	0.0016	0.0038

TEST CASE 1, 2 and 3 correspond to measurements made in the mid-spa plan of a 30cm chord NACA 0012 airfoil
placed in a wind tunnel with AOA=12^{o}. The pressure gradient was maintained at zero win the measurement region of the tunnel with the help of a false wall. In TEST CASE 1 the hot-wire was placed at 5.77cm from the upper surface and 4.0cm from leading edge of the airfoil. The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 3

Data and Characteristics of TEST CASE 2

$\overline{U}$ = 32.22 m/s I_{ACTUAL} = 0.0354 n=35 A= 0. 98355565

u'=135 cm/s $I_{n=30}$= 0.0352 P(1) = 0.4088

f	ΔG	x	k	L	$P_R(x)$	$P_L(x)$
c/s	-	-	cm^{-1}	cm	-	-
0		-	-	-	0.4088	0.4088
	0.02041					
1		0.4369	0.1308	7.645	0.4562	0.3051
	0.100					
2		0.4968	0.2305	4.338	0.4540	0.2901
	0.294					
5		0.6219	0.4801	2.083	0.4407	0.2597
	0.0237					
10		0.6983	0.8327	1.20 1	0.4267	0 2418
	0.0452					
20		0.8136	1.448	0.6904	0.3971	0.2160
	0.0820					
50		0.9734	3.011	0.3321	0.3412	0.1833
	0.0.866					
100		1.116	5.239	0.1909	0.2810	0.1572
	0.1710					
200		1.345	9.022	0.1108	0.1778	0.1213
	0.2430					
500		1.642	18.37	0.0544	0.0699	0.0851
	0.0926					
1000		1.812	31.84	0.0314	0.0330	0.0691
	0.0878					
2000		2.006	56.23	0.078	0.0108	0.0541
	0.1140					
5000		2.305	121.0	0.008	0.0010	0.0369

TEST CASE 1, 2 and 3 correspond to measurements made in the mid-spa plan of a 30cm chord NACA 0012 airfoil
placed in a wind tunnel with AOA=12^{o}. The pressure gradient was maintaine at zero win the measurement region of the tunnel with the help of a false wall. In TEST CASE 2 the hot-wire was placed at 0.67cm from the upper surface and 4.0cm from leading edge of the airfoil.

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 4

Data and Characteristics of TEST CASE 3

$\overline{U}$ =25.35 m/s I_{ACTUAL} = 0.11430 n=10 A= 0.965121

u'= 290cm/s $I_{n=30}$= 0.1131 P(1) = 0.4033

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4033	0.4033
	0.0315					
1		0.4996	0.06530	15.31	0.4175	0.3059
	0.0306					
2		0.6306	0.1036	9.648	0.3996	0.2761
	0.0355					
5		0.7401	0.1985	5.038	0.3775	0.2514
	0.050					
10		0.8620	0.3337	2.9966	0.3459	0.2248
	0.0631					
20		0.9861	0.5682	1.760	0.3076	0.1989
	0.102					
50		1.151	1.176	0.8500	0.2504	0.1671
	0.0809					
100		1.274	2.070	0.4832	0.2063	0.1457
	0.1650					
200		1.491	3.607	0.2750	0.1332	0.1126
	0.224					
500		1.774	7.619	0.01313	0.0607	0.0785
	0.0481					
1000		1.869	13.57	0.0737	0.0437	0.06790
	0.0907					
2000		2.47	24.64	0.0406	0.0215	0.0539
	0.0787					
5000		2.237	54.98	0.0182	0.0085	0.0409

TEST CASE 1, 2 and 3 correspond to measurements made in the mid-spa plan of a 30cm chord NACA 0012 airfoil
placed in a wind tunnel with AOA=12^{o}. The pressure gradient was maintaine at zero win the measurement region of the tunnel with the help of a false wall. In TEST CASE 3 the hot-wire was placed at 0.07cm from the upper surface and 4.0cm from leading edge of the airfoil.

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 5

Data and Characteristics of TEST CASE 4

$\overline{U}$ =32.00 m/s I_{ACTUAL} =0.0385 n=32 A= 0.984412

u'=123cm/s $I_{n=30}$= 0.0384 P(1) = 0.4076

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4076	0.4076
	0.046					
1		0.5783	0.1360	7.356	0.4441	0.2705
	0.0499					
2		0.7372	0.2136	4.681	0.4156	0.2335
	0.0890					
5		0.9274	0.3977	2.514	0.3575	0.1930
	0.0748					
10		1.058	0.6550	1.527	0.3056	0.1681
	0.0657					
20		1.164	1.115	0.897	0.2593	0.1497
	0.0813					
50		1.287	2.365	0.4228	0.2045	0.1304
	0.0689					
100		1.392	4.272	0.2341	0.1591	0.1154
	0.127					
200		1.575	7.716	0.1296	0.0919	0.0930
	0.219					
500		1.882	16.16	0.0603	0.0241	0.0637
	0.0725					
1000		2.061	29.54	0.0339	0.0082	0.0508
	0.05 98					
2000		2.24	53.26	0.0188	0.0019	0.0399
	0.0357					
5000		2.424	118.9	0.0084	0.0004	0.0316

