# VIBRATIONS OF STRUCTURES CONTAINING CRACKS

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## ABSTRACT

In this work, crack contained structures especially the cantilever beam structures and their vibration response relations are analysed from the literature. Moreover, the effects of crack size, the crack location and its depth to the vibrational behavior of the structure is investigated. The effects of the cracks to the natural frequencies, mode shapes, vibration amplitudes and to the higher order harmonics are studied.  Different vibration methods used for damage detection and health monitoring of mechanical systems are also reviewed from the literature within the context of this study and it is observed that many works has been performed to study linear and non-linear vibration using various signal processing techniques. Finally, some conclusions are derived from the literature surveysfor the vibrations of structures containing cracks.

**Keywords:** Crack size, crack depth, vibration, natural frequencies, eigenvalues, mode shapes, eigenvectors.

## INTRODUCTION

[Bouraou and Gelman,1997], [Gelman,2000] and [Cuc, 2002] expressed the equations of motion of a structure with crack as follows:

<table>
<tr><td>$\ddot{X} + 2\zeta_s\omega_s\dot{X} + \omega_s X = A\sin(\omega_f t + \varphi)$,<br>$x \geq 0.$ (Eqn.1)</td><td rowspan="4">Where;<br>x : Displacement<br>m: Object mass<br>$k_s$: Stiffness at stretching (crack opening)<br>$k_c$: Stiffness at compression (crack closing)<br>$k$ : Material stiffness without crack<br>$\zeta_s$ : Damping ratio at stretching (crack opening)<br>$\zeta_c$ : Damping ratio at compression(crack closing)<br>$c$ : Damping coefficient<br>$\omega_f$: Constant excitation frequency;<br>φ : Random initial phase is uniformly distributed in the interval [0;2π]</td></tr>
<tr><td>$\ddot{X} + 2\zeta_c\omega_c\dot{X} + \omega_c X = A\sin(\omega_f t + \varphi)$,<br>$x < 0.$ (Eqn.2)</td></tr>
<tr><td>$X = \frac{x}{m}$ (Eqn.3)</td></tr>
<tr><td>$\omega_s = \sqrt{\frac{k_s}{m}}, \omega_c = \sqrt{\frac{k_c}{m}},$<br>$\zeta_s = \frac{c}{2\sqrt{k_s m}}, \zeta_c = \frac{c}{2\sqrt{k_c m}}$ (Eqn.4)</td></tr>
</table>

At compression state, the crack is closed and the material behaves like a continuum. Therefore, the compression stiffness ($k_c$) becomes equal to the material stiffness without crack at that compression state as stated by [Cuc, 2002].
At stretching state, on the other hand, the crack is opened and the material becomes discontinuous. Therefore, the stiffness decreases in amount ($\Delta k$) as stated by [Cuc, 2002] in Figure 1 below.

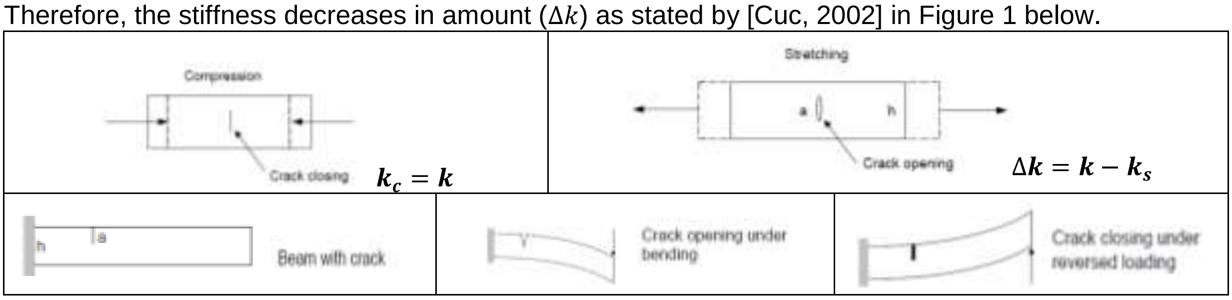


Figure 1: Compression and stretching states in axial loading and bending [Cuc, 2002]

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In [Gelman, 2000], it is showed that in a structure including crack, the stiffness is nonlinear and the level of nonlinearity changes with the crack size. For the axial loading case, the stiffness change due to the crack is related to the width of the object (cross section area). Therefore according to [Cuc, 2002], the linear approximation holds but for bending, the linear approximation is not true anymore.
[Gelman 2000] showed that the frequency of the cracked object depends on the crack size (a) as stated below:

| | |
|---|---|
| $\frac{{\omega_s}^2}{{\omega_c}^2} = 1 - \frac{a}{h} = 1 - r$ (Eqn. 3) | Where;<br>$\omega_s$ : Frequency at stretching (crack opening)<br>$\omega_c$ : Frequency at compression (crack closing)<br>$a$ : Crack size<br>$h$ : Height of the beam<br>$r$ : Relative crack size. |
| $r = \frac{a}{h}$ (Eqn. 4) | |

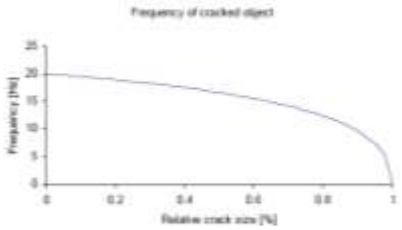
The period of oscillation and the natural frequency of the cracked object are stated by [Gelman, 2000] as follows:

| | |
|---|---|
| $T = \frac{2\pi}{\omega} = \frac{T_1}{2} + \frac{T_2}{2} = \frac{1}{2}\left(\frac{2\pi}{\omega_s}\right) + \frac{1}{2}\left(\frac{2\pi}{\omega_c}\right) = \frac{\pi}{\omega_s} + \frac{\pi}{\omega_c}$ | (Eqn.5) |
| $\omega = 2\frac{\omega_s . \omega_c}{\omega_s + \omega_c}$ | (Eqn.6) |

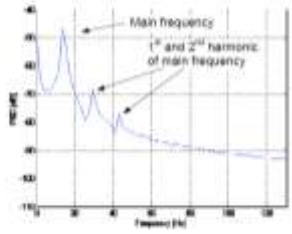
[Gelman, 2000] and [Cuc, 2002] stated the frequency of the crack structure as:

| | |
|---|---|
| $\omega = \omega_n \frac{2.\sqrt{1-r}}{1+\sqrt{1-r}}$ (Eqn. 7) | Where;<br>$\omega_n$ : Natural frequency of the un-cracked object, $\omega_n = \sqrt{\frac{k}{m}}$ ;<br>$r$ : Relative crack size |

The relation between the frequency and the relative crack size is stated by [Cuc, 2002] as shown in Figure 2. As seen while the crack size increases, the frequency of the structure decreases as a consequence of the change in stiffness. Inother words, crack presence in the structure modifies (decreases) the effective stiffness and hence the main frequency. In addition, the nonlinear character of the equations leads to parametric resonances, which appear as additional features in the frequency spectrum as higher order harmonics of the main frequency.