TEST CASE4, 5, 6 and 7 were performed within a pipe of 9cm diameter, the fluid being again air. The hot wire was at the center of the pipe in TEST CASE 4, therefore at 4.5cm from the wall of the Perspex pipe.

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary Unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 6

Data and Characteristics of TEST CASE 5

$\overline{U}$ =31.42 m/s I_{ACTUAL} = 0.0499 n=24 A= 0.980239

u'=158cm/s $I_{n=30}$= 0.0504 P(1) = 0.4004

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0					0.4004	0.4004
	0.0405					
1		0.5538	0.1067	9.368	0.4346	0.2767
	0.0396					
2		0.6949	0.1704	5.867	0.4141	0.2445
	0.1320					
5		0.9738	0.3134	3.101	0.3357	0.1866
	0.0823					
10		1.111	0.5042	1.983	0.2831	0.1617
	0.0882					
20		1.244	0.8419	1.188	0.2279	0.1399
	0.0924					
50		1.378	1.752	0.5708	0.1728	0.1203
	0.0623					
100		1.475	3.146	0.3179	0.1358	0.1076
	0.1100					
200		1.637	5.702	0.1754	0.0829	0.0888
	0.1430					
500		1.861	12.52	0.0799	0.0339	0.0676
	0.1070					
1000		0.081	22.61	0.0442	0.0105	0.0512
	0.05 08					
2000		2.235	41.04	0.0244	0.0037	0.0420
	0.0524					
5000		2.419	92.30	0.0108	0.0008	0.0330

TEST CASE 4, 5, 6 and 7 were performed within a pipe of 9cm diameter, the fluid being again air. The hot wire was at 2.5cm from the wall of the Perspex pipe in TEST CASE 5..

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary Unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 7

Data and Characteristics of TEST CASE 6

$\overline{U}$ = 26.62 m/s I_{ACTUAL} = 0.0661   n=18   A= 0.975410

u'=176cm/s $I_{n=30}$= 0.0661   P(1) = 0.4031

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4031	0.4031
	0.0529					
1		0.6022	0.1258	7.949	0.4245	0.2703
	0.1020					
2		0.8689	0.1743	5.736	0.3642	0.2113
	0.1100					
5		1.064	0.2851	3.507	0.2959	0.1730
	0.0865					
10		1.199	0.4429	2.258	0.2422	0.1495
	0.1030					
20		1.347	0.7233	1.383	0.1826	0.1265
	0.1050					
50		1.498	1.476	0.6774	0.1269	0.1061
	0.0683					
100		1.608	2.626	0.3808	0.0919	0.0929
	0.0957					
200		1.766	4.742	0.2109	0.0528	0.0766
	0.1240					
500		1.988	10.45	0.0957	0.0194	0.0575
	0.0618					
1000		2.149	19.08	0.0524	0.0078	0.0465
	0.05 62					
2000		2.326	35.04	0.0285	0.0023	0.0367
	0.0340					
5000		2.463	79.76	0.0125	0.0007	0.0304

TEST CASE 4, 5, 6 and 7 were performed within a pipe of 9cm diameter, the fluid being again air. The hot wire was at 1.0cm from the wall of the Perspex pipe in TEST CASE 6.

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary Unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 8

Data and Characteristics of TEST CASE 7

$\overline{U}$ = 25.25 m/s I_{ACTUAL} = 0.1000 n=11 A= 0.966623

u'=252cm/s $I_{n=30}$= 0.1038 P(1) = 0.3850

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.3850	0.3850
	0.0492					
1		0.5904	0.0689	14.51	0.3950	0.2758
	0.0575					
2		0.7721	0.1055	9.478	0.3649	0.2383
	0.0939					
5		0.9695	0.1914	5.224	0.3156	0.1995
	0.0652					
10		1.084	0.3128	3.197	0.2809	0.1785
	0.0767					
20		1.205	0.5306	1.885	0.2415	0.1578
	0.1060					
50		1.355	1.115	0.8970	0.1912	0.1344
	0.0755					
100		1.464	1.999	0.5002	0.1560	0.1190
	0.1160					
200		1.623	3.614	0.2767	0.1094	0.0990
	0.1490					
500		1.829	7.947	0.1258	0.0616	0.0771
	0.0912					
1000		1.987	14.48	0.0691	0.0360	0.0631
	0.0 644					
2000		2.122	26.62	0.0376	0.0209	0.0529
	0.0550					
5000		2.258	60.77	0.0165	0.0112	0.0440

TEST CASE 4, 5, 6 and 7 were performed within a pipe of 9cm diameter, the fluid being again air. The hot wire was at 0.5cm from the wall of the Perspex pipe in TEST CASE 7.