Frequency change with relative crack size



Additional features present

Figure 2: Effects of crack to natural frequency and vibration spectrum [Cuc,2002]

Damage detection and failure prevention of mechanical systems is of critical importance. Due to the crack involvement in the structure; the changes in the magnitude of the amplitude, the shift of the main frequency (to the lower values due to crack size increase) and the generation of the higher harmonics in the frequency spectrum have been investigated as stated by [Cuc,2002].Stiffness and nonlinearities change with the crack size. After computing the PSD,peaks in the frequency spectrum were observed.
The MATLAB model is generated by [Cuc,2002] to investigate the crack effects. The signal generated is a constant amplitude sine wave. The frequency of the signal changes as the crack progresses. The natural frequency of the object without crack is $f_n$ = 20 Hz ($w_n$=40 rad/s) and the damping coefficients for stretching and compression are $\zeta\omega_s=\zeta\omega_c$=10 rad/s. The time domain data for two cases, pristine (r = 0), and damaged (r = 0.6) is presented by [Cuc,2002] in amplitude vs. time plot as given in Figure 3:

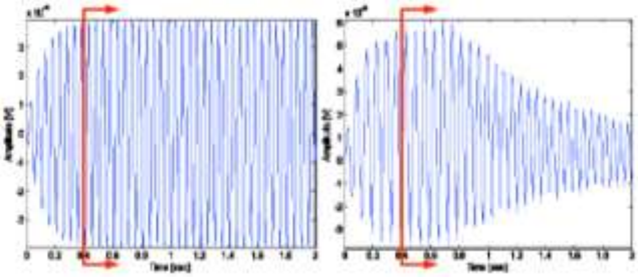
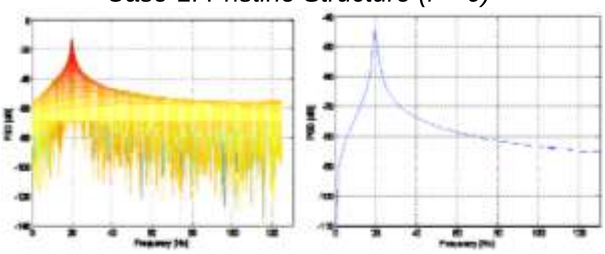
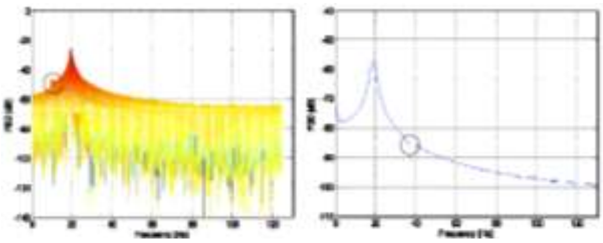
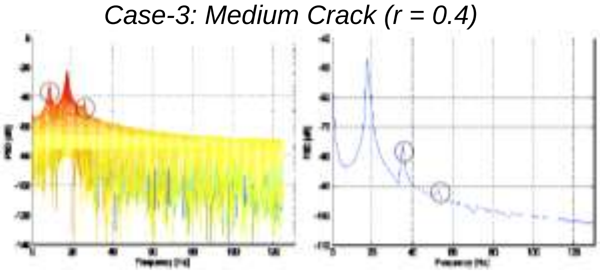
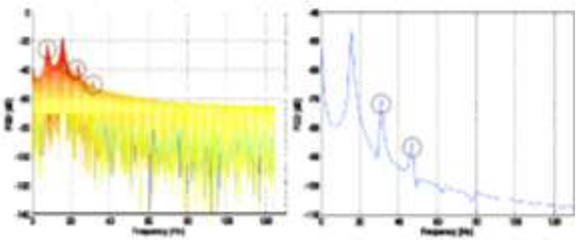


Figure 3: Time domain data for pristine (r = 0) and damaged (r = 0.6) cases [Cuc,2002]

After the time domain signal was obtained, frequency domain analysis techniques were used to investigate the change in frequency as a result of the crack growth by [Cuc, 2002]. The feature of interest was the amplitude of the main frequency as well as shifting of the other frequencies in the frequency domain. A simulation for a cracked component was designed and carried out using MATLAB-SIMULINK by [Cuc, 2002]. The forced oscillation method to generate the time response of the structure for axial loading was used. The crack was simulated as a change in stiffness. Although the traditional methods, i.e. fast Fourier transform and short time Fourier transform are still used, the focus of this recent research is shifting towards more advanced methods and signal processing techniques such as wavelet transform and Wigner-Ville distribution. Hence, the signal processing techniques used for damage detection were short time Fourier transform and Wigner-Ville distribution. The reason for using joint time-frequency methods was the non-linearity of the structure response. The crack presence in the structure modifies the effective stiffness and hence the main frequency. The relative crack sizes were r = 0 (pristine case), r = 0.1 (small crack), r = 0.4 (medium crack), and r = 0.6 (large crack). The results showed that both methods (STFT and WVD) predicted the presence and progress of damage. The feature used to predict damage were, the change in the amplitude of the main frequency and emerging higher order harmonics of the main frequency in the frequency spectrum. The STFT method gives a better indication of the higher harmonics of the main frequency due to the crack growth. On the other hand, WVD method gives stronger sensitivity to the crack presence based on the changes in the main frequency as stated by [Cuc,2002] in Table 1below.

Table 1: WVD and STFT methods comparison with the crack sizes [Cuc,2002]

| WVD & STFT Comparison by [Cuc,2002] | Crack Sizes (r = 0; 0.1; 0.4; 0.6) |
|---|---|
| *Case-1: Pristine Structure (r = 0)*  | Main frequency of undamaged object is the same as the natural frequency of the pristine object, $f_{pristine}$ = 20 Hz. WVD and STFT predict this value correctly. No other effects (interference or harmonics) are present in the frequency spectrum, which is parallel with the predictions of linear analysis. |
| *Case-2: Small Crack (r = 0.1)*  | As the crack grows, the main frequency is changing. Calculated frequency for the relative crack size of 0.1 is f0.1 = 19.5 In WVD and STFT, there is a small change in the peak of the main frequency, indicating a shift of the frequency toward a smaller value. For STFT, the higher harmonic effects start to be noticeable. The first harmonic of the main frequency appears in the spectrum ($f_2$ ≅39 Hz). |
| *Case-3: Medium Crack (r = 0.4)*  | At higher relative crack size, r = 0.4, the calculated main frequency is $f_{0.4}$ = 17.5 Hz. Main frequency changed and well-defined additional features appeared in WVD and STFT. The amplitude of the features is also more prominent. |
| *Case-4: Large Crack (r = 0.6)*  | When the relative crack size reach 0.6, calculated main frequency is $f_{0.6}$= 15.5 Hz. Main frequency decreased and additional features appeared in WVD and STFT. It is also noticeable that the amplitude of the harmonics has grown considerable in comparison with the previous cases that has a smaller relative crack size. |

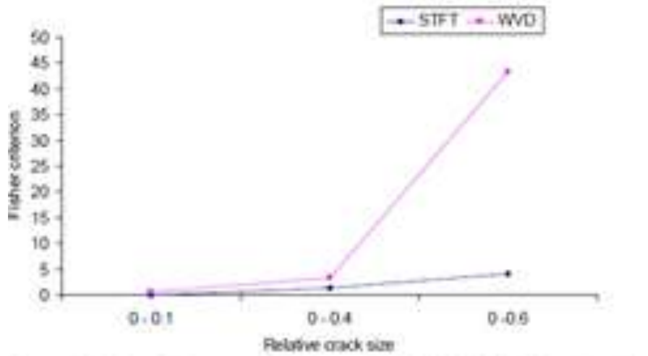
[Cuc,2002] showed that as the crack progresses, the frequency spectra changes. “According to the non-linear analysis when damage is present, the frequency spectra will change and higher harmonics and subharmonics of the main frequency will appear. The results clearly indicate the changes due to the presence of the crack. The appearance of higher harmonics as the crack grows is well indicated by the STFT. The 1st and 2nd harmonic of the main frequency ($f_2 \cong$ 35 Hz, $f_3 \cong$53 Hz) could be easily identified on the power spectra for r=0.4 and r=0.6. While the STFT method can detect well the higher harmonics of the main frequency, the WVD method is less sensitive to this feature. In WVD power spectrum, the 1st higher harmonic of the main frequency appears only for large crack size (relative crack size r = 0.6). It is interesting to notice that subharmonics generated by the interference terms (cross-terms), are present in the frequency spectra of the WVD but do not appear in the STFT. As the crack grows, the cross-terms are more visible and their amplitudes are larger. This phenomenon led us to the idea of using the cross-terms as an indicator of crack propagation. An interesting outcome of the Wigner-Ville distribution was observed, related to the cross-terms apparition. For small crack size and medium crack size subharmonics generated by the cross-terms are present in the frequency spectrum. Apparition of the cross-terms in the frequency spectrum can be related to the apparition of damage in the structure. Further work needs to be done in order to better understand the outcome of the Wigner-Ville distribution and to relate the apparition of cross-terms and the presence of damage. In conclusion, both WVD and STFT methods predict the presence of the damage. The STFT method gives a better indication of the higher harmonics of the main frequency due to the crack growth. On the other hand, WVD method gives stronger sensitivity to the crack presence based on the changes in the main frequency. The Short Time Fourier Transform (STFT) method is a relatively known method and is widely use in machinery damage detection and health monitoring. Wigner-Ville Distribution (WVD) is a relatively new method and the full advantages it can offer are yet to be discovered.
To classify the outcome of the two methods, a statistical tool was needed and this latter observation has also been substantiated by statistical analysis using Fisher criterion. The Fisher linear discriminant method (Fisher criterion) was applied to classify the outcome of the two damage detection techniques. The spectral features used in this classification were chosen to be the change in amplitude of the main frequency for the pristine cases (r = 0) and for the damaged case (r = 0.1), (r = 0.4), (r = 0.6). The main frequencies for the damaged cases where compared one by one with the main frequency for the pristine case, and the Fisher criterion was calculated for each case. The Fisher criterion was calculated for pairs of classes representing the frequency of the main harmonic for the pristine case and the frequency of the main harmonic for various damage cases. The change in the criterion as the damage progresses was examined. Table 2 presents the values of the Fisher criterion for the 2 methods, short time Fourier transform (STFT) and Wigner-Ville distribution (WVD).”

Table 2: Fisher criterion data [Cuc,2002]

| Method | Fisher criterion value | | |
|---|---|---|---|
| | Pristine vs. 0.1 relative crack | Pristine vs. 0.4 relative crack | Pristine vs. 0.6 relative crack |
| STFT | 0.001 | 1.476 | 4.119 |
| WVD | 0.666 | 3.297 | 43.367 |

Fisher criterion calculated for STFT and WVD methods and the feature comparison. Each of the the damaged cases were compared with the pristine case

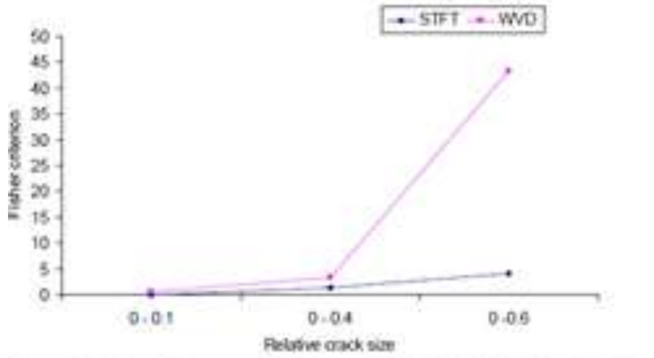
“Fisher criterion method was applied to the data obtained using STFT and WVD. The results were plotted on the same graph and presented in Figure 4. It can be noticed that damage detection capabilities of WVD are definitely better than those of STFT.”