The measurement system was composed of 55DO1 DISA constant temperature hot-wire anemometer, high and low filters of DISA 55D25 Auxiliary Unit, 55D35 r.m.s meter and HP 322 Dual Channel recorder.

TABLE 9

Dara and Characteristics of TEST CASE 8

$\overline{U}$ =12.25 m/s I_{ACTUAL} = 0.0200 n=63 A= 0.991485

u'=24.5cm/s $I_{n=30}$= 0.0199 P(1) = 0.4104

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4104	0.4104
	0.00298					
1		0.2270	1.666	0.6002	0.4502	0.3563
	0.00298					
2		0.2841	2.668	0.3748	0.4565	0.3418
	0.00893					
5		0.3854	4.963	0.2015	0.4630	0.3157
	0.0149					
10		0.4862	7.900	0.1266	0.4625	0.2899
	0.0298					
20		0.6157	12.55	0.797	0.4504	0.2579
	0.0893					
50		0.8469	23.05	0.0434	0.3945	0.2055
	0.130					
100		1.069	36.41	0.0275	0.3053	0.1623
	0.186					
200		1.317	57.87	0.0173	0.1879	0.1227
	0.290					
500		1.657	109.50	0.0091	0.0607	0.0821
	0.134					
1000		1.893	181.7	0.0055	0.0177	0.0614
	0.0 595					
2000		2.058	311.3	0.0032	0.0057	0.0500
	0.0513					
5000		2.231	669.4	0.0015	0.0012	0.0401

TEST CASE 8, 9 and 10 are calculated for data obtained from available literature as reported for each case in the pertinent page of the TEST CASE.

Data of TEST CASE 8 is drawn from the Figure1.18 on page 61 of the article cited below*. The information belongs to a turbulent flow behind a grid made of 5mm cylindrical bars and a having a mesh size of 2.5cm.The data in the Figure belongs to Favre.

* Hinze. Turbulence. Pp. 61 Figure1.18. McGraw–Hill,1959.

TABLE 10

Data and Characteristics of TEST CASE 9

$\overline{\mathrm{U}}$ = 30.00m/s I_{ACTUAL} = 0.0287 n=43 A= 0.987950

u'=86cm/s $I_{n=30}$= 0.0289 P(1) = 0.4050

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4050	0.4050
	0.00462					
1		0.2646	0.4079	2.451	0.4463	0.3445
	0.00462					
2		0.3310	0.6533	1.531	0.4515	0.3281
	0.013					
5		0.4494	1.215	0.8231	0.4542	0.2988
	0.0462					
10		0.6535	1.877	0.5326	0.4355	0.2498
	0.0436					
20		0.7753	2.900	0.3448	0.4093	0.2224
	0.105					
50		0.9805	5.397	0.1853	0.3413	0.1806
	0.247					
100		1.305	8.593	0.1164	0.1988	0.1266
	0.142					
200		1.499	13.80	0.0725	0.1186	0.1012
	0.155					
500		1.724	27.40	0.0365	0.0514	0.0774
	0.152					
1000		1.991	47.07	0.0212	0.0123	0.0557
	0.0 401					
2000		2.115	82.66	0.0121	0.0052	0.0477
	0.0318					
5000		2.231	183.5	0.0055	0.0020	0.0412

TEST CASE 8, 9 and 10 are calculated for data obtained from available literature as reported for each case in the pertinent page of the TEST CASE.

Data of TEST CASE 9 is drawn from the Figure 4.3 on page 269 of the reference cited below*. The information belongs to a turbulent in a pipe of 24.7cm in diameter. The measurements belong to a point at half the radius, therefore 6.2cm from the all. The data in the Figure belong to Laufer.

*Hinze. Turbulence. Pp. 269, Figure 4.3. McGraw–Hill, 1959.