Damage detection effectiveness of the two methods, STFT and WVD, based on the Fisher criterion

Figure 4: Fisher Criterion

Figure 4 shows that, for small cracks the detection of damage in the structure is done poorly by both methods (STFT and WVD). As the crack size increases, the detection of the damage becomes better, as indicated by the increase in the Fisher criterion values. As expected, the best damage detection is achieved when the crack size is large as shown for r = 0.6. For small and medium crack sizes (r = 0.1 and r = 0.4) the two methods are comparable. For medium crack size (r = 0.4), the Fisher criterion for STFT and WVD being FSTFT 0.4 = 1.5 and FWVD 0.4 = 3.3. For larger crack size (r = 0.6), the WVD prediction is much better than the STFT, FSTFT 0.6 = 4.2 and FWVD



Damage detection effectiveness of the two methods, STFT and WVD, based on the Fisher criterion

Figure 4: Fisher Criterion

0.6 = 43.4. The results showed that the Fisher criterion values for the Wigner-Ville distribution method were slightly higher than those for the short time Fourier for small and medium crack size (FSTFT 0.4 = 1.5 and FWVD 0.4 = 3.3), and there was a significant difference between the values for large crack size (FSTFT 0.6 = 4.2 and FWVD 0.6 = 43.4). This is an indication that the WVD method predictions are more accurate and sensitive than the predictions of STFT.

In [Pierre and Shen,1986] the free vibrations of rectangular cross section, simply supported cracked beam is studied. The frequencies and deflection shapes (mode shapes) are obtained for the first three modes by finite element method and then compared with the Bernoulli-Euler theory as well as the experimental results. The natural frequencies and mode shapes of the beam for the crack depth of 1/3, 1/2 and 2/3 of the total of the beam were computed. Finite element, experimental and analytical results for the fundamental mode are compared by [Pierre and Shen,1986] as given in the Table 3. The results are presented in the form of FR and CR, where;
Freqeuncy Ratio (FR), that is the ratio of the frequency of the cracked beam to that of the uncraked and Crack depth Ratio (CR), that is the ratio of the depth of crack to the half depth of the beam thickness.

Table 3: Theoretical and finite element results for cracked beam [Pierre and Shen,1986]

UNCRACKED BEAM

| | Theoretical | Finite Element | |
|---|---|---|---|
| | Natural frequency (Hz) | Natural frequency (Hz) | Total strain energy |
| 1st mode | 9.87 | 9.846 | 198 |
| 2rd mode | 39.5 | 39.100 | 807 |
| 3rd mode | 88.9 | 87.191 | 1840 |

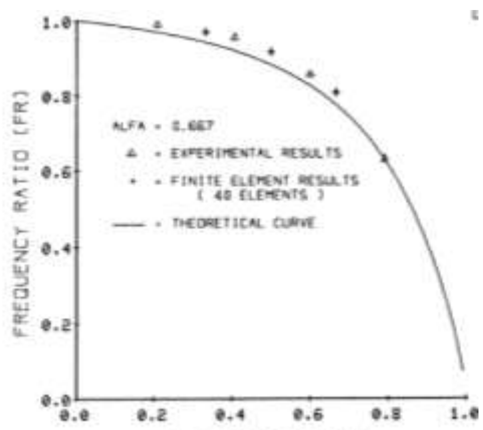
The material properties are as follows:

| | | | |
|---|---|---|---|
| E | 4.1472 | $\times 10^{3}$ | psi |
| $\nu$ | 0.3 | | |
| Density | 2.4 | $\times 10^{-4}$ | $\text{lb-sec}^2/\text{in}^4$ |

CRACKED BEAM

| | Theoretical | | Finite Element | | |
|---|---|---|---|---|---|
| | CR | FR | Natural frequency (Hz) | FR | Total strain energy |
| 1st mode | 1/3 | .9435 | 9.5497 | .9699 | 192 |
| | 1/2 | .8840 | 9.0300 | .9171 | 184 |
| | 2/3 | .7760 | 7.9596 | .8040 | 173 |

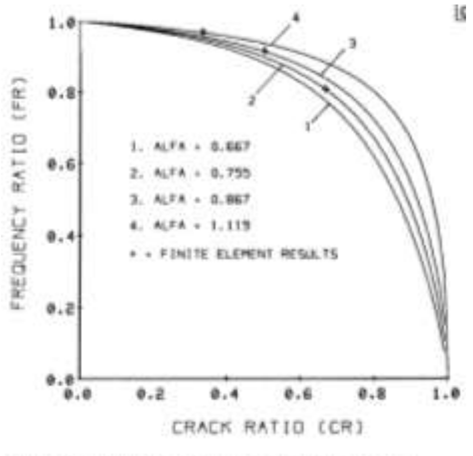
*** Note: CR : crack ratio , FR : frequency ratio



COMPARISON OF FINITE ELEMENT, EXPERIMENTAL AND THEORETICAL VALUES OF FREQUENCY RATIO. ( FIRST MODE )

CRACKED BEAM

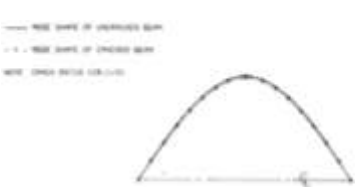
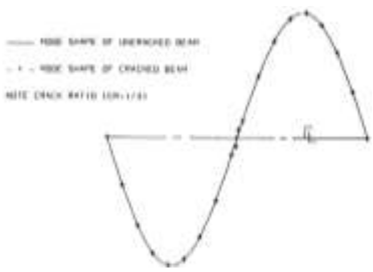
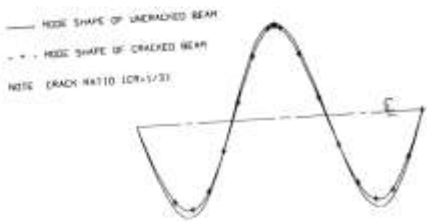
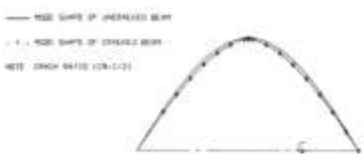
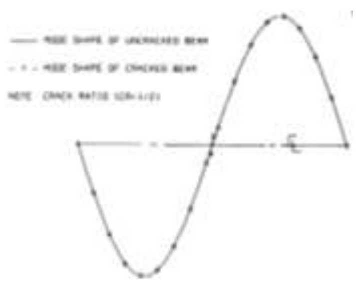
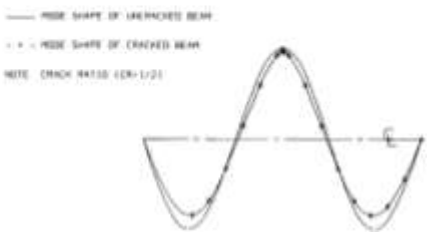
| | Theoretical | | Finite Element | | |
|---|---|---|---|---|---|
| | CR | FR | Natural frequency (Hz) | FR | Total strain energy |
| 2rd mode | 1/3 | | 39.100 | 1.0 | 805 |
| | 1/2 | | 39.100 | 1.0 | 804 |
| | 2/3 | | 39.100 | 1.0 | 803 |
| 3rd mode | 1/3 | | 84.7136 | .9715 | 1600 |
| | 1/2 | | 80.9534 | .9284 | 1400 |
| | 2/3 | | 74.9430 | .8595 | 1220 |




THEORETICAL CURVE DUE TO VARIOUS ALFA (FIRST MODE).

[Pierre and Shen,1986] showed in Table 4 that, "The changes of the first and third mode shapes between cracked and uncracked beams are significant. But the second mode shapes are almost the same for uncracked and cracked beams with all the crack ratios. The reason behind this is due to the position of the crack, which is located at the mid-span of the beam. Therefore, the stress singularity occurs severely when the cracks under compression or tension in the first or third mode vibrations. Since the crack at the mid-span is not under severe compress or tension at the second mode, the strain in neutral axis direction is almost zero except very small inplane strain occurs due to the in-plane displacement. Therefore, stress singularity vanish and the cracks does not affect the eigenfrequency and eigenmode for the second mode vibration]."

Table 4: Mode shape comparison of cracked/uncracked beam [Pierre and Shen,1986]

| First Mode Shapes of Uncracked and Cracked Beams | Second Mode Shapes of Uncracked and Cracked Beams | Third Mode Shapes of Uncracked and Cracked Beams |
|---|---|---|
|  |  |  |
|  |  |  |

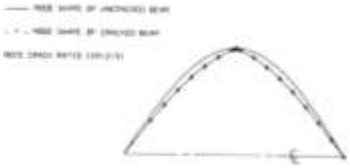
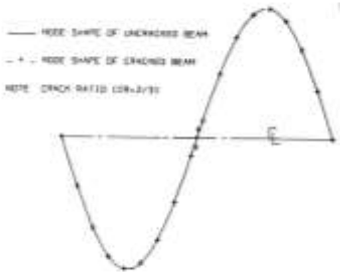
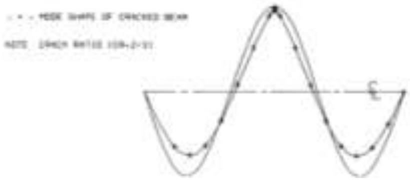
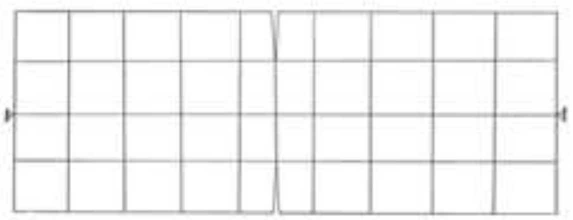
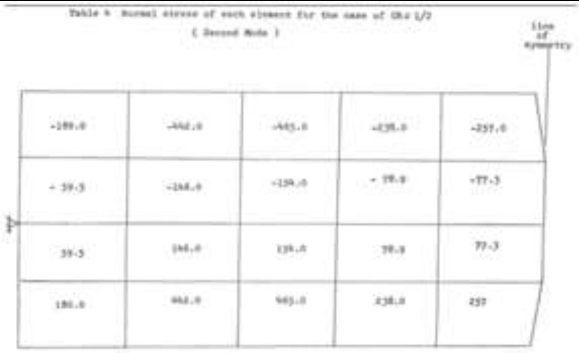
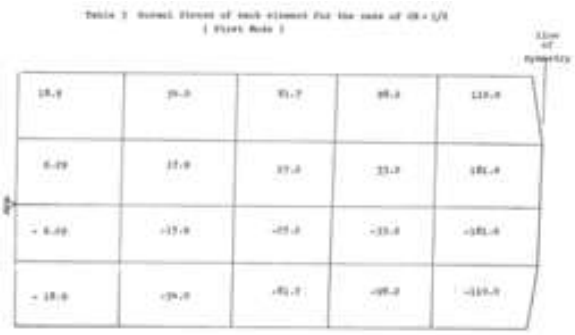
| | | |
|---|---|---|
|  |  |  |

Table 5 shows the stress level of each element of the cracked beam. “Stress singularity occurs at the elements near the crack tip for the 1st and 3rd modes. Almost the same stress level of each element between cracked and uncracked beams is observed for the case of 2nd mode” as stated by [Pierre and Shen,1986].

Table 5: Stress levels of the cracked beam by [Pierre and Shen,1986]

| | |
|---|---|
|  Finite element gridwork used for the simply supported cracked beam |  Normal stress of each element for the case of CR=1/2 (Second Mode) |
|  Normal stress of each element for the case of CR=1/2 (First Mode) |  Normal stress of each element for the case of CR=1/2 (Third Mode) |

According to [Rao, 2009], “a crack in a structural member introduces local flexibility that would affect vibration response of the structure. This property may be used to detect existence of a crack together with its location and depth in the structural member. The presence of a crack in a structural member alters the local compliance that would affect the vibration response under external loads” as shown in Table 6.