TABLE 11

Data and Characteristics of TEST CASE 10

$\overline{U}$ =13.15m/s I_{ACTUAL} = 0.0228 n=55 A= 0.990351

u'= 30cm/s $I_{n=30}$= 0.0225 P(1) = 0.4094

f c/s	ΔG -	x -	k cm^{-1}	L cm	$P_R(x)$ -	$P_L(x)$ -
0		-	-	-	0.4094	0.4094
	0.01282					
1		0.3740	0.8198	1.220	0.4606	0.3186
	0.0126					
2		0.4659	1.319	0.7584	0.4610	0.2953
	0.0354					
5		0.6236	2.472	0.4045	0.4470	0.2564
	0.0491					
10		0.7654	3.980	0.2513	0.4172	0.2237
	0.0769					
20		0.9264	6.456	0.1549	0.3646	0.1898
	0.191					
50		1.204	12.35	0.0809	0.2420	0.1403
	0.149					
100		1.405	20.38	0.0491	0.1501	0.1113
	0.234					
200		1.703	33.86	0.0295	0.0509	0.780
	0.132					
500		1.945	68.30	0.0146	0.0135	0.0578
	0.0561					
1000		2.109	120.0	0.0083	0.0041	0.0471
	0.0 257					
2000		2.209	217.0	0.0046	0.0017	0.0415
	0.0256					
5000		2.316	494.7	0.0020	0.0006	0.0362

TEST CASE 8, 9 and 10 are calculated for data obtained from available literature as reported for each case in the pertinent page of the TEST CASE.

Data of TEST CASE 10 is drawn from the Figure 7.23 on page 501 of the reference cited below*. The information belongs to a very thick turbulent boundary layer of 7.5cm at the point of measurement. The measurements belong to a point 3.75cm from the wall. The data in the Table is constructed from a Figure due to Klebanoff.

*Hinze. Turbulence. Pp. 501, Figure 7.23. McGraw–Hill, 1959.

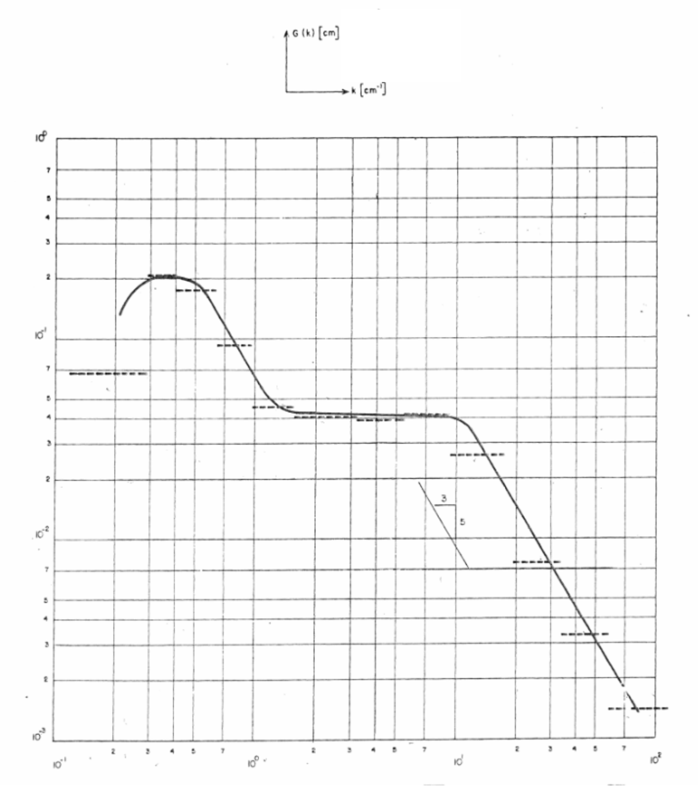

Figure 1: One dimensional spectrum for Test Case #1.

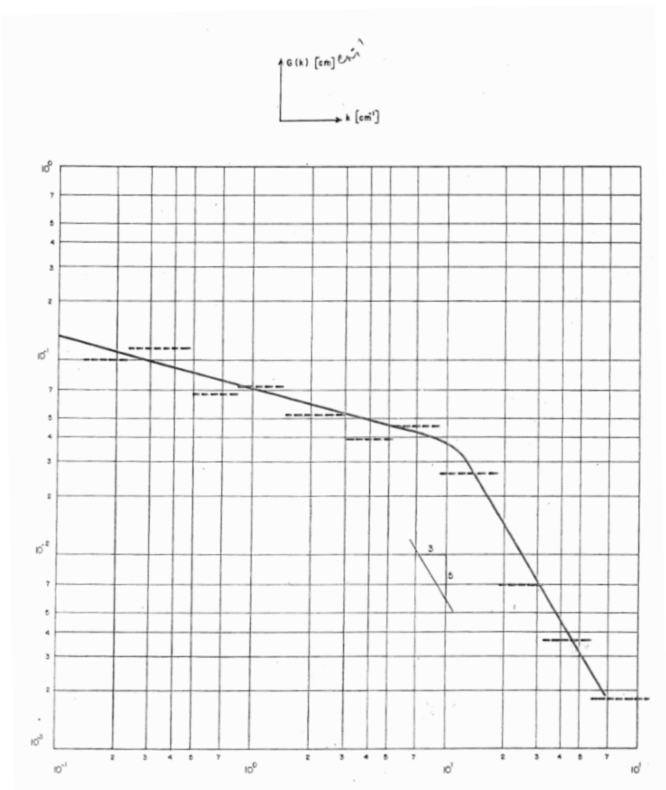

Figure 2: One dimensional spectrum for Test Case #2.