Table 6: Experimental results for cracked beam by [Rao, 2009]

Beam Specification:

| | |
|---|---|
| Software used | FFT analyzer and accessories, Pulse lab shop version 9.0 |
| parameter | frequency |
| Length of cantilever | 20cm |
| Section dimensions | 0.0095X0.0095m$^2$ |
| Boundary conditions | One end fixed and another free |
| Material | Aluminium |
| Mass density | 2659kgm$^{-3}$ |
| Elastic modulus | 68.0E09Nm$^{-2}$ |
| Poison's ratio | 0.205 |

Natural frequency of the beam was theoretically computed using the Fortran program. Experimental results for uncracked beam: Aluminum beam (fixed-free condition) :

| Mode | Frequency (by theoretical method) | Frequency (by practical method) | Percentage of error |
|---|---|---|---|
| first | 197.42 Hz | 187.00 Hz | 5.27% |
| second | 1227.66 Hz | 1180.00 Hz | 3.88% |
| Third | 3393.65 Hz | 3308.00 Hz | 2.52% |
| Fourth | 6547.61 Hz | 6425.00 Hz | 1.87% |

Single crack experimental results:
Aluminum beam (fixed-free)

| | | | 1st mode | 2nd mode | 3rd mode | 4th mode |
|---|---|---|---|---|---|---|
| 1 | Crack at centre | 2mm | 144 | 928 | 2088 | 2792 |
| 2 | | 6mm | 136 | 887 | 2064 | 2744 |
| 3 | | 8mm | 104 | 560 | 1448 | 2744 |
| 4 | Crack at 0.25L | 2mm | 128 | 960 | 1936 | 2832 |
| 5 | | 6mm | 112 | 876 | 1720 | 2456 |
| 6 | | 8mm | 88 | 448 | 856 | 1736 |
| 7 | No crack | nill | 187 | 1180 | 3308 | 6425 |

Multi crack experimental results:
Steel beam (free-free condition)
Length = 25 cm, Height = 9.2mm

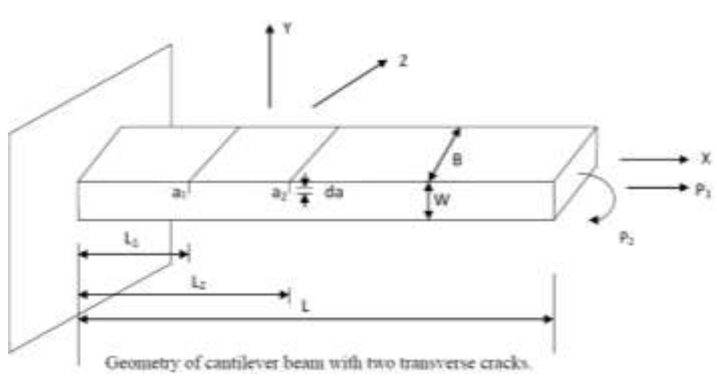
| S.no. | | Crack depth | 1st mode | 2nd mode | 3rd mode | 4th mode |
|---|---|---|---|---|---|---|
| 1 | Cracks at 10mm, 5mm | 2mm | 144 | 928 | 2088 | 2792 |
| 2 | | 6mm | 136 | 887 | 2064 | 2744 |
| 3 | | 8mm | 48 | 560 | 1448 | 2744 |
| 4 | Cracks at 7.5mm, 2.5mm | 2mm | 128 | 960 | 1936 | 2832 |
| 5 | | 6mm | 112 | 876 | 1720 | 2456 |
| 6 | | 8mm | 88 | 448 | 856 | 1736 |
| 7 | No crack | nill | 118 | 756 | 2125 | 4165 |

To determine the structural dynamic response, the following parameters such as the physical dimensions, the boundary conditions and the material properties of the structure play very important roles. Since the presence of a crack in structures modifies the dynamic behavior / characteristics of the structure; the following aspects of the crack greatly influence the dynamic response of the structure.

- The position of crack
- The depth of crack
- The orientation of crack
- The number of cracks

[Prabhakar,2009] calculated the natural frequencies and mode shapes for cantilever beam without a crack and with two cracks of different crack depths. Aluminum material is used for the cracked Bernoulli-Euler beam with the following properties.

Table 7: Geometry and properties of cantilever beam with two transverse cracks [Prabhakar,2009]

| | |
|---|---|
| <br>Geometry of cantilever beam with two transverse cracks. | Width of the beam = 0.05 m<br>Depth of the beam = 0.006 m<br>Length of the beam = 0.8 m<br>Elastic modulus of the beam = 0.724x1011 N/m2<br>Poisson's Ratio = 0.334<br>Density = 2713 kg/m3<br>End condition of the beam = One end fixed and other end free (Cantilever beam). |

[Prabhakar,2009] obtained the followings from the analysis and simulations results:

Table 8: Transverse vibration results for three modes with different depth of cracks [Prabhakar,2009]