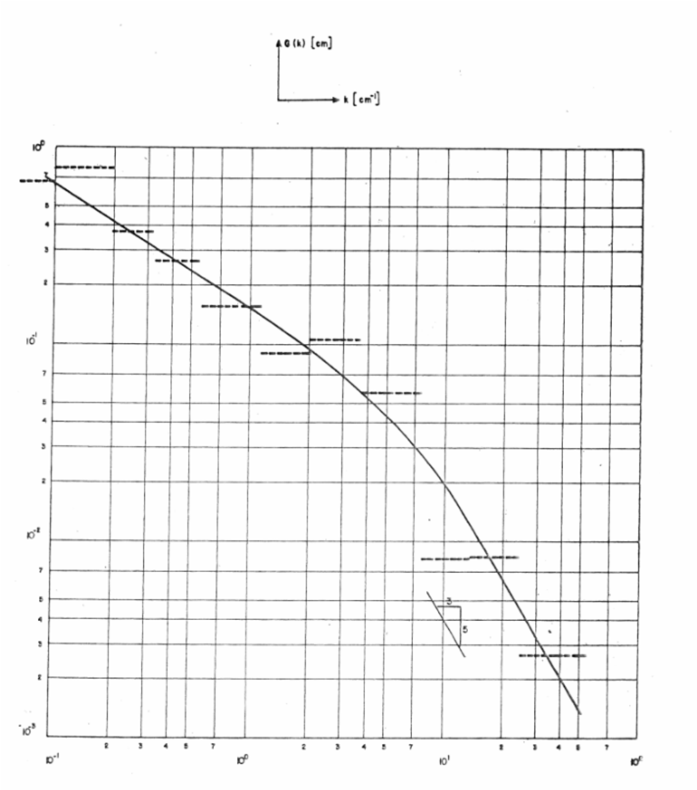

Figure 3: One dimensional spectrum for Test Case #3.

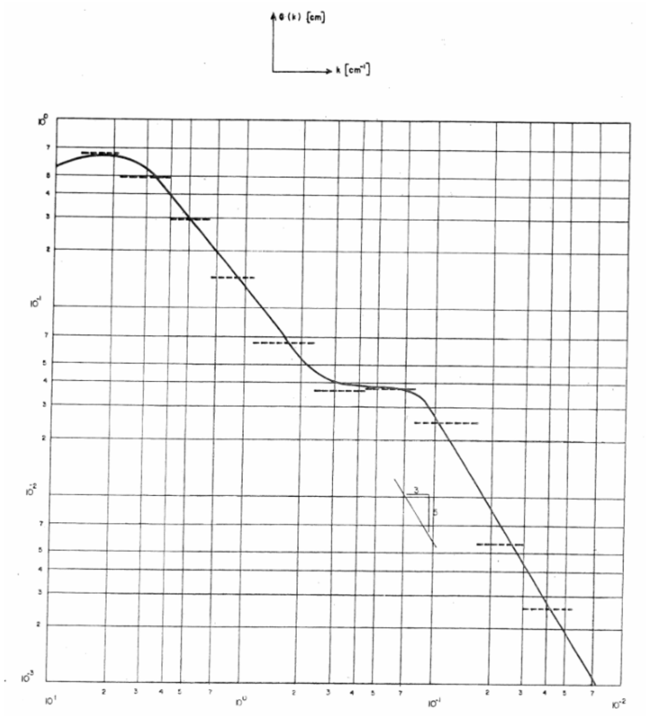

Figure 4: One dimensional spectrum for Test Case #4.

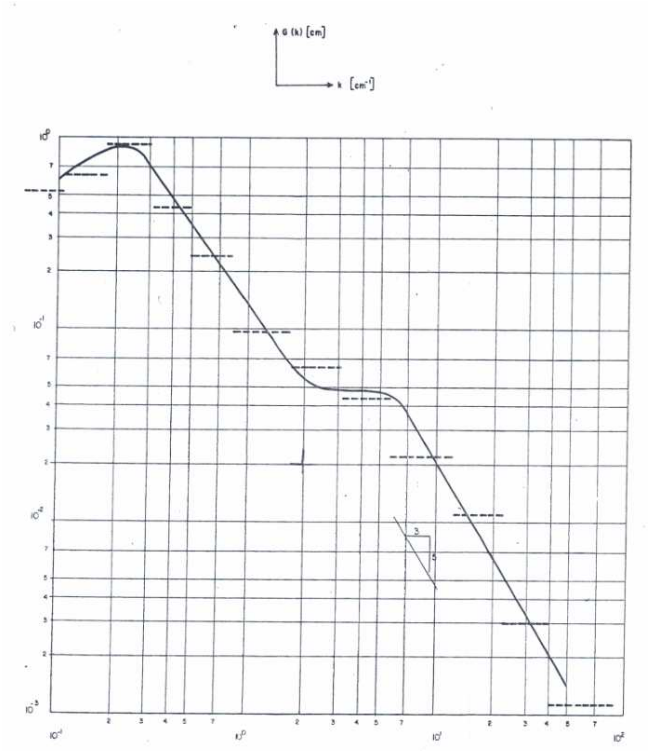

Figure 5: One dimensional spectrum for Test Case #5.

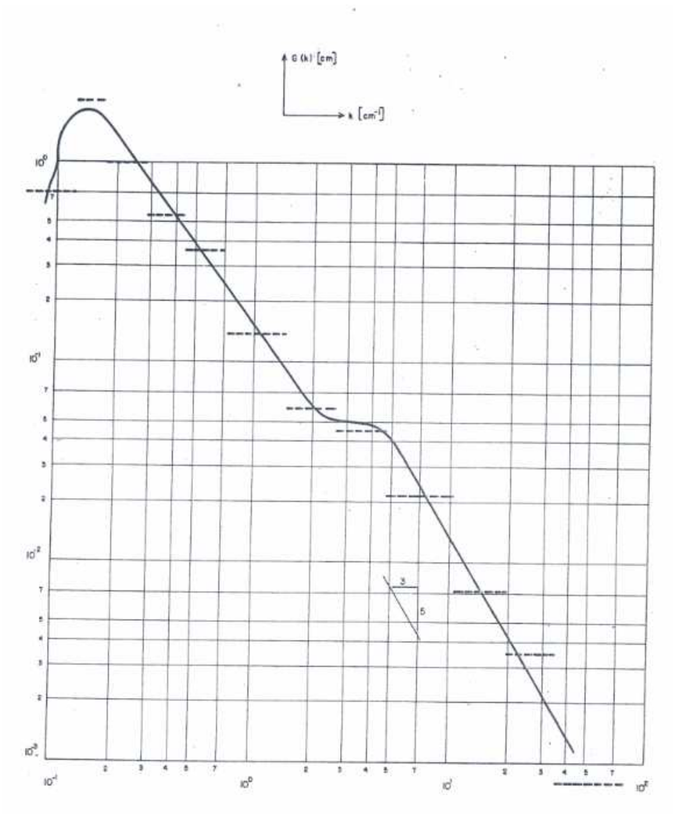

Figure 6: One dimensional spectrum for Test Case #6

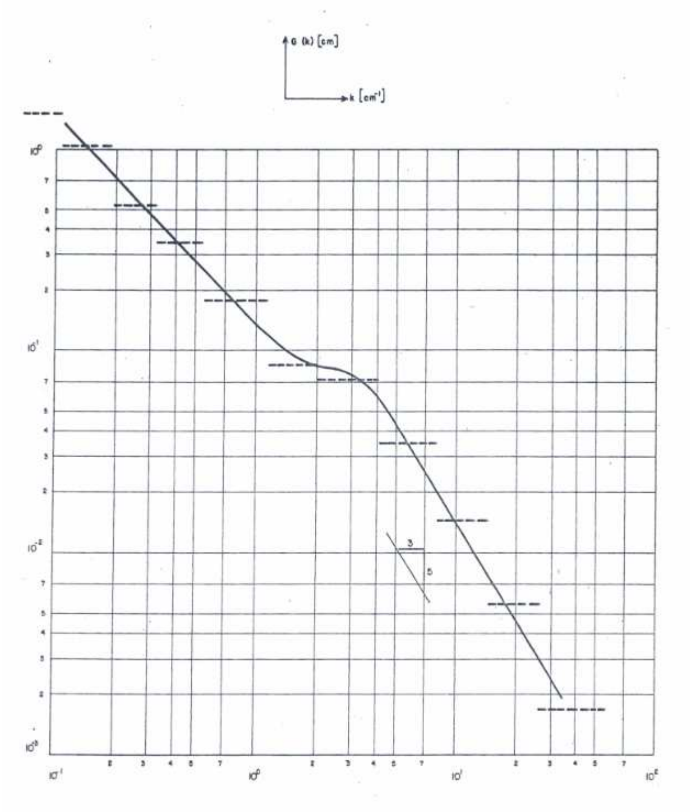

Figure 7: One dimensional spectrum for Test Case #7.