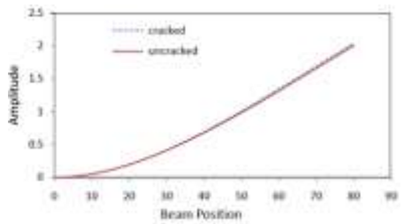
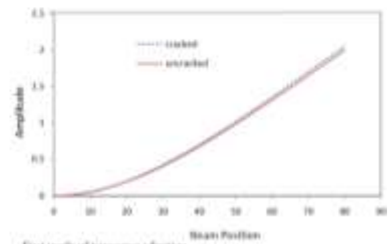
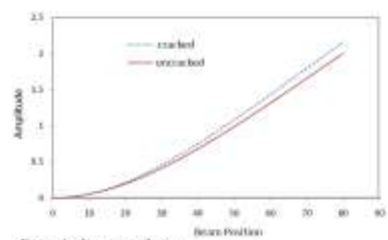
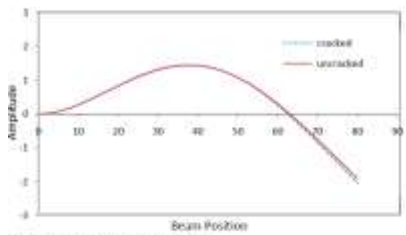
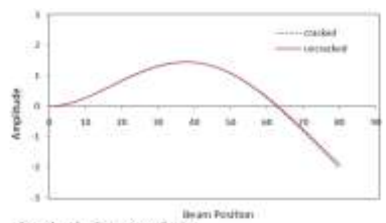
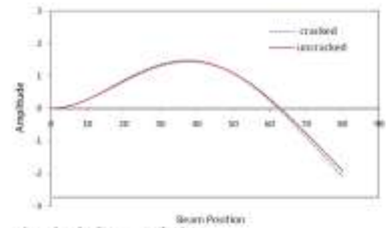
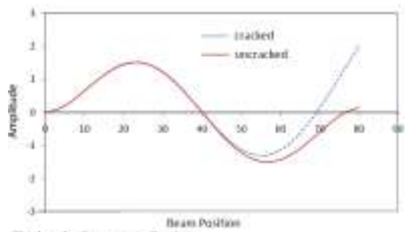
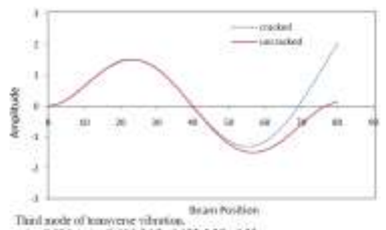
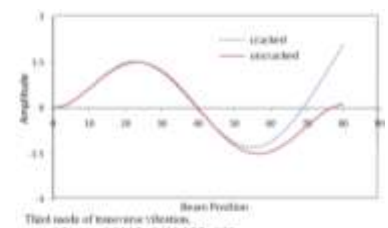
| a1/w=0.1667, a2 /w=0.1667,<br><br>L1/L=0.125, L2/L=0.25 | a1/w= 0.334, a2 /w=0.334,<br><br>L1/L=0.125, L2/L=0.25 | a1/w=0.5, a2/w=0.5,<br><br>L1/L=0.125, L2/L=0.25 |
|---|---|---|
| <br>First mode of transverse vibration.<br>$a_1$/w=0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>First mode of transverse vibration.<br>$a_1$/w=0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 | <br>First mode of transverse vibration.<br>$a_1$/w=0.5, $a_2$ /w=0.5, L1/L=0.125, L2/L=0.25 |
| <br>Second mode of transverse vibration.<br>$a_1$/w= 0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>Second mode of transverse vibration.<br>$a_1$/w=0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 | <br>Second mode of transverse vibration.<br>$a_1$/w=0.5, $a_2$ /w=0.5, L1/L=0.125, L2/L=0.25 |
| <br>Third mode of transverse vibration.<br>$a_1$/w= 0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>Third mode of transverse vibration.<br>$a_1$/w=0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 | <br>Third mode of transverse vibration.<br>$a_1$/w=0.5, $a_2$ /w=0.5,L1/L=0.125, L2/L=0.25 |

Table 9: Longitudinal vibration results for three modes with different depth of cracks [Prabhakar,2009]

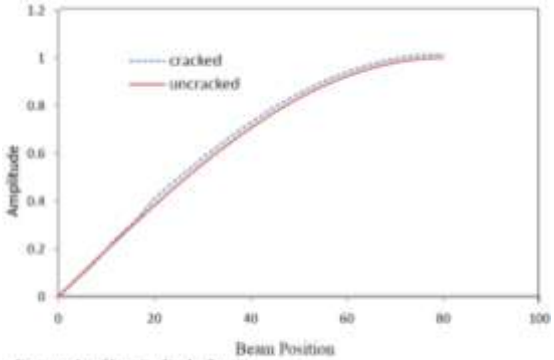
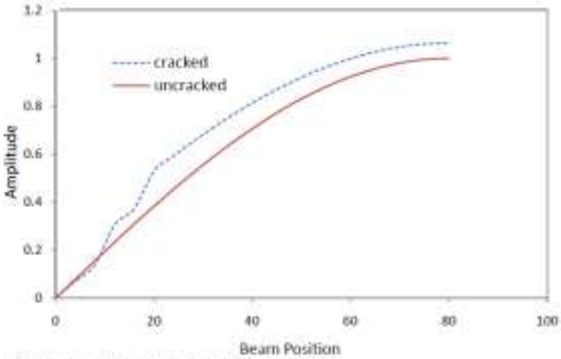
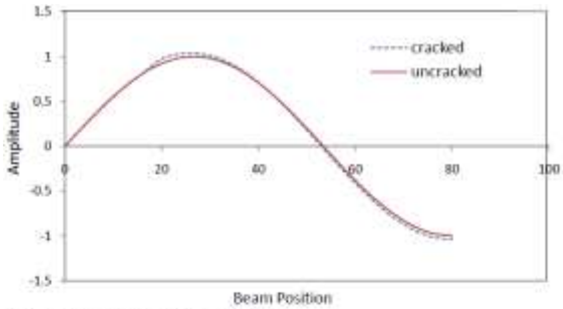
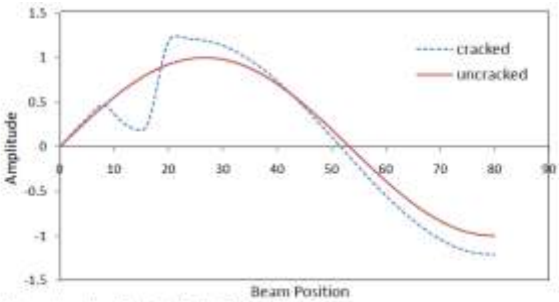
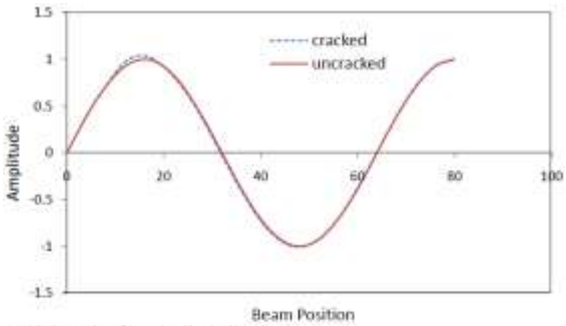
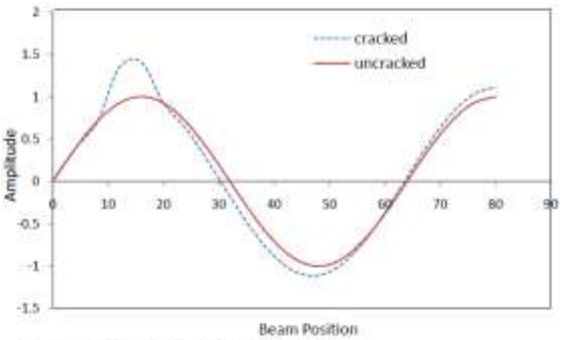
| a1/w=0.1667, a2 /w=0.1667,<br>L1/L=0.125, L2/L=0.25 | a1/w= 0.334, a2 /w=0.334,<br>L1/L=0.125, L2/L=0.25 |
|---|---|
| <br>First mode of longitudinal vibration,<br>$a_1/w$=0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>First mode of longitudinal vibration,<br>$a_1/w$= 0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 |
| <br>Second mode of longitudinal vibration,<br>$a_1/w$= 0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>Second mode of longitudinal vibration,<br>$a_1/w$=0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 |
| <br>Third mode of longitudinal vibration,<br>$a_1/w$= 0.1667, $a_2$ /w=0.1667, L1/L=0.125, L2/L=0.25 | <br>Third mode of longitudinal vibration,<br>$a_1/w$=0.334, $a_2$ /w=0.334, L1/L=0.125, L2/L=0.25 |