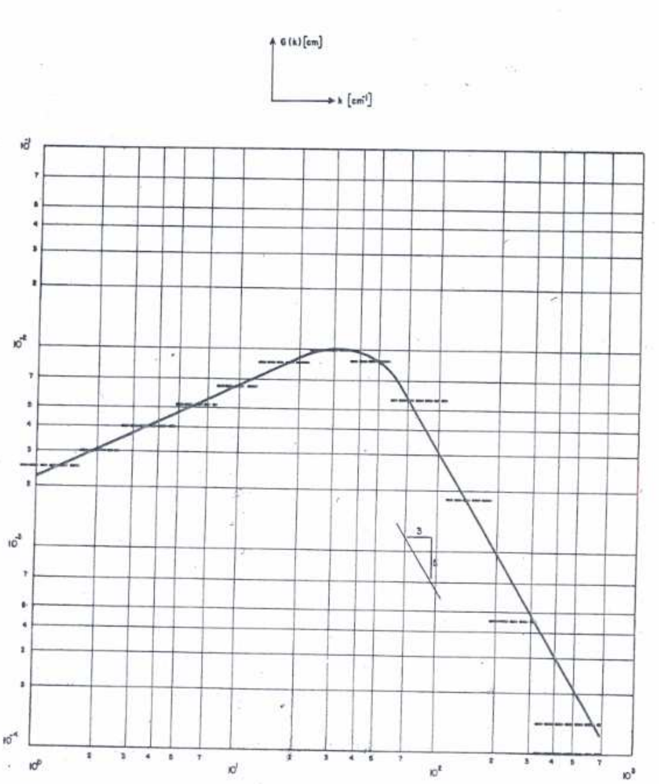

Figure 8: One dimensional spectrum for Test Case # 8

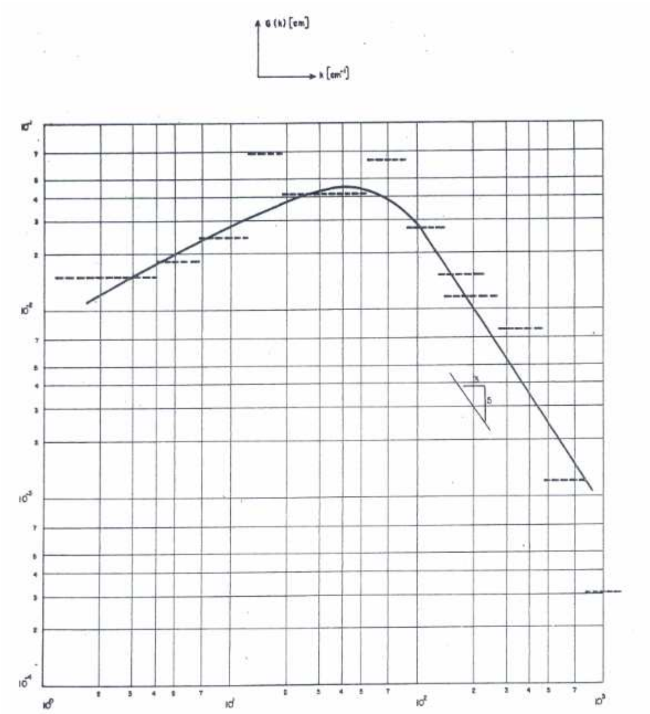

Figure 9: One dimensional spectrum for Test Case #9.

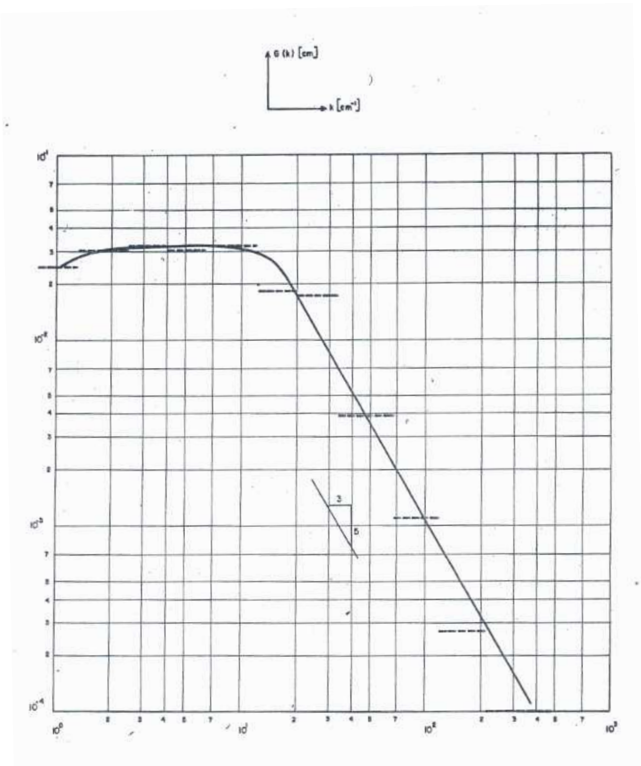

Figure 10: One dimensional spectrum for Test Case #10

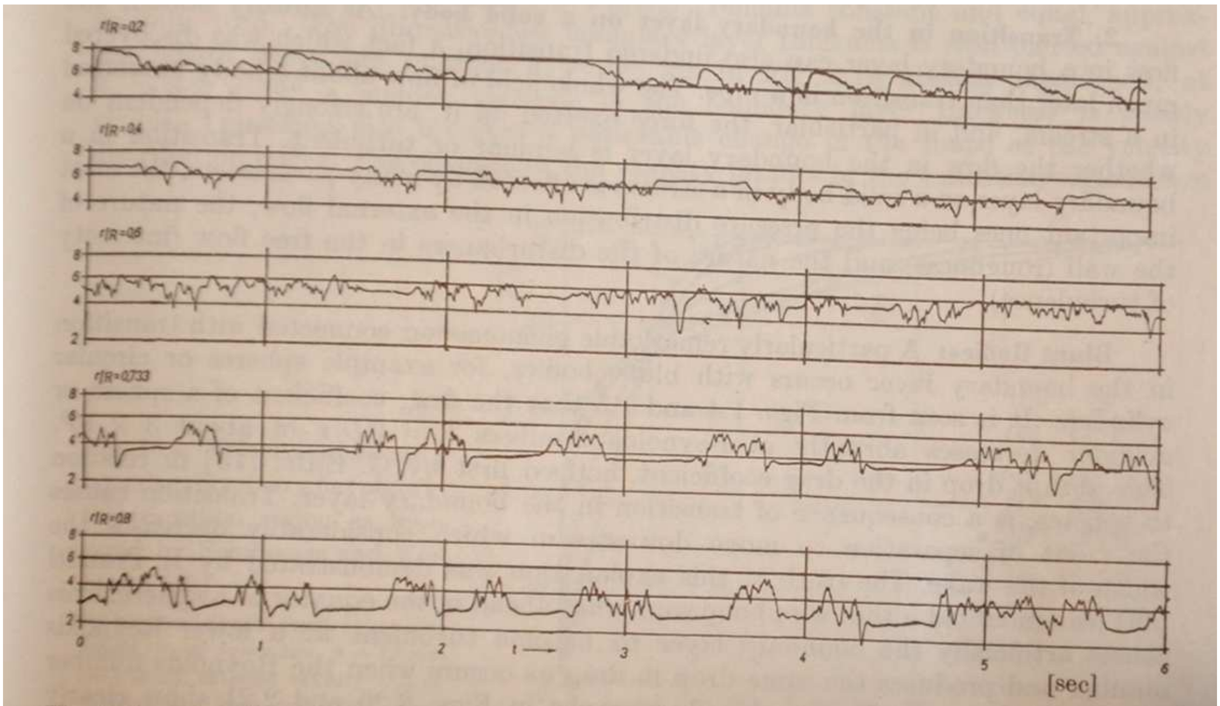

Fig. 16.2. Variation of flow velocity in a pipe in the transition range at different distances r from pipe axis, as measured by J. Rotta [45]

Reynolds number $R = \bar{w}d/\nu = 2550$; axial distance $x/d = 322$; $\bar{w} = 4{\cdot}27$ m/sec ($= 14{\cdot}0$ ft/sec); velocities given in m/sec. These velocity plots, obtained with the aid of a hot-wire anemometer, demonstrate the intermittent nature of the flow in that periods of laminar and turbulent flow succeed each other in time

Figure 11.

This Figure is copied from H.Schlichting's Boundary Layer Theory, fourth edition, McGraw Hill Co.INC,1960, page 377. It is reported to be from J.Rotta's paper which appeared in Ing. Archives **24**,258-281(1956).

This is a very low Reynolds number air flow (R_E=2550) in a pipe of approximately 0.6cm in diameter. The five graphics depict the instantaneous velocity consecutively at r/R= 0.8; 0.733; 0.6; 0.4; 0.2 versus time. Low probability of occurrence for high frequency but large amplitude velocity fluctuations are clearly noticeable.