Table 10: Variation of natural frequencies at different relative crack depths when relative crack location at L1/L=0.125, L2/L=0.25 [Prabhakar,2009]

| Relative crack depth | Frequency first mode HZ | Frequency Found in ANSYS HZ | Frequency second mode HZ | Frequency Found in ANSYS HZ | Frequency Third mode HZ | Frequency Found in ANSYS HZ |
|---|---|---|---|---|---|---|
| 0.08335 | 7.754 | 7.844 | 49.220 | 49.263 | 137.812 | 137.902 |
| 0.1667 | 7.684 | 7.773 | 49.119 | 49.191 | 137.452 | 137.543 |
| 0.250 | 7.588 | 7.631 | 48.950 | 49.045 | 136.651 | 136.744 |
| 0.334 | 7.463 | 7.502 | 48.825 | 48.891 | 135.980 | 136.111 |
| 0.5 | 7.285 | 7.307 | 48.573 | 48.655 | 134.931 | 135.125 |
| 0.667 | 6.988 | 7.059 | 48.266 | 48.346 | 133.050 | 133.225 |
| 0.8 | 6.553 | 6.642 | 47.719 | 47.791 | 131.216 | 131.398 |

[Prabhakar,2009] states that "The natural frequency changes substantially due to the presence of cracks. The changes depending upon the location and size of cracks. The position of the cracks can be predicted from the deviation of the fundamental modes between the cracked and uncracked beam. The frequency of the cracked cantilever beam decreases with increase in the crack depth for the all modes of vibration. For moderate cracks (a1/w=a2/w=0.1667) appreciable changes in mode shapes are noticed and for deep cracks (a1/w=a2/w=0.5) the change in mode shapes are quite substantial.

Results show that there is an appreciable variation between natural frequency of cracked and uncracked cantilever beam. With increase in mode of vibration this difference increases. For moderate cracks appreciable changes in mode shapes are noticed and for deep cracks the changes in mode shapes are quite substantial. However remarkable changes are observed in longitudinal mode shapes at the crack positions. The numerical results indicate that the deviation between the fundamental mode shapes of the cracked and uncracked beam is always sharply changed at the crack location."

## CONCLUSION

The existence of cracks are the most common defects in the structures and they are present in structures due to various reasons. Cracks may be caused by fatigue under service conditions as a result of the limited fatigue strength. They may also occur due to mechanical defects. Other type of cracks are initiated during the manufacturing processes and they are generally small in sizes. Such small cracks are known to propagate due to fluctuating stress conditions. If these propagating cracks remain undetected and reach their critical sizes, then a sudden structural failure may occur. The presence of a crack could not only cause a local variation in the stiffness but it could affect the mechanical behavior of the entire structure to a considerable extent. Therefore, it is possible to use vibration techniques such as natural frequency measurements or mode shape analysis to detect the cracks in the structures.

After performing the literature survey, following observations and results are obtained within the scope of this work: While the crack size increases the natural frequency of the structure decreases as a consequence of the decrease in stiffness. If the crack location is near the edges of the structure, then the natural freqeuncy of the structure again decreases. Because, the crack near the ends would modify the boundary constraints, and thus decreases the frequencies significantly. However, if the crack location coincides with the vibration node of one mode, the frequency for that mode remains almost unchanged. If there are multiple cracks in the structure, the natural frequency also decreases. It has been also noticed that in the case of two cracks of different depths, the larger crack has the more significant effect on the natural frequency. In addition, the nonlinear character of the equations leads to parametric resonances, which appear as additional features in the frequency spectrum as higher order harmonics of the main frequency. It is also observed that when the size of the crack increases the number of the observed harmonics increases while their amplitudes also increasing in the spectrum.

The magnitudes of the mode shape curves decreases or increases, if there is a crack in the structure. When there is a crack in the structure, the change in the first and third mode shapes between cracked and uncracked beams are significant whatever the location of the crack. Therefore, the stress singularity occurs severely when the cracks under compression or tension in the first or third mode vibrations. But the second mode shapes are almost the same for uncracked and cracked beams, for all the crack ratios if the crack is located epecially at the mid-span of the beam. Since the cracks at the mid-span are not under severe compression or tension at the second mode, the strain in neutral axis direction is almost zero except very small inplane strain occurs due to the in-plane displacement. Therefore, stress singularity vanish and the cracks does not affect the eigenfrequency and eigenmode for the second mode vibration. Briefly, stress singularity occurs at the elements near the crack tip for the first and third modes. And for the second mode, almost the same stress level is observed for each element of the cracked and uncracked beams. In another literature review it is also obseerved that if there is a crack on the structure, the mode shapes show the approximate location of the crack in the structure.

As a summary, a crack in a structural member introduces local flexibility and alters the local compliance that would affect vibration response of the structure. This property may be used to detect existence of a crack together with its location and depth in the structural member. In addition, new techniques can also be developed for the damage detection and health monitoring of the structures based on the vibration responses of the cracks.

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